

T.R.
GEBZE TECHNICAL UNIVERSITY
INSTITUTE OF ENERGY TECHNOLOGIES

**NUMERICAL INVESTIGATION ON FATIGUE CRACK
PROPAGATION FOR GAS TURBINE ENGINE PARTS**

ŞEVKET ERTEKİN

**A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE IN APPLIED PROPULSION SYSTEM DESIGN &
ENGINEERING FOR AEROSPACE TECHNOLOGIES**

**GEBZE
2022**

T.R.
GEBZE TECHNICAL UNIVERSITY
INSTITUTE OF ENERGY TECHNOLOGIES

**NUMERICAL INVESTIGATION ON FATIGUE CRACK
PROPAGATION FOR GAS TURBINE ENGINE PARTS**

ŞEVKET ERTEKİN

**A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE IN APPLIED PROPULSION SYSTEM DESIGN &
ENGINEERING FOR AEROSPACE TECHNOLOGIES**

THESIS SUPERVISOR
PROF. DR. MEHMET ALİ ARSLAN

GEBZE

2022

 GEBZE TEKNİK ÜNİVERSİTESİ	YÜKSEK LİSANS JÜRİ ONAY FORMU
---	--------------------------------------

GTU Enerji Teknolojileri Enstitüsü Yönetim Kurulu'nun tarih ve/..... sayılı kararıyla oluşturulan jüri tarafından 23/06/2022 tarihinde tez savunma sınavı yapılan Şevket ERTEKİN'in tez çalışması Enerji Teknolojileri Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

JÜRİ

ÜYE

(TEZ DANIŞMANI)

: Prof. Dr. Mehmet Ali ARSLAN

ÜYE

: Prof. Dr. Osman Saim DİNÇ

ÜYE

: Prof. Dr. Ata MUĞAN

ONAY

GTU Enerji Teknolojileri Enstitüsü Yönetim Kurulu'nun tarih ve/..... sayılı kararı.

İMZA/MÜHÜR

SUMMARY

One of the critical phases of gas turbine engine development program is life management of the parts. Life management process involves investigation of cracks in terms of both fatigue and fracture mechanics. This whole process, from the initiation of the crack to the final fracture, requires excessive effort on the application of the fracture mechanics due to fact that engine parts operating under severe conditions and the complexity of Stress Intensity Factors (SIF). In this study, fracture mechanics theory for gas turbine engine components and its application on numerical solutions are investigated. Crack growth algorithms, flowcharts are created. Finally, this study also leads to software development process. Fracture mechanics results are compared for surface semi-elliptical crack and compact test (CT) specimen crack with non-commercial fracture mechanics software calculations in terms of cyclic crack growth, integration interval, plastic zone, constraint loss and mean stress effect (Walker). The CT specimen type crack model predicts crack propagation life better than the surface semi-elliptical crack type. An in-house working algorithm is created and affordable results in terms of crack dimension, stress intensity factor and final cycle number are obtained. Difference in results are discussed and accordingly, further actions are taken for improvement in the results accuracy.

Key Words: Fatigue, Fracture Mechanics, Stress Intensity Factor, Crack Growth, Algorithm

ÖZET

Gaz türbinli motor geliştirme programının kritik aşamalarından biri de parçaların ömür yönetimidir. Ömür yönetimi süreci, hem yorulma (çatlak başlangıcı) hem de kırılma mekaniği (çatlak ilerlemesi) açısından çatlakların araştırılmasını içerir. Çatlağın başlangıcından nihai kırılmaya kadar olan tüm bu süreç, motor parçalarının ağır koşullar altında çalışması ve gerilme şiddet faktörlerinin (SIF) karmaşıklığı göz önüne alındığında, kırılma mekaniği hesaplarının uygulanmasında yoğun çaba gerektirir. Bu çalışmada, gaz türbinli motor bileşenleri için kırılma mekaniği teorisi ve sayısal çözümler üzerindeki uygulaması incelenmiştir. Çatlak ilerleme algoritmaları ve akış şemaları oluşturulmuştur. Nihayetinde, bu çalışma özgün bir yazılım geliştirme sürecine de öncülük etmiştir. Yapılan hesaplara; çevrimsel çatlak büyümesi, integral adım aralığı, çatlak dibindeki plastik bölge, yüzey ve derinlikte ilerleme hızı farklılığı ve ortalama gerilme (Walker) etkileri dâhil edilmiştir. Yüzey yarı eliptik çatlak ve kompakt test (CT) numune çatlağı için elde edilen kırılma mekaniği sonuçları, ticari olmayan bir kırılma mekaniği yazılımı hesaplamaları ile karşılaştırılmıştır. CT numune tipi çatlak modeli, çatlak yayılma ömrünü yüzey yarı eliptik çatlak tipinden daha iyi tahmin ettiği gözlemlenmiştir. Özgün bir çalışma algoritması oluşturularak, çatlak boyutu, gerilme yoğunluk faktörü ve çatlak ömür sayısı açısından makul sonuçlar elde edilmiştir. Sonuçlardaki farklılıklar tartışılmış ve sonuçların doğruluğunun iyileştirilmesi için daha ileri adımlar belirlenmiştir.

Anahtar Kelimeler: Yorulma, Kırılma Mekaniği, Gerilme Şiddet Faktörü, Çatlak İlerlemesi, Algoritma

ACKNOWLEDGEMENTS

I would like to express my sincere thanks to my supervisor, Prof. Dr. Mehmet Ali Arslan, who gave me practical suggestions and guidance. I am deeply grateful to my wife for her profound belief in my work and patience that cannot be underestimated. I also wish to thank to “TEI – Tusaş Engine Industries” for their support and motivation in this study.



TABLE OF CONTENTS

SUMMARY	iv
ÖZET	v
ACKNOWLEDGEMENTS	vi
TABLE OF CONTENTS	vii
LIST OF ABBREVIATIONS AND ACRONYMS	ix
LIST OF FIGURES	x
LIST OF TABLES	xii
1. INTRODUCTION	1
2. LITERATURE SURVEY	2
2.1 Fracture Mechanism	2
2.1.1 History	2
2.1.2 Why we need fracture mechanics? Damage tolerance basis	4
2.1.3 Fracture Mechanics Approaches; Griffith's energy criterion and Irwin's SIF	5
2.1.4 Nucleation and propagation mechanism of cracks	10
2.1.5 Propagation laws; Paris, Forman, NASA etc.	13
2.2 Local Yielding and and Plastic Zone Near Crack Tip	16
2.2.1 Basics of Plastic Zone	16
2.2.2 Plastic Zone Shape:	20
2.2.3 Cyclic Plastic Zone:	21
2.2.4 Plane Stress vs. Plane Strain:	23
2.2.5 Specimen Thickness:	24
2.2.6 Constraint Loss:	26
2.3 Mean Stress Effects	29
2.3.1 Walker Model	31
2.3.2 Dual Walker Model	31
2.4 Stress Gradient Effect on Propagation	32
2.4.1 Weight functions	35
2.5 Other Parameters	38
2.5.1 Retardation Effect	39
2.5.2 Time-dependent crack growth, dwell-creep-oxidation	41
3. LEFM TESTS	43
3.1 ASTM standards and Specimen Types	43
3.2 Kth Tests	46

3.3	Propagation Tests	47
3.4	K _{IC} Tests	49
3.5	Fractured Specimen Investigations	51
4.	ALGORITHM	53
4.1	Main Algorithm	53
4.2	Sub-Algorithms	55
5.	VALIDATION AND RESULTS	63
5.1	Comparison of Other OEM Company's Fracture Mechanics Calculations and Applications on Life Methods.	63
5.2	Validation of Crack Initiation and Crack Growth	64
5.2.1	Surface semi-elliptical crack:	64
5.2.2	Compact Test (CT) specimen crack:	67
5.3	Validation of Plastic Zone and Constraint Loss Models	69
5.4	Validation of Walker Correction Method	80
5.5	Summary of Results	81
5.6	Further Directions	82
6.	APPENDIX	83
	REFERENCES	86
	BIOGRAPHY	89

LIST OF ABBREVIATIONS AND ACRONYMS

<u>Abbreviations and Acronyms</u>	<u>Explanations</u>
da/dN	: Crack growth rate (crack length increment/cycle increment)
ΔK	: Stress intensity factor range
G	: Energy release rate per unit material thickness
E	: Young's modulus
ν	: Poisson ratio
λ, Y, β, F_0	: Geometry correction factor
a	: Crack length
C, m, B, P, Q, D	: Material constants
r_p	: Plastic zone size
m, m^+, m^-	: Walker coefficients

LIST OF FIGURES

2.1 S.S. Schenectady-The Liberty Ship failed due to fracture before leaving shipyard, 1943 [1].	2
2.2 Comet accident - Origin of the crack [3]	3
2.3 Damage Tolerance Approach	5
2.4 The broad field of fracture mechanics	5
2.5 Crack extension of loading modes	6
2.6 Elastic stress state near the crack tip ($r/a \ll 1$) with polar coordinate components	9
2.7 Possible model for fatigue crack growth	12
2.8 Transition of fatigue crack growth in sheet from tensile mode to shear mode a) single shear b) double shear	13
2.9 Sigmoidal curve representing three stages of crack propagation	16
2.10 First estimate of plastic zone size	17
2.11 Irwin's final plastic zone correction	18
2.12 Dugdale Plastic Zone approach	19
2.13 Comparison of plastic zone corrections for a through crack in plain strain	20
2.14 Plastic zone shapes according to Von Mises and Tresca yield criteria a. Von Mises criterion; b. Tresca criterion	21
2.15 Schematic of plastic zone at the crack tip a) Loading cycle b) Monotonic plastic zone c) Cyclic plastic zone	22
2.16 Three-dimensional plastic zone.	24
2.17 Effect of specimen thickness on stress and mode of fracture	25
2.18 Toughness as a function of thickness.	26
2.19 Crack growth rate curves reduced from both crack length and depth of Inconel 718 surface crack specimen at 538°C (1000°F) [22].	27
2.20 Mean stress effect in fatigue crack growth.	30
2.21 Example of Stress Gradient.	33
2.22 Tabulated Example of Stress Gradient	34
2.23 Surface semi-elliptical crack in a finite thickness plate	35
2.24 Schematic of Overload.	39
2.25 Residual stress after overload (compressive).	40
2.26 Retardation effect in crack growth rate.	40
2.27 da/dt Sigmoidal curve	41
3.1 Fracture mechanics test specimens: (a) C(T) specimen, (b) disk-shaped compact specimen, (c) single-edge notched bend (SE(B)) specimen, (d) MT specimen, and (e) arc-shaped specimen	44
3.2 ASTM notation for specimens extracted from rolled plate and forgings.	45
3.3 Fatigue Precracking	46

3.4 Sigmoidal Crack Growth Curve and Regions - Ti-62222 Mill Annealed.	48
3.5 Test setup configuration for C(T) specimen	49
3.6 K _{IC} test - Load–displacement behaviour.	50
3.7 Schematic view of fatigue fracture surface marks with circular cross sections [34].	51
3.8 Schematic view of fatigue fracture surface marks with square and rectangular cross sections	52
4.1 Main Algorithm Flowchart	54
4.2 Plastic zone sub-algorithm	56
4.3 Constraint Loss Sub-algorithm Flowchart	57
4.4 Walker Correction Sub-algorithm Flowchart	58
4.5 Stress Gradient Sub-algorithm Flowchart	59
4.6 Main Algorithm with Sub-algorithms-1	60
4.7 Main Algorithm with Sub-algorithms-2 (cont.)	61
4.8 Main Algorithm with Sub-algorithms-3 (cont.)	62
5.1 Plastic Zone Tests Comparison Graph	73
5.2 Plastic Zone Tests Comparison Graph-2	73

LIST OF TABLES

2.1 Surface modification factors of crack growth curves [22].	28
2.2 Mean stress effect - possible scenarios	32
3.1 The list of available ASTM standards for fracture mechanics.	45
3.2 Threshold testing related ASTM procedures	46
3.3 The list of current applicable crack propagation related test standards	47
5.1 ÇİTA Settings for surface semi-elliptical crack	65
5.2 ÇİTA output for surface semi-elliptical crack @ N=0 cycle	65
5.3 Comparison for surface semi-elliptical crack @N=0	65
5.4 ÇİTA output for surface semi-elliptical crack @ N=Final cycle	66
5.5 Comparison for surface semi-elliptical crack @N=Final cycle	66
5.6 ÇİTA Settings for Compact Test (CT) specimen crack	67
5.7 ÇİTA output for Compact Test (CT) specimen crack @ N=0 cycle	67
5.8 Comparison for Compact Test (CT) specimen crack @N=0	67
5.9 ÇİTA output for Compact Test (CT) specimen crack @ N=Final cycle	68
5.10 Comparison for Compact Test (CT) specimen crack @N=Final cycle	68
5.11 ÇİTA Plastic Zone-on - Settings for surface semi-elliptical crack	69
5.12 Comparison of plastic zone-on results	69
5.13 ÇİTA Plastic Zone-off - Settings for surface semi-elliptical crack	70
5.14 Comparison of plastic zone-off results	70
5.15 ÇİTA Plastic Zone-on/off - Settings for surface semi-elliptical crack	71
5.16 Test Case 1 cycle results	71
5.17 Test Case 1 SIF results	72
5.18 ÇİTA Plastic Zone-on/off - Settings for surface semi-elliptical crack-2	74
5.19 Test Case 2 cycle results	74
5.20 Test Case 2 SIF results	75
5.21 ÇİTA Plastic Zone-on/off Settings for Compact Test (CT) specimen crack	76
5.22 Test Case 3 cycle results	76
5.23 Test Case 3 SIF results	76
5.24 ÇİTA Plastic Zone-on/off Settings for Compact Test (CT) specimen crack - 2	77
5.25 Test Case 4 cycle results	77
5.26 Test Case 4 SIF results	77
5.27 ÇİTA Plastic Zone-on/off Settings for surface semi-elliptical crack-3	78
5.28 Test case 5 cycle results	78
5.29 Test case 5 SIF results	79
5.30 ÇİTA Walker Settings for Compact Test (CT) specimen crack	80

5.31 Walker Validation Test Cases	80
5.32 Walker Validation Cycle and SIF results	81
5.33 In718 sigmoidal equation material properties (*Normalised)	84
5.34 Variated sigmoidal equation material properties, max and min	84
5.35 Material property variation and its effect in ÇİTA	85



1. INTRODUCTION

One of the critical aspects of gas turbine engine development programs is life management of the parts. Aviation and gas turbine engine designer companies use fracture mechanics software in all of the design, manufacturing, certification and in-service logistic support phases of projects.

Life management programs for aviation parts also force companies to show the engine parts capable to stand steady and integrity is kept in promised life cycle. Most of the engine parts need to be evaluated in terms of fracture mechanics.

Due to complex SIF solutions and many iteration steps, fracture mechanics calculations are commonly made by the help of softwares. Gas turbine engine manufacturers in the world had developed these kinds of in-house softwares by more than 35 years to overcome crack propagation solutions. Each software has a living architecture that helps developer to modify and improve codes day by day.

This thesis aimed to show fracture mechanics theory and application on numerical solution with the help of new developed software. Software development, algorithms are also mentioned. Validation and results are represented for various test cases.

2. LITERATURE SURVEY

2.1 Fracture Mechanism

2.1.1 History

Fracture has been an issue in structures from the beginning of the first structural parts, even starting from ancient Egypt pyramids or Roman bridges. This problem increases with the demand on the complex structural parts up to today.

Recent developments in fracture mechanics have helped to mitigate some of the risks posed by rising technological complexity. Studies are focused on how to prevent failures and how materials fail. Consequently, investing in fracture mechanics could reduce the cost of fractures enormously.

The Liberty Ship:

Most of the Liberty ships built during World War II suffered serious and dramatic fracture failures due to crack initiation and propagation on the deck, where there was a local stress concentration near welds. Weld were also defected and in poor condition. Steel that is used for Liberty ships were made of unexpected poor fracture toughness value. See Figure 2.1 [1] for failed Liberty ship, split in half before leaving the shipyard.

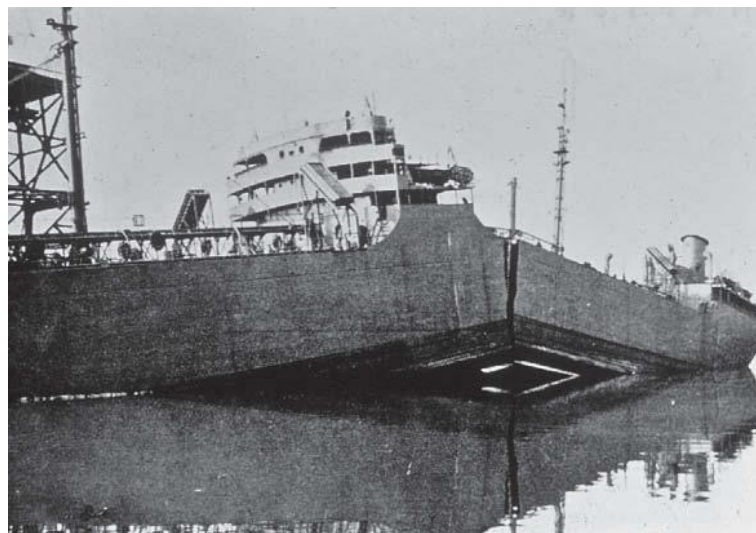


Figure 2.1 S.S. Schenectady-The Liberty Ship failed due to fracture before leaving shipyard, 1943 [1].

Another well-known failed program for fracture is Comet aircraft, which entered into service in 1952. Comet aircrafts are designed for pressurized cabin at high altitude and flying at high speed. After 2 years being in the service, two Comet aircrafts catastrophically crashed in flight in January and April 1954 killing many people. Investigations on accidents and following studies showed that a crack had initiated in the square corners of windows that produced a severe stress concentration. Combination of circumstances, this firstly led to crack propagation and then fracture due to metal fatigue in front of the cabin fuselage near window, and finally aircraft crashed [2]. Following Figure 2.2 [3] shows schematic illustration of crack location in Comet aircraft.

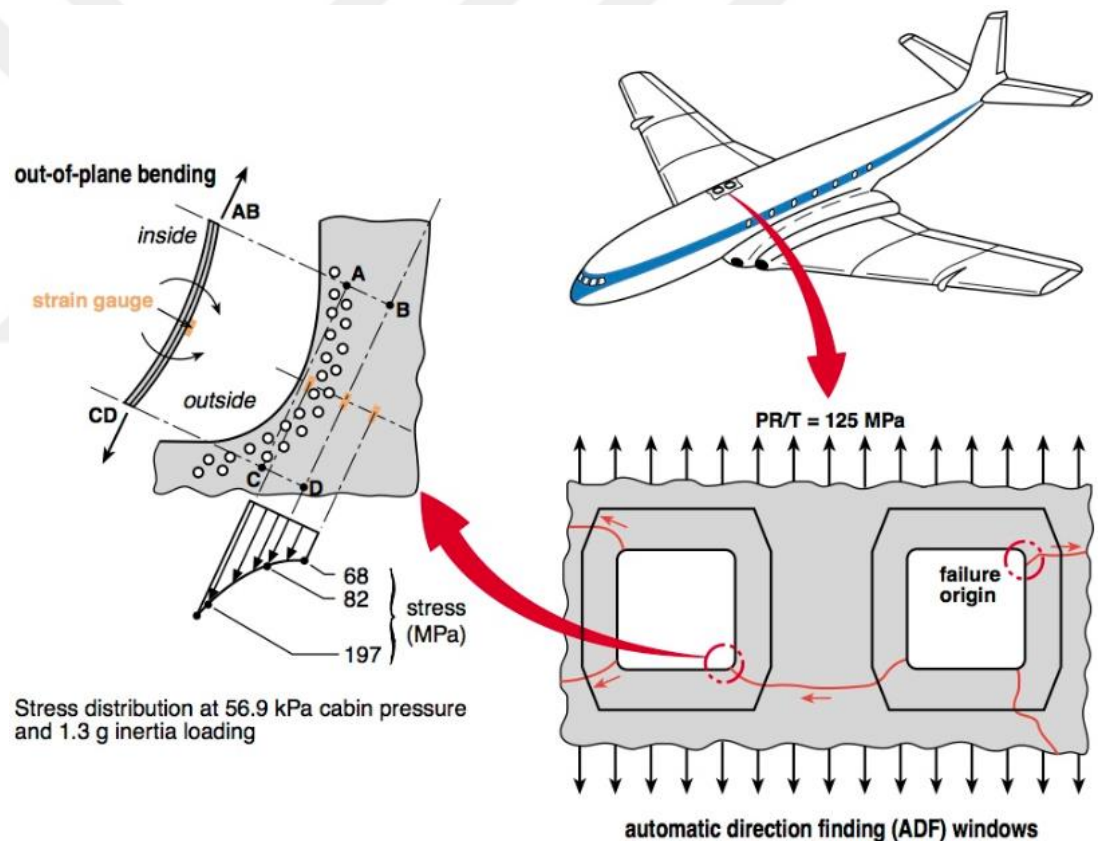


Figure 2.2 Comet accident - Origin of the crack [3]

Firstly, Inglis and Griffith [4,5] studied for energy balance model of the flaw. After the World War II, LEFM and fracture mechanics gained its deserved attention by scientists and companies. Irwin [6], Dugdale [7], Rice [8], etc. were studied yielding and plastic zone on the tip of the crack. Paris [9] also focused on the propagation of the crack under cyclic loading. Wells [10] developed crack tip

opening displacement (CTOD) parameter. Studies have been continued with plasticity on crack tip, mixed mode crack propagation, time-dependent nonlinear crack growth, 3D finite element analyses of cracks etc.

As a brief history of fatigue and failure, the nominal stress-life (S-N) model, first formulated between the 1850s and then, the local strain-life (ϵ -N) model and the fatigue crack growth ($da/dN - \Delta K$) model, first formulated in the 1960s. Subsequently, the two-stage models developed, which consists of combining models 2 and 3 to incorporate both macroscopic fatigue crack formation (nucleation) and fatigue crack growth.

2.1.2 Why we need fracture mechanics? Damage tolerance basis

Since fracture is a significant concern in the recent decades for complex structural elements, it is for sure, there is a need for theoretical and practical studies for designing against fracture. Following questions arise during the design phase of the engine components with damage tolerance aspects.

Questions:

1. What loads can a structure or engine component carry as a function of crack size, crack orientation/configuration, and time if it has a pre-existing defect and/or crack-like flaw?
2. How fast and in what directions would a crack in a structure propagate with load history and environmental history?
3. When will the crack grow catastrophically, and how many cycles of loading will it take?
4. What size of crack and which location can be allowed in a structure while still allowing it to function safely?

Figure 2.3 [11] shows crack size increment with time during in-service conditions of components. Damage tolerance topic is deeply open to several considerations related with inspection intervals, inspection techniques (FPI etc.), useful service life definition, minimum induced crack, manufacturing anomalies etc.

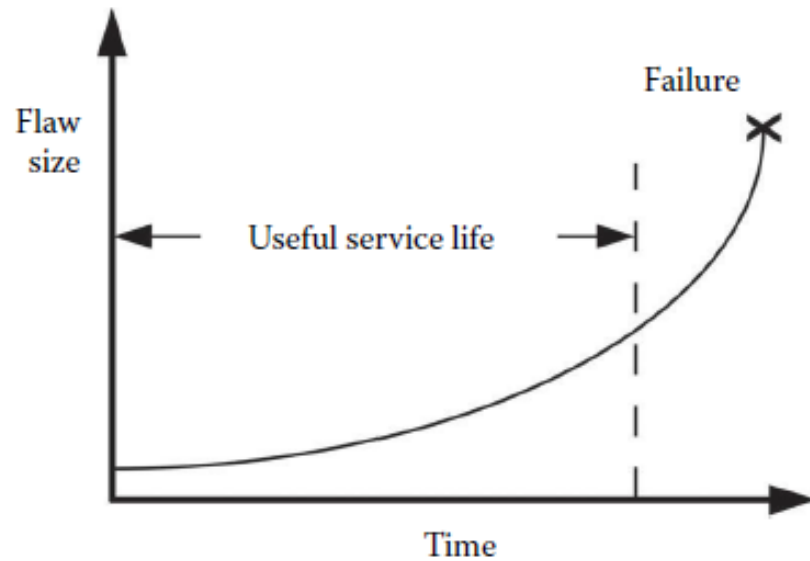


Figure 2.3 Damage Tolerance Approach

Apart from the damage tolerance aspects, it is necessary to have a systematic understanding of fracture mechanics in order to apply it effectively in engineering problems. In Figure 2.4 [12], vertical axis of the chart shows the unit of measure as meter which corresponds to micro/macro aspect of fracture mechanics.

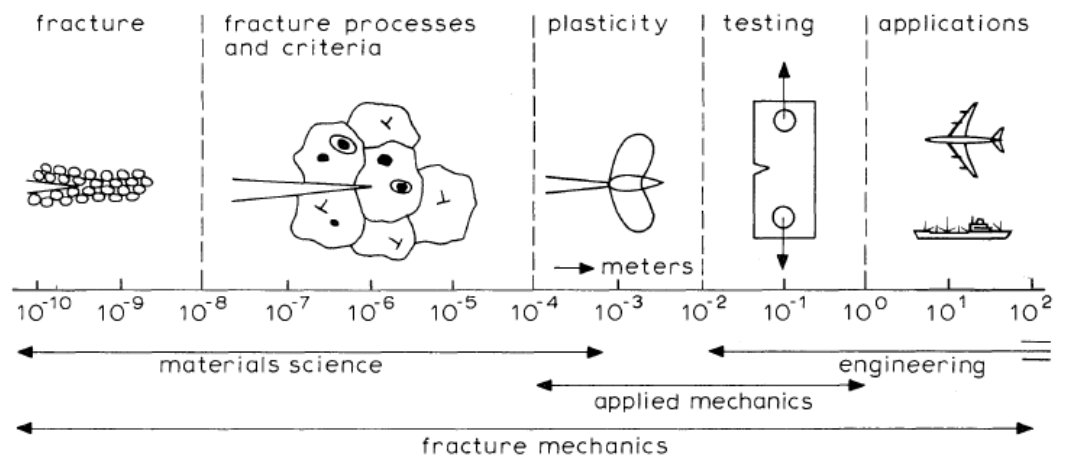


Figure 2.4 The broad field of fracture mechanics

2.1.3 Fracture Mechanics Approaches; Griffith's energy criterion and Irwin's SIF

The fundamental of fracture mechanics rely on the combinations of these three parameters.

- Crack size
- Stress
- Fracture Toughness

Loading Modes:

Figure 2.5 shows three modes by which a crack can extend. Mode I is the opening mode, which is the most common, particularly in fatigue, and has received the greatest amount of investigation. Mode II is the in-plane shearing or sliding mode and Mode III is the tearing or antiplane shear mode.

Figure 2.5 depicts three different orientations where crack opens. The first form is the opening Mode I. In-plane shearing or sliding is the Mode II. Tearing or antiplane shear is Mode III, which is in a pure shear condition.

Because of flaws have a tendency to form on the plane of maximal tensile stress, Mode I crack extension type is the most prevalent in fatigue. However, combination of the crack modes might occur during multi-axial loading conditions, mostly Mode I and Mode III. Microscopic fatigue fractures tend to form in planes of highest shear.

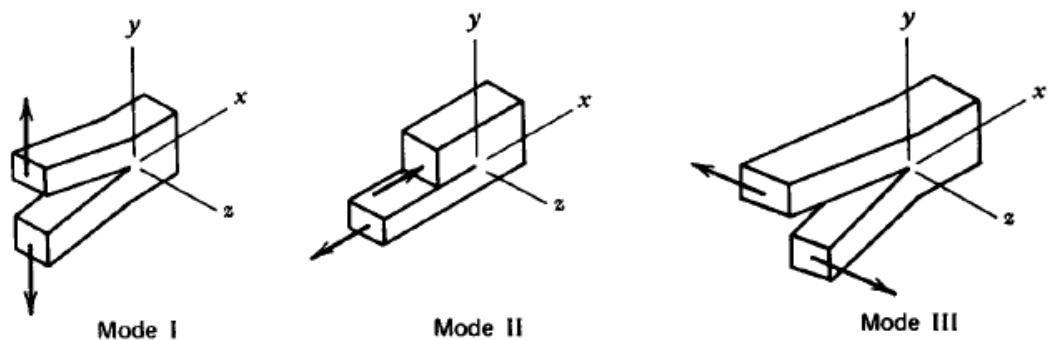


Figure 2.5 Crack extension of loading modes

Strain energy criterion:

In 1920s, the energy criterion, which was developed by Griffith [5], was mainly based on the brittle fracture of glass. Concept of energy balance between theoretical cohesive strength and observed fracture strength of a glass was used in the Griffith theory. When a crack propagates, on one hand, there is a release of

elastic strain energy at the tip. On the other hand, new surfaces are created which increase overall surface energy.

Griffiths study implied that Equation 1.1 gives the equilibrium condition needed for a crack to grow in an ideally brittle material:

$$G = \frac{\pi\sigma^2 a}{E} \quad (1.1)$$

where σ is remotely applied stress, a is the half crack length and E is plane stress Young's modulus. G is a measure of the energy release rate per unit material thickness. So, a crack will grow when the rate of decrease in elastic strain energy is at least as great as the rate of increase of energy required to create the new crack surface.

Stress Intensity Factor (SIF) criterion:

From the history of fracture mechanics perspective, Griffith is called the “early father” and Irwin is called the “modern father” of fracture mechanics. In 1960s, Irwin suggested that the Griffith's theory could be modified to include metals and following Stress Intensity Factor (SIF): K was introduced. K is an indication of the magnitude of the elastic stress field.

$$K = \sigma\sqrt{\pi a} \quad (1.2)$$

To modify Equation 1.1 for critical value of K_c ;

$$K_c = \sigma\sqrt{\pi a_c} \quad (1.3)$$

where a_c is the critical crack length for fast fracture and K_c is fracture toughness of the material. For critical crack depth calculations, Equation 1.3 can be used.

$$a_c = \frac{1}{\pi} \left(\frac{K_c}{\sigma} \right)^2 \quad (1.4)$$

Relationship between G and K is as follows:

$$G = \frac{\pi\sigma^2 a}{E} \text{ and } K_c = \sigma\sqrt{\pi a_c} \quad (1.5)$$

Hence;

$$G = \frac{K^2}{E} \quad (1.6)$$

The energy and stress intensity factor methods are basically equivalent for linear elastic materials in critical condition and R curve check. Thus, same relationship in Equation 1.5 is valid for G_c and K_c .

In the Equation 1.5, E is for plane stress. In case of plane strain, E' :

$$E' = \frac{E}{1 - \nu^2} \quad (1.7)$$

Figure 2.6 [11] shows a through-thickness crack in a linear elastic isotropic body. Definition of stress in the vicinity of the crack tip for Mode I loaded crack, with polar coordinates r (distance to crack tip) and θ (angular location to crack plane) is shown.

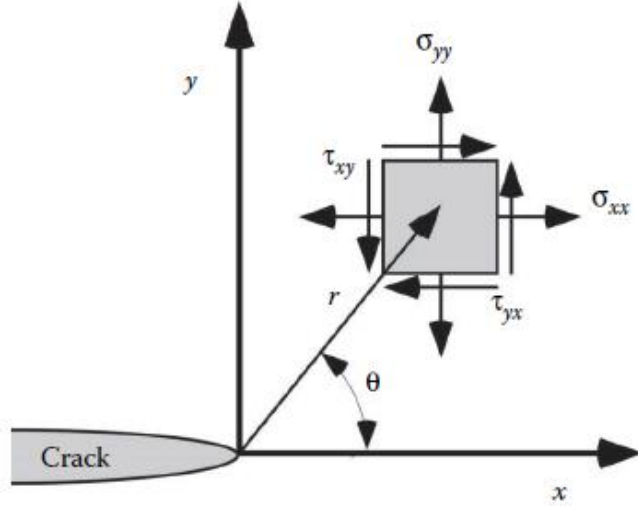


Figure 2.6 Elastic stress state near the crack tip ($r/a \ll 1$) with polar coordinate components

The stress field definition at any point near the crack tip can be expressed in following form by using stress function and the mathematical theory of linear elasticity:

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 - \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \quad (1.8)$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \left[1 + \sin\left(\frac{\theta}{2}\right) \sin\left(\frac{3\theta}{2}\right)\right] \quad (1.9)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right) \cos\left(\frac{3\theta}{2}\right) \quad (1.10)$$

In case of K_I is known, all stress distribution in the vicinity of the crack tip can be computed with Equation 1.8, Equation 1.9 and Equation 1.10. Hence, K_I which is called the stress intensity factor, entirely describes the crack tip stress conditions in an isotropic linear elastic material.

Geometry Correction Factor for SIF:

Tada, Paris and Irwin [13] revealed a handbook to cover geometry correction factor and usage on SIF solutions extensively. With the help of empirical, test-based equations, various type of weight functions are equated.

$$K = \lambda \sigma \sqrt{\pi a} \quad (1.11)$$

λ is a function of crack dimensions. (i.e. a, c, w, b)

2.1.4 Nucleation and propagation mechanism of cracks

Cracks exist in or on the materials due to dislocations in the micro structure. Due to cyclic load history or combination of loads and environmental attack, the crack will propagate gradually in the structure. The more increase in the length, the more growth rate occurs. Stress concentration and geometry compliance factor increase by growing the length of the crack.

Most of the micro cracks occurs on the surface of the material. However, location of the fatigue crack propagation may vary with following parameters: Material impurities, type of applied stress, type of loading, stress concentration, geometric parameters, mechanical properties etc.

Microstructural parameters directly affects fracture mechanics parameters like K, J or CTOD. A specimen and a cracked structure experience similar crack tip conditions at failure, but failure rates are not identical due to imperfection rates are higher in the actual manufactured structural component rather than laboratory specimens.

Studying only continuum models for nucleation of crack is not enough to find solutions. Microstructural aspects are also need higher attention. Understanding of microstructural events during all manufacturing steps (drilling, welding etc.) plays important role for crack initiation. Material inclusions might be shaped during melting process or surface impurities might occur in manufacturing process.

Due to manufacturing experience and catastrophic failures in the aviation, following assumption is made during design process for safety of critical parts: Small crack-like impurities exist in advance of the first use of the aircraft or engine.

Each stage of cyclic fatigue crack growth is illustrated in Figure 2.7 [12]. Stress concentration occurs at crack tip due to sharp crack edge. Slip occurs in direction of

the maximum shear stress in 1st and 2nd stages. Slipping process continues to extend crack depth into another direction in 3rd stage.

As work hardens and stress rises locally, other local slip planes are triggered, resulting in a blunt tip in 4th stage then stress level is stabilised. The crack has grown by Δa during the rising load stage of the cycle.

In a small area surrounded by elastic surroundings, plastic deformation has happened till 5th stage. When load decreases through 5th stage, the elastic surrounds created upon load release, and the plastically deformed region near crack tip no longer fits in its surroundings. There is a huge compressive plastic zone which acts like a residual plastic zone and it will resharpen the crack tip in the end of 5th stage. This term is called retardation effect.

In 6th and 7th stages, similar mechanism to previous stages happens with cyclic loading. This cyclic loading will create a classic ripple pattern, with each loading cycle creating a new ripple. Fatigue striations are ripples that appear on the fracture surface under an electron microscope.

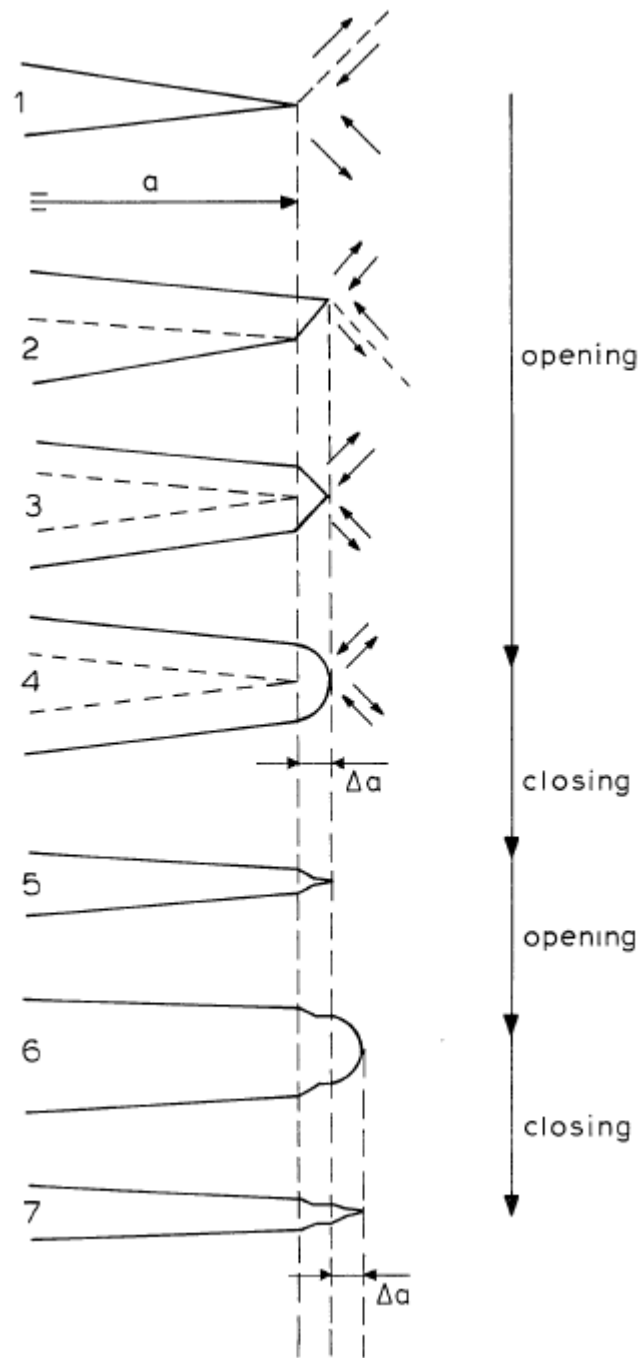


Figure 2.7 Possible model for fatigue crack growth

As crack grows by cyclic loading to a certain point where the remaining material no longer support the loading condition, then final fracture occurs suddenly. Schematic illustrations of crack growth paths are shown in Figure 2.8. According to Broek [12], crack tip plastic zone size is small at short crack lengths and the crack path is usually flat. With plastic zone size increase by growth of the crack,

crack plane can turn to a 45° shear or slant mode. This can be either single shear, as shown in Figure 2.8.a or double shear, as shown in Figure 2.8.b.

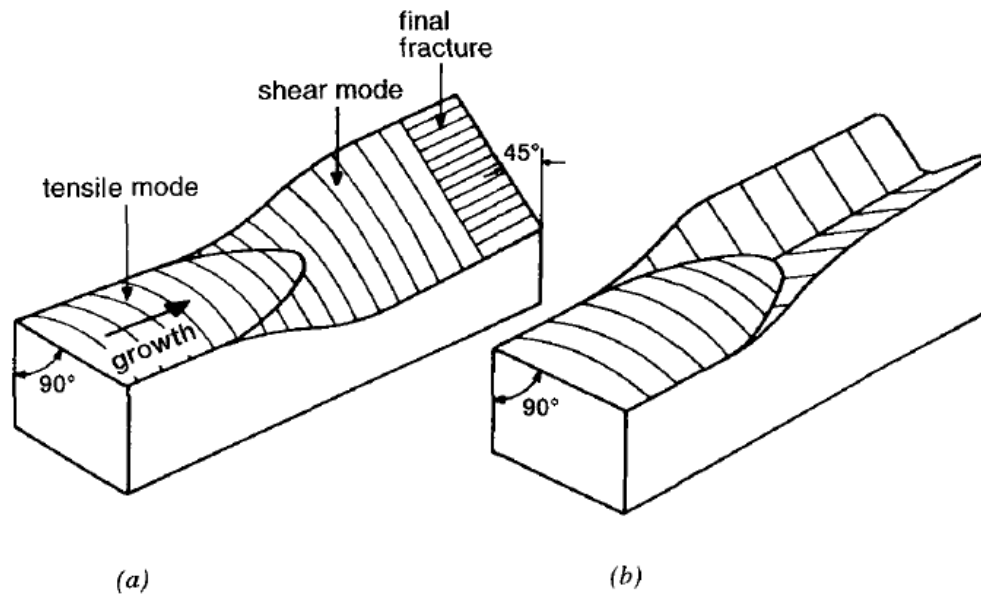


Figure 2.8 Transition of fatigue crack growth in sheet from tensile mode to shear mode a) single shear b) double shear

2.1.5 Propagation laws; Paris, Forman, NASA etc.

When it comes to propagation of cracks in fracture mechanics, Paris and Erdogan [14] showed propagation law in Equation 1.12. This method is useful for describing crack growth behavior under cyclic loads. It can be modified by adding various effects.

$$\frac{da}{dN} = f_1(\Delta K, R) \quad (1.12)$$

Integrating Equation 1.12 gives:

$$N = \int_{a_0}^{a_f} \frac{da}{f_1(\Delta K, R)} \quad (1.13)$$

where da is a crack length increment between $a_{n+1} - a_n$, $\Delta K = (K_{max} - K_{min})$, R is and stress ratio. In order to integrate Equation 1.13, combination of Equations 1.15 and 1.16 is needed.

$$\frac{da}{dN} = C\Delta K^m \quad (1.14)$$

where C and m are material constants. Stress Intensity Factor is defined as:

$$\Delta K = Y\Delta\sigma\sqrt{\pi a} \quad (1.15)$$

Substituting Equation 1.15 into Equation 1.14 and integrating;

$$N_f = \int_0^{N_f} dN = \int_{a_i}^{a_c} \frac{da}{C(\Delta K)^m} = \frac{1}{CY^m(\Delta\sigma)^m\pi^{m/2}} \int_{a_i}^{a_c} \frac{da}{a^{m/2}} \quad (1.16)$$

where N_f is final cycle number to fracture, a_c is critical crack length for fracture, a_i is initial crack length.

$$N_f = \frac{1}{C(Y\Delta\sigma\sqrt{\pi})^m} \cdot \frac{2}{2-m} \cdot \left[a^{\frac{2-m}{2}} \right]_{a_i}^{a_c} \quad (1.17)$$

$$N_f = \frac{2}{(2-m)C(Y\Delta\sigma\sqrt{\pi})^m} \cdot \left[a_c^{\frac{2-m}{2}} - a_i^{\frac{2-m}{2}} \right] \quad (1.18)$$

Equation 1.18 calculates final cycle number of propagation between initial length and critical length of the crack where $m \neq 2$ and $R = 0$ condition. Basis of the crack propagation relies on this integration and integrand depends on a in Equation 1.16.

Main assumption in Equations 1.15 and 1.16 is that geometry correction factor (Y) is assumed as a constant value. However, this factor increases with crack growth, and it is a function of crack dimensions. (i.e. a , c , w , b). So, complexity of the integral increases by adding geometry correction factor (Y) as an integrand in the Equation 1.16. Hence, numerical solution is needed for better integration and improved divergence of the solution.

Modified Forman Equation (NASGRO):

$$\frac{da}{dN} = C \left[\left(\frac{1-f}{1-R} \right) \Delta K \right]^m \frac{(1 - \Delta K_{th}/\Delta K)^p}{(1 - K_{max}/K_c)^q} \quad (1.19)$$

$\frac{da}{dN}$ is the crack growth rate where a is the crack length, N is the cycle number. C , m , p , and q are empirical material constants from laboratory tests. R is the stress ratio, ΔK is range of the stress intensity factor and f is related with crack opening function. K_{th} is the stress intensity factor threshold value, and K_c is critical stress intensity factor. Equation 1.19 which is used in NASGRO software, provides crack growth formulation with mean stress effects (R Ratio).

Coles et al. [15] were developed a new equation based on IN718 material tests considering gas turbine engine parts loading and environmental conditions. Equation 1.20 is called sigmoidal equation which represent all stages of crack propagation (Stage 1, Stage 2 and Stage 3). See Figure 2.9.

$$\frac{da}{dN} = e^B (K_{eff}/K_{th})^P [\ln(K_{eff}/K_{th})]^Q [\ln(K_{crit}/K_{eff})]^D \quad (1.20)$$

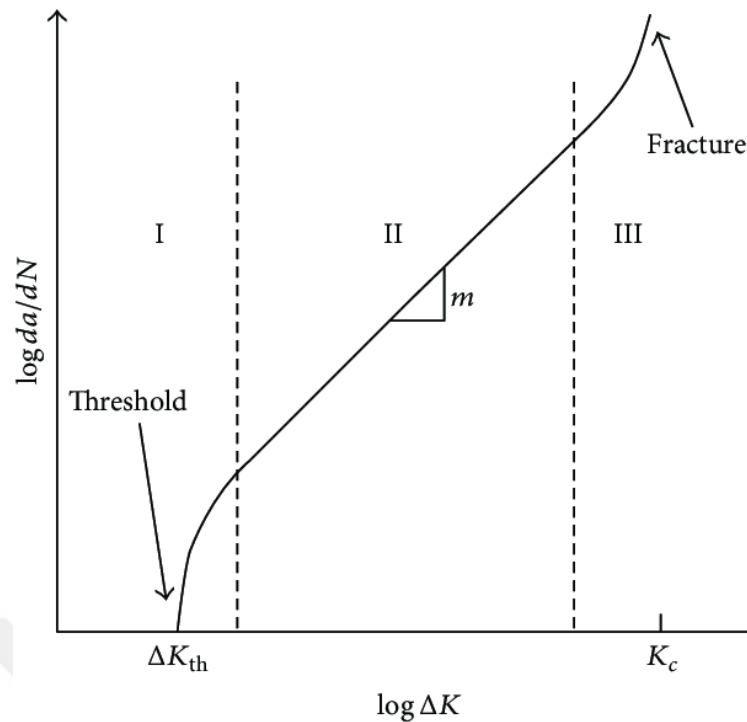


Figure 2.9 Sigmoidal curve representing three stages of crack propagation

2.2 Local Yielding and and Plastic Zone Near Crack Tip

Plastic deformation ahead of the crack tip leads local yielding. This local yielding generates a zone called the plastic zone at the crack tip. Plastic zone behaves as an additional effect on both static and cyclic condition. In other words, crack behaves larger than actual (physical) size due to plastic zone addition.

2.2.1 Basics of Plastic Zone

When the loading is assumed elastic, (general rule: applied stress per cyclic yield stress is lower than 0.8) LEFM - Linear Elastic Fracture Mechanics rules are in effect. In elastic condition, this effect is called small scale yielding (SSY). For elastic-plastic fracture mechanics (EPFM), plasticity on the crack tip is extensively large compared with LEFM plastic zone. This effect is called large-scale yielding (LSY).

In order to take into consideration of plastic zone effect for LEFM calculations, this zone is calculated and added to crack length shown in below equation. This

calculation called plastic zone correction. This correction for effective crack length causes a very small but systematic increase in stress intensity factor.

$$a_{eff} = a + r_p \quad (1.22)$$

There are some theoretical models for predicting plastic zone size ahead of the crack tip. Firstly, Irwin [16] shows some approaches for plastic zone near a crack tip, and its effects in fracture toughness of the material. According to elastic stress field solutions, stress concentrated at the crack tip. This singularity causes an infinite stress value at the very tip of the crack. However, due to plastic deformation at the tip of the crack, yielding occurs.

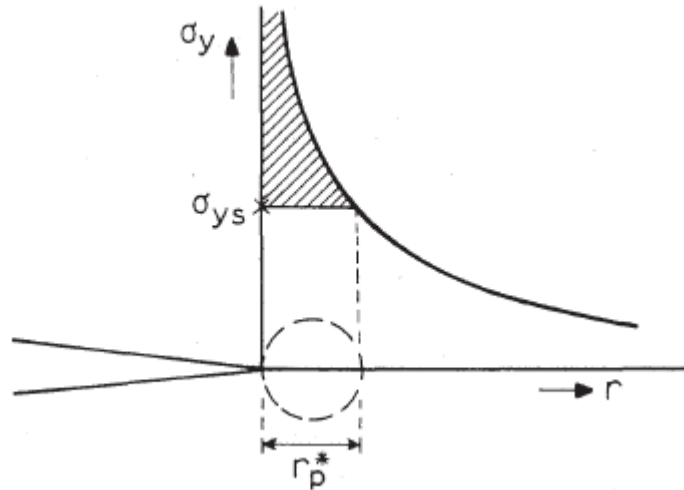


Figure 2.10 First estimate of plastic zone size

In Figure 2.10 from Irwin [16], estimation of the plastic zone for plane stress condition is shown. Since crack tip is sharp, stress concentration becomes infinite. Thus magnitude of the stress levels also goes infinite in the plane $\theta = 0$. According to Irwin's solution, there is a distance, r_p^* from the crack tip that stress level at this distance equals to yield stress, σ_{ys} , theoretically. This distance r_p^* is the size of the plastic zone in a first glance. Irwing also substituted σ_{ys} in the equation for σ_y .

Then r_p^* can be calculated:

$$\sigma_y = \frac{K_I}{\sqrt{2\pi r_p^*}} = \sigma_{ys} \text{ or } r_p^* = \frac{K_I^2}{2\pi\sigma_{ys}^2} = \frac{\sigma^2 a}{2\sigma_{ys}^2} \quad (1.23)$$

The load shown by the shaded area in Figure 2.10 should be carried in the material somehow. But how? This can be accomplished by the fact if the real plastic zone size must be bigger than r_p^* . This term is called r_p , which present all plastic field above yield stress. See Figure 2.11 [16].

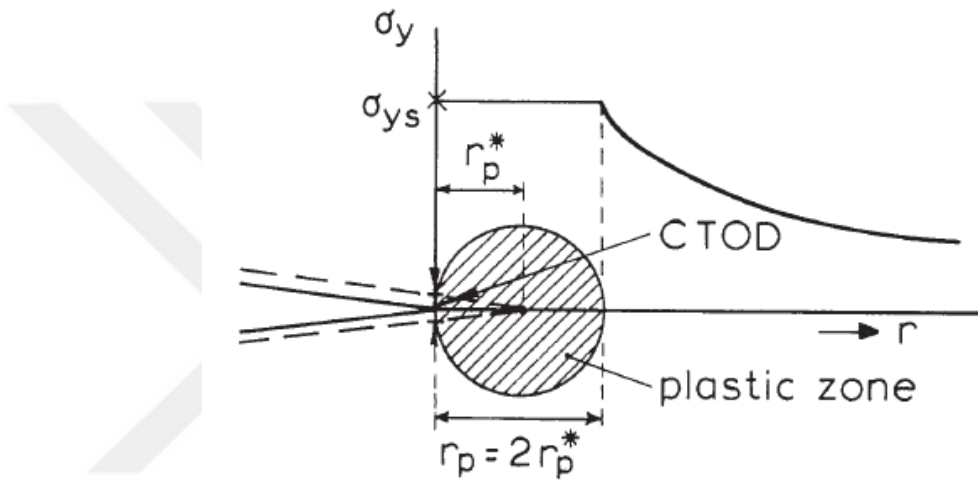


Figure 2.11 Irwin's final plastic zone correction

Plastic zone correction directly affects stress intensity factor equation. With plastic zone size addition to crack size, corrected SIF which is ΔK_p , can be generated with following equations:

$$a_{eff} = a + r_p \quad (1.24)$$

$$\Delta K_p = \beta \Delta \sigma \sqrt{\pi a_{eff}} \text{ or } \Delta K_p = \beta \Delta \sigma \sqrt{\pi(a + r_p)} \quad (1.25)$$

$$\Delta K_p = \Delta K \sqrt{1 + \left(\frac{r_p}{a}\right)} \quad (1.26)$$

In addition to Irwin's approach, Dugdale [17] and Barenblatt [18] proposed the strip yield model. Their approach assumes a long, slender shape plastic zone at the crack tip in plain stress condition. Similar to Irwin's approach, Dugdale also considers an effective crack that is longer than the physical crack size. ρ is the plastic zone length. Total crack length is $2a+2\rho$.

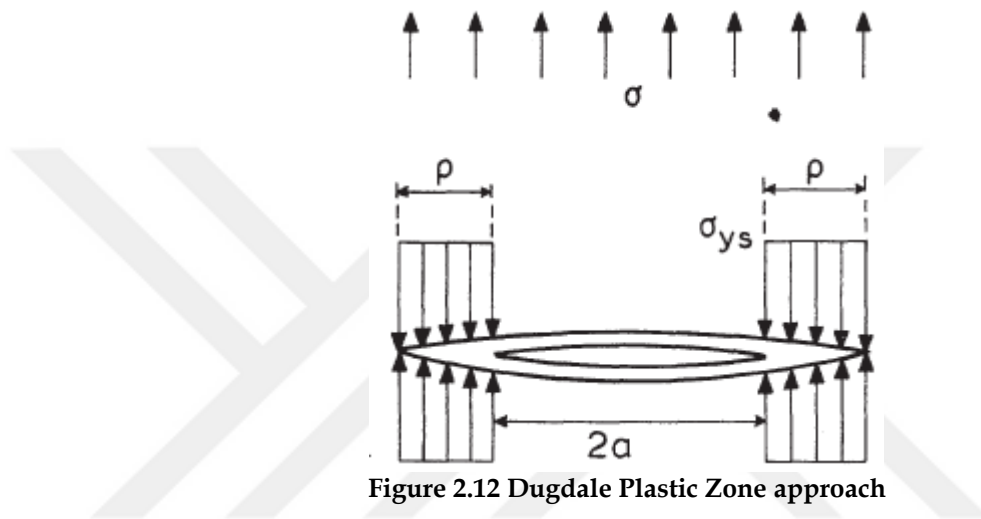


Figure 2.12 Dugdale Plastic Zone approach

According to Dugdale's approach, ahead of the actual crack, there is a crack edge, ρ , which can hold or carry the excessive yield stress σ_{ys} . In the vicinity of the crack tip, the part ρ , can still bear the yield stress. The part ρ is not cracked yet and trying to close the crack.

The size of ρ is decided when the stress level is below the yield stress which the stress singularity vanishes. K should be zero at the end of the part ρ . As a result of Dugdale approach [17], plastic zone size can be deduced as Equation 1.27.

$$\rho = \frac{\pi K^2}{8\sigma_{ys}^2} \quad (1.27)$$

The strip yield correction does not apply to other configurations rather than for a through crack in an infinite plate. Applicability of Dugdale approach and

Equation 1.27 is limited for thin specimens with surface crack in pure elastic region where $\sigma \ll \sigma_{ys}$.

Anderson [11] compared the Irwin and Dugdale approaches in Figure 2.13 that shows comparison of plastic zone estimation approaches. Nondimensionalised $\sigma_{ys}\sqrt{\pi a}$ effective stress intensity is plotted alongside the normalized stress. After the $0.5 \sigma_{ys}$, both the Irwin's approach and Dugdale's strip yield corrections slightly deviate from LEFM theory. However, the two plasticity corrections agree with each other up to approximately $0.85 \sigma_{ys}$.

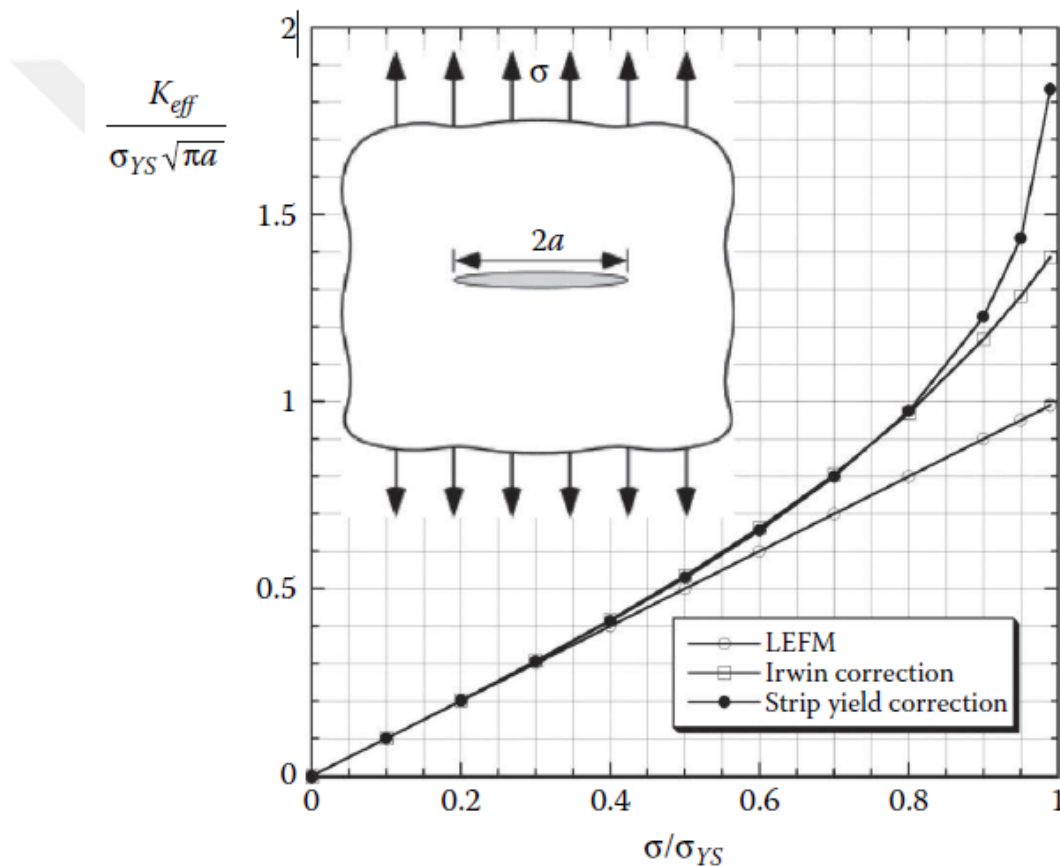


Figure 2.13 Comparison of plastic zone corrections for a through crack in plain strain

2.2.2 Plastic Zone Shape:

Plastic zone shape at the crack tip depends on which type of yielding criteria is used. Among all, Von Mises and Tresca are most widely used yielding criteria.

According to Tresca yielding criteria, yielding occurs if τ_{max} (maximum yield stress) exceeds the yield stress in shear, which is $\sigma_{ys}/2$. However, according to Von Mises criterion, yielding occurs in terms of the principal stresses shown in Equation 1.28.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_{ys}^2 \quad (1.28)$$

According to yielding criteria, plastic zone shape can be estimated by θ changes. Following Figure 2.14 [11] shows plastic zone shapes for Von Mises and Tresca yielding criteria for Mode 1 loading.

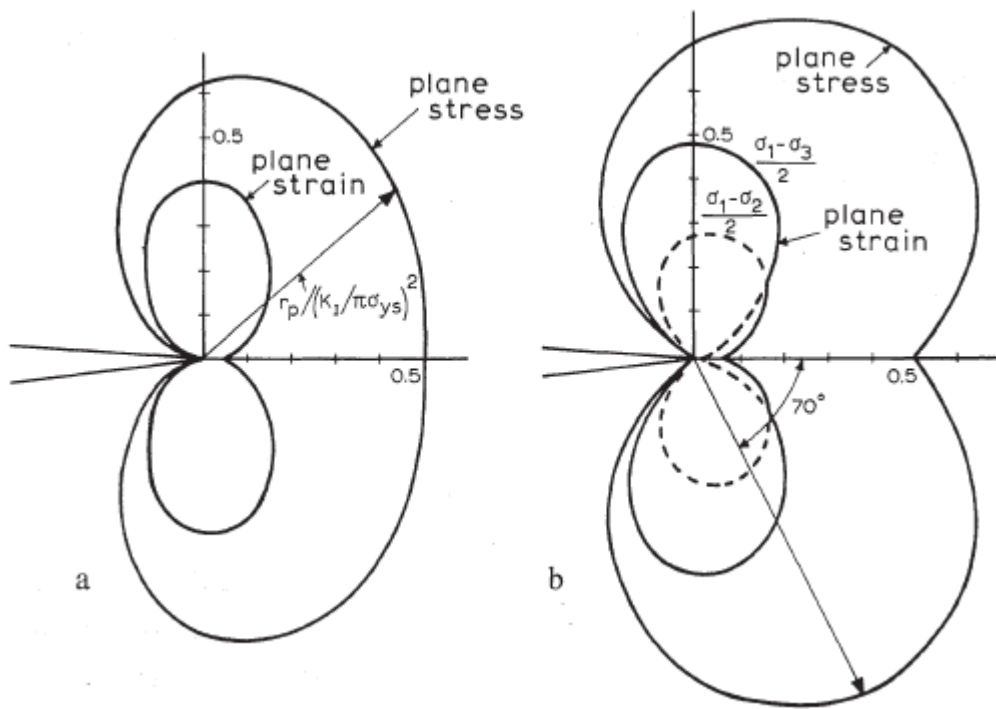


Figure 2.14 Plastic zone shapes according to Von Mises and Tresca yield criteria a. Von Mises criterion; b. Tresca criterion

2.2.3 Cyclic Plastic Zone:

Monotonic plastic zone size is explained in the previous sections. Following plastic zone term, which is cyclic plastic zone, developed during cyclic loading. It shows cyclic force driven plastic zone.

Rice [8] showed cyclic plastic zone by LEFM based approximate solutions. Schematic view of plastic zone size change at the crack tip shown in Figure 2.15 [19]. Monotonic (b) and cyclic (c) plastic zone shape and sizes as per changing load cycle are presented.

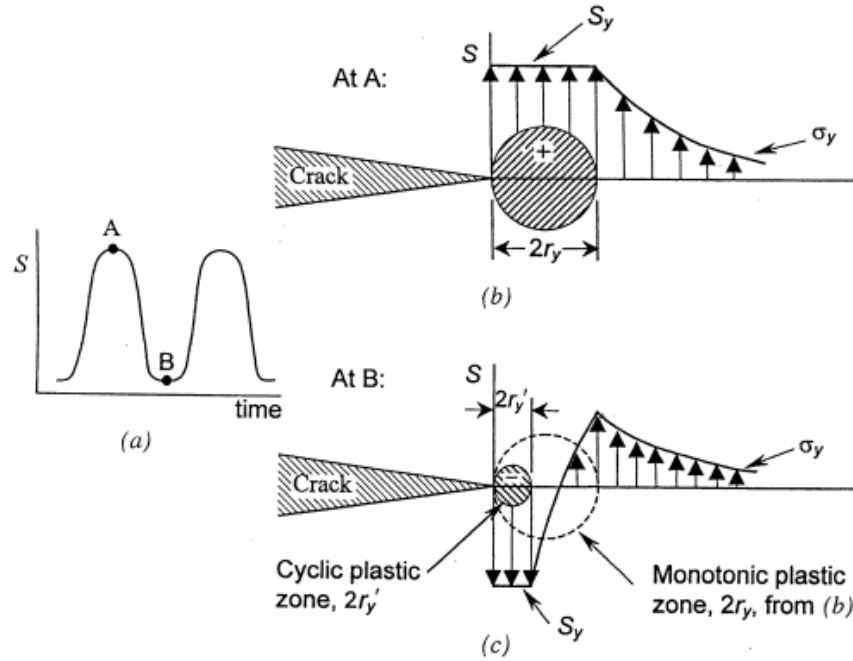


Figure 2.15 Schematic of plastic zone at the crack tip a) Loading cycle b) Monotonic plastic zone c) Cyclic plastic zone

Rice [8] also indicated following equations for cyclic plastic zone (r'_y) for plane stress and plane strain condition, respectively.

- For plane stress condition:

$$r'_y = \frac{1}{4\pi} \left(\frac{\Delta K}{\sigma_{ys}} \right)^2 \quad (1.29)$$

- For plane strain condition:

$$r'_y = \frac{1}{12\pi} \left(\frac{\Delta K}{\sigma_{ys}} \right)^2 \quad (1.30)$$

When using σ_{ys} in Equation 1.29 and 1.30, σ_{ys} should be cyclic yield strength due to softening or hardening characteristic of the material might affect yield strength in cyclic condition.

Besel et al. [13] investigated plastic zones under cyclic loading found at a crack tip. Study is based on 3D finite element solutions. They also supported their plastic zone correction study with mechanical testing.

Besel et al. [13] concluded that cyclic plastic zone exist in max and min loading. A primary plastic zone (PPZ) developing during the first loading, a backward cyclic plastic zone (BCPZ) found during unloading phases, and a forward cyclic plastic zone (FCPZ) found during subsequent loading phases. FCPZ and BCPZ form the so-called cyclic plastic zone (CPZ). The FCPZ is always much larger than the BCPZ.

Chikh et al. [20] composed recent cyclic plastic zone solutions in literature. Generalization of formulae for all crack type does not fit for all loading condition, and formulae also depends on the elastic loading or elasto-plastic loading. As a result, 3D FE solutions or specimen testing methods are to be used each crack case in order to define more accurate estimation on the cyclic plastic zone.

2.2.4 Plane Stress vs. Plane Strain:

Plane stress refers to thin specimens with large plastic zone compared to the plate thickness. The stress state can be considered bi-axial, stress perpendicular to the outer surface are non-existent, hence $\sigma_3 = 0$. On the other hand, plane strain refers to thicker specimens where the plastic zone size is small compared to plate thickness due to presence of tri-axial stresses, $\sigma_3 = \nu(\sigma_1 + \sigma_2)$.

When our interest goes to the interior of the specimen, strain along the σ_3 direction, which is, ε_3 decreases. As a result, σ_3 gradually increases from zero to the plane strain value and prevails in the interior. Accordingly, the plastic zone size progressively decreases from surface to interior of the plate that means, the plane stress plastic zone size at the surface gradually decrease to the plane strain plastic zone size in the interior of the plate, shown schematically in Figure 2.16.

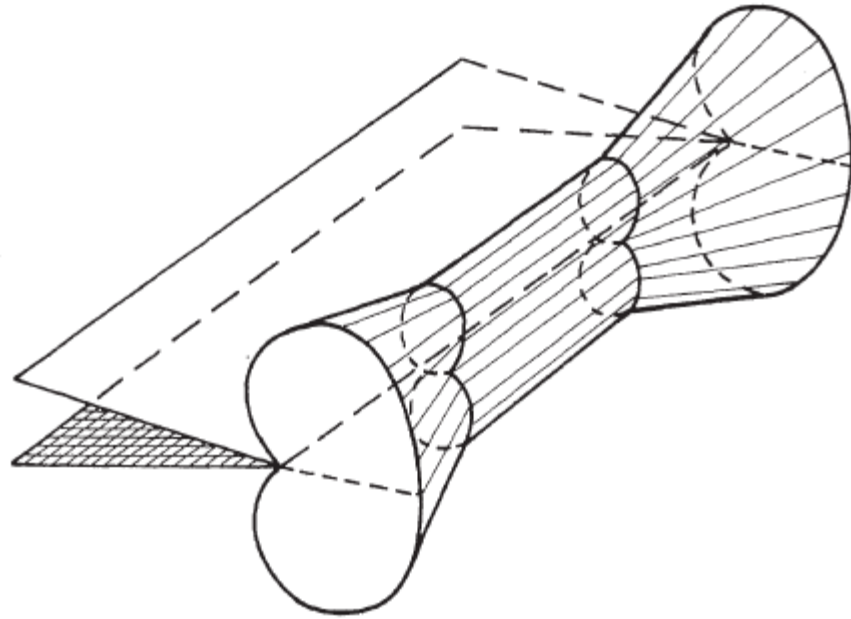


Figure 2.16 Three-dimensional plastic zone.

2.2.5 Specimen Thickness:

Specimen thickness plays significant role in testing of fracture toughness or crack growth. Thin specimens show higher fracture toughness compared to thick specimens. Following Figure 2.17 illustrates effect of specimen thickness. Thin specimens are governed by plane stress phenomena while thick is by plane strain.

While going from surface to interior in the thick specimen, creation of shear lips at the free surface decreases and stress in the σ_3 direction increases gradually. This mechanism happens with the help of the enough material along the σ_3 direction. If there is not enough material to resist σ_3 , which is called thin part, plane strain cannot be created.

Shear lips can be seen in Figure 2.17 for thin and medium specimens. Shear lip formation in free surface mainly caused by plane stress condition. Figure 2.18 additionally shows the fracture toughness value as a function of specimen thickness.

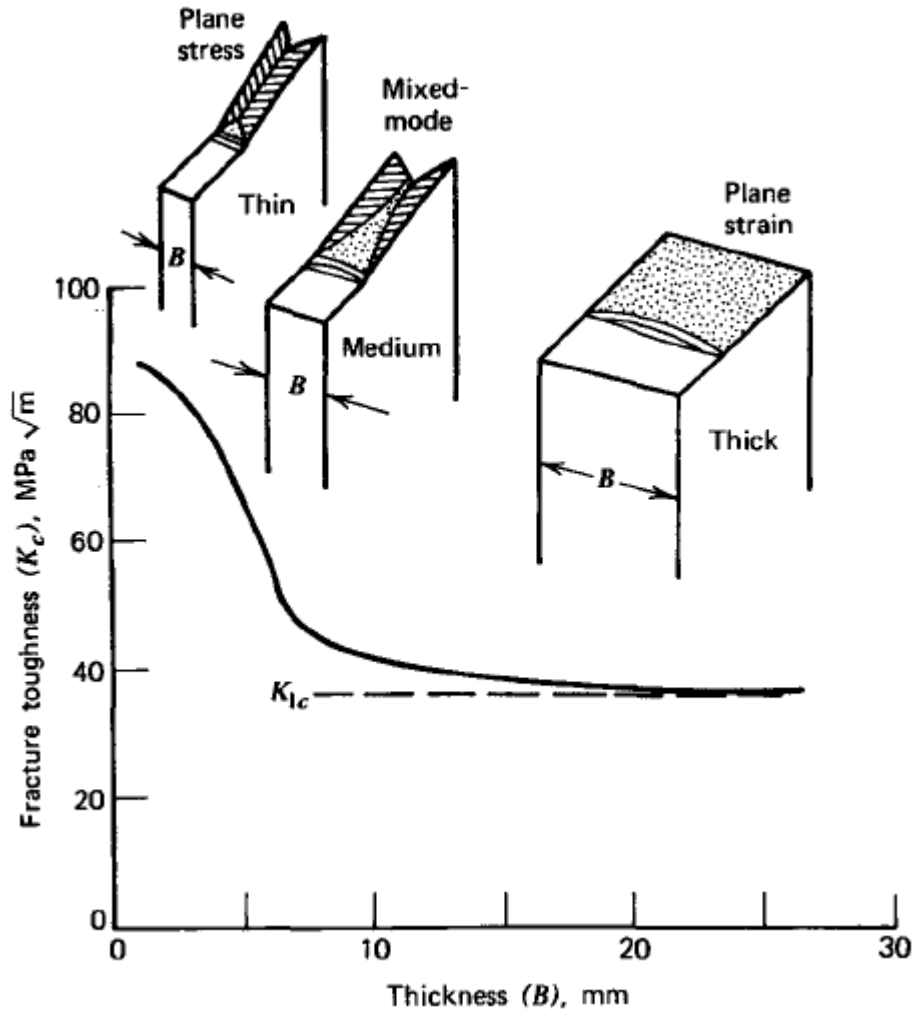


Figure 2.17 Effect of specimen thickness on stress and mode of fracture

ASTM E399 offers following equation for plane strain condition check for test specimens.

$$a_{\min}, B_{\min} \geq 2.5 \left(\frac{K_{IC}^2}{\sigma_{ys}^2} \right) \quad (1.31)$$

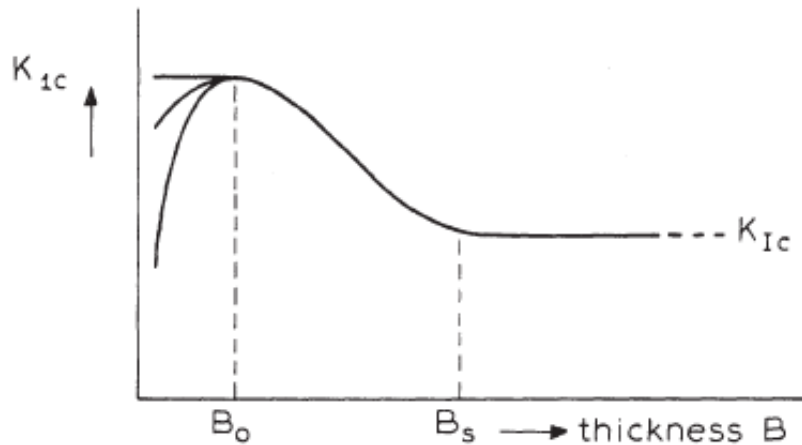


Figure 2.18 Toughness as a function of thickness.

2.2.6 Constraint Loss:

As plane strain and plane stress terms are described in the previous sections, following term “constraint loss” is explained below.

Since plane stress condition is more dominant at the surface, plastic zone is produced more easily at the surface of the crack. This difference is due to more extensive yielding at the surface than at interior locations. This phenomenon is described by the term “constraint loss”, indicating loss of plane strain constraint.

Andrews and Shih [21] indicated that the stress-intensity factor (SIF) in compact (through-crack) specimens is lower at the specimen surface than at its depth direction (midplane).

Yau and Van Stone et al. [22, 23] investigated nickel-base superalloys for constraint loss behaviors in fracture mechanics. The study has revealed that the growth in the depth direction is faster than the crack growth along the surface direction, which results in changes in crack aspect ratio (a/c) as the crack propagates.

Yau [22] proposed a constraint loss model to shift approximately 12% dc/dN curves into the da/dN curve to be in line for value of fracture toughness. Figure 2.19 shows the difference between dc/dN curves and da/dN for In718 material test.

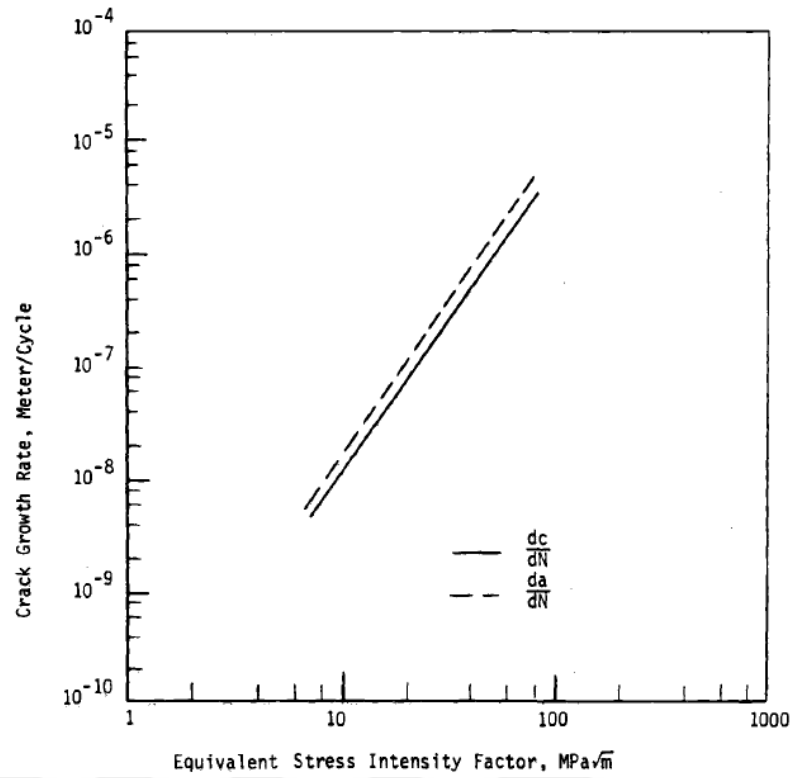


Figure 2.19 Crack growth rate curves reduced from both crack length and depth of Inconel 718 surface crack specimen at 538°C (1000°F) [22].

Van Stone et al. [23] proposed following equation to deal with constraint loss modification. β is approximately 0.88 and α can be taken as 0 if there is no performed tests.

$$\frac{K}{K_{el}} = \beta - \alpha K^2 \quad (1.32)$$

where

- K = actual K at surface,
- K_{el} = K at surface as calculated by the elastic solution,
- β, α = material constants

Table 2.1 [22] shows constraint loss surface factor for various temperatures for Inconel 718 and Ti 6-4 material. Each factor is generated by specific testing procedure that compares surface crack growth rate versus depth crack growth rate.

Table 2.1 Surface modification factors of crack growth curves [22].

Material	Temperature	Surface Factor
Inconel 718	300	0.88
	600	0.88
	800	0.89
	1000	0.89
	1200	0.89
Ti-6Al-4V (annealed)	300	0.90
	600	0.90
Ti-6Al-4V (double solution treated and aged)	75	0.90
	300	0.91
	600	0.91

2.3 Mean Stress Effects

One of the key effects, which is the mean stress effect, plays significant role in fatigue crack growth. Figure 2.20 shows da/dN curve for some stress ratios for $R \geq 0$.

$$R = \frac{K_{min}}{K_{max}} = \frac{\sigma_{min}}{\sigma_{max}} \quad (1.33)$$

When the mean stress increases (increase in R ratio), crack growth rate increases in all regions of sigmoidal curve. According to Figure 2.20, in Region I, K_{th} value vary with R ratio. Region III also shows the similar tendency in mean stress effect for fracture toughness K_{IC} . However, in Region II, the slope of the sigmoidal curve is almost identical which means exponential constant in Paris equation would be same for different R ratios.

It is to be noted that magnitude of mean stress effect depends on material type. So, these shifting amounts in sigmoidal curve strongly vary with material type.

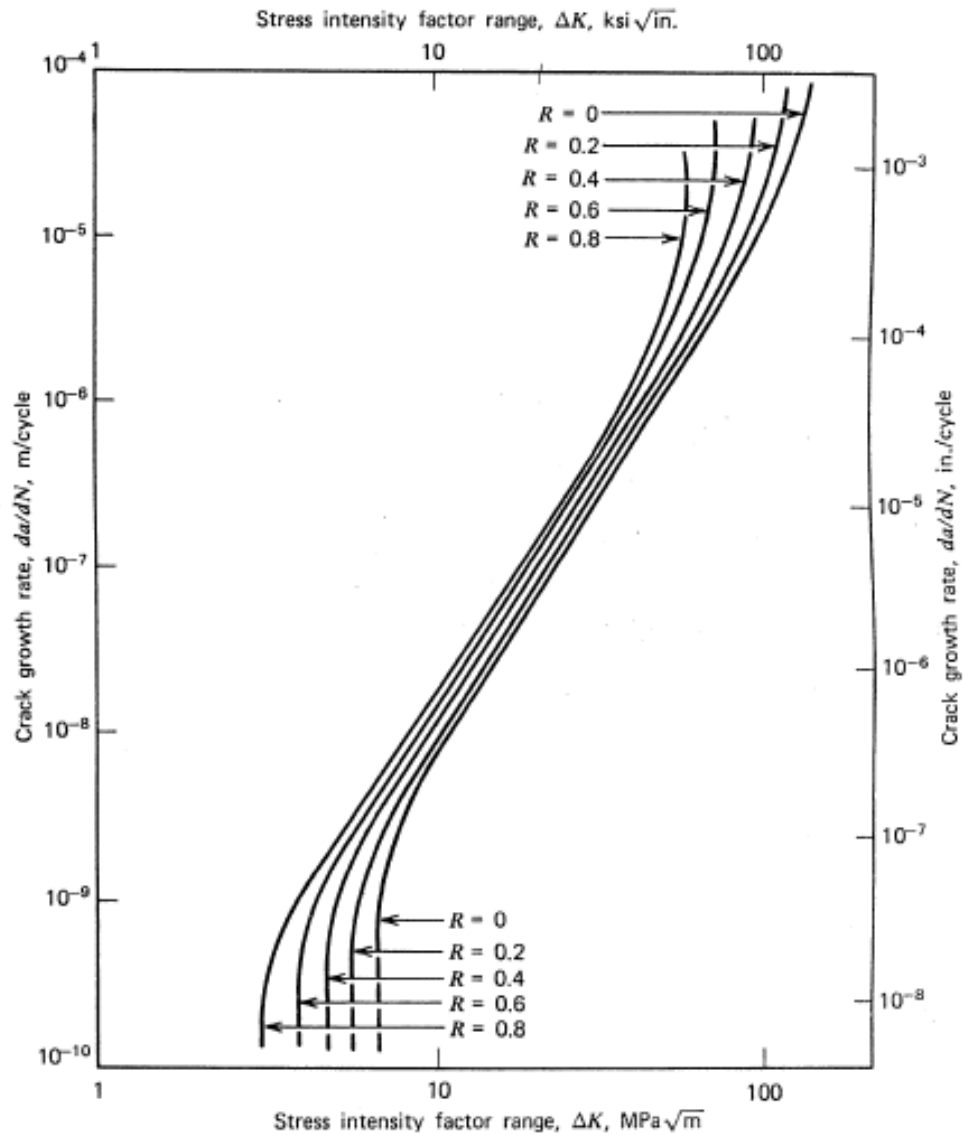


Figure 2.20 Mean stress effect in fatigue crack growth.

Walker [24], Forman [25] and Newman [26] are proposed empirical equations for mean stress effect in fatigue crack growth. Each method can be adequate in a particular area for a limited set of data. The distinctions between these expressions are minor. However, in terms of general applicability, none of them is inclusive for all R ratios and all regions of crack growth.

2.3.1 Walker Model

Walker [24] aimed to present a simple equation accounting for stress-ratio effects in fatigue crack growth. He proposed a method to take into an account mean stress effects. This method is called an effective stress modification (lately known as walker modification).

$$\bar{\sigma} = (\sigma_{max})^{1-m} \Delta\sigma^m = \sigma_{max}(1 - R)^m \quad (1.34)$$

$$\bar{K} = (K_{max})^{1-m} \Delta K^m = K_{max}(1 - R)^m \quad (1.35)$$

where m is a material constant. This constant is called as walker constant. Then, the crack growth curve can then be defined in terms of K_{eff} rather than ΔK .

When m is zero, $\bar{K}$ or K_{eff} equals K_{max} . The walker constant m is an empirical factor that take into an account mean stress effect and crack closure effects. However, the walker correction does not consider hold time effects or retardation effects.

It is to be noted that, since there are some differences on material testing procedures for fatigue test and crack growth test, m value for same material type might be different for fatigue usage and crack growth usage.

2.3.2 Dual Walker Model

The Walker approach was modified by Van Stone [27]. He proposed a separate Walker exponents for positive and negative R-ratios (m^+ and m^- , respectively). This method is called “Dual Walker Model”. Since the large amount of crack closure occurs when the minimum stress is compressive, different exponents (m^+ and m^-) are required for the different R ratios. Two different walker constants (m^+ and m^-) are generated from crack growth tests.

- $m = m^+$ when $R > 0$
- $m = m^-$ when $R < 0$

As an example, m^+ is around 0.5 to 0.7 for most of the metallic materials while m^- is around 0.2 to 0.3. When using m^- , it means that there is a compression stress on the crack which results decrease in crack growth rate.

m^+ value can be generated with the raw test results for $R>0$ test setups with the help of at least two different R ratio test conditions (i.e. $R=0.3$ nad $R=0.5$). Upon finishing the tests, crack growth values are computed by solver to fit the data according to Equation 1.35. Aim of the computing process is to find out the correct m^+ value.

m^- value can also be generated with the raw test results for $R<0$ test setups same as m^+ . If the crack growth measurement method is crack tip opening displacement, $R<0$ test setups would not be feasible in terms of test equipment feasibility. Attachment of test parts may slip or disassemble during the test due to negative stress. Thus, test procedure will fail. Potential drop measurement in the crack tip would be more feasible for $R<0$ test scenario.

Table 2.2 shows the possible loading scenarios and applicable methods in line with mean stress effect theory.

Table 2.2 Mean stress effect - possible scenarios

Case No:	Kmin	Kmax	Method
1	+	+	Standad Walker Keff with m^+
2	-	+	Standad Walker Keff with m^- or (in case of m^- is not available, $m^- = 0$)
3	-	-	No crack growth will occur
4	-	0	No crack growth will occur
5	0	+	Standad Walker Keff with m^+

2.4 Stress Gradient Effect on Propagation

Ideal cracks are common in LEFM testing specimen whereas those specimens are subjected to simple, constant loading. However, natural cracks do not meet

exact conditions of specimens. One of the major differences is the varying stress in the cracked body through the thickness. Structural cracks are usually located somewhere through the thickness of the section. Furthermore, in real-world conditions, stress acting on a crack plate is not constant, variable stress gradients are common. Whereas LEFM specimens are subjected to relatively gentle loading.

For a given place and time point, the stress gradient or stress field specifies the fluctuation in stress. It varies away from the surface that as you move away from the crack origin, the stress on the uncracked component decreases or increases. See Figure 2.21.

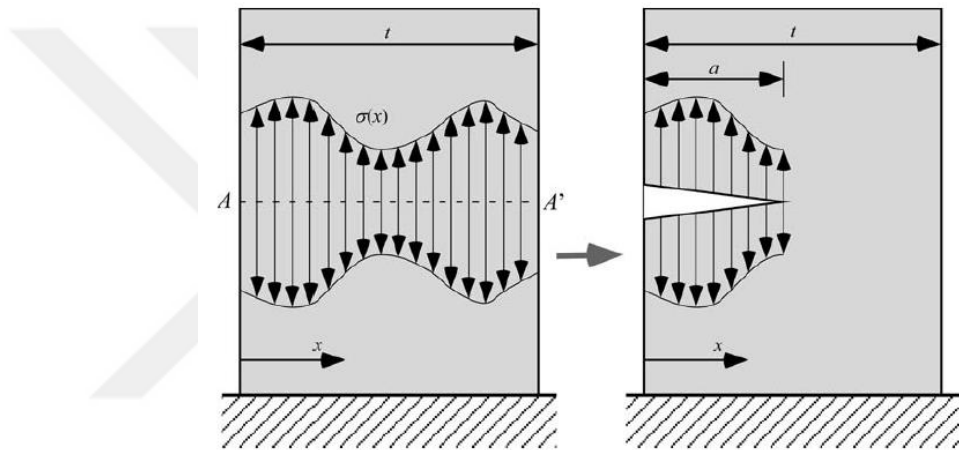


Figure 2.21 Example of Stress Gradient.

In order to eliminate time dependency of stress state in SIF solutions and da/dN integrals, it is assumed that stress state in crack does not change as the crack propagates. For example, if user defines a stress gradient once, it is assumed that this gradient and stress values does not change with crack propagation.

Since Mode 1 crack opening is the most severe one for crack propagation, stress fields shall be defined in the specific direction which is perpendicular to the major principle stress (σ_1) direction (the plane of prospective crack advance).

A polynomial or a series of distance-stress pairings can be used to represent the elastic-stress gradient. See Figure 2.22.

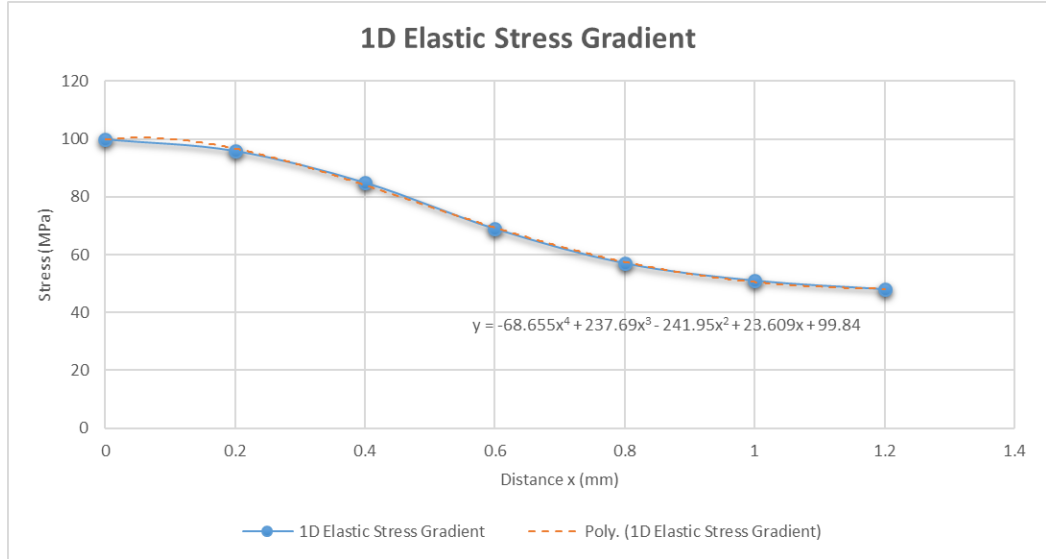


Figure 2.22 Tabulated Example of Stress Gradient

Tabulated example stress gradient is formulated by excel polynomial fit with 4th order expression given in Equation 1.36.

$$\sigma(x) = -68.655x^4 + 237.69x^3 - 241.95x^2 + 23.609x + 99.84 \quad (1.36)$$

It is known that calculation of the stress intensity factor for a given crack face stress distribution $\sigma_r(x)$ requires integration of the stress distribution and the appropriate weight function. Following Equation 1.37 shows the need for integration of the stress gradient distribution.

$$K_I = \int_0^a \sigma(x)m(x,a)dx \quad (1.37)$$

$\sigma(x)$ is known from tabulated data or user input. However, weight function, $m(x,a)$, needs to be defined to calculate integral. Following equations shows derivation process of empirical weight function model.

2.4.1 Weight functions

In order to encounter complex stress gradients, Bueckner and Rice [28,29] proposed weight function method. Glinka and Shen [30] extended this approach to surface semi-elliptical crack weight functions for 1D and 2D cracks.

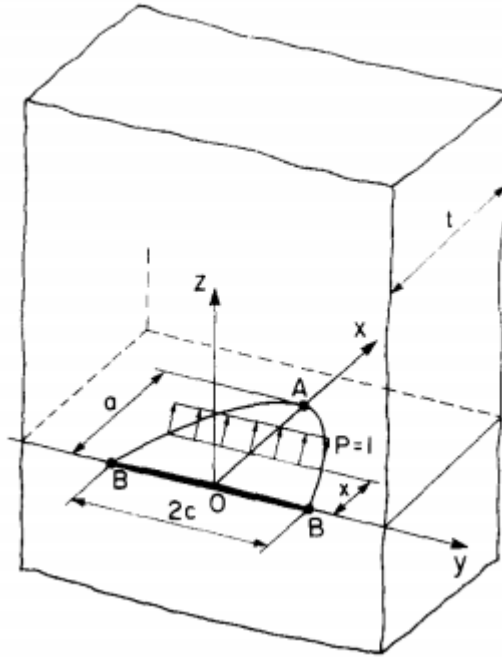


Figure 2.23 Surface semi-elliptical crack in a finite thickness plate

According to Glinka and Shen [30], following Equation 1.38 shows the weight function for surface semi-elliptical crack type. See Figure 2.23 [30]

For crack depth direction (a):

$$m_a(x, a) = \frac{2}{\sqrt{2\pi(a-x)}} \left(1 + M_{1a} \left(1 - \frac{x}{a} \right)^{1/2} + M_{2a} \left(1 - \frac{x}{a} \right) + M_{3a} \left(1 - \frac{x}{a} \right)^{3/2} \right) \quad (1.38)$$

By substituting reference stress field equations into weight function and apply double differentiation of the weight function, M_{1a}, M_{2a}, M_{3a} are determined below.

$$M_{1a} = \frac{\pi}{\sqrt{2Q}}(4Y_0 - 6Y_1) - \frac{24}{5} \quad (1.39)$$

$$M_{2a} = 3 \quad (1.40)$$

$$M_{3a} = 2 \left(\frac{\pi}{\sqrt{2Q}} Y_0 - M_{1A} - 4 \right) \quad (1.41)$$

For the first reference solution:

$$Y_0 = B_0 + B_1 \left(\frac{a}{t} \right)^2 + B_2 \left(\frac{a}{t} \right)^4 \quad (1.42)$$

$$B_0 = 1.10190 - 0.019863(a/c) - 0.043588(a/c)^2 \quad (1.43)$$

$$B_1 = 4.32489 - 14.9372(a/c) + 19.4389(a/c)^2 - 8.52318(a/c)^3 \quad (1.44)$$

$$Q = 1. + 1.464(a/c)^{1.65} \text{ for } 0 \leq a/c \leq 1 \quad (1.45)$$

For the second reference solution:

$$Y_1 = A_0 + A_1(a/t)^2 + A_2(a/t)^4 \quad (1.42)$$

$$A_0 = 0.456128 - 0.114206(a/c) - 0.046523(a/c)^2 \quad (1.43)$$

$$A_1 = 3.022 - 10.8679(a/c) + 14.94(a/c)^2 - 6.8537(a/c)^3 \quad (1.44)$$

$$A_2 = -2.28655 + 7.88771(a/c) - 11.0675(a/c)^2 + 5.16354(a/c)^3 \quad (1.45)$$

For crack surface direction (c):

Similar procedure from the depth direction is applied for the crack surface direction (c).

$$m_c(x, a) = \frac{2}{\sqrt{2\pi x}} \left(1 + M_{1c} \left(\frac{x}{a} \right)^{1/2} + M_{2c} \left(\frac{x}{a} \right) + M_{3c} \left(\frac{x}{a} \right)^{3/2} \right) \quad (1.46)$$

By substituting reference stress field equations into weight function and apply double differentiation of the weight function, M_{1c}, M_{2c}, M_{3c} are determined below.

$$M_{1c} = \frac{\pi}{\sqrt{4Q}} (30F_1 - 18F_0) - 8 \quad (1.47)$$

$$M_{2c} = \frac{\pi}{\sqrt{4Q}} (60F_0 - 90F_1) + 15 \quad (1.48)$$

$$M_{3c} = -(1 + M_{1c} + M_{2c}) \quad (1.49)$$

For the first reference solution:

$$F_0 = \alpha(a/c)^\beta \quad (1.50)$$

$$\alpha = 1.14326 + 0.0175996(a/t) + 0.501001(a/t)^2 \quad (1.51)$$

$$\beta = 0.458320 - 0.102985(a/t) - 0.398175(a/t)^2 \quad (1.52)$$

For the second reference solution:

$$F_1 = \gamma(a/c)^\delta \quad (1.53)$$

$$\gamma = 0.976770 - 0.131975(a/t) + 0.484875(a/t)^2 \quad (1.54)$$

$$\delta = 0.448863 - 0.173295(a/t) - 0.267775(a/t)^2 \quad (1.55)$$

F_0, F_1 and Y_0, Y_1 are normalized Stress intensity factors (reference solutions). F_0, F_1 are obtained at the c-tip, and Y_0, Y_1 are at the a-tip.

Glinka and Shen [31] also revealed that the four-term universal weight function is applicable into numerous one-dimensional edge cracks and through cracks of any depth (with accuracy better than 1%).

2.5 Other Parameters

Main parameters for crack growth are discussed in the previous sections. However, those parameters are not limited with them. There are also some additional concepts that effects crack propagation which will be explained briefly in this section. Those are retardation effect, time dependent mechanisms, static crack growth (dwell), oxidation on crack tip, creep-temperature effect.

2.5.1 Retardation Effect

When the loading condition vary with the time, load history effects might affect crack growth behavior. As explained in previous sections, plastic zone size at the crack tip affects crack growth. In real life applications loading rates vary with the time, it has peak conditions rarely. Consequently, plastic zone size is affected significantly from overloads.

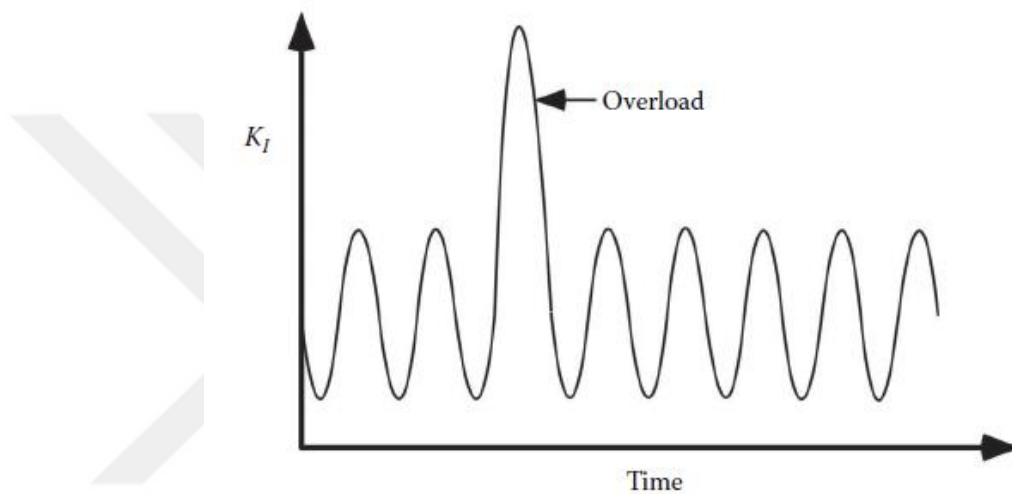


Figure 2.24 Schematic of Overload.

A single overload interrupts the constant-amplitude ($R=\text{constant}$) loading during cyclic loading in Figure 2.24. During the constant amplitude loading regime, there is a steady-state small plastic zone. However, overload brings extensively larger plastic zone size. After that, loading decreases very low value but crack growth behavior changed due to fact that there is still significant residual stress that come from previous overload plastic zone See Figure 2.25.

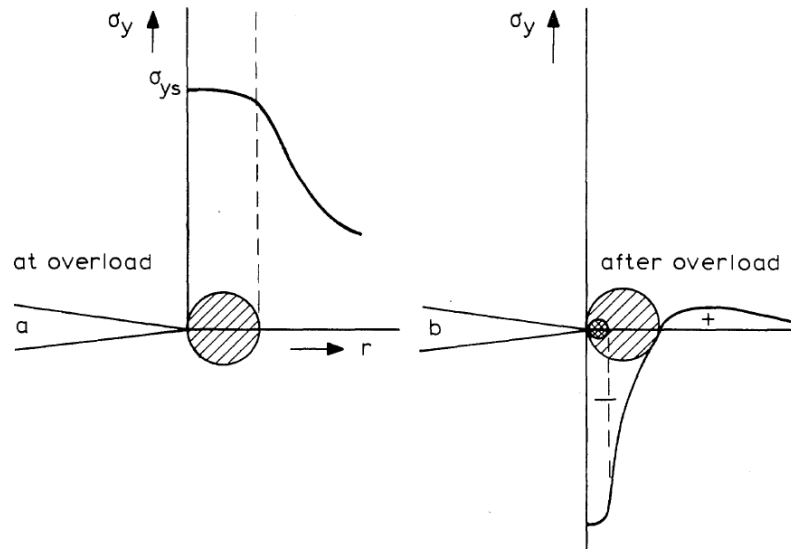


Figure 2.25 Residual stress after overload (compressive).

The surrounding elastic material will force crack tip to create compressive plastic zone. Compressive plastic zone retards the crack growth rate until tensile plastic zone overcome compressive one. See Figure 1.26. As a result, overload may arrest completely or partially crack propagation during low amplitude cycling.

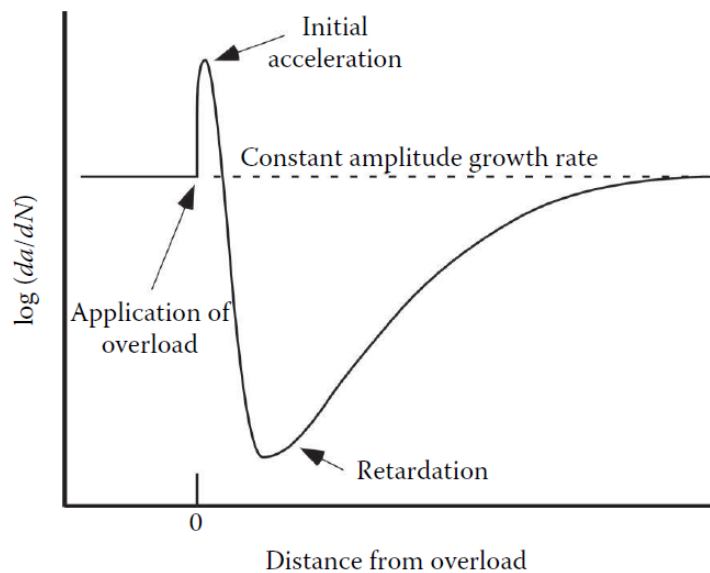


Figure 2.26 Retardation effect in crack growth rate.

2.5.2 Time-dependent crack growth, dwell-creep-oxidation

Gas turbine engine components are subject to operate at high temperature. When the applied load remains constant for prolonged period at elevated temperature, crack propagation rates might be affected by time dependent mechanisms such as creep, oxidation and corrosion. This time dependent crack growth effect is called with different titles that are dwell fatigue, creep crack growth or static crack growth.

$$\frac{da}{dt} = e^B (K_{eff}/K_{th})^P [\ln(K_{eff}/K_{th})]^Q [\ln(K_{crit}/K_{eff})]^D \quad (1.56)$$

If crack still grows when there is no cycle increment, new term in Equation 1.56, which is static crack growth, da/dt needs to be defined. Sigmoidal curve for da/dt , similar to cyclic crack growth (da/dN), can be defined in Figure 2.27.

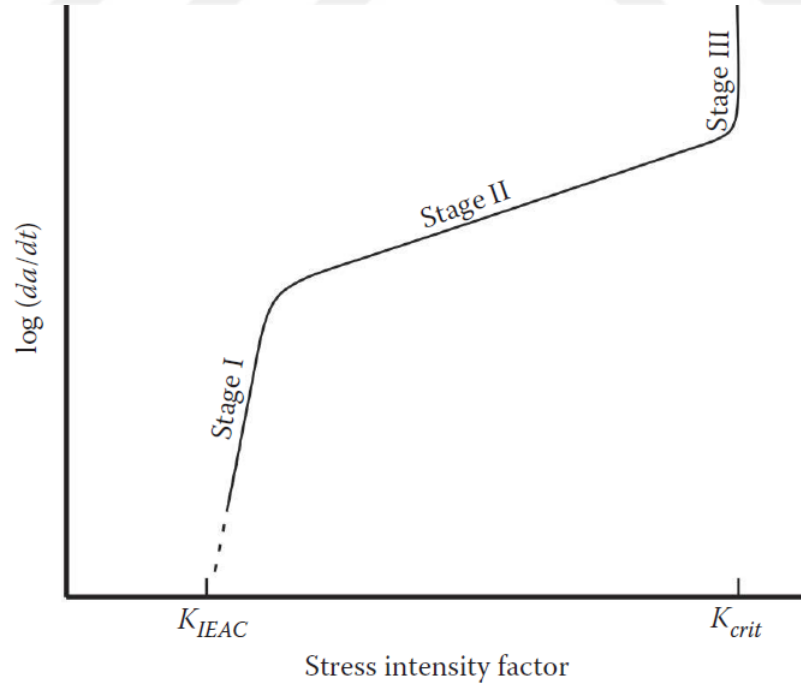


Figure 2.27 da/dt Sigmoidal curve

Environmentally assisted crack growth at high temperature application is mainly depends on laboratory testing results which indicates high temperature, harsh environment, dwell time and combinations of these effects. Linear superposition of time dependent and independent crack growth rates is possible.



3. LEFM TESTS

3.1 ASTM standards and Specimen Types

The American Society for Testing and Materials-ASTM, the British Standards Institution-BSI and the International Organization for Standardization-ISO are just the organizations that issue standards for fracture mechanics measurements. ASTM created the first K and J testing standards in 1970.

Test types are determined according to needs and aim is to examine required properties such as K_{IC} , J_{IC} , K_{th} , $K-R$ curve, $J-R$ curve, CTOD, testing, creep crack growth etc.

Specimen configurations and orientations depends on the needs and extend of the test method. ASTM standardized the specimen geometries in related standard number. Following Figure 3.1 shows some of the orientation types of fracture mechanics testing specimens [11].

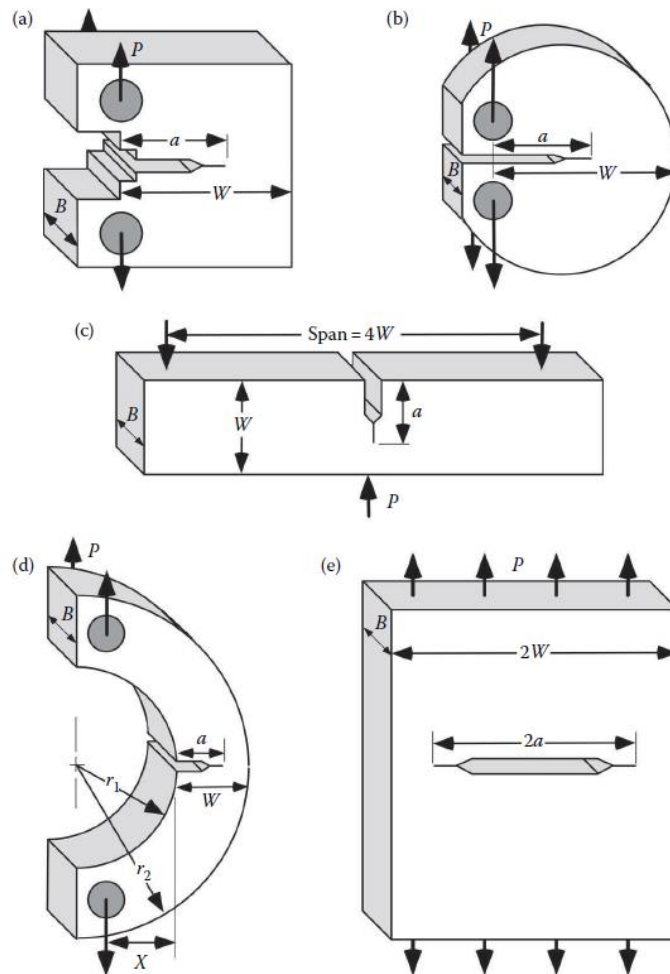


Figure 3.1 Fracture mechanics test specimens: (a) C(T) specimen, (b) disk-shaped compact specimen, (c) single-edge notched bend (SE(B)) specimen, (d) MT specimen, and (e) arc-shaped specimen

Geometrical orientation of the specimen preparing process in the tested part plays important role in test results. Therefore, ASTM E1823-13 shows the standard terminology relating to fatigue fracture testing.

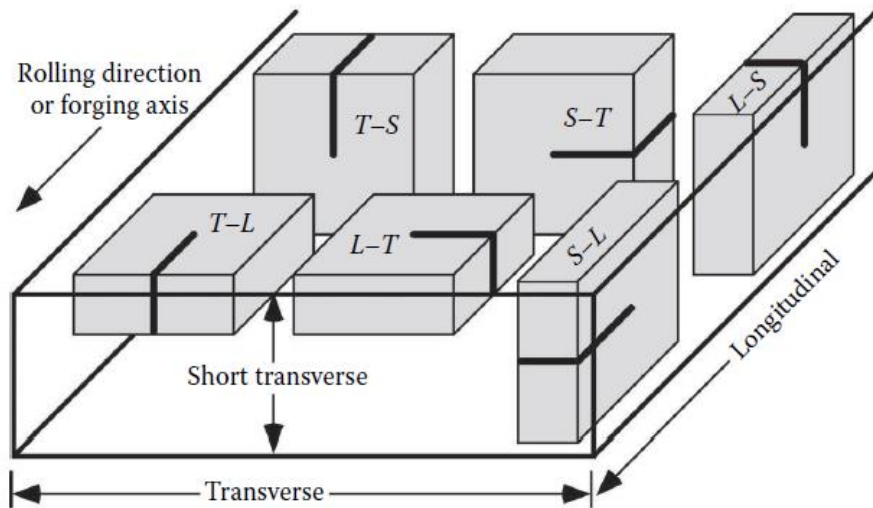


Figure 3.2 ASTM notation for specimens extracted from rolled plate and forgings.

Following Table 3.1 shows the list of available ASTM standards for general fracture mechanics.

Table 3.1 The list of available ASTM standards for fracture mechanics.

Designation	Title
ASTM E399 - 20a	Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness of Metallic Materials
ASTM E561 - 20	Standard Test Method for KR Curve Determination
ASTM E740 / E740M - 03(2016)	Standard Practice for Fracture Testing with Surface-Crack Tension Specimens
ASTM E1221 - 12A(2018)e1	Standard Test Method for Determining Plane-Strain Crack-Arrest Fracture Toughness, K_{Ia} , of Ferritic Steels
ASTM E1820 - 20b	Standard Test Method for Measurement of Fracture Toughness
ASTM E1921 - 20	Standard Test Method for Determination of Reference Temperature, T_0 , for Ferritic Steels in the Transition Range
ASTM E2472 - 12(2018)e1	Standard Test Method for Determination of Resistance to Stable Crack Extension under Low-Constraint Conditions
ASTM E2818 - 11(2019)	Standard Practice for Determination of Quasistatic Fracture Toughness of Welds
ASTM E2899 - 19e1	Standard Test Method for Measurement of Initiation Toughness in Surface Cracks Under Tension and Bending
ASTM E3039 - 20	Standard Test Method for Determination of Crack-Tip-Opening Angle of Ferritic Steels Using DWTT-Type Specimens

3.2 Kth Tests

K_{th} , threshold stress intensity factor value is the lowest point at which a fatigue crack propagation will not occur. The threshold value is determined experimentally via a load shedding process either constantly or by a series of decremented steps. K is gradually reduced until there is no measurable crack growth, $da/dN \cong 0$.

Following table shows threshold testing related ASTM procedures of K_{th} .

Table 3.2 Threshold testing related ASTM procedures

Designation	Title
ASTM E647 - 15e1	Standard Test Method for Measurement of Fatigue Crack Growth Rates
ASTM E1681 - 03	Standard Test Method for Determining Threshold Stress Intensity Factor for Environment-Assisted Cracking of Metallic Materials

LEFM principles and fracture mechanics rules rely on the infinitely sharp crack tip. As explained in detail in the ASTM E647 standard [32], prior to the actual test, fatigue precracking for all specimens is required to provide a sharp crack tip.

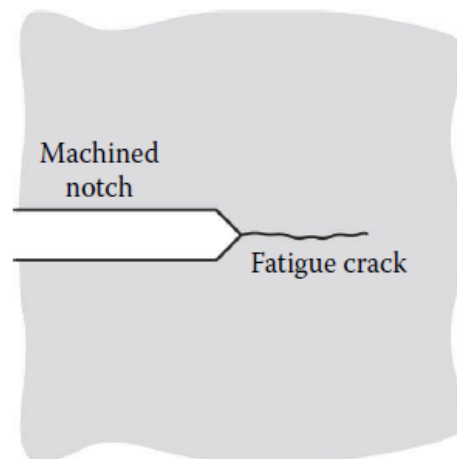


Figure 3.3 Fatigue Precracking

In order to avoid retardation effects, the initial K_{max} in the fatigue test should be greater than the K_{max} at the end of fatigue precracking process. If not, retardation effects might have an impact on plastic zone size and da/dN rates.

According to ASTM Standard E647 [32], there are two methods to conduct the test. K_{max} or the R ratio is held constant while K decreases in a typical K decreasing test. Due to the R ratio effect on K_{th} , these two methods generally result in a different behavior in the threshold region, K_{th} .

It is to be noted that retardation effect in K -decreasing tests might play significant role in plastic zone size and da/dN rates. Since previous cycles produce larger plastic zones, which might slow crack formation, the K -decreasing test is more likely to create historical effects in crack tip.

Retardation is not an issue because the plastic zone formed by each cycle is slightly larger than the preceding cycle's in a K -increasing test.

3.3 Propagation Tests

As explained in K_{th} test section, prior to propagation test, precracking is also required for sharp crack tip in crack growth (propagation) test.

In order to measure crack length and crack growth periodically during the test, measurement techniques are developed. Application of the measurement method depends on material type, test scenario and required data type etc. Those techniques are optical, compliance, ultrasonic, eddy current, acoustic emission and potential drop methods.

Following Table 3.3 shows the list of current applicable crack propagation related test standards.

Table 3.3 The list of current applicable crack propagation related test standards

Designation	Title
ASTM E647 - 15e1	Standard Test Method for Measurement of Fatigue Crack Growth Rates
ASTM E1457 - 19e1	Standard Test Method for Measurement of Creep Crack Growth Times in Metals
ASTM E1681 - 03	Standard Test Method for Determining Threshold Stress Intensity Factor for Environment-Assisted Cracking of Metallic Materials
ASTM E2760 - 19e1	Standard Test Method for Creep-Fatigue Crack Growth Testing

Scanning the full range of the sigmoidal curve in only one conducted test is not possible. Firstly, K decreasing test is performed to understand K_{th} and Region I behaviour. Then, a constant load amplitude (K increasing test) fatigue crack development test is performed. Results of the da/dN crack growth rate testing are generally above $\frac{10^{-8}m}{cycle} \left(4 \times \frac{10^{-7}in}{cycle} \right)$ which corresponds to Region II and Region III fatigue crack growth.

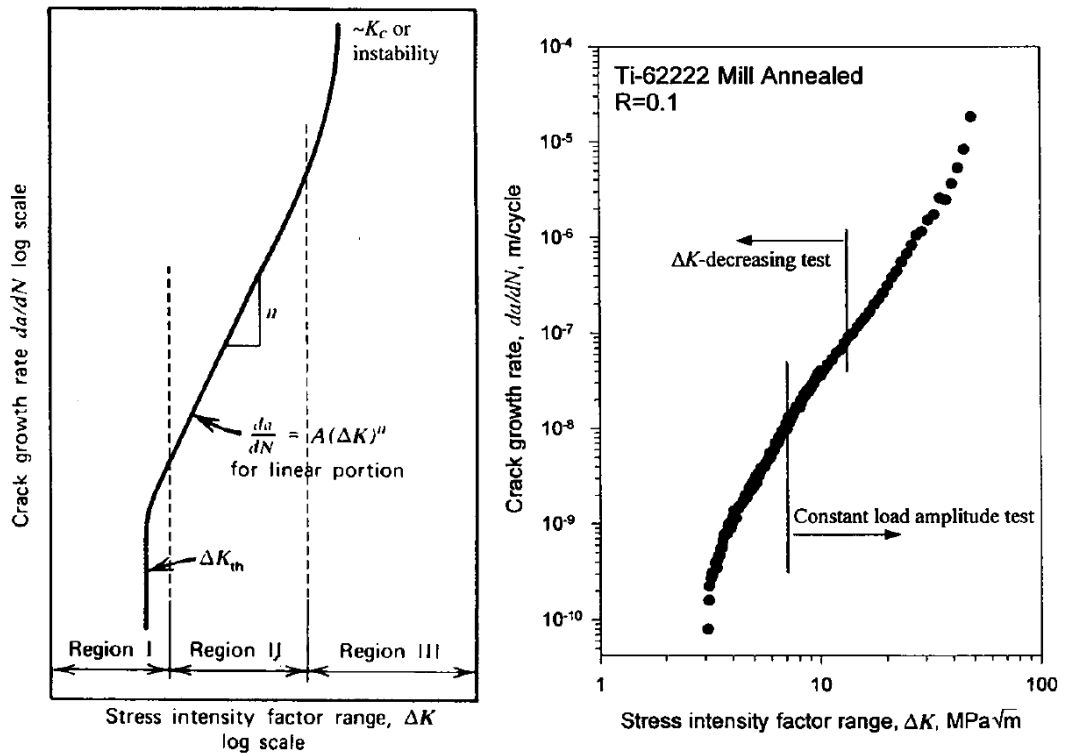


Figure 3.4 Sigmoidal Crack Growth Curve and Regions - Ti-62222 Mill Annealed.

With the aim of generating propagation rate, da/dN data, firstly, the a versus N data is taken from test machines. Then, data reduction is required to generate da/dN data. ASTM Standard E647 [32] recommends two techniques, point to point (secant) and successive points (incremental polynomial) data reduction techniques. Two of them are applicable for K -increasing while secant technique is more preferable in K -decreasing test.

3.4 K_{IC} Tests

ASTM Standard E399 [33], shows K_{IC} testing procedure details. It is only applicable for plane-strain condition tests. Plastic zone size shall be small compared with the width or length of the specimen cross section. There is a specific requirement in standard for plane strain condition, which is shown in Equation 3.1.

$$W - a \geq 2.5 \left(\frac{K_{IC}^2}{\sigma_{ys}^2} \right) \quad (3.1)$$

Rectangular Compact Test (C(T)) or Single-edge notched bend (SE(B)) specimens are the most typical specimen types for fracture toughness tests. Fatigue precracking is also required for need of sharp edged crack. This fatigue precracking should be small enough to not create large crack tip plasticity.

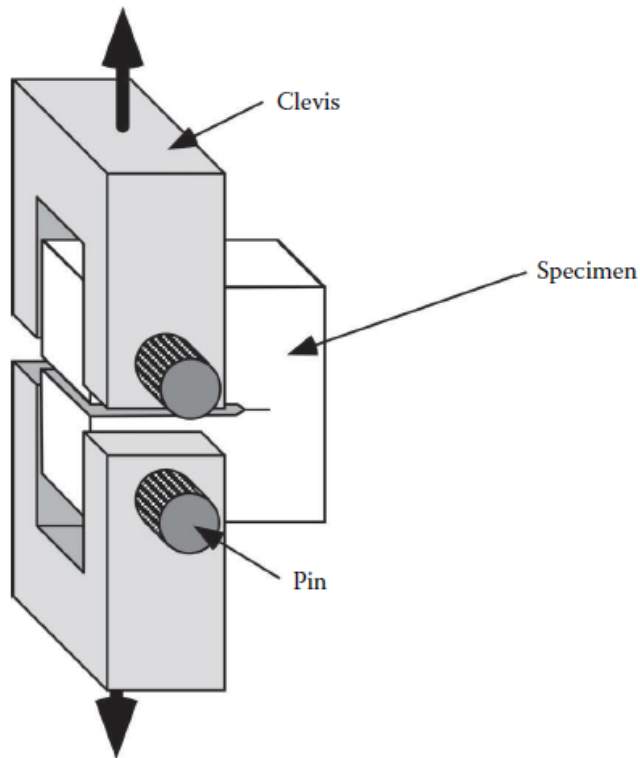


Figure 3.5 Test setup configuration for C(T) specimen

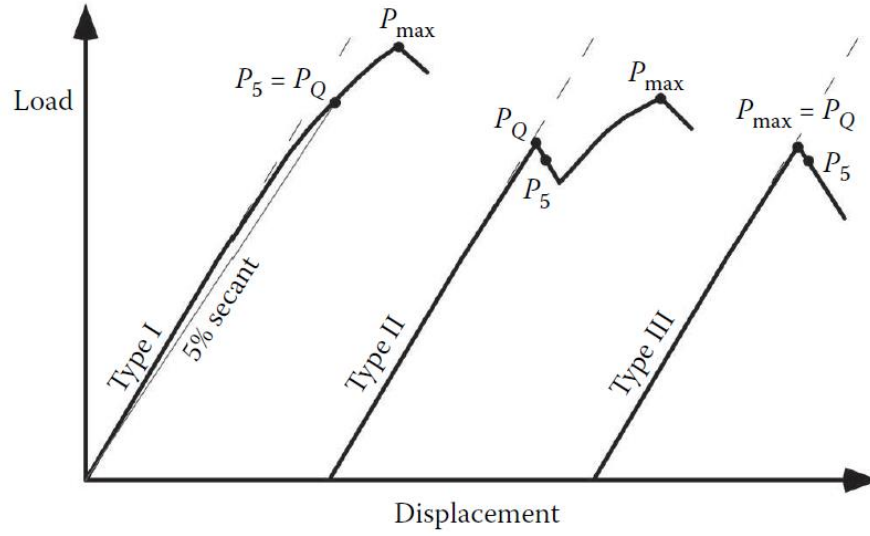


Figure 3.6 KIC test - Load-displacement behaviour.

Applied load P_Q reaches a critical value from one of the three types of loading method. Then, test is terminated.

$$K_Q = \frac{P_Q}{B\sqrt{W}} f(a/W) \quad (3.2)$$

After the test, K_Q calculation follows. If the test results and specimen configuration meets all of the necessities below from ASTM E399 [33], then $K_Q = K_{IC}$.

Requirements:

- $0.45 \leq a/W \leq 0.55$
- $W - a \geq 2.5 \left(\frac{K_Q}{\sigma_{YS}} \right)^2$
- $P_{max} \leq 1.10P_Q$

3.5 Fractured Specimen Investigations

After the failure part or after the completing the crack propagation or fracture toughness tests, fractured specimen investigation is essential to understand growth behavior. Visual examination is one of the key techniques. SEM “Scanning Electron Microscopy” is most widely used visual examination tool in laboratories. Light source and magnifiers are also used to detect beachmarks or fatigue striations on fractured surface. Length of the fatigue striations are the sign of magnitude in loading cycle and crack growth rate corresponding to this cycle repetition.

Figures 3.7 and Figure 3.8 provide the most detailed schematics for understanding fatigue fractures in round and rectangular cross sections, respectively. Each one includes examples of loading condition (high and low nominal stress), three type of stress concentration (no, mild, severe), and five different forms of loading: (tension-tension or tension-compression, unidirectional bending, reversed bending, rotating bending, and torsional bending) [34].

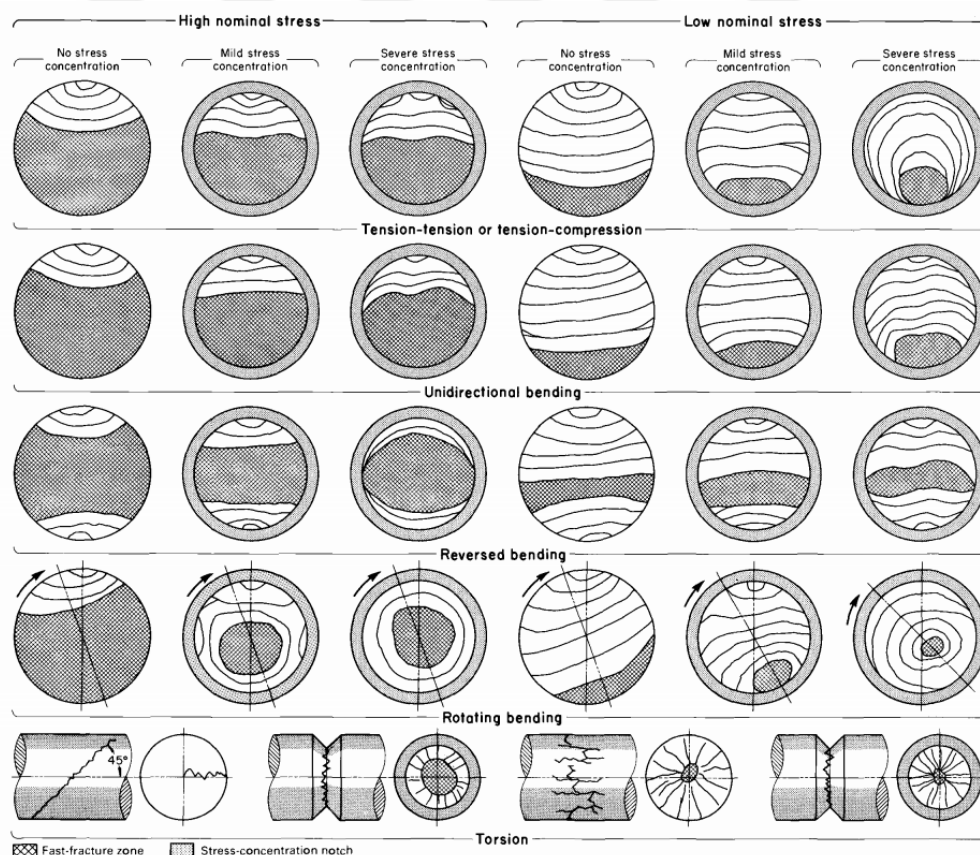


Figure 3.7 Schematic view of fatigue fracture surface marks with circular cross sections [34].

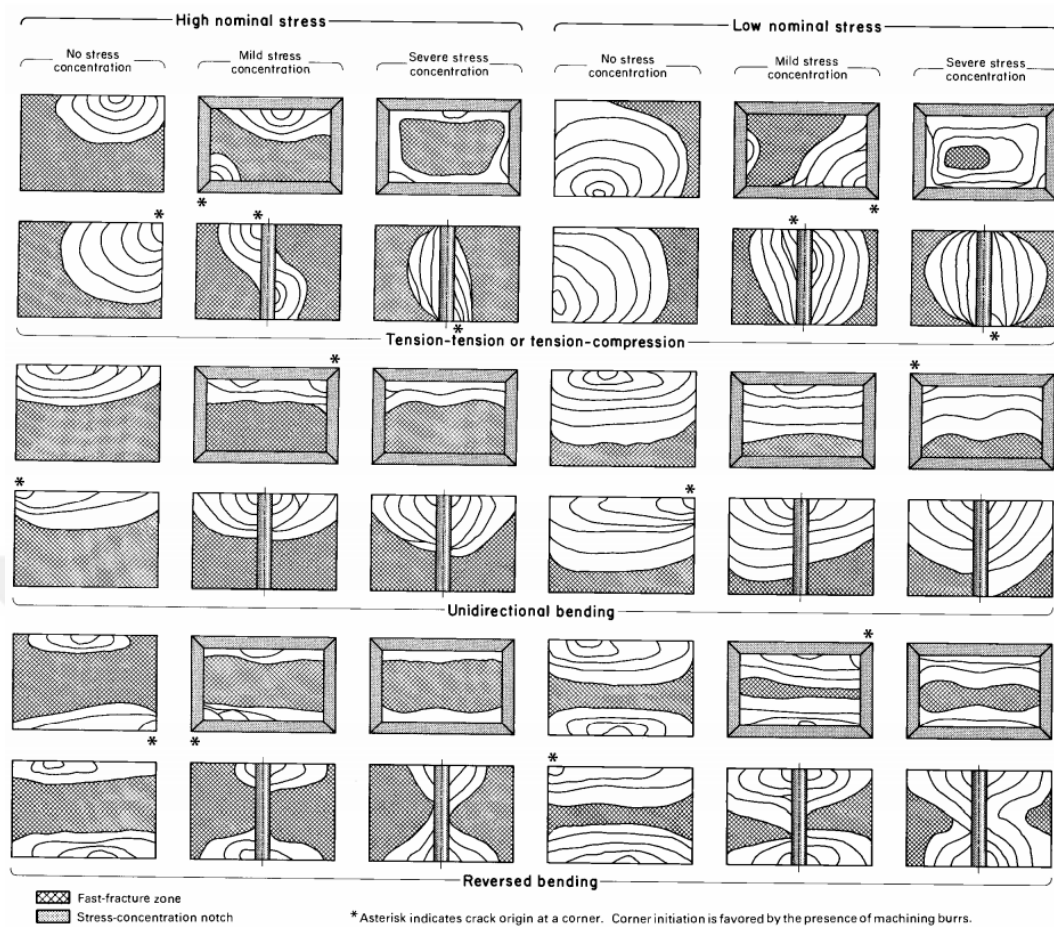


Figure 3.8 Schematic view of fatigue fracture surface marks with square and rectangular cross sections [34].

4. ALGORITHM

Generally, crack growth calculation algorithms are mainly based on Paris and Erdogan's propagation law (Equation 2.14), which is called power law. In this study, extended version is used (Equation 1.20). This version has some modifications in the the formulae in order to include all sigmoidal curve, Stage 1 and Stage 3 crack growth. The sigmoidal curve allows user to include all stages of the crack growth in calculations.

Initial crack dimensions and loading conditions are main input parameters. Algorithm is designed to calculate crack size increment in each iteration step. Propagation is allowed until one of the termination criteria is met. After the termination, final crack dimensions and final cycle number is written.

- Crack size reaches its dimensional limits
- Calculated SIF reaches 96% of fracture toughness

4.1 Main Algorithm

The main algorithm consist of following steps; input parameters, geometry factor calculation, SIF calculation, decision of crack stage (stage 1, stage 2 or stage 3), crack size increment integral calculation, generating new crack dimensions. Following Figure 4.1 shows the basic concept of the main algorithm flowchart without any sub-algorithm steps.

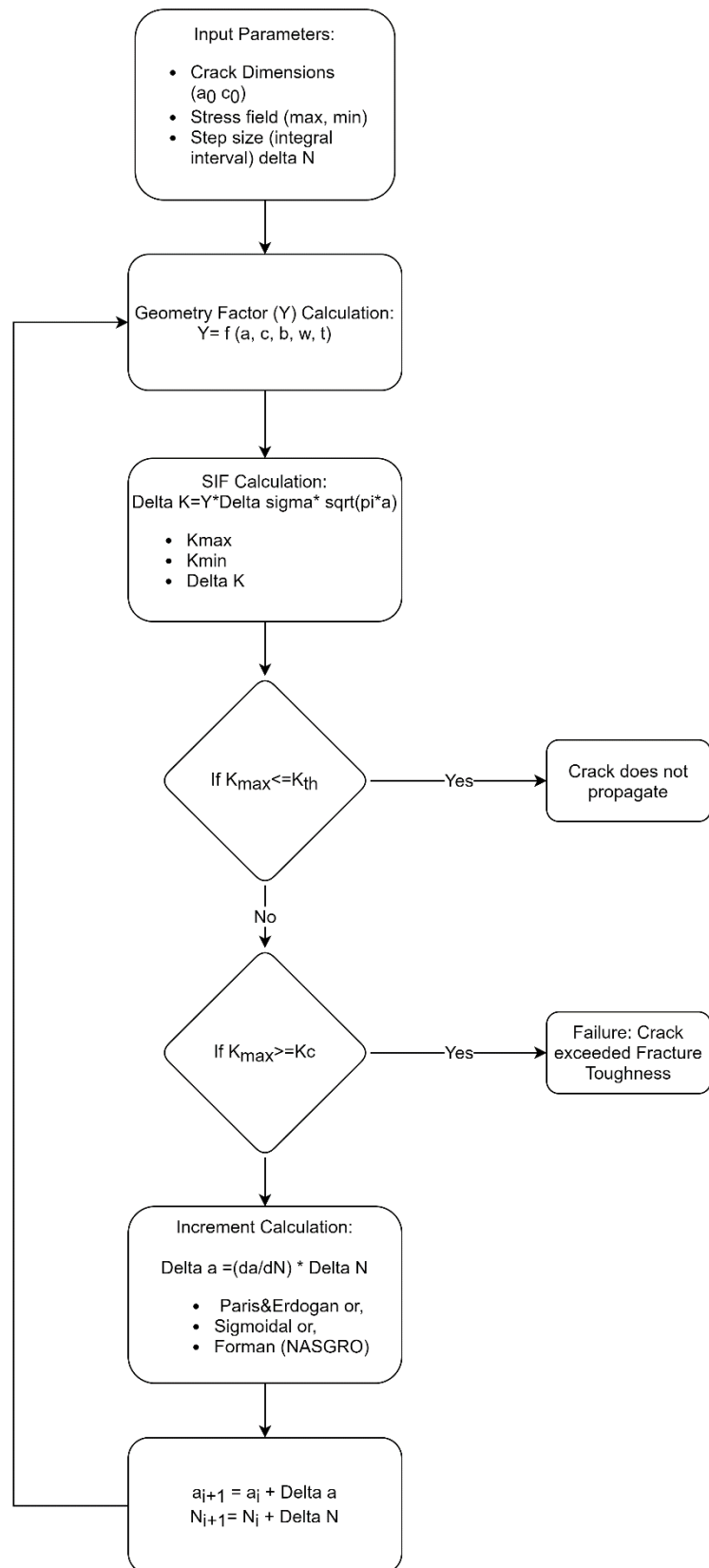


Figure 4.1 Main Algorithm Flowchart

4.2 Sub-Algorithms

Beside of the main algorithm, there are also sub-algorithms which acts like a supporting algorithm in order to include additional features into the main algorithm. This concept is called as object-oriented programming that depends on classes and objects. It helps to divide coding and algorithm into simple and modifiable pieces.

Sub-algorithms are plastic zone correction, constraint loss, Runge-Kutta 2, Walker and stress gradient. Sub-algorithms are also illustrated in Figure 4.2 to Figure 4.6. After all of the sub-algorithm steps are included into flowchart, it is too long and far beyond a page size. See Figure 4.6, Figure 4.7 and Figure 4.8.



Plastic zone sub-algorithm:

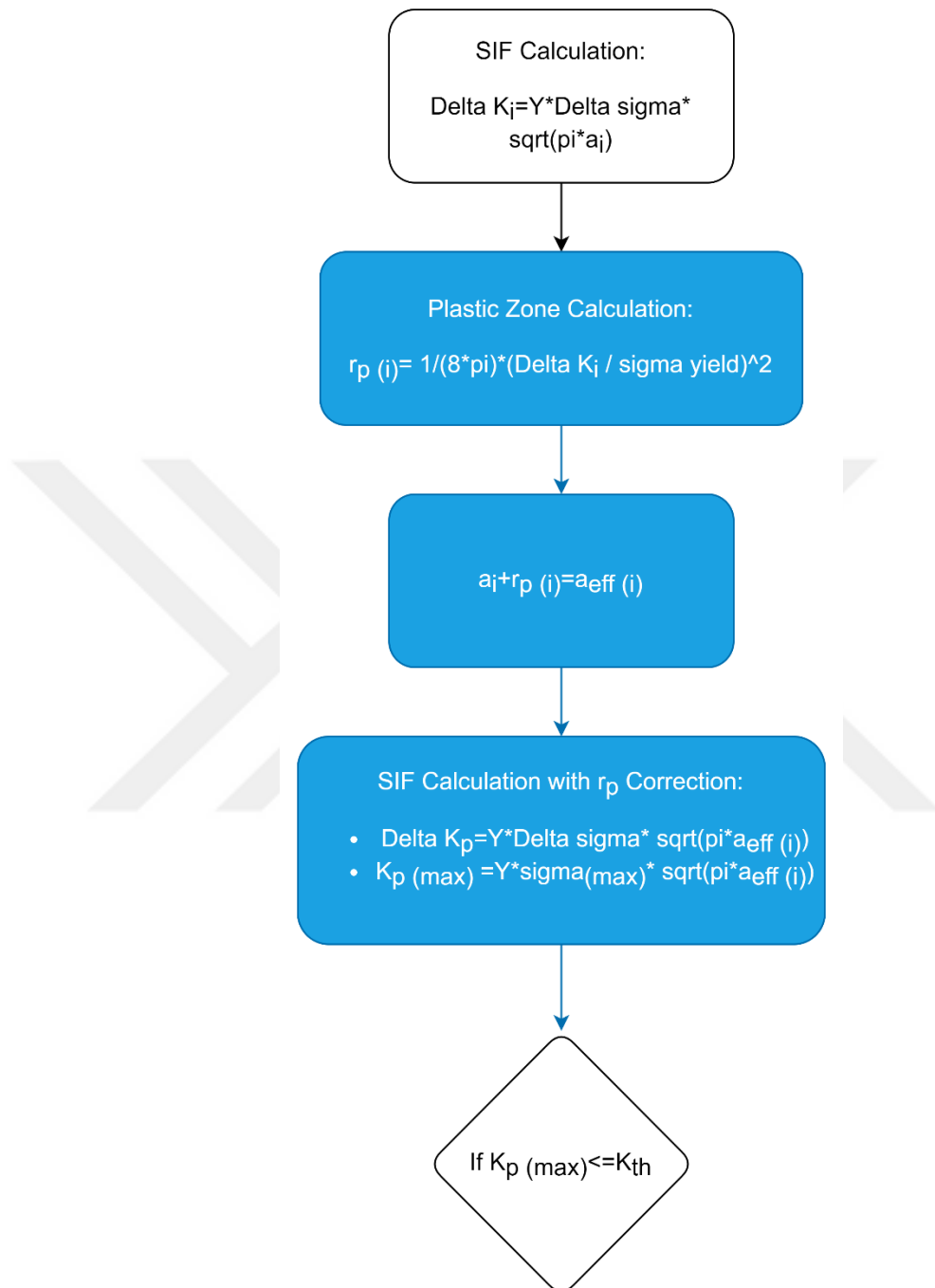


Figure 4.2 Plastic zone sub-algorithm

Constraint loss sub-algorithm:

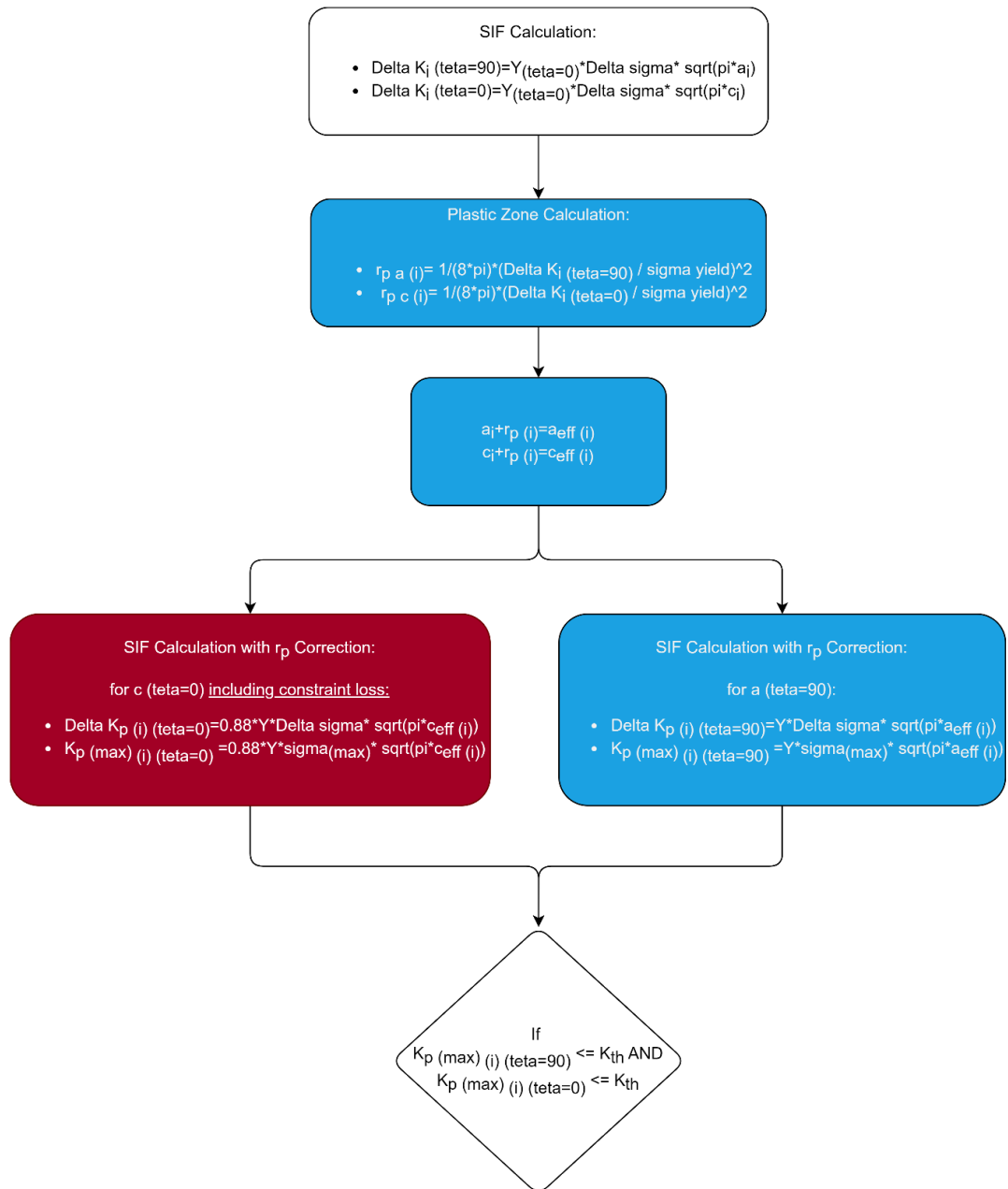


Figure 4.3 Constraint Loss Sub-algorithm Flowchart

Walker correction sub-algorithm:

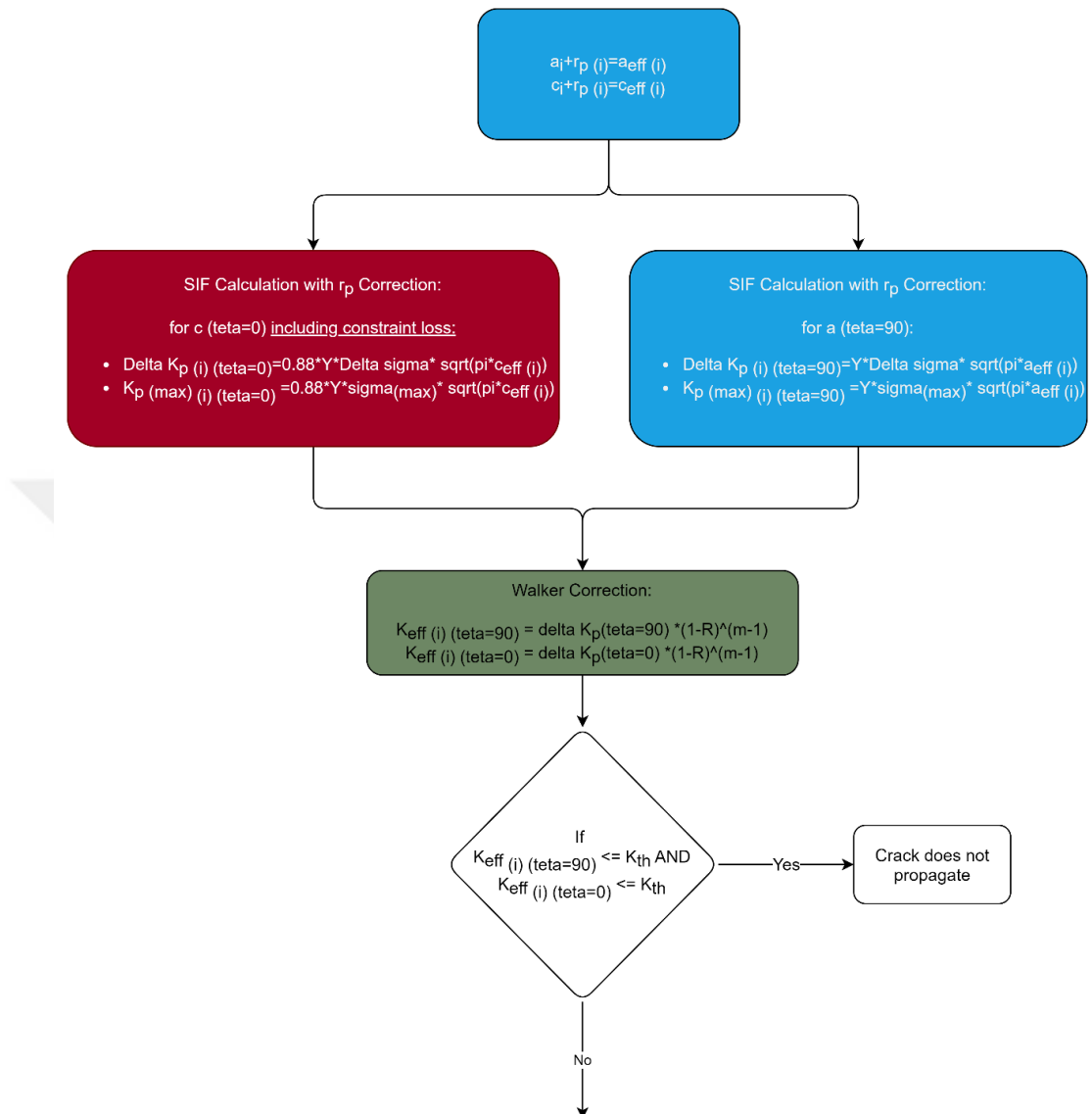


Figure 4.4 Walker Correction Sub-algorithm Flowchart

Stress gradient sub-algorithm:

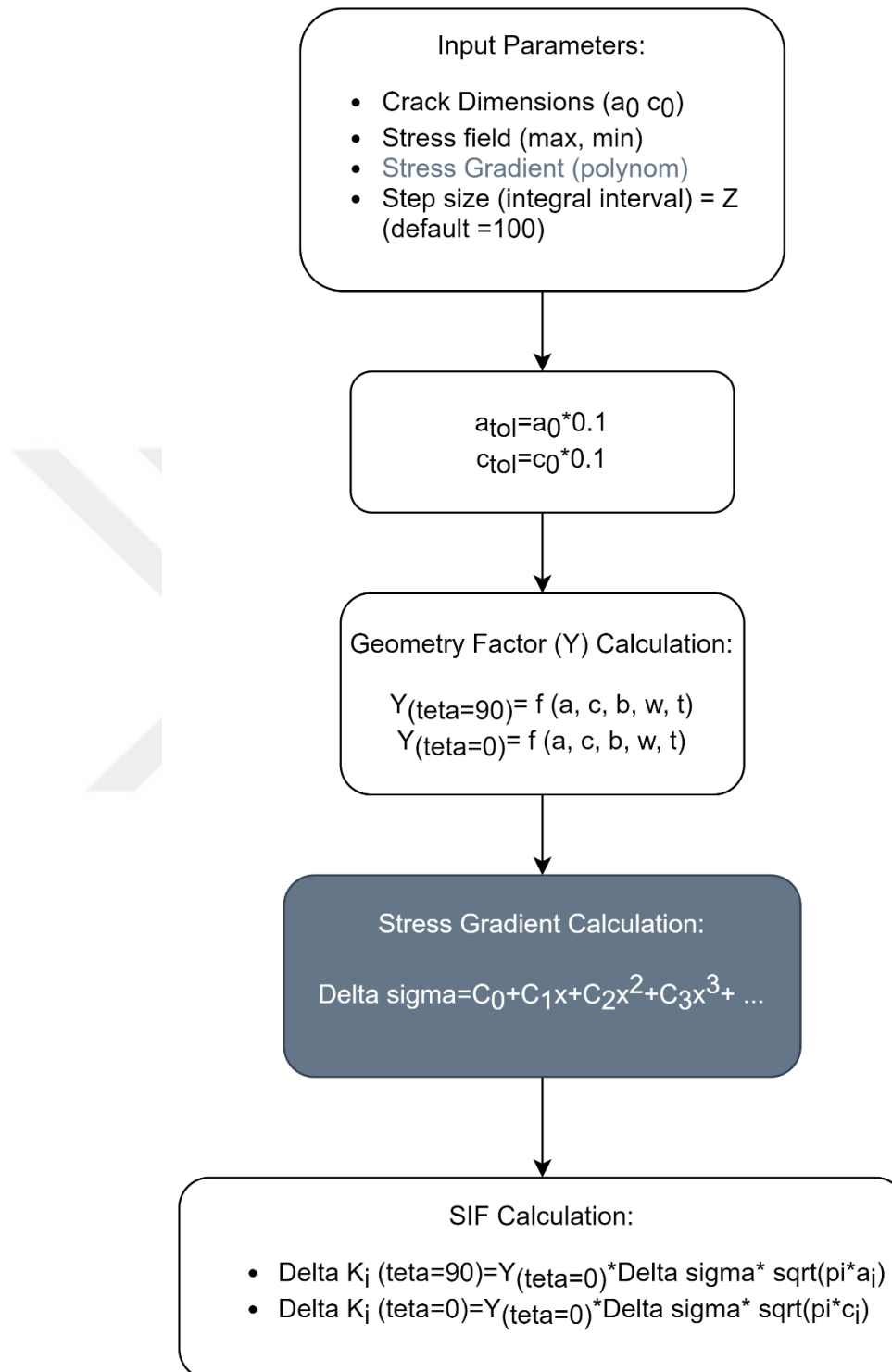


Figure 4.5 Stress Gradient Sub-algorithm Flowchart

Main algorithm-1:

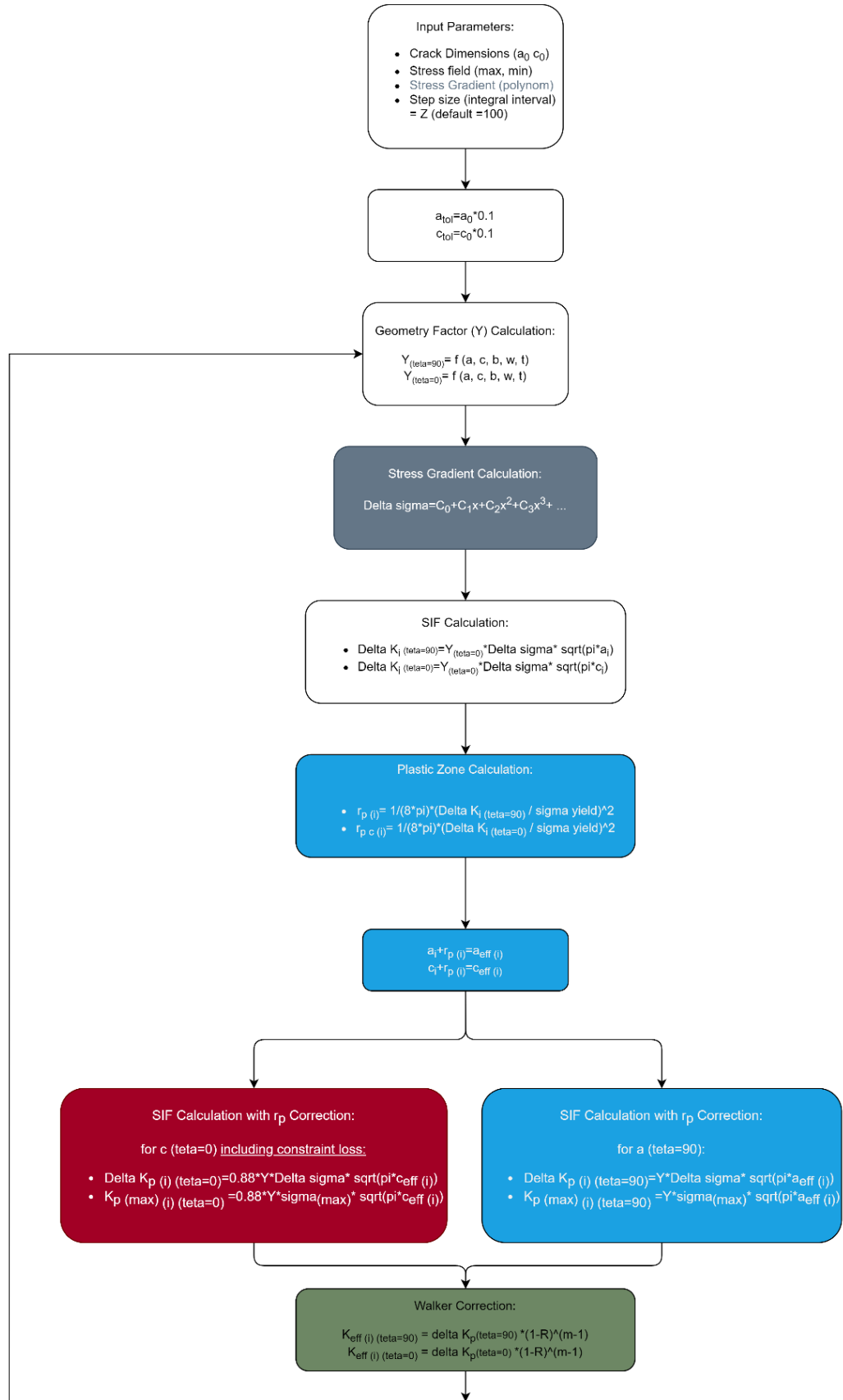


Figure 4.6 Main Algorithm with Sub-algorithms-1

Main algorithm-2 (cont.):

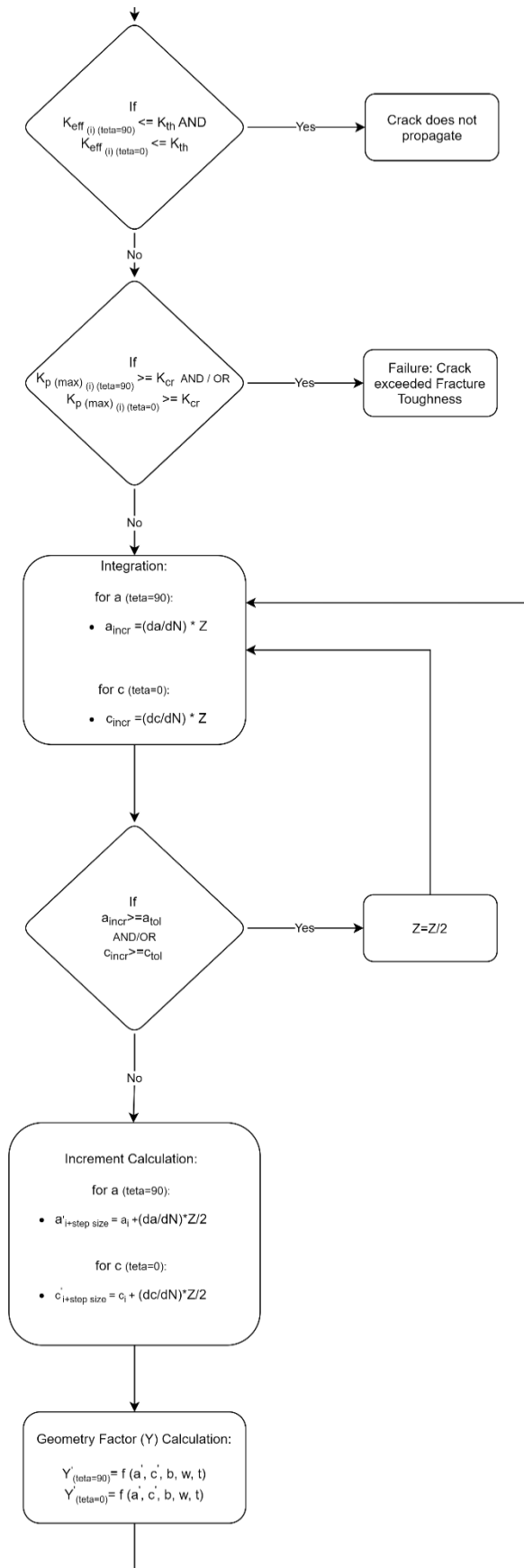


Figure 4.7 Main Algorithm with Sub-algorithms-2 (cont.)

Main algorithm-3 (cont.):

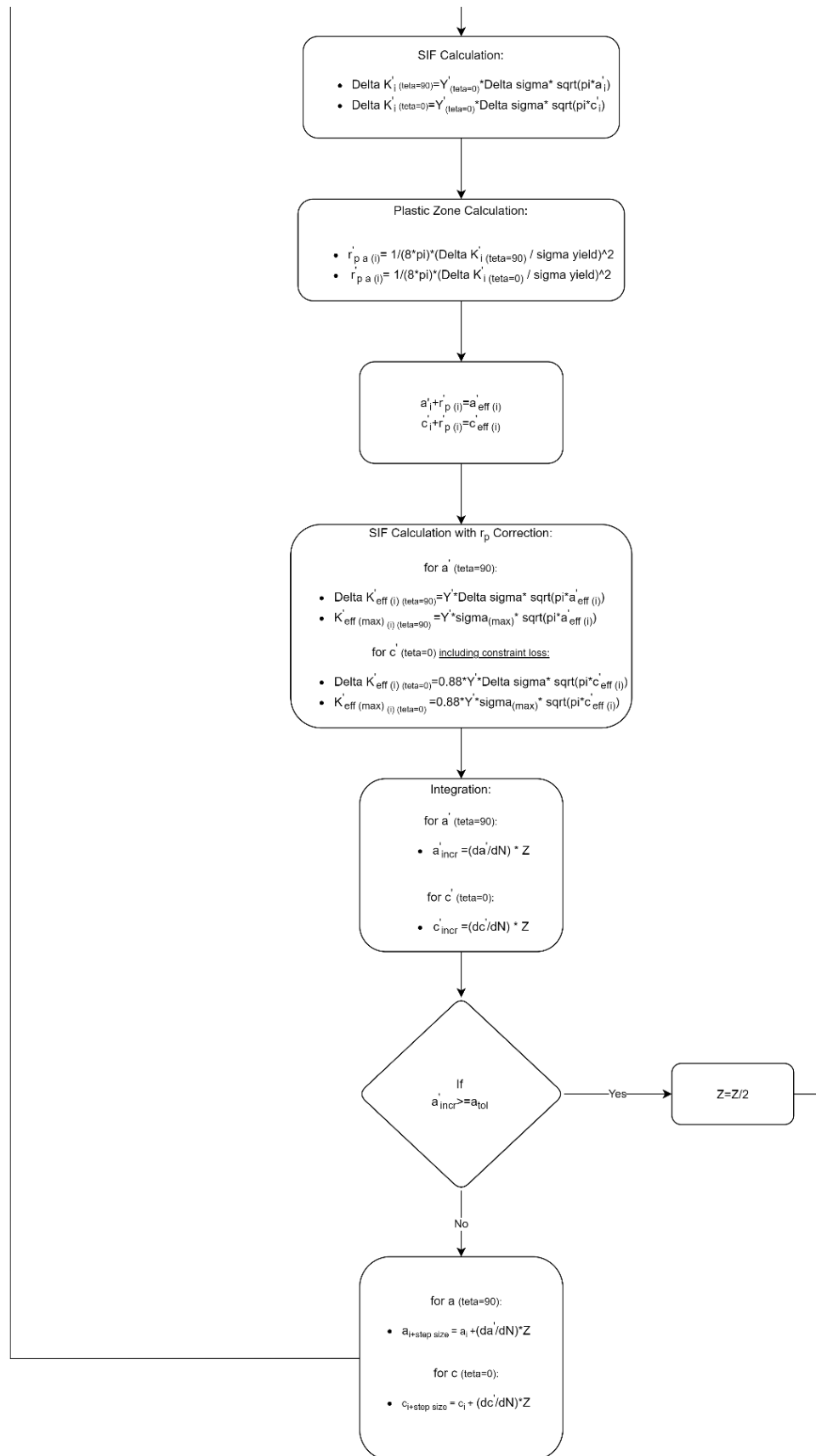


Figure 4.8 Main Algorithm with Sub-algorithms-3 (cont.)

5. VALIDATION and RESULTS

5.1 Comparison of Other OEM Company's Fracture Mechanics Calculations and Applications on Life Methods.

Aviation and gas turbine engine designer companies use fracture mechanics software in all of the design, manufacturing, certification and in-service logistic support phases of projects. One of the critical aspects of gas turbine engine development programs is life management of the parts.

Life management programs for aviation parts also force companies to show the engine parts capable of stand steady and integrity is kept in promised life cycle. Most of the engine parts need to be evaluated in terms of fracture mechanics.

Due to complex SIF solutions and many iteration steps, fracture mechanics calculations are commonly made by the help of softwares. Gas turbine engine manufacturers in the world had developed this kind of in-house softwares by more than 35 years to overcome crack propagation solutions. Each software has a living architecture that helps developer to modify and improve codes day by day.

In order to validate the crack growth solutions implemented in ÇİTA, crack initiation and growth results has been compared to the other aviation OEM companies fracture mechanics fatigue crack growth calculation tools. Those softwares are used in many engine programs with the scope of certification, manufacturing, in-service support etc. over 30 years. In order to avoid conflict, these softwares will be called as "Benchmark Software".

Capabilities of the other OEM's softwares depends on their need. Most of the aviation companies use this kind of softwares for fracture mechanics calculations and damage tolerance purpose. It includes SIF solutions for various crack geometries. (e.g. surface semi-elliptical, corner, bolt hole surface, bolt hole corner, embedded semi-elliptical.) Each crack type has its own SIF solutions. SIF solutions depends on various studies with empirical equations in the literature. Studies are mainly based on the FEA results of each geometry and development of equations.

Laboratory results are another main contributor to the SIF and crack growth solutions. There are many parameters developed in the laboratory results which are explained in the literature section of thesis. These parameters are produced with experimental data models like plastic zone correction, constraint loss model, walker correction, stress gradient effect, shotpeen effect, Neuber plasticity correction, static crack growth etc.

Compact test specimens are the most commonly used fracture mechanics test specimen as explained in the LEFM test section of thesis. Main crack growth parameters derived from CT specimen test like K_{th} , K_{IC} , C , n , *sigmoidal equation constants* (B, P, Q, D) etc.

Benchmark softwares have more than 10 crack types in terms of geometrical orientation of the flaw. Studies in this thesis are focused on the surface semi-elliptical and compact test (CT) specimen crack types. Other crack types are the future directions of the thesis.

ÇİTA software is validated in this thesis in terms of:

- Crack initiation, K_0
- Crack propagation rate, $\frac{da}{dN}$
- Crack termination, K_{IC}
- Runge-Kutta 2 numerical iteration ve Integral interval (step size)
- Plastic zone correction
- Constraint loss correction
- Walker correction

5.2 Validation of Crack Initiation and Crack Growth

5.2.1 Surface semi-elliptical crack:

In ÇİTA, a semi elliptical surface crack SIF solution is available, it is the most used defect type in the aviation industry. The solution and results in ÇİTA and other OEM's fracture mechanics software have been compared. Following parameters are set in Table 5.1.

Table 5.1 ÇİTA Settings for surface semi-elliptical crack

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.015 in or 0.381 mm
Plate width	1 in or 25.4 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	off
Constraint Loss	off
Integration algorithm	RK2
Integration interval (step size)	100
Applied Load	100 ksi or 689.5 MPa
Applied Temp	1000 F° (537.8 C°)

For given settings in the Table 5.1, ÇİTA gives following output in Table 5.2 for the start of propagation (@ N=0 cycle).

Table 5.2 ÇİTA output for surface semi-elliptical crack @ N=0 cycle

ÇİTA				
Cycle	Depth	Length	Ka	Kc
0	0.015	0.015	14.38263	15.82118

Following Table 5.3 shows the comparison for surface semi-elliptical crack solutions and results @N=0 cycle. There is 0.06911 % and 0.06913% difference in SIF solution results, for Ka (depth direction) and Kc (surface direction) respectively.

Table 5.3 Comparison for surface semi-elliptical crack @N=0

ÇİTA				
Cycle	Depth	Length	Ka	Kc
0	0.015	0.015	14.38263	15.82118
Benchmark				
Cycle	Depth	Length	Ka	Kc
0	0.015	0.015	14.3727	15.81025
Comparison				
Difference @ Cycle	Difference @ a%	Difference @ c%	Difference @ Ka%	Difference @ Kc%
0	0	0	0.06911	0.06913

Table 5.4 shows final output results in ÇİTA. Calculated final cycle number is 15189. Crack growth is terminated due to K_c reached %96 of the K_{IC}

Table 5.4 ÇİTA output for surface semi-elliptical crack @ N=Final cycle

Cycle	Depth	ÇİTA		
		Length	Ka	Kc
15189	0.533396293	0.627598425	97.12475	100.7224

Following Table 5.5 shows the comparison for final state surface semi-elliptical crack solutions and results @N=Final cycle. There is -0.14797 % and -0.1529 % difference in SIF solution results, for Ka (depth direction) and Kc (surface direction) respectively. Since SIF values at final cycle is too close, the difference in cycle number and critical crack dimensions @final cycle is up to %7.

Main cause of the discrepancy at final cycle is that da/dN curve slope is increasing rapidly in 3rd Zone of the da/dN curve. While increasing SIF value in the Zone 3, crack growth rate (da/dN) becomes infinite at K_{IC} . In order to taking into account this divergence problem in sigmoidal curve, %96 of the K_{IC} value is taken as the crack termination criteria.

Table 5.5 Comparison for surface semi-elliptical crack @N=Final cycle

ÇİTA				
Cycle	Depth	Length	Ka	Kc
15189	0.533396293	0.627598425	97.12475	100.7224
Benchmark				
Cycle	Depth	Length	Ka	Kc
15032	0.4992543	0.584151	97.26868	100.8766
Comparison				
Difference @ Cycle	Difference @ a%	Difference @ c%	Difference @ Ka%	Difference @ Kc%
1.0444	6.8386	7.4377	-0.14797	-0.1529

5.2.2 Compact Test (CT) specimen crack:

Table 5.6 ÇİTA Settings for Compact Test (CT) specimen crack

Crack Type	Compact Test (CT) Specimen
Crack Depth (a)	0.3 in or 7.62 mm
Crack Length (c)	0.25 in or 6.35 mm
Plate width	0.5 in or 12.7 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	ASTM E647
Stress Gradient	off
Plastic Zone	off
Constraint Loss	off
Integration algorithm	RK2
Integration interval (step size)	300
Applied Load	Uniform - 0 to 1.35 kips (0 to 6005 N)
Applied Temp	1000 F° (537.8 C°)

For given settings in the Table 8 for CT specimen type crack, ÇİTA gives following output in Table 5.7 for the start of propagation (@ N=0 cycle).

Table 5.7 ÇİTA output for Compact Test (CT) specimen crack @ N=0 cycle

ÇİTA				
Cycle	Depth	Length	Ka	Kc
0	0.3	0.25	15.17641	-

Following Table 5.8 shows the comparison for rectangular Compact Test (CT) crack solutions and results @N=0 cycle. There is 0.000087 % difference in SIF solution result for Ka (depth direction). Since the crack orientation and specimen geometry is rectangular in CT specimen, there is no propagation and no need to calculate SIF value for Kc.

Table 5.8 Comparison for Compact Test (CT) specimen crack @N=0

ÇİTA				
Cycle	Depth	Length	Ka	Kc
0	0.3	0.25	15.17641	-
Benchmark				
Cycle	Depth	Length	Ka	Kc
0	0.3	0.25	15.1764	-
Comparison				
Difference @ Cycle	Difference @ a%	Difference @ c%	Difference @ Ka%	Difference @ Kc%
0	0	0	0.000087	-

Table 5.9 shows final output results in ÇİTA. Calculated final cycle number is 76812 cycle. Crack growth is terminated due to K_a reached %96 of the K_{IC}

Table 5.9 ÇİTA output for Compact Test (CT) specimen crack @ N=Final cycle

Cycle	Depth	ÇİTA		
		Length	Ka	Kc
76812	0.787119179	-	100.7185461	-

Following Table 5.10 shows the comparison for final state of rectangular Compact Test (CT) crack solutions and results @N=Final cycle. There is -0.30946 % difference in SIF solution result for K_a (depth direction). Difference in cycle numbers is 76835-76812=23 cycle, which results -0.026%.

Table 5.10 Comparison for Compact Test (CT) specimen crack @N=Final cycle

ÇİTA				
Cycle	Depth	Length	Ka	Kc
76812	0.7871192	-	100.7185	-
Benchmark				
Cycle	Depth	Length	Ka	Kc
76835	0.7875327	-	101.0312	-
Comparison				
Difference @ Cycle %	Difference @ a%	Difference @ c%	Difference @ Ka%	Difference @ Kc%
-0.026	-0.0525	-	-0.30946	-

5.3 Validation of Plastic Zone and Constraint Loss Models

Plastic zone is a zone that extend crack depth and length due to localized stress in the crack tip. Following results shows plastic zone on and off options for both ÇİTA, Benchmark and hand calculations.

Table 5.11 ÇİTA Plastic Zone-on - Settings for surface semi-elliptical crack

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.015 in or 0.381 mm
Plate width	2 in or 50.8 mm
Wall thickness	2 in or 50.8 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	on
Constraint Loss	off
Integration algorithm	RK2
Integration interval (step size)	100
Applied Load	10 ksi or 68.95 MPa
Applied Temp	1000 F° (537.8 C°)

Following Table 5.12 shows the plastic zone-on comparison for surface semi-elliptical crack solutions and results @N=0 cycle. There are 0.07 % difference in SIF solution results for K_a (depth direction) and K_c (surface direction).

Table 5.12 Comparison of plastic zone-on results

	K_a (ksi.in ^{0.5})	K_c (ksi.in ^{0.5})
Benchmark	1.4375	1.3915
Çita	1.4385	1.3925
Difference @ SIF	0.07%	0.07%

Table 5.13 ÇİTA Plastic Zone-off - Settings for surface semi-elliptical crack

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.015 in or 0.381 mm
Plate width	2 in or 50.8 mm
Wall thickness	2 in or 50.8 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	off
Constraint Loss	off
Integration algorithm	RK2
Integration interval (step size)	100
Applied Load	10 ksi or 68.95 MPa
Applied Temp	1000 F° (537.8 C°)

Following Table 5.14 shows the plastic zone-on comparison for surface semi-elliptical crack solutions and results @N=0 cycle. There are 0.07 % and 11.92% difference in SIF solution results for K_a (depth direction) and K_c (surface direction). Reason for the significant difference in K_c values (11.92%) is constraint loss is not taken into consideration in the calculations.

Table 5.14 Comparison of plastic zone-off results

	K_a (ksi.in ^{0.5})	K_c (ksi.in ^{0.5})
Benchmark	1.43727	1.581025
Çita	1.4383	1.3925
Difference @ SIF	0.07%	11.92%

There are 3 test scenarios for plastic zone and constraint loss combination:

1. K_0 = Constraint Loss Model included, Plastic Zone Correction not included
2. K_1 = Constraint Loss Model included, Plastic Zone Correction included
3. K_2 = Constraint Loss Model not included, Plastic Zone Correction not included

Test case 1:**Table 5.15 ÇİTA Plastic Zone-on/off - Settings for surface semi-elliptical crack**

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.015 in or 0.381 mm
Plate width	1 in or 25.4 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	on/off
Constraint Loss	on/off
Integration algorithm	RK2
Integration interval (step size)	100
Applied Stress	Uniform - 0 to 100 ksi (0 to 689.5 MPa)
Applied Temp	1000 F° (537.8 C°)

Table 5.16 Test Case 1 cycle results

TC #1	Cycle		
	Benchmark	Çita	Difference %
K_0	19063	19220	0.8236
K_1	18110	18279	0.9332
K_2	15032	15189	1.0444

Table 5.17 Test Case 1 SIF results

TC #1	Final Crack Size - a (Depth)			Ka		
	Benchmark	Çita	Difference %	Benchmark	Çita	Difference %
K_0	0.6199	0.6736	8.6603	100.2590	99.9681	-0.2901
K_1	0.6035	0.6536	8.2931	100.2620	99.9221	-0.3390
K_2	0.4993	0.5334	6.8386	97.2687	97.1248	-0.1480
	Final Crack Size - c (Surface)			Kc		
	Benchmark	Çita	Difference %	Benchmark	Çita	Difference %
K_0	0.6091	0.6659	9.3214	100.8440	100.7962	-0.0474
K_1	0.5962	0.6492	8.8813	100.9398	100.7547	-0.1834
K_2	0.5842	0.6276	7.4377	100.8766	100.7224	-0.1529

In order to see plastic zone correction and constraint loss models effect on crack propagation, crack length/cycle graphs are illustrated below.

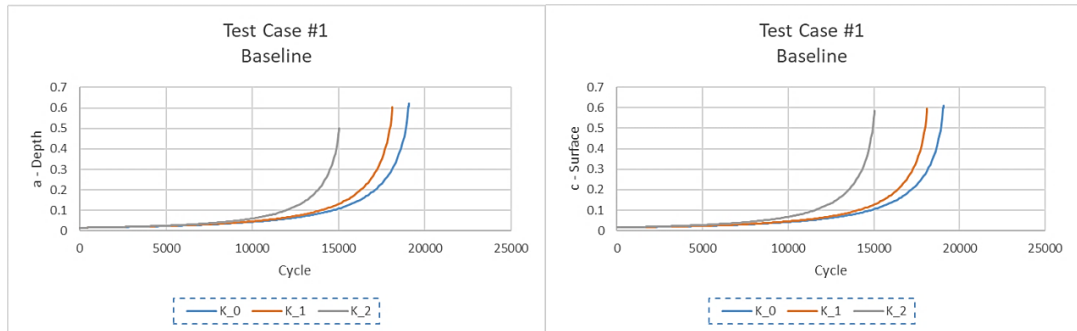


Figure 5.1 Plastic Zone Tests Comparison Graph

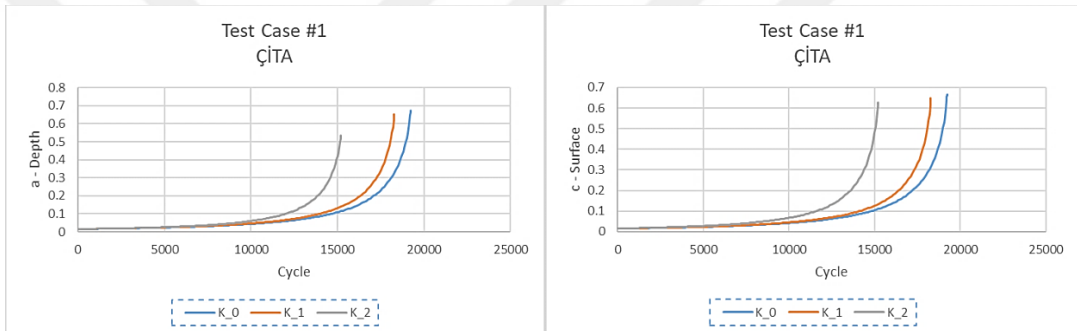


Figure 5.2 Plastic Zone Tests Comparison Graph-2

Test case 2:

Table 5.18 ÇITA Plastic Zone-on/off - Settings for surface semi-elliptical crack-2

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.015 in or 0.381 mm
Plate width	1 in or 25.4 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	on/off
Constraint Loss	on/off
Integration algorithm	RK2
Integration interval (step size)	1
Applied Stress	Uniform - 0 to 100 ksi (0 to 689.5 MPa)
Applied Temp	1000 F° (537.8 C°)

Table 5.19 Test Case 2 cycle results

TC #2	Cycle		
	Benchmark	Çita	Difference %
K_0	19058	19217	0.8343
K_1	18104	18277	0.9556
K_2	15028	15188	1.0647

Table 5.20 Test Case 2 SIF results

TC #2	Final Crack Size - a (Depth)			Ka		
	Benchmark	Çita	Difference %	Benchmark	Çita	Difference %
K_0	0.6205	0.6731	8.4722	100.3567	99.9108	-0.4443
K_1	0.6026	0.6534	8.4290	100.1588	99.8629	-0.2955
K_2	0.4994	0.5337	6.8674	97.3085	97.0912	-0.2234
	Final Crack Size - c (Surface)			Kc		
	Benchmark	Çita	Difference %	Benchmark	Çita	Difference %
K_0	0.6099	0.6653	9.0809	100.9360	100.7435	-0.1907
K_1	0.5954	0.6486	8.9322	100.8236	100.7295	-0.0933
K_2	0.5845	0.6272	7.3093	100.9005	100.7453	-0.1539

Test case 3:**Table 5.21 ÇİTA Plastic Zone-on/off Settings for Compact Test (CT) specimen crack**

Crack Type	Compact Test (CT) Specimen
Crack Depth (a)	0.3 in or 7.62 mm
Crack Length (c)	0.25 in or 6.35 mm
Plate width	0.5 in or 12.7 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	ASTM E647
Stress Gradient	off
Plastic Zone	On/off
Constraint Loss	On/off
Integration algorithm	RK2
Integration interval (step size)	1
Applied Load	Uniform - 0 to 1.35 kips (0 to 6005 N)
Applied Temp	1000 F° (537.8 C°)

Table 5.22 Test Case 3 cycle results

TC #3	Cycle		
	Benchmark	Çita	Difference %
K_0	76768	76768	0.0000
K_1	76496	76495	-0.0013
K_2	76768	76768	0.0000

Table 5.23 Test Case 3 SIF results

TC #1	Final Crack Size - a (Depth)			Ka		
	Baseline	Çita	Difference %	Baseline	Çita	Difference %
K_0	0.7881	0.7868	-0.1626	101.4355	100.4656	-0.9562
K_1	0.7865	0.7850	-0.1925	101.5835	100.3876	-1.1773
K_2	0.7881	0.7868	-0.1626	101.4355	100.4656	-0.9562

Test case 4:

Table 5.24 ÇİTA Plastic Zone-on/off Settings for Compact Test (CT) specimen crack

- 2

Crack Type	Compact Test (CT) Specimen
Crack Depth (a)	0.3 in or 7.62 mm
Crack Length (c)	0.25 in or 6.35 mm
Plate width	0.5 in or 12.7 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	ASTM E647
Stress Gradient	off
Plastic Zone	On/off
Constraint Loss	On/off
Integration algorithm	RK2
Integration interval (step size)	300
Applied Load	Uniform - 0 to 1.35 kips (0 to 6005 N)
Applied Temp	1000 F° (537.8 C°)

Table 5.25 Test Case 4 cycle results

TC #1	Cycle		
	Benchmark	Çita	Difference %
K_0	76835	76812	-0.0299
K_1	76574	76547	-0.0353
K_2	76835	76812	-0.0299

Table 5.26 Test Case 4 SIF results

TC #1	Final Crack Size - a (Depth)			Ka		
	Baseline	Çita	Difference %	Baseline	Çita	Difference %
K_0	0.7875	0.7871	-0.0525	101.0312	100.7185	-0.3095
K_1	0.8111	0.7853	-3.1767	90.5060	100.6240	11.1794
K_2	0.7875	0.7871	-0.0525	101.0312	100.7185	-0.3095

Test case 5:

Table 5.27 ÇITA Plastic Zone-on/off Settings for surface semi-elliptical crack-3

Crack Type	Surface Semi-elliptical
Crack Depth (a)	0.015 in or 0.381 mm
Crack Length (c)	0.25 in or 6.35 mm
Plate width	2 in or 50.8 mm
Wall thickness	2 in or 50.8 mm
SIF Solution	Newman and Raju
Stress Gradient	off
Plastic Zone	on/off
Constraint Loss	on/off
Integration algorithm	RK2
Integration interval (step size)	1
Applied Stress	Uniform - 0 to 100 ksi (0 to 689.5 MPa)
Applied Temp	1000 F (537.8 C)

Table 5.28 Test case 5 cycle results

TC #1	Cycle		
	Benchmark	Çita	Difference %
K_0	5266	5373	2.0319
K_1	4780	4905	2.6151
K_2	4830	4889	1.2215

Table 5.29 Test case 5 SIF results

TC #1	Final Crack Size - a (Depth)			Ka		
	Baseline	Çita	Difference %	Baseline	Çita	Difference %
K_0	0.6128	0.6692	9.1978	100.9454	100.5971	-0.3450
K_1	0.5968	0.6504	8.9760	100.8590	100.5439	-0.3124
K_2	0.4961	0.5326	7.3582	98.4670	98.1011	-0.3716
	Final Crack Size - c (Surface)			Kc		
	Baseline	Çita	Difference %	Baseline	Çita	Difference %
K_0	0.6147	0.6716	9.2499	100.4810	100.6692	0.1873
K_1	0.6006	0.6545	8.9857	100.5190	100.6748	0.1550
K_2	0.5953	0.6385	7.2667	100.8130	100.7755	-0.0372

5.4 Validation of Walker Correction Method

Mean stress correction significantly affects the crack growth behaviour. Walker method is the most common type of mean stress correction method. There is an advanced version, called as Dual Walker model, represents more accurate correction for wider R ratio.

In the validation of the Walker method, test cases are created for different R ratios. Hand calculations, benchmark results and ÇİTA results are compared.

Table 5.30 ÇİTA Walker Settings for Compact Test (CT) specimen crack

Crack Type	Compact Test (CT) Specimen
Crack Depth (a)	0.3 in or 7.62 mm
Crack Length (c)	0.25 in or 6.35 mm
Plate width	0.5 in or 12.7 mm
Wall thickness	1 in or 25.4 mm
SIF Solution	ASTM E647
Stress Gradient	off
Plastic Zone	On
Constraint Loss	On
Integration algorithm	RK2
Integration interval (step size)	100
Applied Load	Varying (kip)
Applied Temp	1000 F° (537.8 C°)

Table 5.31 shows the test case loading condition. R ratio varies between -1 to +0.25.

Table 5.31 Walker Validation Test Cases

Case No	S min	S max	Delta S	S mean	R
1	0	3	3	1.5	0
2	1	4	3	2.5	0.25
3	-1.5	1.5	3	0	-1
4	-0.5	2.5	3	1	-0.2

Table 5.32 illustrates the ÇİTA results and comparison with respect to benchmark results. There are only slight difference in final cycle number (N) and SIF numbers.

Table 5.32 Walker Validation Cycle and SIF results

Case No	Benchmark N	Ka0	ÇİTA N	ÇİTA Ka0	N% diff	Ka0 %diff
1	5951	33.85677	5928	33.8557	0.386%	0.003%
2	4401	36.97232	4427	36.9712	-0.591%	0.003%
3	41149	18.47333	41156	18.4727	-0.017%	0.003%
4	9984	28.86962	9985	28.8687	-0.010%	0.003%

5.5 Summary of Results

Fracture mechanics related topics are investigated, an in-house working algorithm is formed and affordable results in terms of crack dimension, stress intensity factor and final cycle number are obtained. Differences in results are discussed and accordingly, further actions are taken for improvement in the results accuracy. To sum up, in terms of;

Cyclic crack growth: There is less than 0.1% approximation for SIF values in cycle=0 condition while there is less than a 1% in SIF and total life calculations for both two types of crack orientation (surface semi-elliptical type and CT specimen type). CT specimen type showed more accurate results than surface semi-elliptical type due to it has only one direction of crack growth and simpler calculation steps.

Plastic zone correction and constraint loss: There is less than 0.1% difference in SIF values after the plastic zone correction for CT Specimen type crack. Surface semi-elliptical crack shows close approximation for a direction. However, there is 12% difference in SIF values for c direction. This difference leads author to study a new parameter that is a constraint loss parameter. This parameter is in effect for crack types that has a crack growth in a surface direction. After the investigation and new parameter is added into calculations. Result shows less than 1% difference in SIF and final cycle number results for both a and c directions.

Mean stress correction (Walker): Load cases are created with different R ratios and results showed that less than 1% difference in SIF and final cycle numbers are obtained for four different loading conditions.

As all of the test results indicate that ÇİTA agrees well with hand calculations and non-commercial fracture mechanics software calculations.

5.6 Further Directions

Gas turbine engine parts suffers many different types of loading and temperature environment. The topic, fatigue crack growth, is widely open to many further studies and discussions. Some of them are covered in this thesis. However, for further directions, more focus on other crack types is planned. The following topics; fine tuning of integration interval, stress gradient effect, retardation effect, crack type transition, small cracks growth, probabilistic crack growth, crack growth on welded structures, mixed mode loading, EPFM (Elastic-Plastic Fracture Mechanics), FEM or BEM for 3D behaviour of cracks and static crack growth are scheduled.

6. APPENDIX

Material Properties – Sensitivity Analysis

Material constants in the sigmoidal equation (B, P, Q, D) plays important role in the crack growth rate. These parameters are generated by LEFM tests as explained in LEFM test section of the thesis. While using these constants in numerical calculations, one question arises. Is there any numerical digit error? This study is aimed to understand digit error effect on crack propagation.

Following study shows sensitivity analysis according to the material constants (B, P, Q, D) in the sigmoidal equation. Each value is increased/decreased by it's last digit after comma. For example, B value varies between -15.10065 and -15.10074.

Mathematical Model: Coles- Sigmoidal Equation as explained in the literature section of the thesis:

$$\frac{da}{dN} = e^B (K_{eff}/K_{th})^P [\ln(K_{eff}/K_{th})]^Q [\ln(K_{crit}/K_{eff})]^D \quad A.1$$

In terms of crack growth rate ($\frac{da}{dN}$), the worst combination of each parameter is found by excel solver for all stages of the crack growth. Nominal constants are listed below. Material type is In718 → @1000 F° (@ 537.8 C°).

Table 6.1 In718 sigmoidal equation material properties (*Normalised)

	Nominal*
B	-14.1007
P	3.6060
Q	1.2920
D	-1.4040
K_{th} (ksi.in ^{0.5})	8.0000
K_{IC} (ksi.in ^{0.5})	106.0000

Table 6.2 Variated sigmoidal equation material properties, max and min

	Max%	Min%
B	-14.10074	-14.10065
P	3.60550	3.60640
Q	1.29150	1.29240
D	-1.40350	-1.40440
K_{th} (ksi.in ^{0.5})	8.00000	8.00000
K_{IC} (ksi.in ^{0.5})	106.00000	106.00000

There are two cases; max and min. their singular effect in one of the iteration cycle is calculated.

$$\Delta_{max} \left(\frac{da}{dN} \right) = 0.53\%$$

$$\Delta_{min} \left(\frac{da}{dN} \right) = -0.43\%$$

$$\Delta_{total} \left(\frac{da}{dN} \right) = 0.96\%$$

Total difference in crack growth rate of max and min cases is 0.96% for one cycle. It is to be noted that according to results from sensitivity analysis, Stage 3 crack growth is more sensitive to material property variation due to crack growth rate are higher and goes to infinity in sigmoidal curve.

In order to see cumulative effect on whole stages, two cases are run separately in ÇITA to see their effect in crack propagation rate in following table.

Table 6.3 Material property variation and its effect in ÇİTA

ÇİTA	Nominal	Min	% Deviation	Max	% Deviation
a	0.53367	0.53405	0.071609	0.53363	-0.00695
c	0.62723	0.62779	0.090697	0.62718	-0.00662
Cycle	15188	15179	-0.05926	15202	0.092093

According to results from ÇİTA, there is minor deviation between min and max cases, 0.092093% for final cycle. Since there are four digits after comma, (e.g. 14.1006 or 3.6064) there will be no excessive values on final cycle and final crack dimensions. However, this sensitivity should be taken into consideration while processing specimen test results into material database.

REFERENCES

- [1] Parker E.R., (1957), "Brittle Behavior of Engineering Structures, National Academy of Sciences", National Research Council, John Wiley & Sons, 1st Edition, New York.
- [2] Wells, A.A., (1955), "The condition of fast fracture in aluminum alloys with particular reference to comet failures.", British Welding Research Association Report, 1st Edition.
- [3] Swift T., (1987), "Damage tolerance in pressurized fuselages, 11th Plantema Memorial Lecture", New Materials and Fatigue Resistant Aircraft Design, (ed. D L Simpson), pp. 1-77, Engineering Materials Advisory Services Ltd., Warley, UK.
- [4] Inglis, C.E., (1913), "Stresses in a plate due to the presence of cracks and sharp corners" Transactions of the Institute of Naval Architects, 55, 219–241.
- [5] Griffith, A.A., (1920), "The phenomena of rupture and flow in solids", Philosophical Transactions, Series A, 221, 163–198.
- [6] Irwin, G.R., (1961), "Plastic zone near a crack and fracture toughness", Sagamore Research Conference Proceedings, (4), Syracuse University Research Institute, Syracuse, NY, pp. 63–78.
- [7] Dugdale, D.S., (1960), "Yielding in steel sheets containing slits", Journal of the Mechanics and Physics of Solids, (8), 100–104.
- [8] Rice, J.R., (1968), "A path independent integral and the approximate analysis of strain concentration by notches and cracks", Journal of Applied Mechanics, (35), 379–386.
- [9] Paris, P.C., Gomez, M.P., and Anderson, W.P., (1961), "A rational analytic theory of fatigue", The Trend in Engineering, (13), 9–14.
- [10] Wells, A.A., (1961), "Unstable crack propagation in metals: Cleavage and fast fracture", Proceedings of the Crack Propagation Symposium, (1) , Paper 84, Cranfield, UK.

- [11] Andreson T.L.,(2017), "Fracture Mechanism: Fundamentals and Applications, CRC Press LLC", 4th Edition, Boca Raton-Florida.
- [12] Broek D., (1986), "Elementary Engineering Fracture Mechanics", 4th Edition, Kluwer Academic Publications, Dordrecht, the Netherlands.
- [13] Tada, H., Paris, P. C., Irwin, George R. (2000), "The Stress Analysis of Cracks Handbook", 3rd Edition, ASME Press.
- [14] Paris P.C. and Erdogan F., (1960), "A critical analysis of crack propagation laws", Journal of Basic Engineering, (85), 528–534.
- [15] Coles, A., Johnson, R. E., Popp, H. G., (1976), "Utility of Surface-Flawed Tensile Bars in Cyclic Life Studies", Journal of Engineering Materials and Technology, 98(4): 305-315.
- [16] Irwin G.R., (1960), "Plastic Zone Near a Crack and Fracture Toughness", *Proceedings of Seventh Sagamore Conference*, P. IV-63.
- [17] Dugdale, D. S., (1960), "Yielding of steel sheets containing slits", J. Mech. Phys. Sol., (8), pp. 100-108.
- [18] Barenblatt, G. I., (1962), "The mathematical theory of equilibrium of cracks in brittle fracture", *Advances in Appl. Mech.*, (7), pp. 55-129.
- [19] Stephens, R. I., & Fuchs, H. O. (2001), "Metal fatigue in engineering", 2nd Edition, New York Wiley.
- [20] Chikh B. O., Imad A., Benguediab M., (2008) "Influence of the cyclic plastic zone size on the propagation of the fatigue crack in case of 12NC6 steel", *Computational Materials Science*, (43), pp. 1010-1017.
- [21] Andrews, W. R. and Shih, C. F , (1979), "Elastic Plastic Fracture", ASTM STP 668, American Society for Testing and Materials, Philadelphia, 426-450.
- [22] Yau J. F., (1986), "Fracture Mechanics", (7), ASTM STP 905, American Society for Testing and Materials, Philadelphia, pp. 601-624.
- [23] Van Stone, R. H. and Richardson, T. L., (1985), "Automated Test Methods for Fracture and Fatigue Crack Growth", ASTM STP 877, American Society for Testing and Materials, Philadelphia, pp. 148-166.
- [24] Walker, K., (1970), "The Effect of Stress Ratio During Crack Propagation and Fatigue for 2024-T3 and 7075-'1"6 Aluminum", *Effects of Environment and*

Complex Load History on Fatigue Life, ASTM STP 462, American Society for Testing and Materials, pp. 1-14.

[25] Forman, R. G., and Mettu, S. R., (1992), "Behavior of Surface and Corner Cracks Subjected to Tensile and Bending Loads in Ti-6Al-4V Alloy", *Fracture Mechanics: Twenty-second Symposium, (1)*, ASTM STP 1131, H. A. Ernst, A. Saxena, and D. L. McDowell, eds., American Society for Testing and Materials, Philadelphia, pp. 519-546.

[26] Newman, Jr., J. C., (1984), "A Crack Opening Stress Equation for Fatigue Crack Growth", *International Journal of Fracture*, (24), No. 3, pp. R131-R135.

[27] Van Stone, R. H., (1982), "The Influence of Temperature and Stress Ratio on the Fatigue Crack Growth Behavior of Rene 95", General Electric Aircraft Engine Group Report TM 82-640, General Electric Co., Cincinnati, OH 45215.

[28] Bueckner H.F., (1970), "A novel principle for the computation of stress intensity factors", *Z. Angew. Math. Mech.* (50), 129-146.

[29] J. Rice, (1972), "Some remarks on elastic crack-tip stress field", *Int. J. Solids Struct.* (8), 751-758.

[30] G. Shen, G. Glinka, (1991), "Weight functions for a surface semi-elliptical crack in a finite thickness plate", *Theoretical and Applied Fracture Mechanics*, (15), Issue 3, Pp. 247-255.

[31] G. Glinka, G. Shen, (1991), "Universal features of weight functions for cracks in mode I", *Engineering Fracture Mechanics*, (40), Issue 6, pp. 1135-1146.

[32] ASTM Standard E647, Standard Test Method for Measurement of Fatigue Crack Growth Rates.

[33] ASTM Standard E399, Standard Test Method for Linear-Elastic Plane-Strain Fracture Toughness of Metallic Materials.

[34] Vander Voort, George F. (1987), "Visual Examination and Light Microscopy", ASM International.

BIOGRAPHY

The author, Şevket ERTEKİN, has started his career as a Mechanical Design Engineer on gas turbine engine industry. He currently works as a Structural Analysis Engineer in fracture mechanics and life calculations for gas turbine engine parts.

