

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL

**QUASI-STATIC FDTD MODELING OF DISPERSIVE
SUPERPARAMAGNETIC SOILS FOR TIME DOMAIN
ELECTROMAGNETIC METHOD**

TALHA SAYDAM

A THESIS OF DOCTORATE
DEPARTMENT OF ELECTRONICS ENGINEERING

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GEBZE TEKNİK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

ZAMAN UZAYINDA ELEKTROMANYETİK YÖNTEM
İÇİN DİSPERSİF SÜPERPARAMANYETİK
TOPRAKLARIN KUVAZİ-STATİK ZUSF İLE

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DOKTORA TEZİ
ELEKTRONİK MÜHENDİSLİĞİ ANABİLİM DALI

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ABSTRACT

Time Domain Electromagnetic (TDEM) or Transient Electromagnetic (TEM) method is one of the most common techniques for subsurface geophysical prospecting and sounding problems such as hydrocarbon exploration, hydrogeological search, mineral deposit investigation, and deep crustal exploration and so on. However, the performance of the TDEM systems significantly affected by Magnetic Viscosity (MV) of the soil. The MV effect is mostly caused by single domain superparamagnetic (SPM) particles in soil such as ferrimagnetic particles, maghemite and magnetite. Since this SPM effect causes magnetic dispersion, it effect makes the analysis of the TDEM signals difficult. It may mask the desired signal especially at late times. To model the SPM effect in soil, a log-uniform model is generally used for the distribution of magnetic particles in soil. This log-uniform model results in a uniform distribution of time constants in a limited time range. Therefore, analytical and numerical modeling of the TDEM problems with the SPM soils is an important topic for better understanding the behavior of this effect.

In this thesis, a 3D Quasi-Static Finite Difference Time Domain (QS-FDTD) numerical algorithm is developed to investigate the SPM soil effect on the TDEM systems. For modeling the SPM soil in FDTD method, an Auxiliary Differential Equation (ADE) technique is used to implement the log-uniform model. In this sense, first, a 2D homogeneous lossy half-space problem is solved. After that, a thin layer added at the top of the half-space and a conductive anomalous body buried inside the half-space. As the last of the 2D problems, a lossy SPM half-space problem is solved. The results from the 2D problems are confirmed with the knowledge from literature. For 3D problems, first, homogeneous lossy and SPM half-space problems are solved. The SPM half-space results are validated with an analytical solution in literature. Then, a lossy half-space problem with SPM thin layer and buried anomalous conductive body problem is solved. Finally, a more complex 3D problem with the SPM thin layer problem is considered. The results obtained in this thesis can be useful for evaluating the performance of the TDEM systems for complex problems with the dispersive SPM soils.

Keywords: TDEM/TEM, Low Frequencies, Superparamagnetic (SPM) Soil, QS-FDTD Method, Auxiliary Differential Equation (ADE).

ÖZET

Zaman Uzayı Elektromanyetik (ZUEM) yöntemi, hidrokarbon arama, hidrojeolojik araştırmalar, mineral yatağı araştırması ve derin kabuk araştırması gibi yeraltı jeofizik arama ve sondaj problemleri için yaygın olarak kullanılan tekniklerden biridir. Ancak, ZUEM sistemlerinin performansı toprağın Manyetik Viskozitesinden (MV) önemli ölçüde etkilenir. MV etkisi çoğunlukla topraktaki ferrimanyetik parçacıklar, maghemit ve manyetit gibi tek alanlı süperparamagnetik (SPM) parçacıklardan kaynaklanır. Bu SPM etkisi manyetik dispersiyona neden olur ve bu etki TDEM sinyallerinin analizini zorlaştırır. Bu etki özellikle geç zamanlarda istenen sinyali maskeleyebilir. Topraktaki SPM etkisini modellemek için genellikle topraktaki manyetik parçacıkların dağılımı için log-üniform modeli kullanılır. Bu log-üniform model sınırlı bir zaman aralığında zaman sabitlerinin düzgün bir dağılımıyla sonuçlanır. Bu nedenle SPM topraklarıyla ZUEM problemlerinin analitik ve nümerik modellemesi bu etkinin davranışını daha iyi anlamak için önemli bir konudur.

Bu tezde ZUEM sistemleri üzerindeki SPM toprak etkisini araştırmak için 3B Kuvazi-Statik Zaman Uzayı Sonlu Farklar (KS-ZUSF) nümerik algoritması geliştirilmiştir. SPM toprağını ZUSF yönteminde modellemek için, log-uniform modeli uygulamak için Yardımcı Diferansiyel Denklem (YDD) tekniği kullanılmıştır. Bu anlamda ilk olarak 2B homojen kayıplı yarı-uzay problemi çözülmüştür. Daha sonra yarı-uzayın tepesine ince bir katman eklenmiş ve yarı-uzayın içine iletken bir anormal gövde gömülmüştür. 2B problemlerin sonuncusu olarak kayıplı bir SPM yarı-uzay problemi çözülmüştür. 2B problemlerin sonuçları literatürdeki bilgilerle doğrulanmıştır. 3B problemler için önce homojen kayıplı ve SPM yarı-uzay problemleri çözülmüştür. SPM yarı-uzay sonuçları literatürdeki analitik bir çözümle doğrulanmıştır. Daha sonra SPM ince tabaka ve gömülmüş anormal iletken gövde içeren kayıplı bir yarı-uzay problemi çözülmüştür. Son olarak, SPM ince tabaka problemi olan daha karmaşık bir 3B problem ele alınmıştır. Bu tezde elde edilen sonuçlar, dispersif SPM topraklar içeren karmaşık problemlerde TDEM sistemlerinin performansının değerlendirilmesinde yararlı olabilir.

Anahtar Kelimeler: ZUEM, Düşük Frekanslar, Süperparamanyetik (SPM) Toprak, KS-ZUSF Yöntemi, Yardımcı Diferansiyel Denklem (YDD).

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LIST OF SYMBOLS AND ABBREVIATIONS

α	: Quasi-static FDTD scaling constant (dimensionless)
α_k	: Scaling constant from propagation constant (dimensionless)
α_{MQS}	: Scaling constant from MQS condition (dimensionless)
α_τ	: Scaling constant from time constants (dimensionless)
α'	: Quasi-static rescaling constant (dimensionless)
∇	: Operator Nabla
ε	: Dielectric permittivity (F/m)
ε_0	: Dielectric permittivity of free space (F/m)
ε_r	: Relative dielectric permittivity (dimensionless)
μ	: Magnetic permeability (H/m)
μ_0	: Magnetic permeability of free space (H/m)
μ_r	: Relative magnetic permeability (dimensionless)
μ'	: Real part of magnetic permeability (H/m)
μ''	: Imaginary part of magnetic permeability (H/m)
μm	: Micrometer
σ	: Conductivity (S/m)
δ	: Skin depth (m)
Δx	: Spatial cell size on x axis (m)
Δy	: Spatial cell size on y axis (m)
Δz	: Spatial cell size on z axis (m)
Δt	: Time step size (s)
Δt_{max}	: Maximum time step size (s)
χ	: Magnetic susceptibility (dimensionless)
χ'	: Real part of magnetic susceptibility (dimensionless)
χ''	: Imaginary part of magnetic susceptibility (dimensionless)
χ_0	: Magnetic susceptibility before turning off the DC source (dimensionless)
χ_{off}	: Magnetic susceptibility for turn-off type source (dimensionless)
χ_{on}	: Magnetic susceptibility for turn-on type source (dimensionless)
χ_∞	: High frequency limit of magnetic susceptibility (dimensionless)
τ	: Time relaxation constant (s)
τ_t	: Relaxation time (s)
τ_e	: Transmit time (s)
τ_m	: Diffusion time (s)
ω	: Angular frequency (rad/s)
a	: Loop radius (m)
b	: Radius of wire (m)
$\vec{B}$	: Magnetic induction vector (Weber/m ² or Tesla)
B_x	: x component of magnetic induction (Weber/m ² or Tesla)
B_y	: y component of magnetic induction (Weber/m ² or Tesla)
B_z	: z component of magnetic induction (Weber/m ² or Tesla)
c	: Electromagnetic wave propagation velocity (m/s)
c_0	: Electromagnetic wave propagation velocity in free space (m/s)
d	: Thickness of soil layer (m)
$\vec{D}$	: Electric displacement vector (C/m ²)

$\vec{E}$	: Electric field vector (V/m)
E_x	: x component of electric field (V/m)
E_y	: y component of electric field (V/m)
E_z	: z component of electric field (V/m)
f	: Frequency (Hz)
f_{max}	: Maximum frequency (Hz)
$f(\tau)$	: Time constant distribution function (dimensionless)
F	: Fröhlich distribution (dimensionless)
H	: Magnetic field strength before turning off the DC source (A/m)
$\vec{H}$	: Magnetic field vector (A/m)
H_x	: x component of magnetic field (A/m)
H_y	: y component of magnetic field (A/m)
H_z	: z component of magnetic field (A/m)
I	: Current (A)
i	: Spatial mesh index on x axis (dimensionless)
j	: Spatial mesh index on y axis (dimensionless)
$\vec{J}$	: Current density vector (A/m ²)
J_x	: x component of current density (A/m ²)
J_y	: y component of current density (A/m ²)
J_z	: z component of current density (A/m ²)
k	: Spatial mesh index on z axis, propagation constant (rad/m)
k_0	: Propagation constant in free space (rad/m)
k^N	: Numerical propagation constant (rad/m)
k_{real}	: Real part of propagation constant in free space (rad/m)
k_{imag}	: Imaginary part of propagation constant in free space (rad/m)
L	: Characteristic length of the object (m)
m	: Meter
$\vec{M}$	: Magnetization vector (A/m)
M_x	: x component of magnetization (A/m)
M_y	: y component of magnetization (A/m)
M_z	: z component of magnetization (A/m)
M_0	: Magnetization before turning off the DC source (A/m)
n	: Time step index (dimensionless)
$P(\tau)$	: After-effect function (dimensionless)
$\vec{r}$	: Position vector (m)
s	: Second
S	: Siemens
t	: Time (s)
v	: Voltage (V)
x	: Axis of the 3D Cartesian coordinate system (m)
y	: Axis of the 3D Cartesian coordinate system (m)
z	: Axis of the 3D Cartesian coordinate system (m)
Z	: Electromagnetic wave impedance (Ω)
2D	: 2 Dimensional
3D	: 3 Dimensional
ABC	: Absorbing Boundary Condition
ADE	: Auxiliary Differential Equation
AEM	: Airborne Electromagnetic

CC	: Cole-Cole
CFL	: Courant–Friedrichs–Lewy
CN FDTD	: Crank-Nicholson FDTD
DC	: Direct Current
EM	: Electromagnetic
EMI	: Electromagnetic Induction
EQS	: Electro-Quasi-Static
FDEM	: Frequency Domain Electromagnetic
FDFD	: Finite Difference Frequency Domain
FDTD	: Finite Difference Time Domain
FEM	: Finite Element Method
GPR	: Ground Penetrating Radar
IE	: Integral Equation
IQ	: Inphase-Quadrature
IP	: Induced Polarization
MQS	: Magneto-Quasi-Static
PML	: Perfectly Matched Layer
QS-FDTD	: Quasi-Static FDTD
RC	: Recursive Convolution
SEA	: Spherical Excitation Approach
SPM	: Superparamagnetic
TDEM	: Time Domain Electromagnetic
TEM	: Transient Electromagnetic
UXO	: Unexploded Ordnance
VRM	: Viscous Remanent Magnetization

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1. INTRODUCTION

1.1. Overview

Electromagnetic (EM) methods play an important role in geophysical research such as hydrocarbon exploration, hydrogeological search, mineral deposit investigation, deep crustal exploration and so on. The EM methods are used to determine the electromagnetic properties of the underground (conductivity, dielectric permittivity and magnetic permeability) through data obtained by measuring the electromagnetic fields induced underground.

In EM systems used in geophysical research, loop antennas or long wires fed with time-dependent current are generally used as transmitters and receivers. The electromagnetic fields produced by the transmitter penetrate the soil and cause the formation of secondary electromagnetic fields originating from conductive environments. These secondary magnetic fields are measured in the receiver antenna and the measured data is used to determine the underground electrical properties [1].

Frequency Domain Electromagnetic (FDEM) method and Time Domain Electromagnetic (TDEM or Transient Electromagnetic, TEM) method are two main types of the EM methods in geophysical research which are based on low frequency electromagnetic induction phenomena. DC resistivity and GPR devices are also examples of electromagnetic-based geophysical methods, but they are not discussed in this thesis because their working principles and areas of use are very different from the FDEM and the TDEM systems. An illustration for working principle of the TDEM systems is shown in **Figure 1.1**.

In the FDEM systems, the transmitter is fed by a continuous sinusoidal current signal. Since the secondary field measured in the receiver is too small in amplitude compared to the primary field, separation of primary and secondary parts from the total signal in the receiver is difficult. On the other hand, a step-off type current signal is generally used in the TDEM systems. Therefore, after turning off the input signal, only the secondary field can be measured in the receiver. Moreover, in the TDEM systems a wideband response can be obtained by a single measurement while it is necessary a

frequency-sweep in the FDEM systems. The TDEM methods provide a higher resolution than the FDEM methods [2]. For these main reasons, the TDEM systems are more preferable than the FDEM systems [3].

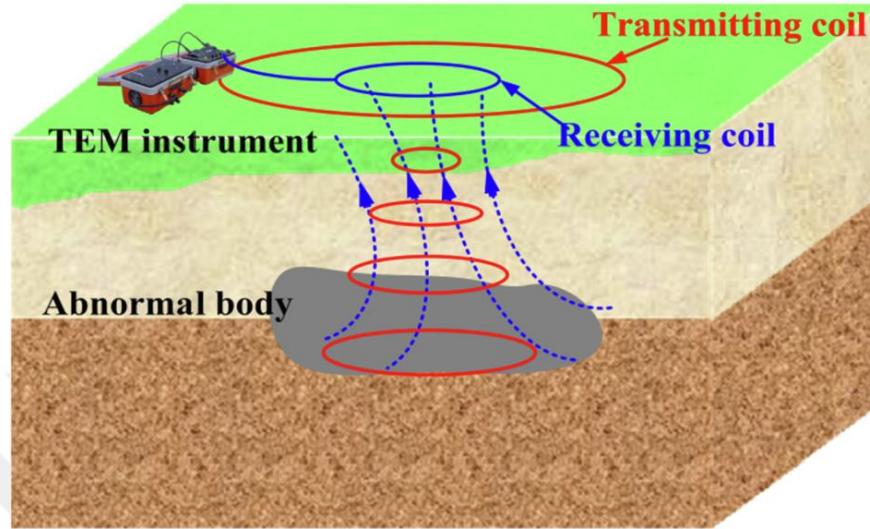


Figure 1.1: Working principle of the TDEM/TEM systems [4].

In the TDEM systems, sounding and prospecting can be challenging if the underground media exhibit frequency-dependent properties. This frequency dependence typically arises from two phenomena: Induced Polarization (IP) and Viscous Remanent Magnetization (VRM). The IP effect results from the frequency-dependent behavior of the soil's electrical conductivity, while the VRM, generally caused by superparamagnetic (SPM) particles have frequency dependency of the soil's magnetic susceptibility [2], [5], [6]. In this thesis, the SPM effect was focused on, while the IP effect was not studied.

There are three types of magnetic susceptibility that contribute to the electromagnetic response detected by an EMI sensor: (i) permanent remanent, (ii) induced, and (iii) VRM. Permanent remanent magnetization exists independently of any external magnetic field. Induced magnetization arises in the presence of the applied magnetic field. The VRM, on the other hand, is characterized by time-dependent changes in magnetization relative to the applied field [7]. The SPM effect can be very significant for the late-time period and sometimes even may exceed the conductivity effect [8], [9].

Sattel and Mutton conducted a detailed investigation of the SPM effect using data from Airborne Electromagnetic (AEM) surveys. Their field tests demonstrated that the SPM responses can be mistaken for those of deep conductors, and vice versa. Insights into distinguishing the SPM effects are derived from field distributions and time-decay curves:

- At lower system elevations, smaller sensor separations are sufficient due to the steep gradient of the SPM response.
- Using larger transmitter loops and increasing the offset between transmitter and receiver loops helps reduce the SPM effects.
- In systems with in-loop geometry, the x -component of the response over the SPM-layered earth is zero; however, x -component data are generally more susceptible to noise.
- In the frequency domain systems, the SPM response resembles that of elevated magnetic permeability, becoming noticeable mainly at lower frequencies for high magnetic susceptibility χ_0 values.
- The SPM response becomes negative during the on-time of the transmitted pulse [10], [11].

Understanding the SPM effect is also important for magnetic modeling, magnetic petrology, rock magnetism and magnetic permeability measurements and inversion of magnetic susceptibility [12], [13].

To better understand the SPM effects on the TDEM systems, it is necessary to use numerical tools. Unfortunately, there are very few numerical solutions for the TDEM problems with the dispersive SPM media in the literature [14]-[17]. All these studies have some approximations and/or do not solve full-wave Maxwell's equations.

1.2. Purpose and the Scope of the Thesis

The main purpose of this thesis is to develop a numerical tool to solve the TDEM problems consisting of the dispersive SPM soils. In this manner, a Quasi-Static Finite Difference Time Domain (QS-FDTD) algorithm is presented with the solution of Maxwell's equations in 2D and 3D Cartesian coordinates. In this algorithm, the displacement currents are kept under the Quasi-Static condition. Therefore, with this

full-wave solution, the wave propagation behavior with a finite velocity is preserved. First, a 2D lossy half-space problem, then a 2D half-space problem with a buried anomalous conductive body and a thin upper layer problem is solved. A 3D lossy half-space problem is also considered.

To evaluate the dispersive media in the FDTD method, there are two main techniques. The first one is a Recursive Convolution technique which is a recursive method applied to solve convolution of susceptibility with fields in time domain [18]. The other technique is an Auxiliary Differential Equation (ADE) technique which is based on a solution of a differential equation for polarization or magnetization with the finite difference method along with the Maxwell's equations [19].

In this thesis, for the dispersive SPM materials, the log-uniform model is implemented with the ADE technique (without any convolution or any other approximations) in the FDTD algorithm. The proposed method is validated with an analytical solution for a 3D SPM half-space problem. Then, the effect of the SPM is investigated with solution of a 3D half-space problem with the thin SPM upper layer and the buried conductive anomalous body problem. Finally, a more complex problem with the thin SPM upper layer is solved.

2. LITERATURE REVIEW

2.1. Superparamagnetic Soils

2.1.1. Magnetic Loss Phenomenon and Soil Magnetism

There are five major types of magnetic losses. These are hysteresis, Eddy current, fluctuation after-effect, diffusion after-effect, and Jordan type after-effect.

- At the low frequencies, the loss of hysteresis depends on the amplitude of the magnetic field, and it is less important at the high frequencies.
- The Eddy current loss is defined as a power loss by the Eddy currents caused by magnetization change in magnetic materials. It strongly depends on frequency, conductivity and permeability. It can be modeled by a single exponential.
- The fluctuation after-effect weakly depends on temperature and frequency and strongly depends on non-homogeneity. Its time domain characteristic is modeled as $\log(t)$.
- The diffusion after-effect was also called the Richter-type after-effect. It is modeled by two-time constants. The magnetization is proportional to $\ln t$ in a specific time interval. This effect is attributed as the superparamagnetism of small particle interactions in rocks and soils [20], [21].

In this thesis, the Richter-type after-effect is considered due to its importance of the SPM soil effects on the TDEM systems.

Magnetism in soils is generally a result of multi-domain iron (generally have a weak magnetic viscosity) or single domain (the SPM) ferrimagnetic particles (commonly iron oxide), maghemite and magnetite in soils [22]. The magnetic susceptibility of soil is highly dependent on shape, size and concentration of these minerals [23]. The magnetization of soils shows up as several sources as

- Basic bedrocks give rise to weathered magnetic minerals, and this results in coarse mineral grains ($\sim 10 \mu\text{m}$).

- A fermentation process causes a weathering of iron from the soil materials to form fine grains ($< 0.1 \mu\text{m}$) magnetite and maghemite with the superparamagnetic properties.
- Fire transforms iron-oxides into the fine grain single domain superparamagnetic (viscous) materials.
- Atmospheric pollution and soil erosion may cause the soil to form coarse grains.

Thus, the most common sources of magnetic soils are bedrocks and pedogenic conditions [24]. The list magnetic susceptibilities of some common soil minerals and bedrocks are given in [23]. The SPM effect is also observed in a glacial till [25].

2.1.2. Models for Superparamagnetic Soils

2.1.2.1. Logarithmic Model in Time Domain

Magnetic after-effect is a phenomenon that is defined as the change of magnetization after a magnetic field exposition. This is also called magnetic viscosity [21]. The magnetic viscosity generates a phase shift between the magnetic field and the magnetization [26].

Magnetization of the SPM $M(t)$ is time dependent and is given by

$$M(t) = \chi(t)H \quad (2.1)$$

where, $\chi(t)$ is the time dependent magnetic susceptibility and H is the static magnetic field. The situation of the magnetic field turn-off at $t = 0$ results in decay of the magnetization in the form of

$$M(t) - M_0 = H\chi_0 P(t) \quad (2.2)$$

where M_0 is the magnetization before turn-off, χ_0 is the static magnetic susceptibility and $P(t)$ is the after-effect function. The after-effect $P(t)$ is also equivalent to step response function [27], [28].

The SPM behaviors of minerals can be defined by a weight function $f(\tau)$ (a distribution function). Then the after-effect $P(t)$ is defined as

$$P(t) = \int_0^{\infty} f(\tau) e^{-t/\tau} d\tau \quad (2.3)$$

where, τ is a time constant.

$$f(\tau) = \frac{1}{\tau \ln(\tau_2/\tau_1)}, \quad \tau_1 \leq \tau \leq \tau_2. \quad (2.4)$$

where, τ_1 and τ_2 are lower and upper limits of the time constants, respectively. Gose and Carnes used this exact model to describe the decay of magnetization of lunar breccias which show the SPM behavior in the time domain as

$$M(t) = \frac{M_0}{\ln(\tau_2/\tau_1)} \int_{\tau_1}^{\tau_2} \frac{e^{-t/\tau}}{\tau} d\tau. \quad (2.5)$$

This magnetization decay is interpreted as a Richter-type after-effect. It is reported that no lower limit in the relaxation time distribution and τ_2 is dependent on the exposure time to magnetic field and it is not dependent on material properties [29]. In this respect, $M(t)$ can be rewritten as

$$M(t) = \frac{M_0}{\ln(\tau_2/\tau_1)} \int_{t/\tau_2}^{\infty} \frac{e^{-t/\tau}}{t/\tau} d(t/\tau). \quad (2.6)$$

For $t < \tau_2$, above integral is approximated as.

$$M(t) = \frac{M_0}{\ln(\tau_2/\tau_1)} (\ln \tau_2 - \ln t - 0.577). \quad (2.7)$$

This formula provides a log-linearity in the time domain [29].

Then, the relation between the viscous remanent magnetization and H -field is given by

$$\frac{dM(t)}{dt} = \frac{H\chi_0}{\ln(\tau_2/\tau_1)} \frac{1}{t} [e^{-t/\tau_1} - e^{-t/\tau_2}]. \quad (2.8)$$

Within the $\tau_1 \leq t \leq \tau_2$ range, it is evident [30] that $dM(t)/dt$ becomes

$$\frac{dM(t)}{dt} \cong \frac{-H\chi_0}{\ln(\tau_2/\tau_1)} \frac{1}{t}. \quad (2.9)$$

Therefore, it can be shown that with a single loop antenna configuration, the derivative of the magnetization decay from the SPM soil changes¹ with $1/t$; with two separate loops, the voltage decay changes with $1/t^2$. The apparent resistivity also reduces with $1/t$. It is discussed that a magnetic ground can behave as a nonmagnetic ground of a different conductivity function of magnetic susceptibility [31], [32]. However, the overall time domain response is considered in three time stages as early, intermediate and late time stages. At the early time, the stage time domain response decays very slowly. $1/t^1$ behavior is only valid at the intermediate time stage. Finally, at the late time stage the time domain response behaves exponentially [33], [34].

Candy proposed another simple empirical model with a single time constant τ for time derivative of the magnetization in the magnetic viscous soils as [35]

$$\frac{dM(\tau, t)}{dt} = \ln\left(\frac{t + \tau}{t}\right) - \frac{\tau}{t} \quad (2.10)$$

where, $M(\tau, t)$ is the magnetization and τ is the period of the applied source signal. Barsukov and Fainberg used a magnetic susceptibility model $\chi(t)$ for the SPM soils in time domain (under a step turn off magnetic field) as

¹ It was also shown that some SPM effects have $t^{1 \pm 0.2}$ and $t^{1 \pm 0.4}$ dependences [36], [37].

$$\chi(t) = \chi_0 \frac{1}{t \ln(\tau_1/\tau_2)} [e^{-t/\tau_1} - e^{-t/\tau_2}]. \quad (2.11)$$

Kozhevnikov and Antonov simulated the transient response of uniform conductive and magnetically viscous earth using two different methods. The first method is based on the relation between the viscous magnetization and the magnetic flux with logarithmic time dependent susceptibility model. The second method is based on the solution of the Helmholtz equation in a boundary-value problem using the Fourier transform with a frequency dependent logarithmic susceptibility model. Time decays for different loop configurations are calculated with two methods. In that study, several important conclusions are highlighted;

- On-time and off-time behaviors of the time dependent susceptibility model are related as $\chi_{off}(t) = \chi_0 - \chi_{on}(t)$.
- As the loop size increased, the eddy current effect increased as well but the magnetic viscosity effect decreased.
- Noncoincident-loop configuration causes a sign reversal in the transient response for the SPM [38], [39].

Dabas and Skinner used a simpler logarithmic model to fit time domain the VRM data by a PI metal detector. The model defined for J (A/m) is

$$J(h_0, t) = J(h_0, t_0) + Sh_0 \ln(t/t_0) \quad (2.12)$$

where S is the magnetic viscosity coefficient and it is proportional to the applied magnetic field. The time exponent measurements laid between as -1.30 and -1.42 (it is expected to be -1) and it is claimed that the finite energizing time deviates the time exponent from -1 . Therefore, this shows the importance of the knowledge of pulse shape. It is also discussed that the viscous magnetization is linearly related to the inducing field only in a specific range [8], [40], [41].

Kozhevnikov and Antonov used the log-uniform model in the time domain for the forward modeling of the near surface TDEM problems. The inversion of the TDEM data is shown in terms of the time dependent magnetic susceptibility [42]

Barsukov and Fainberg proposed to use the SPM effect for the exploration of gold and nickel deposits by the variation of the SPM time constant as $t^{-1\pm\delta}$. The value of δ is used as an indicator for the existence of deposits [43]. Candy used the logarithmic model for the viscous SPM soils on his invention on an advanced metal detector [44].

2.1.2.2. Logarithmic Model in Frequency Domain

A model for the magnetic permeability of the SPM ground is proposed as

$$\mu = \mu_0 \left\{ 1 + \chi_0 \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln \frac{(1 + j\omega\tau_2)}{(1 + j\omega\tau_1)} \right] \right\} \quad (2.13)$$

where χ_0 is the DC value of magnetic susceptibility. τ_1 and τ_2 are the lower and upper time constants, respectively [45], [46].

Extraction of this model is possible in two ways; i) from the generally dispersive case in the frequency domain, ii) from the magnetization $M(t)$ in the time domain.

i) Extraction in the frequency domain: The general dispersive case for the magnetic susceptibility is

$$\chi(\omega) = \chi_\infty + (\chi_0 - \chi_\infty) \int_{\tau=0}^{\infty} \frac{f(\tau)}{1 + j\omega\tau} d\tau \quad (2.14)$$

where χ_∞ is the high frequency limit. The function $f(\tau)$ is the distribution of the relaxation times [47], [48].

Principally, the time constant distribution $f(\tau)$ can be evaluated in three manners as

- ‘Finite Discrete Time Constant’ susceptibility model: The relaxation time distribution is a collection of finite number of discrete time constants.
- ‘Infinite Continuous Distribution of Time Constants’ susceptibility model: General infinite continuous relaxation time distribution model is based on Néel theory.

- ‘Finite Continuous Distribution of Time Constants’ susceptibility model: A continuous relaxation time distribution in the interval of two time constants, τ_1 and τ_2 is defined [33]. This is the most appropriate model to evaluate the SPM soils for metal detectors.

Considering the last model the integral form can be rewritten as

$$\chi(\omega) = \chi_\infty + (\chi_0 - \chi_\infty) \int_{\tau_1}^{\tau_2} \frac{F(\ln(\tau))}{1 + i\omega\tau} d(\ln(\tau)) \quad (2.15)$$

where $\tau_1 < \tau < \tau_2$ and $F(\ln(\tau)) = F_0$. This distribution function is also referred as Fröhlich distribution [27]. Assume that $\chi_\infty \rightarrow 0$, taking the integral, it is obtained that

$$\chi(\omega) = \chi_0 F_0 \ln\left(\frac{\tau_2}{\tau_1} \frac{1 + i\omega\tau_1}{1 + i\omega\tau_2}\right) \quad (2.16)$$

and

$$\chi(\omega = 0) = \chi_0 = \chi_0 F_0 \ln\left(\frac{\tau_2}{\tau_1}\right) \Rightarrow F_0 = \frac{1}{\ln\left(\frac{\tau_2}{\tau_1}\right)} \quad (2.17)$$

finally, the log-uniform model is found as

$$\chi(\omega) = \chi_0 \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln\left(\frac{1 + i\omega\tau_1}{1 + i\omega\tau_2}\right) \right]. \quad (2.18)$$

ii) *Extraction in the time domain*: The magnetization of the material with the presence of a step-off magnetic field can be written in a general form

$$M(t) = M_0 \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \left(\int_{t/\tau_2}^{\infty} \frac{e^{-\tau}}{\tau} d\tau - \int_{t/\tau_1}^{\infty} \frac{e^{-\tau}}{\tau} d\tau \right) \right] \quad (2.19)$$

where M_0 is the final value of the magnetization. Taking the Laplace transform of $M(t)$ yields

$$\bar{M}(s) = \frac{M_0}{s} \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln \left(\frac{1 + \tau_2 s}{1 + \tau_1 s} \right) \right]. \quad (2.20)$$

By the spectrum of the step function source, the transfer function S for the magnetization is

$$S = \bar{M}s. \quad (2.21)$$

If the material is essentially non-magnetic, the susceptibility $\chi_0 = M_0/H$ is small with the presence of the magnetic field H . Then S becomes

$$S = \chi_0 H \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln \left(\frac{1 + \tau_2 s}{1 + \tau_1 s} \right) \right]. \quad (2.22)$$

Then the spectrum of the susceptibility is S/H . If s is replaced by $i\omega$, the susceptibility expression in the frequency domain is obtained as

$$\chi(\omega) = \chi_0 \left[1 - \frac{1}{\ln(\tau_2/\tau_1)} \ln \left(\frac{1 + \tau_2 i\omega}{1 + \tau_1 i\omega} \right) \right] \quad (2.23)$$

and the permeability μ is

$$\mu(\omega) = \mu_0 (1 + \chi(\omega)). \quad (2.24)$$

Under the assumption of $|\omega\tau_2| \gg 1$ and $|\omega\tau_1| \ll 1$, the model can be approximated as

$$\mu(\omega) = \mu_0 \left[1 + \chi_0 \left(1 - \frac{\ln(i\omega\tau_2)}{\ln(\tau_2/\tau_1)} \right) \right]. \quad (2.25)$$

Some studies about $\mu(\omega)$ are given below.

This model is used to investigate the effect of the SPM soils on UXO detection and discrimination. It is highlighted that between the frequencies $\omega = 1/\tau_2$ and $\omega = 1/\tau_1$, the quadrature component of the susceptibility is nearly constant but there is a peak at $\omega = (\tau_1\tau_2)^{-1}$, and the in-phase component decreases linearly with the logarithm of the frequency. It is also reported that the effects of the magnetic susceptibility will appear only in the vertical component of magnetic field [49]-[51].

Vitale et al. obtained the logarithmic model for magnetic permeability in the frequency domain by fitting the data from the complex permeability measurements of Co-based soft ferromagnetic amorphous alloys [52].

Shubitidze et al. used this model with a dipole model based forward method, generalized Spheroidal Excitation Approach (SEA) method. The frequency domain response and the time signatures of the UXOs are calculated. The frequency and time domain results are validated with the measurements using the Geophex GEM-3 frequency domain EMI sensor and the Geonics EM-63 time domain EMI sensor [53].

Lhomme et al used the logarithmic model in the frequency domain for modeling the magnetic viscosity to use inversion of frequency domain EMI sensor data for UXO detection [54].

Das was also used the same logarithmic model to calculate the triangular pulse response of the SPM half-space. The magnetic susceptibility model is extracted to its real and imaginary parts as

$$\begin{aligned}\chi'(\omega) &= \chi_0 \left[1 - \frac{1}{2\ln(\tau_2/\tau_1)} \ln \left(\frac{1 + \omega^2 \tau_2^2}{1 + \omega^2 \tau_1^2} \right) \right] \\ \chi''(\omega) &= \chi_0 \left[-\frac{1}{2\ln(\tau_2/\tau_1)} (\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1)) \right].\end{aligned}\tag{2.26}$$

Then, under $\omega\tau_1 \ll 1$ and $\omega\tau_2 \gg 1$ conditions, $\chi'(\omega)$ and $\chi''(\omega)$ are approximated as

$$\begin{aligned}\chi'(\omega) &\approx -\frac{\chi_0}{\ln(\tau_2/\tau_1)} \ln \omega + \chi_0 \left[1 - \frac{\ln \tau_2}{2\ln(\tau_2/\tau_1)} \right] \\ \chi''(\omega) &\approx -\frac{\chi_0}{\ln(\tau_2/\tau_1)} \frac{\pi}{2}.\end{aligned}\tag{2.27}$$

Then, the following relationship is extracted from the above approximated model as

$$\frac{\partial \chi'}{\partial \ln \omega} = \frac{2}{\pi} \chi'' = -\frac{\chi_0}{\ln(\tau_2/\tau_1)}.\tag{2.28}$$

The homogeneous half-space problem with the above susceptibility model and an aluminum sphere in air problem is solved analytically. The time domain results are obtained by an inverse Fourier transform. It is shown that the sum of the half-space response and the sphere response are equal to the response of the sphere buried in the half-space. This way is chosen due to the oscillatory behavior of the inverse Fourier transform of the direct buried sphere solution. The proposed technique is validated with measurements [55]-[57].

Das investigated the SPM effect on detection of low-metal landmine buried in the soil with an EMI sensor. The logarithmic model is used to solve diffusion equation in the frequency domain and the time domain results obtained by the inverse Fourier transform. Induced voltages from targets buried in the SPM soil are calculated [58].

Kozhevnikov and Antonov used the logarithmic model of the magnetic viscosity in the frequency domain to find the vertical distribution of the magnetic viscosity. It is claimed that there is no way to find the distribution of the susceptibility using a fixed-geometry TDEM system. Therefore, a geometric sounding method is proposed with

an automatic fitting technique based on the inverse Fourier transform solution [59]. Additionally, Kozhevnikov et al. proposed using multi-offset TEM arrays for identification of the SPM effect on the TDEM data [60].

Bournas et al. used the same log model for the magnetic susceptibility as where, $\tau_2 \gg \tau_1$. This model is used to investigate the SPM effects in the geological explorations. A time domain EMI sensor, VTEM, is used for the data collection. The time decay measurements are done and it is shown that the SPM causes a slow decay at late times (order of t^{-1}). It is also claimed that a concentric loop (in-loop) configuration may minimize the SPM effect [61], [62].

If $\ln \tau_1 = \ln \tau_\chi - \ln(\tau_2/\tau_1)/2$, $\ln \tau_2 = \ln \tau_\chi + \ln(\tau_2/\tau_1)/2$ and in the limit $\lim_{\tau_2/\tau_1 \rightarrow 1} \tau_\chi = \tau_1 = \tau_2$, the general equation reduces to the standard Debye dispersion relation as [63]

$$\lim_{\tau_2/\tau_1 \rightarrow 1} \mu = \mu_0 \left\{ 1 + \frac{\chi_0}{1 + j\omega\tau_\chi} \right\}. \quad (2.29)$$

McFadden and Scott used another similar log model to fit the magnetic susceptibility of the soil from measurement data as

$$\chi(\omega) = c(\ln(\omega/\omega_0) + j\pi/2) \quad (2.30)$$

where c and ω_0 are unknown constants. The real and imaginary parts of the measured susceptibilities are extracted and validated by reported measurement data. A multi-frequency sensor in the frequency range from 10 kHz to 90 kHz is used is the measurements [64].

2.1.2.3. Cole-Cole Model

Olhoeft and Strangway used a Cole-Cole model for ferromagnetic (magnetite) sphere to investigate the effect of magnetic losses on in-phase and quadrature signature (I/Q signature). The Cole-Cole model for the complex magnetic permeability is given as

$$\mu = \mu' - j\mu'' = \mu_\infty + \frac{\mu_s - \mu_\infty}{1 + (j\omega\tau)^{1-\alpha}} \quad (2.31)$$

where

$$\mu_\infty = \lim_{\omega \rightarrow \infty} \mu', \quad \mu_s = \lim_{\omega \rightarrow 0} \mu' \quad (2.32)$$

and $1 - \alpha = a$ is the distribution parameter. When $a = 1$, it forms a Debye like relaxation mechanism. It is also highlighted that the complex permeability gives rise to a phase difference between the electrical and magnetic relaxation times [65].

Pasion et al. also mentioned that the in-phase and quadrature components of the magnetic susceptibility of materials with the magnetic viscosity can be modelled by Cole-Cole model with parameters appropriate for a broad distribution of the relaxation times and plotted the in-phase and quadrature component in **Figure 2.1** [49], [50]. They used the Cole-Cole model to construct susceptibility model from the field measurement. Then using this model, the TEM response of the homogeneous soil is calculated by the inverse Fourier transform based analytical forward model. The TEM responses of the soil with static magnetic susceptibility and a conductive and permeable sphere in air also calculated and the results of these three cases are compared [30].

Dabas et al. was also used the Cole-Cole model to investigate the magnetic behavior of the soils both the frequency and the time domain [66].

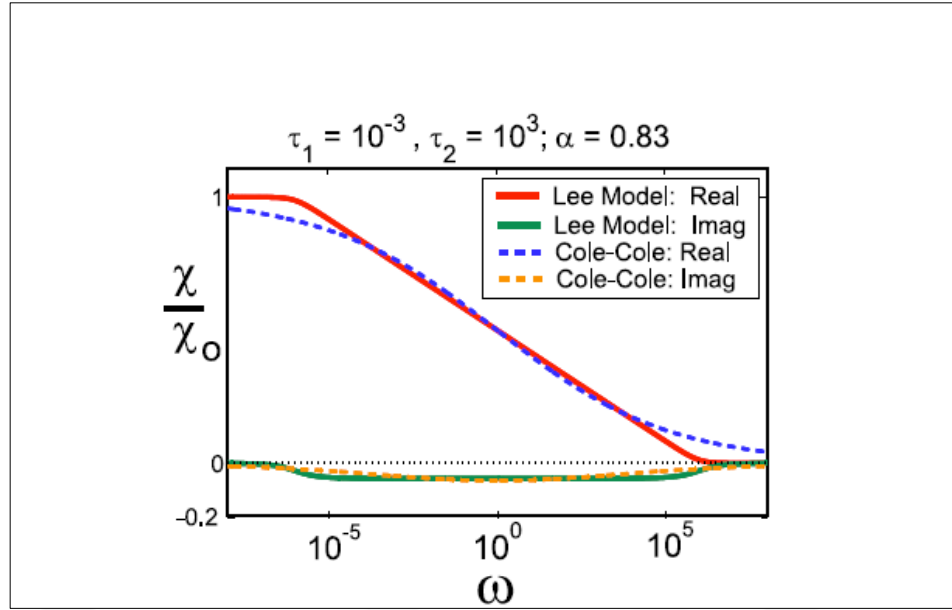


Figure 2.1: Magnetic susceptibility models for the SPM soils [49].

The Cole-Cole model is similar to the logarithmic model, however the band where the magnetic properties change with the frequency is narrower in the Cole-Cole model [67]. The two models are also compared by the after-effect functions [27], [28].

The Cole-Davidson model is also proposed to fit the magnetization data under the magnetic viscosity case [20], [27], [28].

2.1.2.4. SPM Parameters

The lower limit τ_1 of time constants is defined for mathematical reasons and it must be smaller than the shortest response time of the instrument. On the other hand, τ_2 is dependent on the exposure time to the magnetic field and it is not dependent of the material properties [29].

Shubitidze et al. investigated the effect of the SPM soils on the time domain EMI data collected by a MPV sensor. The SPM parameters are estimated by an image method for $\chi_0 = 0.005$, $\tau_1 = 10^{-8}$ s and $\tau_2 = 10^3$ s from time domain sensor data [7]. Kozhevnikov and Antonov chose the model parameters as $\chi_0 = 0.001$, $\tau_1 = 10^{-6}$ s and $\tau_2 = 10^6$ s for simulation of the uniform SPM half-space [39]. Das measured the SPM parameters of Cambodian lateritic soil as $\chi_0 = 0.003169$, $\tau_1 = 2.74 \times 10^{-26}$ s and $\tau_2 = 1$ s. Accordingly, τ_1 value is physically meaningless, and it obeys with the above proposition [57].

For the extreme cases which yield strong late-time responses such as some regions in Africa, it is suggested to use the values of τ_1 and τ_2 as 800 μs and 8 s, respectively, with $\chi_0 = 0.04$ [10], [11]. The other practical values of the time constants for resistive permafrost terrains are 1 μs and 1 s [43], [68].

2.1.3. SPM Effects on Time Domain Electromagnetic Responses

The SPM effect in the electromagnetic induction EMI systems is initially introduced in archaeological research. The voltage measured by the receiver antenna decays in proportion to t^{-1} which is independent of soil conductivity [69].

Butler investigated the effects of the magnetically dispersive soils in UXO detection. It is suggested to use the mean square of the horizontal component of the measured magnetic field. It is also suggested to take the measurements in the areas where no abnormal signals are visible and to subtract the fitted At^{-1} signal from this reference measurement to prevent unwanted abnormal signals [70].

Buselli demonstrated that the SPM effects interfere with accurate extraction of apparent resistivity from the TDEM data when using a coincident loop or single-loop configuration. It is also shown that separation of the transmitter and receiver and increase in height of the transmitter relieves the SPM effect [71].

Ripka et al. observed the SPM effect in pulse induction metal detector measurements in which the measured voltage decayed with t^x (x was between 1.1 and 1.3) and it is concluded that the deviation from t^{-1} is caused by inhomogeneous distribution of the SPM particles [72].

Stognii et al. performed a field experiment at a site in West Yakutia applying a high-resolution array to investigate the effects of receiver location on the SPM response [73].

Kwan et al. analyzed AEM data over barren rocks in Greenland. The SPM anomalies appeared in spatially sharp variations in the vertical component of the derivative of the magnetic field while they are absent in the horizontal component [74].

Macnae analyzed the amplitude and spatial characteristics of AEM data to discriminate the near-surface SPM sources from deeper mineral targets. It is found that the SPM sources could be detected through short-wavelength spatial filtering and typically exhibited smaller amplitudes compared to the responses of conductive sources at similar depths [68].

Zhang et al. proposed a method for selecting the optimal offset to suppress the SPM effect by analyzing the effects of resistivity and susceptibility using a grounded-wire source configuration [75].

Gaucher and Smith observed the SPM effect on the TDEM data from iron oxide minerals embedded in rocks. It is found that the SPM effect may originate not only from magnetite and maghemite particles but also from other iron oxides and magnetic minerals buried in a rock. It is also observed that the SPM effect is visible only on the vertical component when the receiver placed inside the transmitter [76].

Laboratory experiments are also performed for the investigation of the behavior of magnetic viscous soil samples through laboratory experiments. The power law at^{-b} in transient response is analyzed with impulse magnetization response [77], [78]. Borry et al. also performed laboratory measurements to understand how magnetic viscous soils adversely affect the performance of the metal detectors. The soil's magnetic susceptibility is measured at two frequencies [79].

2.1.3.1. Soil Signals in Time Domain

Several formulas induced voltage signal are proposed in the literature. These signal models can be used for the validation of the QS-FDTD results in this thesis.

Wahrlich patented a signal processing technique for PI metal detectors which reduces the effects of SPM soil signals by a fitting algorithm. As the fitted soil signal model, the soil signal behavior supposed as

$$S(t) = k_1 \frac{1}{t} + k_2 \frac{\log t}{t} \quad (2.33)$$

where $S(t)$ is the soil signal, k_1 and k_2 are the constants which characterize the soil [80].

- Coincident Loop Configuration

The induced voltage expressions in the received loop while the transmitting loop coincides with the receiving loop are listed below for the different cases.

- *Non-Magnetic and Non-Dispersive Conducting Soil:*

When the homogeneous lossy half-space medium is non-magnetic and ($\mu = \mu_0$) and nondispersive, Cross and Das found a formula for the induced voltage as

$$v(t) \approx -\frac{\sqrt{\pi}\mu_0^{5/2}\sigma^{3/2}a^4I}{20}t^{-5/2} \quad (2.34)$$

where a is the radius of the loop, I is the source current. In this case, the voltage decays as $t^{-5/2}$. For the conducting and non-dispersive magnetic soil case, no difference is observed in the induced voltage form [81].

- *Non-Conducting and Dispersive Magnetic Soil:*

When the magnetically dispersive SPM soil is non-conducting, Das wrote a for the induced voltage as

$$v(t) \approx -\mu_0 m(h) \frac{\chi_0}{2 \ln(\tau_2/\tau_1)} \frac{1}{t} \quad (2.35)$$

where $m(h)$ is calculated from the integral formula for lossless soil with constant permeability and $\tau_1 \ll t \ll \tau_2$. In this case, the induced voltage decays as t^{-1} [55].

- *Conducting and Dispersive Magnetic Soil:*

Lee found an analytical formula for the induced voltage from the lossy and magnetically dispersive SPM half-space as

$$v(t) \approx \frac{-M_0\mu_0 a}{bt} \frac{\chi_0 d}{\ln(\tau_2/\tau_1)} - \frac{-M_0\mu_0 a \chi_0 d}{\ln(\tau_2/\tau_1)} \frac{\sigma_1\mu_0}{16t^2} (2\pi a) - \frac{\sqrt{\pi a \mu_0}}{20t} M_0 \left(\frac{\sigma_1\mu_0 a^2}{t} \right)^{3/2} \quad (2.36)$$

where b is the radius of the wire and d is the thickness of the magnetic soil layer [31]. The first term in this expression is caused by the SPM effect. Since it is the dominant term, the voltage decays as $1/t$.

Lee expressed transient electric fields from the lossy SPM half-space in a simpler form as

$$E \approx \frac{A_1}{t} + \frac{A_2}{t^2} + \frac{A_3}{t^{5/2}} + \dots$$

where A_1 , A_2 and A_3 are geometric factors. A_1 and A_2 are dependent on the SPM parameters and thickness of the SPM layer. A_3 originates from the lossy ground. It is concluded that the t^{-1} part of the signal dominates at late time [82].

Corbyn claimed that the secondary voltage signal from the magnetic viscous soil is proportional to derivative of the decaying magnetization $g'(t)$ as

$$g'(t) = (1 - P)e^{-t/T_1} + Pe^{-t/T_2} \quad (2.37)$$

where P , T_1 and T_2 are constants defined empirically [83].

- Separate Loop Configuration

In this case, the induce voltage expression is

$$\begin{aligned}
V \approx & \frac{-M_0\mu_0}{2t} \frac{\chi_0 db}{\ln(\tau_2/\tau_1)} \int \frac{4}{3\pi^2} \sqrt{\left(\frac{a}{\rho}\right)} \frac{1}{(\rho-a)^2} \\
& \times \left[\frac{(\rho^2+a^2)}{2\rho a} Q_{-1/2}\left(\frac{\rho^2+a^2}{2\rho a}\right) - Q_{1/2}\left(\frac{\rho^2+a^2}{2\rho a}\right) \right] \cos\psi dl \\
& - \frac{M_0\mu_0 d\chi_0}{\ln(\tau_2/\tau_1)} \frac{\sigma_1\mu_0}{16t^2} \int G(\rho/a, a/\rho) \cos\psi dl \\
& - \frac{M_0\mu_0}{\ln(\tau_2/\tau_1)} \left(\frac{\sigma_1\mu_0 a^2}{t}\right)^{3/2} \int \rho \cos\psi dl.
\end{aligned} \tag{2.38}$$

where $G(\rho/a, a/\rho)$ is equal to ρ/a for $\rho < a$ and equal to a/ρ for $\rho > a$ and $Q_{-1/2}$ and $Q_{+1/2}$ are the ring function. It is concluded from this equation that the presence of b in the numerator reduces the contribution of the first term. Nevertheless, the voltage eventually decays as $1/t$ [31].

2.1.3.2. SPM and IP Effects

The IP effect occurs when the conductivity is frequency dependent. In extreme cases, the SPM effect and the IP effect come across together. The IP effect produces a time reversal at late times on the EMI sensor system. Consequently, it may distort the SPM late time behavior [84], [85]. The IP and the SPM effects together with the frequency dispersion of conductivity and susceptibility can distort the time domain response and even make it unrecognizable [86]. Cross and Das compared the IP effect and the SPM effect in an analytical solution of the half-space. The log-uniform model is used for the SPM and the Cole-Cole model used for the IP [63]. Barsukov and Fainberg defined the time intervals of the SPM and the IP responses plotted together in **Figure 2.2** [36].

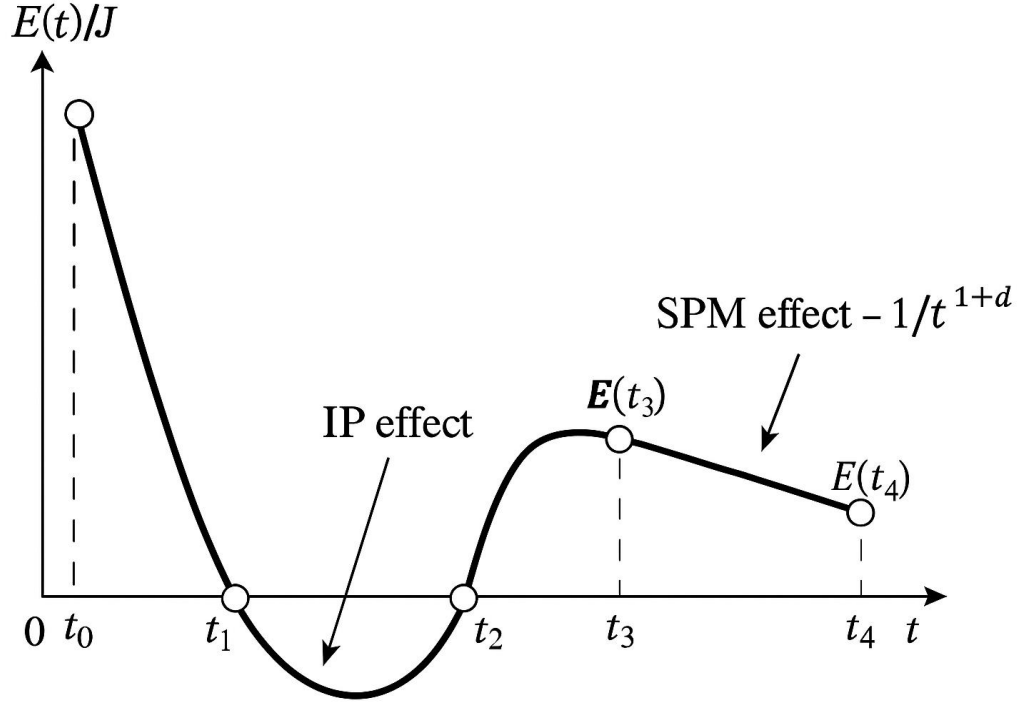


Figure 2.2: The SPM-IP response [36].

Kratzer and Macnae investigated the SPM and the IP effects together on the TDEM systems. The least-squares fitting method is used for determining areas with the SPM and the IP effects. Representative examples of the synthetic SPM, the IP and the EM responses are given as in **Figure 2.3** [87].

Kratzer et al. developed a technique to distinguish the SPM effect from the IP effect. They used a least-squares fitting method with the SPM basis function and subtracted the fitted SPM response from the total field distribution [84]. Macnae proposed an estimation technique for the SPM and the IP effects on the AEM data based on a fitting procedure. The log-uniform model for the SPM effect and the Cole-Cole model for the IP effect is used for the forward modeling. The SPM parameters are inverted from the AEM data [88]. As a result, it is stated in some cases that the SPM and the IP effects could be equal or opposite in magnitude. They could cancel each other or cause larger effect or even have a negligible effect [89].

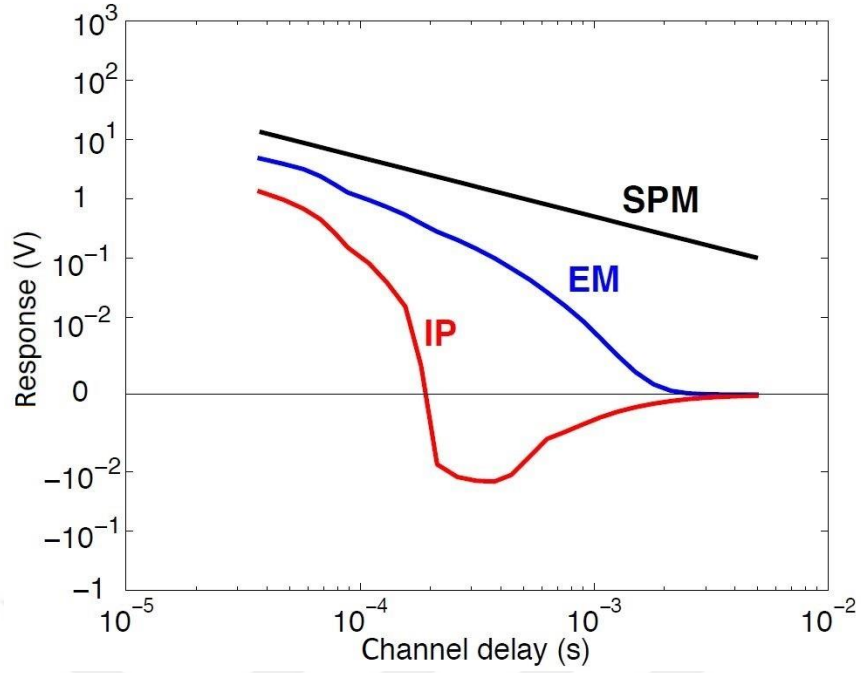


Figure 2.3: The SPM, the IP and the EM responses [87].

2.1.3.3. Reduction of the SPM Effect

To neutralize the SPM effect, different techniques are proposed in the literature as

1. Reduction of the mutual inductance of the transmitter and the receiver,
2. Increment of the radius of the coils,
3. Elevation of the coils [39], [43].
4. A slingram coil configuration of the vertical transmitter and the horizontal receiver with 1 m spacing [90].

Sharlov et al. proposed to use multi-offset arrays to reduce the SPM effect on the TDEM measurements. The SPM effect on the apparent resistivity data for different transmitter-receiver configurations is demonstrated [91].

2.2. Forward Modeling of TDEM Systems for SPM Soils

2.2.1. Analytical Solutions

For the analytical solution of the TDEM problems with the SPM soils, the log-uniform model is widely used. It is convenient for uniform distribution of the SPM particles in

the soil. Lee derived analytical formulations using the log-uniform model for the calculation of transient response of the superparamagnetic half-space using single transmitter loop and separated transmitter-receiver loops. For the single transmitter loop, the voltage decays as proportional to $1/t$ while it is proportional to $1/t^2$ for the separated loops case. It is also shown that the apparent resistivity also reduced with $1/t$ [31].

Kozhevnikov and Antonov solved the Helmholtz equation in a boundary-value problem with the relationship between the viscous magnetization and the magnetic flux. The transient response of the magnetically viscous (dispersive) earth is calculated using the inverse Fourier transform. The effects of the central-loop, the coincident loops and the separated loops configurations and the effect the loop size on the transient response of the SPM half-space and the SPM layered earth are investigated [39], [92], [93].

Kozhevnikov and Antonov calculated the transient response of the layer Earth with the SPM soil using in-loop and coincident loop configurations. The log-uniform model for the layered medium is used. It is shown that the coincident loop configuration is strongly affected by magnetic viscosity [94].

Antonov and Kozhevnikov calculated the transient electric field response of the uniform lossy SPM half-space. A grounded line source is used as the transmitter. The log-uniform model in the frequency domain is used to represent the SPM effect. The time domain results are obtained by numerical integration of the inverse Fourier transform [95].

Cowan et al. presented analytical expressions for the horizontal and vertical components of the time domain response of the SPM half-space with a large circular loop excited by a step-off current source. A cross-over time in which the SPM effect starts to dominate the time domain response is also presented [96].

Druyts et al. proposed Born approximation based on an image theory for modeling the non-conductive dispersive magnetic soil response of the electromagnetic induction sensor. The time domain results are obtained by the inverse Fourier transform. Sensitivity maps for the different sensor configurations are also extracted [97].

2.2.2. Numerical Solutions

For the numerical evaluation of the TDEM systems, the FDTD method is widely used. However, the classical FDTD method is not practical for the TDEM problems because of its computational limitations. To this end, an unconditionally stable explicit FDTD (Digital FDTD) method, unconditionally stable implicit FDTD methods and an FDTD method with a fictitious term are proposed.

– *Unconditionally Stable Explicit FDTD (Digital FDTD):*

In this method, an explicit diffusion equation based 2D unconditionally stable DuFort-Frankel FDTD method is proposed to relax the time steps in the FDTD method. With the DuFort-Frankel scheme, a hyperbolic term is added to the wave equation and the update equation is transformed into the damped wave equation. This gives rise to an unconditional stability [98], [99].

Adhidjaja et al., used the 2D DuFort-Frankel FDTD method with a step-off excitation a double line source. The transient voltage responses of the homogeneous half-space, the half-space with buried large object, the two-layer space and the half-space with overburden and large object are calculated. A scattered field formulation is used because it is five times faster than the total field formulation with the same accuracy. An upward boundary condition is applied to the air region. The results are validated with the Integral Equation (IE) and Finite Element Method (FEM) solutions [100]. Additionally, Ahidjaja and Hohmann discussed the differences of the step-off response and the impulse response in this method [101]. The non-uniform meshes are used to alleviate computational burden [102].

Yue et al. presented a 3D DuFort-Frankel FDTD solution with a Mur absorbing boundary condition. An analytical solution is used as an initial source. Problems with the whole-space and the layered space with the buried objects are considered [103]. Yue et al. used the 3D DuFort-Frankel FDTD method to analyze the whole-space effect on the underground mine TEM problems [104]. Liu et al. used the scattered field formulation in the 2D Dufort-Frankel FDTD solution and compared the results with the total field formulation [105]. Commer et al. used the 3D Dufort-Frankel FDTD method to solve a rock fracture detection problem in the soil with steel-casing borehole with the TDEM system. The non-uniform meshes and an adaptive time-stepping technique are used in the FDTD algorithm [106].

– *Unconditionally Stable Implicit FDTD (Digital FDTD):*

A 3D non-uniform unconditionally stable Backward-Euler Direct-Splitting (BEDS) FDTD [107], and a 3D unconditionally stable Crank-Nicholson (CN) FDTD [108] algorithms are developed for the solution of the TDEM problems. Tajima developed a parallel 3D FDTD algorithm to calculate the induced magnetic field for determining the electrical conductivity anomalies in Earth's mantle [109].

– *Fictitious FDTD:*

In this method, a dynamic fictitious dielectric permittivity term introduced to turn the diffusion problem into the wave equation problem in the FDTD method [110]-[116]. Therefore, an explicit algorithm is obtained with an adaptive time step incrementation ability.

– *Numerical Solution of the TDEM problems with the SPM Soils:*

Adaptations of the above methods for the dispersive problems are not well-developed in the literature. This thesis provides a detailed discussion on the FDTD modeling of the TDEM problems with the dispersive SPM soils. In the FDTD method, two main techniques are used to handle material dispersion in the time domain. The first is a Recursive Convolution (RC) technique, which is based on a discrete convolution of the field and susceptibility in the time domain [18]. The second is an Auxiliary Differential Equation (ADE) technique, which introduces an additional differential equation to model material dispersion [19].

For modeling the SPM soils, only a few number of studies exist in the literature. Zhao et al. applied the RC FDTD technique to the dispersive Cole-Cole (CC) soil model with the log-normal model of the MV soils in the TDEM systems. Their 3D Maxwell's equations based the nonuniform FDTD solution, is an approximation, as the CC model is represented using a rational function approximation to enable the RC technique [14]. Similarly, Liu et al. applied the RC FDTD technique for the dispersive CC soil model to analyze both the IP and the MV effects. Their CC model is approximated by using a Multiple-Zero-Pole method in the 3D diffusion equation solution [15]. Liu et al. extended their study incorporating the initial (static) value of the B-field to enhance the accuracy of the solution when using the CC model with the RC technique for the MV effect [16].

Additionally, Ji et al. proposed a 3D Finite Difference Frequency Domain (FDFD) solution for the diffusion equation for the ATEM problem using the Chikazumi magnetic susceptibility model for the SPM medium. The transformation to the time domain is achieved through a digital filter algorithm [17].

Kingdon et al. used finite volume method in the frequency domain for calculating the SPM half-space response with the buried targets and with a trench and a bump on the surface of the ground. The real and imaginary parts of the magnetic field are calculated and field distributions are obtained [117].

In [15] and [16], the 3D diffusion equation is solved using a forward difference scheme within the FDTD method, achieving the first-order numerical accuracy in the time domain. However, due to the infinite propagation velocity assumption, this method fails to capture the full-wave physics of the wave propagation and complete phase information is lost. Therefore, the FDTD solution for the full-wave Maxwell's equations with the second-order accuracy is preferable.

3. FINITE DIFFERENCE TIME DOMAIN (FDTD) METHOD

The FDTD method is widely used for modeling the TDEM problems. It numerically solves Maxwell's equations by approximating spatial and temporal derivatives using finite difference schemes, namely, central, forward, and backward differences [118]. Among these, the central difference technique is commonly preferred for its superior accuracy. In the FDTD, the numerical derivatives are typically derived from Taylor series expansions, offering the second-order or higher accuracy.

Some strengths of the FDTD method:

- There is no need for matrix inversion.
- Obtaining a broadband pulse or sinusoidal steady-state response in a single run is possible with direct time-domain solutions of Maxwell's equations.
- A wide variety of materials such as lossy, anisotropic, dispersive, time dependent, non-linear materials can be defined.
- Arbitrarily shaped complex geometries can easily be handled.
- Since the electric and magnetic fields are calculated everywhere in the problem domain, the field distributions at desired simulation time can be presented.
- A full-wave solution of Maxwell's equations is offered.

Some weaknesses of the FDTD method:

- Because of the staircase modeling, the geometric modeling errors are present.
- The stability condition may cause the long simulation times.
- Solving large-scale problems requires high memory usage and therefore high computation times.
- Due to the meshing, the numerical dispersion error is present.

3.1. Application to Maxwell's Equations

Maxwell's equations in a simple lossy medium are

$$\begin{aligned}
 \nabla \times \vec{E}(\vec{r}, t) &= -\mu_0 \frac{\partial \vec{H}(\vec{r}, t)}{\partial t} \\
 \nabla \times \vec{H}(\vec{r}, t) &= \varepsilon \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} + \sigma \vec{E}(\vec{r}, t) + \vec{J}(\vec{r}, t) \\
 \nabla \cdot \vec{D}(\vec{r}, t) &= \rho(\vec{r}, t) \\
 \nabla \cdot \vec{B}(\vec{r}, t) &= 0
 \end{aligned} \tag{3.1}$$

where $\vec{E}$, $\vec{H}$, $\vec{D}$, $\vec{B}$, $\vec{J}$ and ρ are electric field, magnetic field, electric displacement, magnetic induction, current density vectors and charge density, respectively. Spatial and temporal derivatives in first and second equations in (3.1) can be evaluated using the finite differences technique. Then, the latest temporal terms are calculated iteratively in the time domain.

If the central difference technique is applied to the time derivative of the first Maxwell's equation, it is written as

$$\nabla \times \vec{E}(\vec{r})|_n = -\mu_0 \frac{|\vec{H}(\vec{r})|^{n+1/2} - |\vec{H}(\vec{r})|^{n-1/2}}{\Delta t} \tag{3.2}$$

where Δt is time step size and n is time step index. If the latest term in the time domain $|\vec{H}(\vec{r})|^{n+1/2}$ is left alone on the left side of the equation, the update equation for the magnetic field is found as

$$|\vec{H}(\vec{r})|^{n+1/2} = |\vec{H}(\vec{r})|^{n-1/2} - \frac{\Delta t}{\mu_0} \nabla \times \vec{E}(\vec{r})|_n. \tag{3.3}$$

Similarly, if the central differences technique is also applied to the second Maxwell's equation at time step $(n + 1/2)$ for the sake of coherence of the two Maxwell's equations, it becomes

$$\nabla \times \vec{H}(\vec{r})|^{n+1/2} = \varepsilon \frac{|\vec{E}(\vec{r})|^{n+1} - |\vec{E}(\vec{r})|^n}{\Delta t} + \sigma |\vec{E}(\vec{r})|^{n+1/2} + |\vec{J}(\vec{r})|^{n+1/2}. \quad (3.4)$$

where getting rid of the half time step term of electric field $|\vec{E}(\vec{r})|^{n+1/2}$, arithmetic mean of $|\vec{E}(\vec{r})|^{n+1}$ and $|\vec{E}(\vec{r})|^n$ terms are taken as

$$|\vec{E}(\vec{r})|^{n+1/2} = \frac{|\vec{E}(\vec{r})|^{n+1} + |\vec{E}(\vec{r})|^n}{2}. \quad (3.5)$$

Finally, the update equation for $|\vec{E}(\vec{r})|^{n+1}$ is written as

$$\begin{aligned} |\vec{E}(\vec{r})|^{n+1} = & \left(\frac{2\varepsilon - \sigma\Delta t}{2\varepsilon + \sigma\Delta t} \right) |\vec{E}(\vec{r})|^n \\ & + \left(\frac{2\Delta t}{2\varepsilon + \sigma\Delta t} \right) \left[|\nabla \times \vec{H}(\vec{r})|^{n+1/2} - |\vec{J}(\vec{r})|^{n+1/2} \right]. \end{aligned} \quad (3.6)$$

Applying the central differences technique to the spatial derivatives in (3.4) and (3.6), the final version of the update equations can easily be obtained. There is no need to solve the divergence equations of Maxwell's equations because they are satisfied inherently in the FDTD method [119]. However, in some cases, such as in the TDEM problems, including the solution of the divergence equation for $\vec{B}$, $\nabla \cdot \vec{B}(\vec{r}, t) = 0$, the FDTD solution accuracy increases. The details of this divergence-free improvement of the FDTD method is explained in **Chapter 3.4**.

3.2. Implementation of ADE Technique

To model the TDEM problems more realistically, the FDTD method must be modified for the frequency dependent material properties. These materials are called the dispersive media which the propagation velocity is dependent on the frequency. To solve the EM problems in the dispersive media using the FDTD method, two fundamental techniques are used:

- Recursive Convolution (RC) technique,
- Auxiliary Differential Equation (ADE) technique.

The RC technique is an approach for the FDTD solution based on the discrete convolution of the field and susceptibility in the time domain [18]. In the ADE technique, an additional differential equation is used to model the material dispersion in the time domain [19].

Since finding a recursive relation for the log-uniform model for SPM media is not an easy topic, ADE technique is preferred in this thesis.

Maxwell's equations in a simple lossy magnetic dispersive medium are

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t} - \mu_0 \frac{\partial \vec{M}}{\partial t} \quad (3.7)$$

$$\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t} + \sigma \vec{E} + \vec{J} \quad (3.8)$$

where an ordinary differential for magnetization $\vec{M}$ is defined for the magnetic viscous media in equation (2.7) and (2.9) as

$$M(t) = \frac{\chi_0 H}{\ln(\tau_2/\tau_1)} (\ln \tau_2 - \ln t - 0.577)$$

$$\frac{dM(t)}{dt} \cong \frac{-\chi_0 H}{\ln(\tau_2/\tau_1)} \frac{1}{t}$$

where H is the initial magnetic field strength which is switched off at $t = 0$. This equation is used as the ADE for the SPM medium. Then, in the three-dimensional Cartesian coordinates, the field components are

$$\begin{aligned} \frac{\partial H_x}{\partial t} &= \frac{1}{\mu_0} \left(\frac{\partial E_y}{\partial z} - \frac{\partial E_z}{\partial y} \right) - \frac{\partial M_x}{\partial t} & \frac{\partial E_x}{\partial t} &= \frac{1}{\varepsilon} \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} - \sigma E_x - J_x \right) \\ \frac{\partial H_y}{\partial t} &= \frac{1}{\mu_0} \left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z} \right) - \frac{\partial M_y}{\partial t} & \frac{\partial E_y}{\partial t} &= \frac{1}{\varepsilon} \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} - \sigma E_y - J_y \right) \\ \frac{\partial H_z}{\partial t} &= \frac{1}{\mu_0} \left(\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) - \frac{\partial M_z}{\partial t} & \frac{\partial E_z}{\partial t} &= \frac{1}{\varepsilon} \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} - \sigma E_z - J_z \right). \end{aligned} \quad (3.9)$$

The FDTD update equations for each magnetic and electric field components are found to be

$$\begin{aligned}
H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} &= H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n-\frac{1}{2}} \\
&+ \frac{\Delta t}{\mu_{i,j+\frac{1}{2},k+\frac{1}{2}}} \left[-\frac{E_z|_{i,j+1,k+\frac{1}{2}}^n - E_z|_{i,j,k+\frac{1}{2}}^n}{\Delta y} \right. \\
&\quad \left. + \frac{E_y|_{i,j+\frac{1}{2},k+1}^n - E_y|_{i,j+\frac{1}{2},k}^n}{\Delta z} \right] - \left[M_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - M_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n-\frac{1}{2}} \right]
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} &= H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n-\frac{1}{2}} \\
&+ \frac{\Delta t}{\mu_{i+\frac{1}{2},j,k+\frac{1}{2}}} \left[-\frac{E_z|_{i+1,j,k+\frac{1}{2}}^n - E_z|_{i,j,k+\frac{1}{2}}^n}{\Delta x} \right. \\
&\quad \left. + \frac{E_x|_{i+\frac{1}{2},j,k+1}^n - E_x|_{i+\frac{1}{2},j,k}^n}{\Delta z} \right] - \left[M_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - M_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n-\frac{1}{2}} \right]
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} &= H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n-\frac{1}{2}} \\
&+ \frac{\Delta t}{\mu_{i+\frac{1}{2},j+\frac{1}{2},k}} \left[-\frac{E_x|_{i+\frac{1}{2},j+1,k}^n - E_x|_{i+\frac{1}{2},j,k}^n}{\Delta y} \right. \\
&\quad \left. + \frac{E_y|_{i+1,j+\frac{1}{2},k}^n - E_y|_{i,j+\frac{1}{2},k}^n}{\Delta x} \right] - \left[M_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - M_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n-\frac{1}{2}} \right]
\end{aligned} \tag{3.12}$$

$$\begin{aligned}
E_x|_{i+\frac{1}{2},j,k}^{n+1} &= \left(\frac{2\varepsilon_{i+\frac{1}{2},j,k} - \Delta t\sigma_{i+\frac{1}{2},j,k}}{2\varepsilon_{i+\frac{1}{2},j,k} + \Delta t\sigma_{i+\frac{1}{2},j,k}} \right) E_x|_{i+\frac{1}{2},j,k}^n \\
&+ \left(\frac{2\Delta t}{2\varepsilon_{i+\frac{1}{2},j,k} + \Delta t\sigma_{i+\frac{1}{2},j,k}} \right) \left[\frac{H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i+\frac{1}{2},j-\frac{1}{2},k}^{n+\frac{1}{2}}}{\Delta y} \right. \\
&\quad \left. - \frac{H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i+\frac{1}{2},j,k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} - J_x|_{i+\frac{1}{2},j,k}^{n+\frac{1}{2}} \right]
\end{aligned} \tag{3.13}$$

$$\begin{aligned}
E_y|_{i,j+\frac{1}{2},k}^{n+1} = & \left(\frac{2\varepsilon_{i,j+\frac{1}{2},k} - \Delta t \sigma_{i,j+\frac{1}{2},k}}{2\varepsilon_{i,j+\frac{1}{2},k} + \Delta t \sigma_{i,j+\frac{1}{2},k}} \right) E_y|_{i,j+\frac{1}{2},k}^n \\
& + \left(\frac{2\Delta t}{2\varepsilon_{i,j+\frac{1}{2},k} + \Delta t \sigma_{i,j+\frac{1}{2},k}} \right) \left[\frac{H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j+\frac{1}{2},k-\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta z} \right. \\
& \left. - \frac{H_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} - H_z|_{i-\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}}}{\Delta x} - J_y|_{i,j+\frac{1}{2},k}^{n+\frac{1}{2}} \right]
\end{aligned} \tag{3.14}$$

$$\begin{aligned}
E_z|_{i,j,k+\frac{1}{2}}^{n+1} = & \left(\frac{2\varepsilon_{i,j,k+\frac{1}{2}} - \Delta t \sigma_{i,j,k+\frac{1}{2}}}{2\varepsilon_{i,j,k+\frac{1}{2}} + \Delta t \sigma_{i,j,k+\frac{1}{2}}} \right) E_z|_{i,j,k+\frac{1}{2}}^n \\
& + \left(\frac{2\Delta t}{2\varepsilon_{i,j,k+\frac{1}{2}} + \Delta t \sigma_{i,j,k+\frac{1}{2}}} \right) \left[\frac{H_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} - H_y|_{i-\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta x} \right. \\
& \left. - \frac{H_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} - H_x|_{i,j-\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}}}{\Delta y} - J_z|_{i,j,k+\frac{1}{2}}^{n+\frac{1}{2}} \right].
\end{aligned} \tag{3.15}$$

Then, the update equations for the ADE of magnetization can be written over equation (2.9) as

$$M_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{-M_0}{\log(\tau_2/\tau_1)} \frac{1}{n} + M_x|_{i,j+\frac{1}{2},k+\frac{1}{2}}^{n-\frac{1}{2}} \tag{3.16}$$

$$M_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n+\frac{1}{2}} = \frac{-M_0}{\log(\tau_2/\tau_1)} \frac{1}{n} + M_y|_{i+\frac{1}{2},j,k+\frac{1}{2}}^{n-\frac{1}{2}} \tag{3.17}$$

$$M_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n+\frac{1}{2}} = \frac{-M_0}{\log(\tau_2/\tau_1)} \frac{1}{n} + M_z|_{i+\frac{1}{2},j+\frac{1}{2},k}^{n-\frac{1}{2}}. \tag{3.18}$$

In this sense, the FDTD iteration can be carried out for solving the H -fields, E -fields and the magnetization, respectively.

3.3. Quasi-Static FDTD Method

In the low-frequency electromagnetic induction applications, such as the TDEM systems, the wavelength is significantly larger than the thickness of earth layers or the dimensions of the buried bodies. As a result, the classical FDTD method requires extremely small unit cell sizes relative to the wavelength. This constraint, dictated by the Courant–Friedrichs–Lewy (CFL) stability criterion, leads to very small time steps and consequently, prohibitively long simulation times, potentially spanning months with current computing capabilities. To address this limitation, a QS-FDTD method has been proposed, which is valid under the quasi-static condition and significantly reduces computational demand [120].

There are two types of quasi-static approximations: Magneto-Quasi-Static (MQS) which neglects displacement currents and Electro-Quasi-Static (EQS) which neglects magnetic induction currents.

In the TDEM systems, the displacement currents on the buried body or on the soil are smaller than the conduction currents. Therefore, the MQS condition is inherently valid. In the time domain, this condition is shown as below

$$\frac{\partial}{\partial t} \vec{D}(\vec{r}, t) \ll \sigma \vec{E}(\vec{r}, t). \quad (3.19)$$

In a simple lossy medium, with frequency domain conversion for the monochromatic wave, (3.19) becomes

$$|i\omega\epsilon\vec{E}(\vec{r})| \ll |\sigma\vec{E}(\vec{r})| \Rightarrow |i\omega\epsilon| \ll |\sigma| \Rightarrow 2\pi f\epsilon \ll \sigma. \quad (3.20)$$

Under these conditions, completely neglecting the displacement currents turns the wave equation into a diffusion equation. However, in the QS-FDTD method, displacement currents are kept under the MQS condition. To reduce the propagation velocity and allow for the larger FDTD time steps, the dielectric permittivity is fictitiously scaled up. This adjustment significantly increases the time step size, enabling the problem to be solved within a reasonable computational time [120].

Effects of the MQS approximation on the fundamental parameters:

1) *Propagation Constant*: The propagation constant in the lossy medium

$$k = k_{real} + ik_{imag} = \sqrt{\omega^2 \varepsilon \mu + i \omega \sigma \mu} \quad (3.21)$$

where the real part k_{real} and imaginary part k_{imag} can be expressed as

$$k_{real} = \underbrace{\sqrt{\varepsilon_0 \mu_0} \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2} + 1 \right] \right)}_{\sqrt{\alpha_k}}^{1/2} \quad (3.22)$$

$$k_{imag} = \underbrace{\sqrt{\varepsilon_0 \mu_0} \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2} - 1 \right] \right)}_{\sqrt{\alpha_k}}^{1/2} \quad (3.23)$$

where k_{real} is $\sqrt{\alpha_k}$ times bigger than the free space propagation constant k_0 which models the wave propagation phenomenon. As a consequence, the propagation velocity c in the lossy medium reduces as

$$c = \frac{\omega}{k_{real}} = \frac{\omega}{\sqrt{\alpha_k} k_0} = \frac{1}{\sqrt{\alpha_k}} c_0 \quad (3.24)$$

where $c \ll c_0$. This shows the propagation velocity in the lossy medium slows down.

If k_{real} and k_{imag} are equal, the displacement currents are completely neglected as

$$k \cong \sqrt{i \omega \mu \sigma} \Rightarrow k_{real} = k_{imag} \cong \sqrt{\frac{\omega \mu \sigma}{2}}. \quad (3.25)$$

This condition leads to the diffusion equation.

2) *Skin Depth*: The formula of the skin depth δ is

$$\delta = \frac{1}{k_{real}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0} \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \left[\sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2} + 1 \right] \right)^{1/2}}. \quad (3.26)$$

With the MQS approximation (3.21) becomes

$$k_{real} \cong \sqrt{\frac{\omega\mu\sigma}{2}} \Rightarrow \delta = \frac{1}{k_{real}} \cong \sqrt{\frac{2}{\omega\mu\sigma}}. \quad (3.27)$$

3) *Wave Impedance*: The wave impedance in the simple lossy medium is

$$Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu}{\varepsilon + i\frac{\sigma}{\omega}}}. \quad (3.28)$$

where a material scaling (ε , μ , σ or ω -scaling) affects the wave impedance, so a rescaling will be necessary. Neglecting the displacement currents under the MQS condition Z can be approximated as

$$Z = \frac{|\vec{E}|}{|\vec{H}|} = \sqrt{\frac{\mu\omega}{j\sigma}}. \quad (3.29)$$

4) *Time Constants and Continuity Equation*: Three fundamental time parameters must be investigated under the MQS condition: Relaxation time τ_e , transmit time τ_t and diffusion time τ_m .

The relaxation time $\tau_e = \sigma/\varepsilon$ is extracted from the continuity equation. It is a constant related to the charge transition from the transient to the steady-state regime. Neglecting the displacement current, the continuity equation is violated. In these circumstances, it is not possible to model the transient charge transportation. Therefore, the diffusion equation is not appropriate for modeling the transient charges.

The transmit time $\tau_t = L/c = L\sqrt{\varepsilon\mu}$ is extracted from Maxwell's equations and related to the electromagnetic wave propagation where L is the characteristic length of the considered object.

The diffusion time $\tau_m = \sigma\mu L^2$ is extracted from the diffusion equation. It is a time constant related to the diffusion process.

In the MQS approximation, the following condition must be inherently satisfied in the lossy medium

$$\tau_e < \tau_t < \tau_m. \quad (3.30)$$

To ensure the physical meaning of the MQS phenomenon, the material scaling technique used in the numerical solution must not violate the above considerations.

First of all, to analyze the numerical behavior of the FDTD method in the lossy medium, it is necessary to extract the numerical wave number k^N

$$k^N = \omega \sqrt{\varepsilon_N \mu} = \omega \sqrt{\left(\varepsilon + i \frac{\sigma \Delta t}{2 \tan(\omega \Delta t/2)} \right) \mu} \quad (3.31)$$

where ε_N is the numerical dielectric permittivity [120]. The real and imaginary parts of k^N are

$$k_{real}^N = k_0 \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2 \left(\frac{\omega \Delta t/2}{\tan(\omega \Delta t/2)} \right)^2} + 1 \right)^{1/2} \quad (3.32)$$

$$k_{imag}^N = k_0 \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2 \left(\frac{\omega \Delta t/2}{\tan(\omega \Delta t/2)} \right)^2} - 1 \right)^{1/2}. \quad (3.33)$$

If $\tan(\omega \Delta t/2) \cong \omega \Delta t/2$ for the small argument approximation is used, k_{real}^N and k_{imag}^N becomes

$$k_{real}^N \cong k_0 \sqrt{\varepsilon_r \mu_r} \left(\frac{1}{2} \sqrt{1 + \left(\frac{\sigma}{\omega \varepsilon_0 \varepsilon_r} \right)^2} + 1 \right)^{1/2} \Rightarrow k_{real}^N \cong k_{real} \quad (3.34)$$

$$k_{imag}^N \cong k_0 \sqrt{\epsilon_r \mu_r} \left(\frac{1}{2} \sqrt{1 + \left(\frac{\sigma}{\omega \epsilon_0 \epsilon_r} \right)^2} - 1 \right)^{1/2} \Rightarrow k_{imag}^N \cong k_{imag} \quad (3.35)$$

where it is clear that the numerical wave number is approximately equal to the analytical wave number. Thus, the MQS problems do not affect the numerical dispersion significantly.

In the lossy medium, the propagation velocity slows down because $k^N > k_0$. However, in the classical FDTD solution, the slowed propagation velocity does not affect Δt because the stability condition depends on c_0 , not the slowed c . Therefore, it is necessary to use a material scaling technique to model this slowdown properly. The material scaling technique can be applied by scaling the parameters ϵ , μ , σ and ω . However, it is shown that only the ϵ -scaling is feasible for the FDTD solution [120].

To find the proper value for the scaling factor, its effects on the following critical parameters and conditions must be analyzed:

- i) QS and MQS conditions,
- ii) Parameters of τ_e , τ_t and τ_m ,
- iii) Parameters of δ , Z and rescaling,
- iv) Numerical dispersion and stability.

3.3.1. Dielectric Permittivity Scaling

In the ϵ -scaling technique, to slow down the propagation velocity, the dielectric permittivity is scaled up as $\epsilon^{QS} = \alpha \epsilon$. Then, the time step size is reduced as $\Delta t^{QS} = \sqrt{\alpha} \Delta t$. Investigation on how the scaling affects the critical parameters is shown below.

- 1) *QS Condition and k* : The QS condition and k are not seriously affected by the ϵ -scaling, because k_{real} does not have strong dependence on ϵ .
- 2) *MQS Condition*: The MQS condition is affected by the scaling. Therefore, the scaling factor must obey the following criterion

$$2\pi f \alpha_{MQS} \varepsilon \ll \sigma \quad \Rightarrow \quad \alpha_{MQS} \ll \frac{\sigma}{2\pi f \varepsilon}. \quad (3.36)$$

3) τ_e , τ_t and τ_m : To model the MQS condition properly in the QS-FDTD method, the modified unit time step Δt^{QS} must satisfy the following condition

$$\Delta t^{QS} < \tau_e^{QS} < \tau_t^{QS} < \tau_m \quad (3.37)$$

where τ_e^{QS} and τ_t^{QS} are the changed forms of τ_e and τ_t after the ε -scaling. Furthermore, τ_m is not affected by the scaling. These limitations can be analyzed one by one as follows.

– $\Delta t^{QS} < \tau_e^{QS}$ case:

$$\Delta t^{QS} = \sqrt{\alpha} \Delta t < \tau_e^{QS} = \alpha \tau_e \quad \Rightarrow \quad \alpha_{\Delta t} > \left(\frac{\Delta t}{\tau_e} \right)^2 \quad (3.38)$$

where τ_e^{QS} reduces for the very high conductive objects. This strongly limits the QS-FDTD unit time step. Therefore, the transient charge calculation using the QS-FDTD method for the high conductive objects is not feasible.

– $\tau_e^{QS} < \tau_t^{QS}$ and $\tau_t^{QS} < \tau_m$ cases have the same limitation:

$$\begin{aligned} \tau_e^{QS} < \tau_t^{QS} &\Rightarrow \alpha \tau_e < \sqrt{\alpha \tau_e \tau_m} \Rightarrow \alpha = \alpha_\tau < \frac{\tau_m}{\tau_e} \\ \tau_t^{QS} < \tau_m &\Rightarrow \sqrt{\alpha \tau_e \tau_m} < \tau_m \Rightarrow \alpha = \alpha_\tau < \frac{\tau_m}{\tau_e} \end{aligned} \quad (3.39)$$

4) δ , Z and Rescaling: The skin depth can be approximated as $\delta = 1/k_{real} \cong (2/\omega\sigma\mu)^{1/2}$. Therefore, δ is weakly affected by the ε -scaling. The ε -scaling weakly affects the wave impedance as

$$Z^{QS} = \frac{|\vec{E}^{QS}|}{|\vec{H}^{QS}|} = \sqrt{\frac{\mu}{\alpha\varepsilon + i\frac{\sigma}{\omega}}}; \quad Z = \sqrt{\frac{\mu}{\varepsilon + i\frac{\sigma}{\omega}}} \quad (3.40)$$

where Z^{QS} is the scaled and Z is the unscaled wave impedance. Under the MQS condition, they can be approximated as

$$\begin{aligned} Z &= \sqrt{\frac{\mu}{\varepsilon}} \sqrt{\frac{1}{1 + i\frac{\sigma}{\omega\varepsilon}}} \cong Z_0 \sqrt{\frac{1}{i\frac{\sigma}{\omega\varepsilon}}} \\ Z^{QS} &= Z_0 \sqrt{\frac{1}{\alpha + i\frac{\sigma}{\omega\varepsilon}}} = Z_0 \sqrt{\frac{1}{j\frac{\sigma}{\omega\varepsilon}}} \sqrt{\frac{1}{1 + \alpha\frac{\omega\varepsilon}{i\sigma}}} \end{aligned} \quad (3.41)$$

where Z_0 is the wave impedance in the free space. Under the MQS condition, $|\alpha\omega\varepsilon/i\sigma| \ll 1$ is true. Z^{QS} can be approximated with the generalized binomial expansion as

$$Z^{QS} = Z \sqrt{\frac{1}{1 + \alpha\frac{\omega\varepsilon}{j\sigma}}} \cong Z \frac{1}{1 + \frac{1}{2}\alpha\frac{\omega\varepsilon}{j\sigma}} = \frac{Z}{\alpha'} \quad (3.42)$$

where α' is the rescaling factor between Z^{QS} and Z . The rescaling factor depends on the problem type. If the source is an electric field type, the magnetic field must be scaled up by $|\alpha'|$. Therefore, the magnetic field must be rescaled by $|\alpha'|$. If the source is the magnetic field type, the electric field must be scaled up by $|\alpha'|$. Therefore, the electric field must be rescaled by $|\alpha'|$.

5) *Numerical Stability and Dispersion:* The QS-FDTD stability criterion is the same as the classical FDTD method below

$$\frac{c^{QS}\Delta t^{QS}}{\Delta x^{QS}} \leq 1 \implies \Delta t^{QS} \leq \frac{\Delta x^{QS}}{c^{QS}} = \sqrt{\alpha} \frac{\Delta x}{c_0} \leq \sqrt{\alpha} \Delta t. \quad (3.44)$$

Nyquist sampling limitation is also important factor in the QS-FDTD method. Δt must be smaller than the Nyquist unit time step Δt_N

$$\Delta t \leq \Delta t_N = \frac{1}{2f_{max}} \quad (3.44)$$

where f_{max} is the maximum frequency value. After the scaling, Δt becomes $\Delta t^{QS} = \sqrt{\alpha}\Delta t$, and the Nyquist criterion must be again satisfied as

$$\Delta t^{QS} = \sqrt{\alpha}\Delta t \leq \Delta t_N = \frac{1}{2f_{max}} \Rightarrow \alpha_N \leq \left(\frac{1}{2f_{max}\Delta t} \right)^2. \quad (3.45)$$

3.4. Adaptive Time-Step Increment

In the FDTD solutions for the TDEM problems, the time step can be gradually increased in late time due to the diffusive behavior of the fields. This technique is used for the solutions of diffusion wave equation [99] and Maxwell's equations with fictitious dielectric permittivity in the literature [110]. In this thesis, an adaptive time stepping technique for the QS-FDTD method is applied for the solution of the full-wave Maxwell's equations.

- *DuFort-Frankel FDTD with adaptive time-stepping:*

The adaptive time stepping technique is applied for the solution of the diffusion equation using the unconditionally stable DuFort-Frankel FDTD method [101]. Accordingly, the time step can be increased until reaching a maximum practical time step, namely,

$$\Delta t_{max} = \sqrt{\mu\sigma t} \frac{\Delta}{2} \quad (3.46)$$

where μ , σ and Δ are the magnetic permeability, the conductivity and the spatial grid size, respectively [99].

- *Fictitious FDTD with adaptive time-stepping:*

For the solution of the TDEM problems under the quasi-static condition, a fictitious dielectric permittivity term is added to the diffusion field. Therefore, a wave-like

equation is obtained. The value of this fictitious permittivity is increased with the time step due to the stability condition as

$$\gamma \geq \frac{3}{\mu_{min}} \left(\frac{\Delta t_n}{\Delta_{min}} \right) \quad (3.47)$$

where μ_{min} and Δ_{min} are the minimum magnetic permeability and the minimum spatial grid spacing. To keep the magneto-quasi-static condition valid, the maximum time step is given as

$$\Delta t_{max} = \alpha \sqrt{\frac{\mu_{min} \sigma t}{8}} \Delta_{min} \quad (3.48)$$

where $\alpha < 1$ is a constant. According to (3.48), the time step can be gradually increased with time [110].

- *QS-FDTD with adaptive time-stepping:*

In the QS-FDTD method, the adaptive time stepping technique can also be used. However, the stability condition does not vary with the time step in the QS-FDTD method. Therefore, the adaptive time-stepping technique must be considered in the stability range. In the early times, a small constant time step must be used because in the early times the current source consists of the high frequency parts which restrict the stability condition. In the late times which have the low frequency parts of the source, the time step size can be gradually increased with (3.48) until the stability criterion.

3.5. Divergence-Free Correction

The divergence-free condition must be considered explicitly in the FDTD solutions. It improves the accuracy of the calculations and yields a faster and memory efficient algorithm [110].

In 2D Cartesian coordinates, the two components of magnetic induction fields are dependent on each other in Maxwell's first equation. One of them can be computed from the other using the divergence equation of magnetic field as

$$\nabla \cdot \vec{B} = 0 \Rightarrow \frac{\partial B_y}{\partial y} = -\frac{\partial B_x}{\partial x}. \quad (3.49)$$

This equation can be solved numerically using central differences algorithm at the same time step as

$$H_{y,i+1,j}^{n+\frac{1}{2}} = H_{y,i+1,j+2}^{n+\frac{1}{2}} + \frac{\Delta y}{\Delta x} H_{x,i+2,j+1}^{n+\frac{1}{2}} - \frac{\Delta y}{\Delta x} H_{x,i,j+1}^{n+\frac{1}{2}}. \quad (3.49)$$

Now, the final equation can be used instead of the classical update equation for the H_y field component. This formulation can be easily extended to 3D Cartesian coordinates. In the problems consist of the SPM medium, the divergence-free correction is not applied in this thesis because of some stability issues.

4. NUMERICAL RESULTS

As the numerical examples, first, the 2D problems are considered. For the validation of the method, the 2D lossy half-space problem with a known analytical solution is solved. After that, the 2D lossy half-space problem consists of the thin overburden layer and the anomalous body is considered. Finally, the lossy SPM half-space problem is solved.

As the 3D numerical solutions, the lossy half-space and the lossy SPM half-space problems are solved and validated with the analytical solution. Then, the thin SPM layer and the lossy anomalous body problem are solved in the 3D. Finally, a more complex problem with the SPM thin layer is considered.

In the 2D problems, the two infinitely long wires placed on the ground are used as the transmitting antenna, while in the 3D problems, the loop antenna is used as the transmitter. A pulsed current source with 0.5 ms on-time and 1 μ s turn-off and turn-on times is used for the excitation of the transmitter. The pulse shape and the frequency response of the source are shown in **Figure 4.1**.

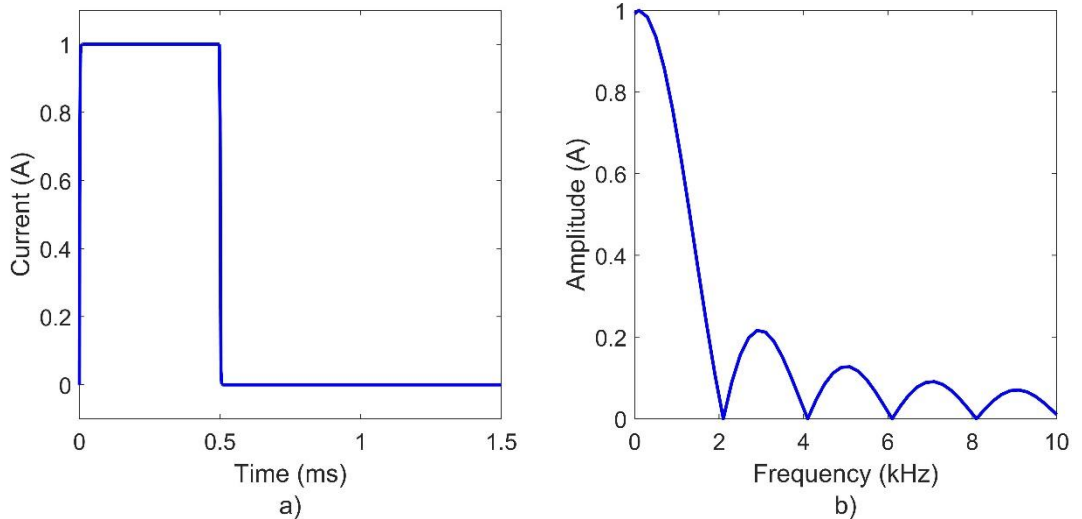


Figure 4.1: a) Current source waveform, b) its frequency spectrum.

4.1. 2D Problems

4.1.1. Homogeneous Lossy Half-Space

The homogeneous lossy half-space problem is a simple and fundamental problem to understand the effectiveness of the applied method. Therefore, first, this problem is considered to evaluate the feasibility of the QS-FDTD method for the solution of the TDEM problems.

The signal from the lossy half-space ground in the TDEM device can be evaluated in early and late-time stages in the time domain. Both the low and the high frequencies are present in the early-time response while the low frequency components are dominant in the late-time response. In the 3D case, when a direct current flowing on a large transmitting loop antenna placed on the lossy half-space ground is switched off at time $t = 0$, the time dependence of the vertical and the horizontal components of the magnetic field induced in the receiver placed at the center of the transmitter are proportional to, according to analytical solutions, $t^{-1.5}$ and t^{-2} at the late-time [121]. In the 2D case, the time dependence of the horizontal component is not known precisely while the vertical component is proportional to t^{-2} according to the analytical solution [122], [123].

The lossy half-space problem is solved in the 2D Cartesian coordinates with the QS-FDTD method. As the transmitter antenna, the two current-carrying wires of the infinite length and 60 m apart are placed on the surface of the soil and excited with the pulsed current in **Figure 4.1**. Considering the source frequency response given in **Figure 4.1**, the highest frequency for the QS-FDTD solution is selected as the 10 kHz. The conductivity value of the soil is taken as 0.01 S/m and the conductivity value of the air is taken as 10^{-6} S/m to meet the MQS condition in the entire problem space for an acceptable calculation time (assuming that it would not affect the results too much). The problem geometry is depicted in **Figure 4.2**.

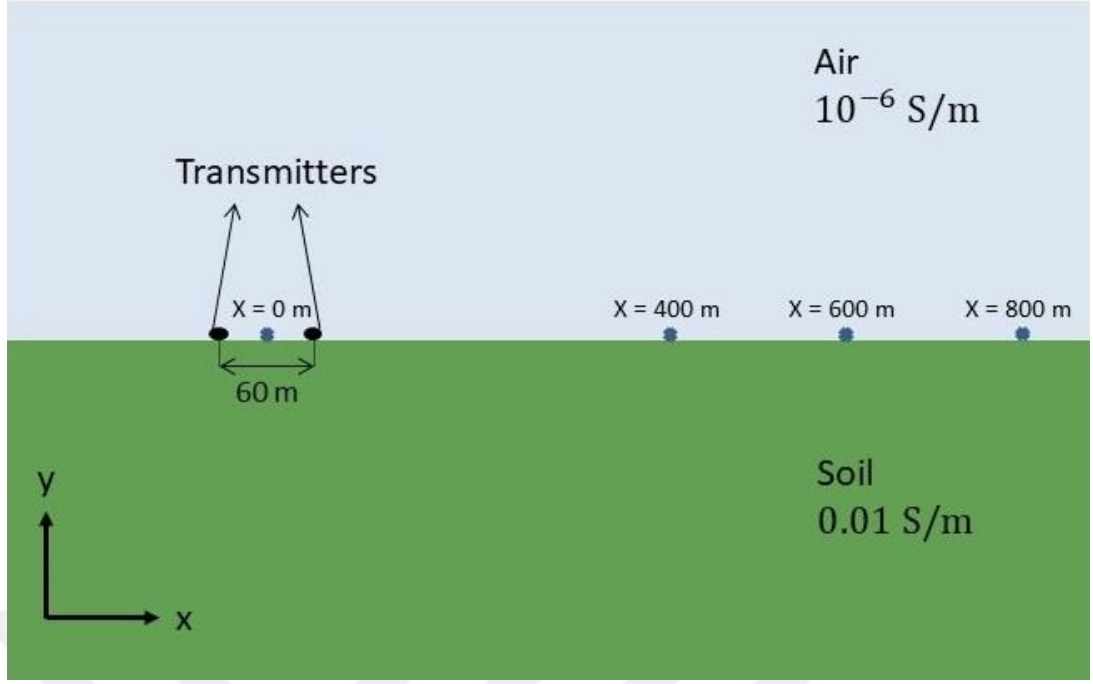
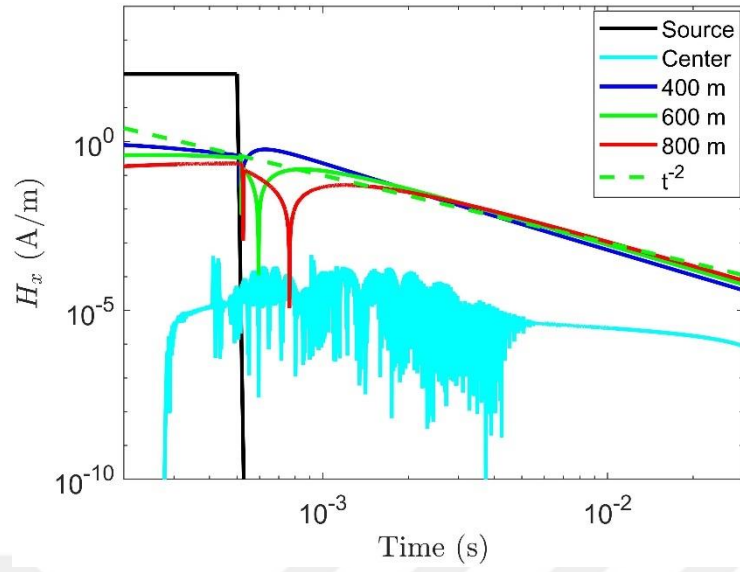


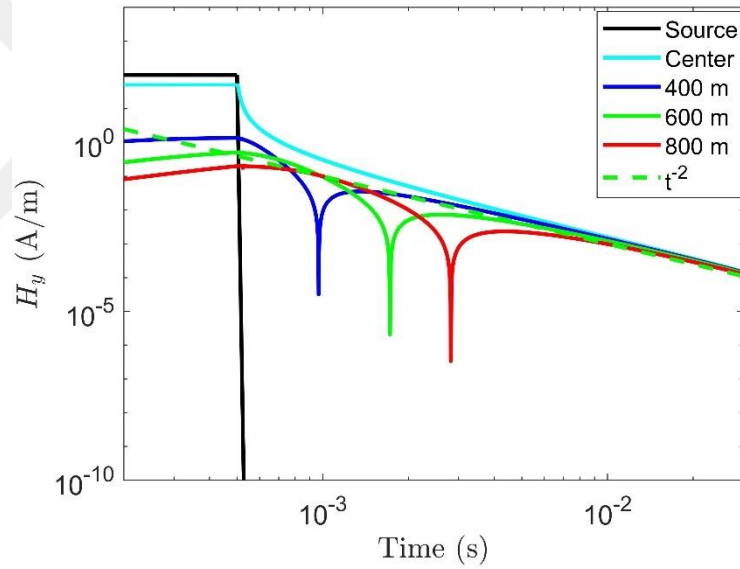
Figure 4.2: Geometry of the lossy half-space TDEM problem.

In this context, the QS scaling factor is calculated as $\alpha_k = 9.45$. The divergence-free condition is also applied. The horizontal and the vertical components of the magnetic fields calculated at the center of the transmitter antenna, 400 m, 600 m and 800 m away from the transmitter are shown in **Figure 4.3**. Accordingly, the vertical component of the magnetic field calculated with the QS-FDTD overlaps with the t^{-2} curve. Again, it can be seen that the horizontal component (x component) of the magnetic field is very weak at the center of the transmitters as expected [121]. It is also observed that as the position of the receiver antenna gets farther away, the time of the sign reversal in the magnetic field intensity is delayed and it fits the t^{-2} curve later.

Therefore, it can be concluded that the QS-FDTD method is an effective method for the solution of the 2D TDEM problems. In the next section, a more complex problem with the buried anomalous body and the thin lossy layer is examined.



a)



b)

Figure 4.3: Induced magnetic field strength for the lossy half-space: a) the horizontal, b) the vertical component.

4.1.2. Thin Layer and Anomalous Body Problem

The calculations in the previous section are repeated by adding the thin conductive layer of 50 meters thick with 0.1 S/m conductivity on the soil surface and placing a 100 m x 100 m object with a conductivity of 1 S/m at a depth of 100 meters. The same transmitter antenna as the homogeneous half-space problem is placed on the ground

and excited by the pulse signal shown in **Figure 4.1**. The geometry of the anomalous problem space is depicted in **Figure 4.4**.

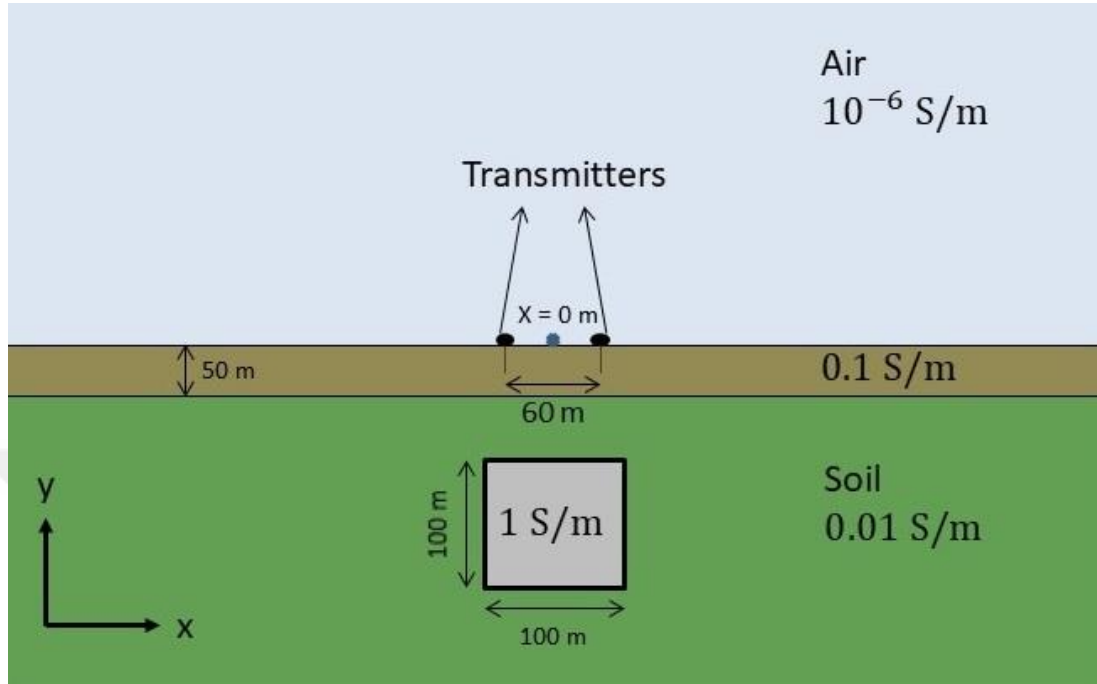
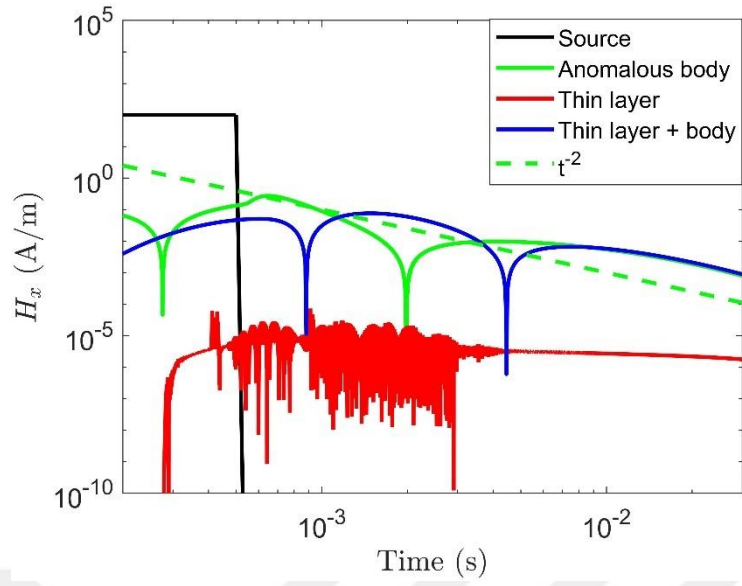


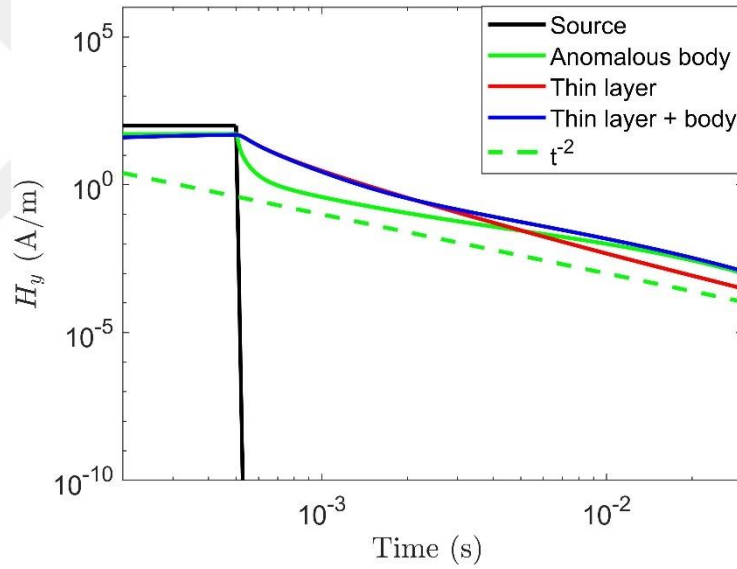
Figure 4.4: Geometry of the anomalous body problem with an overburden.

In **Figure 4.5**, the horizontal and the vertical components of the calculated magnetic fields are shown for three cases, i) the buried conductive anomalous body in the lossy half-space without the thin layer, ii) the thin layer at the top of the half-space, and iii) the anomalous body and the half-space together. According to the results shown in **Figure 4.5** for the anomalous body only case, the horizontal and the vertical components are seriously affected by the body especially at the early-time. The only thin layer case slightly affected both the horizontal and the vertical components. In the thin layer with the anomalous body case, two sign reversals in the horizontal component are observed at the early-time, while the vertical component is very similar to only the thin layer case at the early-time. At the late-time, the only anomalous body and the anomalous body with the thin layer cases coincide for the horizontal and the vertical components both.

It can be concluded from these results that the buried large conductive bodies and the upper layers of the ground may significantly affect the TDEM responses. This situation provides benefits for the detection in the subsurface research.



a)



b)

Figure 4.5: Induced magnetic field strength for the anomalous body problem: a) the horizontal, b) the vertical component.

4.1.3. Homogeneous SPM Half-Space

As the next problem, the lossy SPM half-space problem is solved. The problem geometry is the same as the **Figure 4.2**, only the half-space is the SPM soil. The log-uniform model parameters for the SPM soil are chosen as $\chi_0 = 0.03$, $\tau_1 = 10^{-6}$ s and

$\tau_1 = 10^{-2}$ s. The other physical parameters are set to the same as the lossy half-space problem.

The vertical and the horizontal components of the secondary magnetic fields are calculated at the center (central loop) and 400 m away (separated loop) from the transmitter antenna.

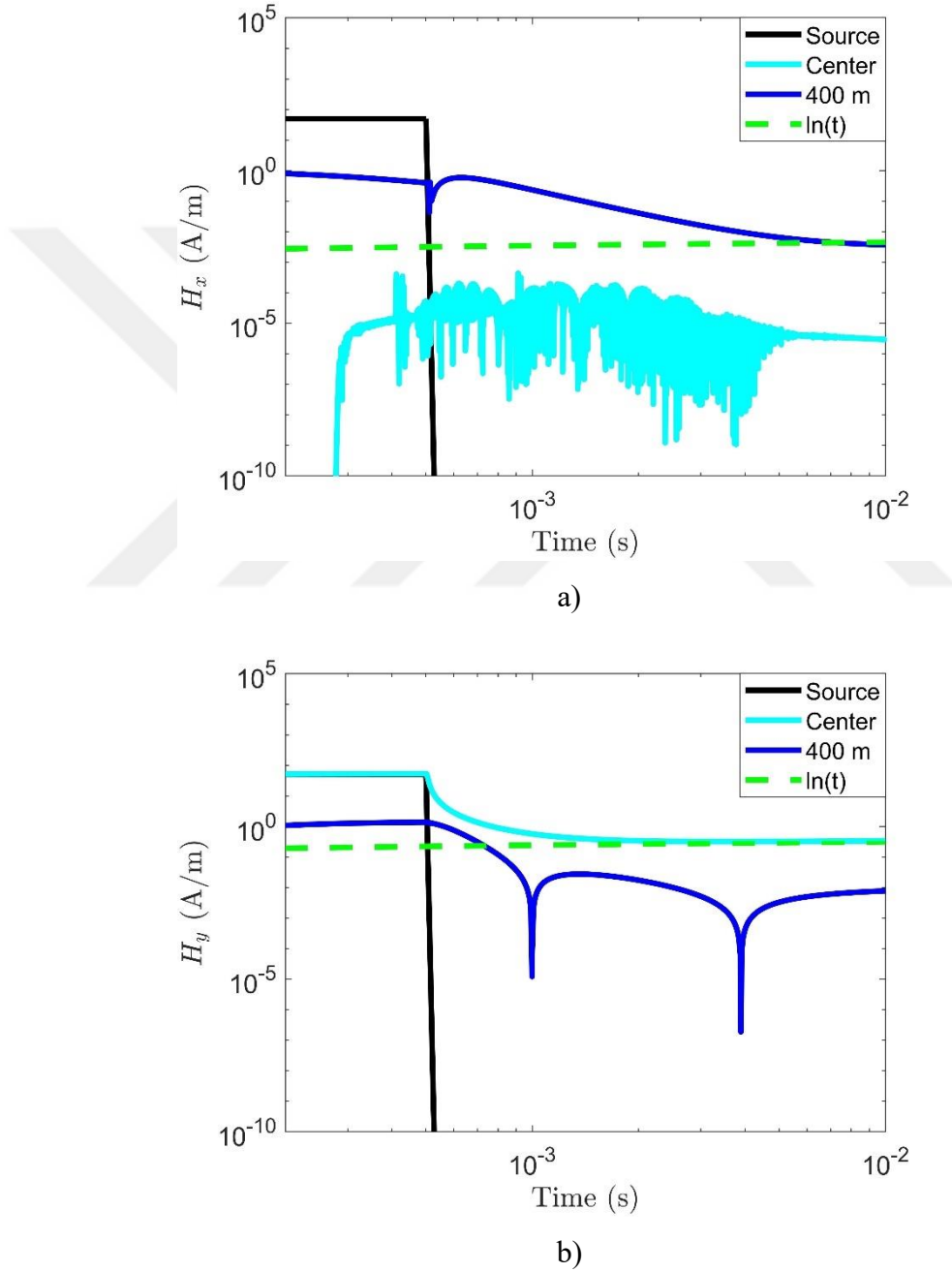


Figure 4.6: Induced magnetic field strength for the lossy SPM half-space problem: a) the horizontal, b) the vertical component.

The calculated horizontal and the vertical components from the SPM half-space problem are shown in **Figure 4.6**. According to the 2D analytical solution for the vertical component of magnetic field is decay as $\ln(t)$ for the central loop case and its effect is reduced for the separated loop case [10]. However, for the horizontal component of the magnetic fields the analytical solution is not clear in the literature [124].

4.2. 3D Problems

4.2.1. Homogeneous Lossy Half-Space

For the solution of the homogeneous conductive half-space problem in the 3D Cartesian coordinates, the same parameters as in the 2D case are used. Only a circular loop is used as a transmitter antenna instead of the two infinitely long wires. According to results presented in **Figure 4.7**, the vertical component (H_z) and the horizontal components (H_x and H_y) of the induced magnetic field at the center of the transmitter loop decayed as $t^{-1.5}$ and t^{-2} at late-times as expected from [121].

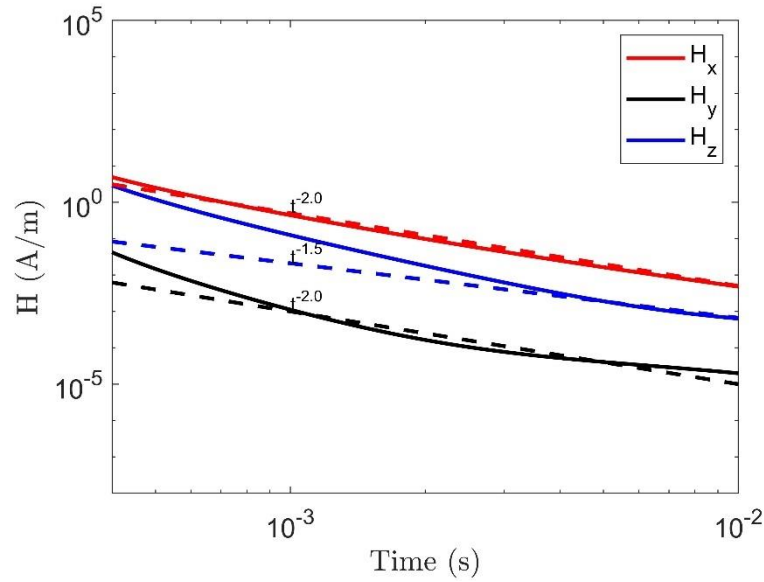


Figure 4.7: Induced magnetic fields from the homogeneous conductive half-space.

4.2.2. Homogeneous SPM Half-Space

For the SPM half-space problem in the 3D Cartesian coordinates, the geometric parameters and the electromagnetic model parameters are chosen the same as in the 2D SPM half-space case. A circular transmitter loop antenna is placed on the ground surface. In **Figure 4.8**, the calculated $\partial B_z/\partial t$ curve at the center of the transmitter loop is shown. The decay curve of the calculated $\partial B_z/\partial t$ field fits the analytical solution in [31] which is proportional to t^{-1} at the late-times. The relative error against the analytical solution is below 5 %. Therefore, the feasibility of the QS-FDTD method with the ADE technique for the solution of TDEM problems is proven.

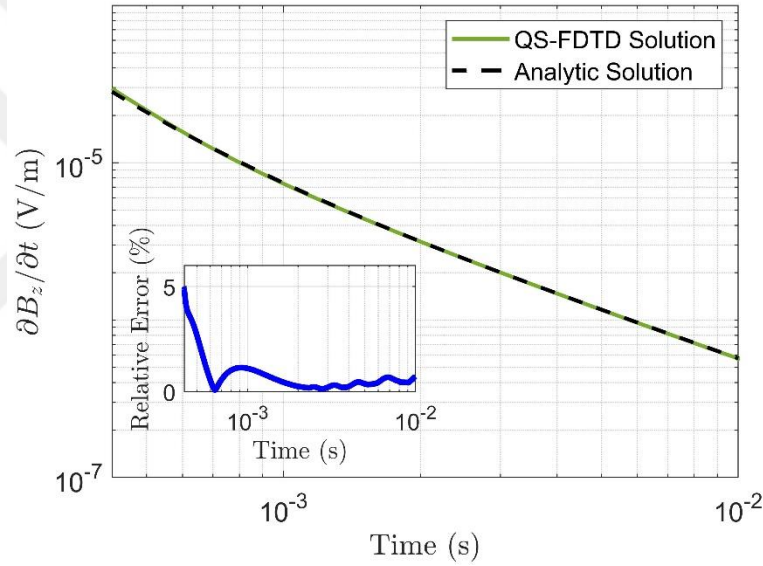


Figure 4.8: Calculated $\partial B_z/\partial t$ signal for the SPM half-space problem.

4.2.3. SPM Thin Layer and Anomalous Body

As a second SPM problem in the 3D Cartesian coordinates, a problem involving the conductive anomalous body buried beneath the thin, spatially homogeneous SPM upper layer is considered. A circular transmitting loop with a radius of 7.5 meters is positioned 30 meters above the ground in the air. The geometry of the considered is illustrated in **Figure 4.9**.

The electromagnetic properties of the thin SPM upper layer with a thickness of 50 meters are identical to those used in the SPM half-space problem. Beneath this layer, the background medium has a conductivity of $\sigma = 0.01$ S/m. The anomalous body measuring $210 \times 210 \times 300$ meters with a conductivity of 5 S/m, is located at a depth of 550 meters.

To evaluate the influence of the thin SPM layer on the detection of the anomalous body, $\partial B_z / \partial t$ signals at the center of the transmitter loop are calculated for three scenarios: i) the presence of only the thin SPM layer, ii) the presence of only the buried conductive anomalous body, and iii) both the SPM layer and the anomalous body present together. Additionally, a case involving the underground SPM layer at 10 m depth is also examined.

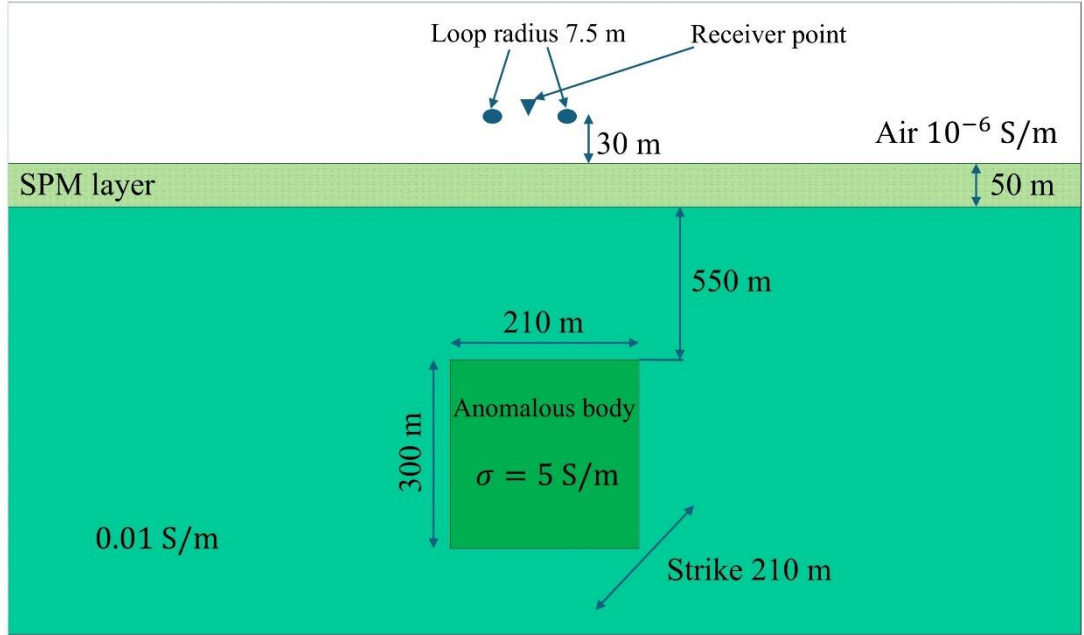


Figure 4.9: The geometry of the anomalous body with the thin SPM layer problem.

As shown in **Figure 4.10**, for both the surface and the underground SPM layer cases, the response from the SPM layer dominates the signal after the early-time period. When both the SPM layer and the anomalous body are present, the SPM response tends to mask the signal from the anomalous body, particularly at the late-times as expected and consistent with theoretical findings in [14]. When the SPM layer is placed at the 10 m depth, the masking effect of the layer is slightly reduced.

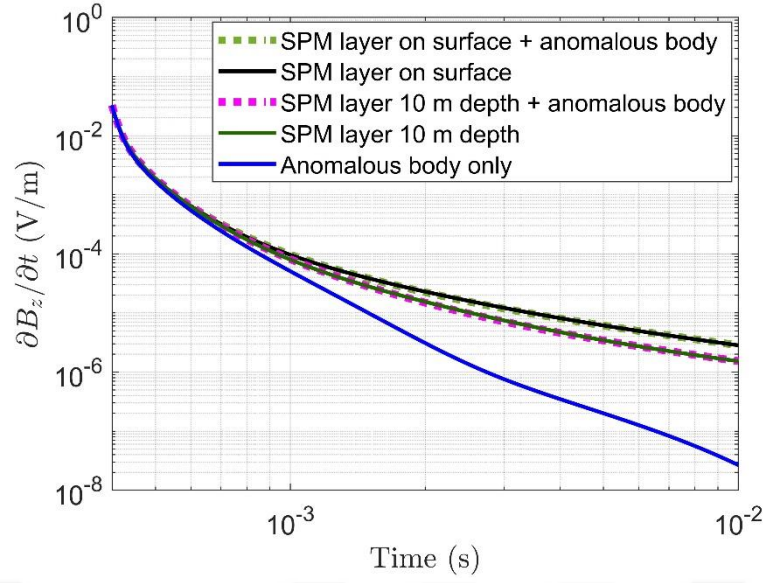


Figure 4.10: Calculated $\partial B_z/\partial t$ signals for the anomalous body with the thin SPM layer problem.

4.2.4. Complex Problem

Finally, a complex problem incorporating the thin SPM layer is analyzed, as illustrated in **Figure 4.11**. A circular transmitter loop of the 50 m radius is placed on the ground. The underground media is divided into four regions: the SPM upper layer of the 50 m thickness, the lossy soil region of 0.01 S/m conductivity on the left, another lossy soil region of 0.0333 S/m conductivity on the right, and the complex shaped anomalous conductive body of 1 S/m conductivity.

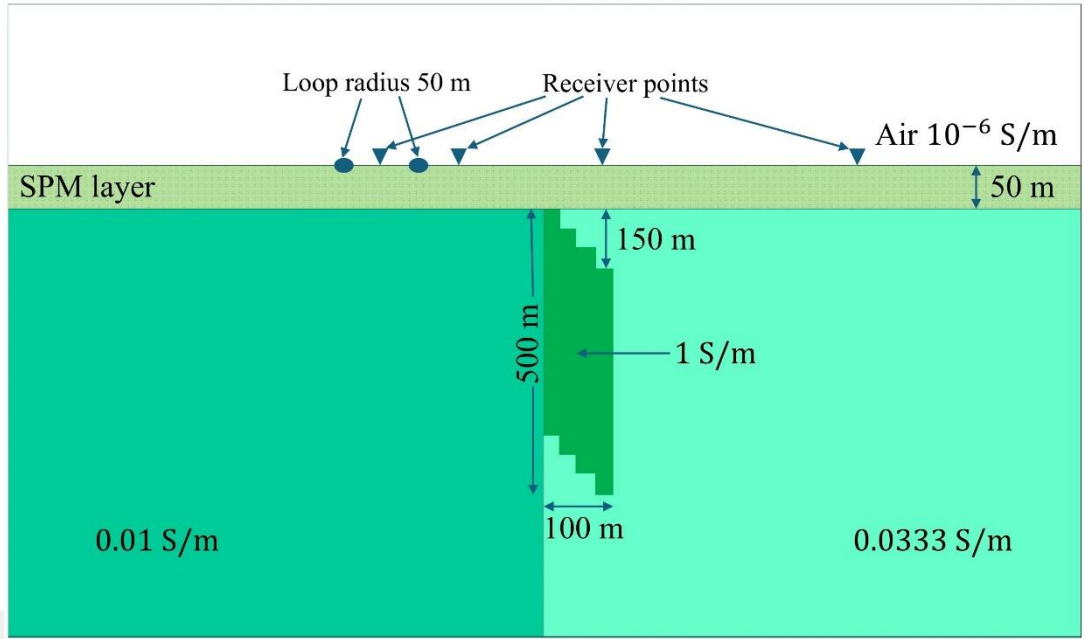


Figure 4.11: The geometry of the complex problem with the thin SPM layer.

$\partial B_z / \partial t$ signals are calculated at the four receiver locations: directly beneath the center of the transmitting loop, and at the distances of 100 m, 400 m, and 1000 m from the loop center.

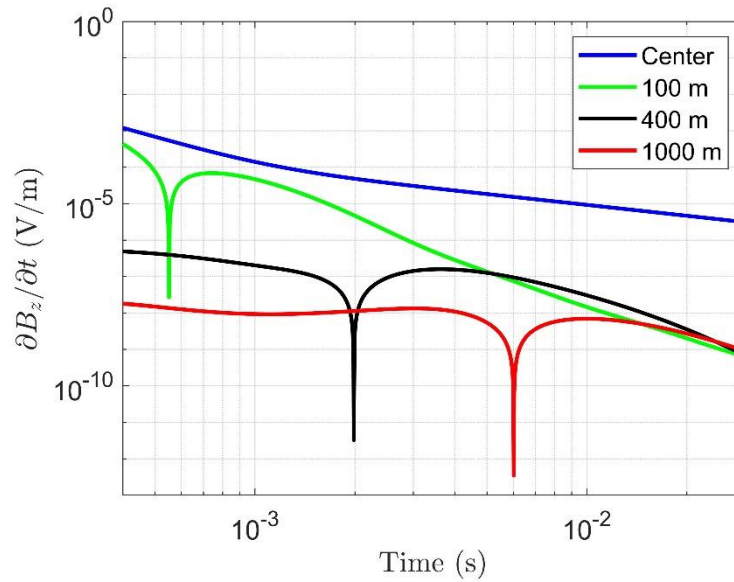


Figure 4.12: $\partial B_z / \partial t$ signals for the complex problem.

As shown in **Figure 4.12**, when the receiver is positioned at the center of the transmitter loop, the SPM layer's influence becomes dominant at the late-times.

However, at the other receiver locations, no significant effect from the SPM layer is observed due to the separation between the transmitter and the receiver.



5. CONCLUSION AND FUTURE WORKS

In this thesis, the effects of the SPM soil on the performance of the TDEM systems are investigated by the QS-FDTD method. In this sense, firstly, the 2D problems are considered without the SPM soils to confirm the feasibility of the QS-FDTD method for the TDEM problems. The transient magnetic fields are calculated for the homogeneous lossy half-space problem and obtained decay curves are validated with the knowledge from literature. Then, the more complex problem with the buried conductive anomalous body and the thin overburden layer is solved using the QS-FDTD method. Therefore, the validity and feasibility of the QS-FDTD method for the solution of the 2D TDEM problems are confirmed.

For the solution of the TDEM problems with the SPM soils, the log-uniform model is used for the modeling of the dispersive magnetic susceptibility. Implementation of the log-uniform model with the QS-FDTD method is performed with the ADE technique for the first time in the literature. Its applicability is confirmed by the solution of the 2D lossy SPM half-space problem.

As the 3D problems, first, the lossy half-space problem is solved with the QS-FDTD method. An adaptive time step increment technique is applied to speed up the simulations. For the problems without the SPM media, the divergence-free correction is also applied to the QS-FDTD method. Then, the 3D lossy SPM half-space is solved and validated with the analytical solution. To investigate the effects of the SPM soils on the TDEM systems, the thin SPM layer with and without the buried conductive anomalous body and a more complex problem with a thin lossy SPM layer problems are solved. The numerical results show that the SPM layer masks the late-time signals from the anomalous body for the central loop configuration. It is also concluded that one way to reduce the SPM effects on the TDEM systems is to separate the receiver coil from the transmitter loop.

As a conclusion, the method used in this thesis offers a better understanding of the effects of difficult dispersive soils, such as the SPM soils, on the TDEM data. This numerical tool can also be useful for the apparent resistivity/conductivity and the magnetic susceptibility extraction from the TDEM data.

As future works, first, a Perfectly Matched Layer (PML) Absorbing Boundary Condition (ABC) algorithm is planned to be developed for the TDEM problems with the magnetic dispersive SPM soils. The different problem configurations such as a cave under the SPM layer, problems with underground water sources, tunnel problems etc. are also planned to be investigated with the QS-FDTD method. By utilizing the masking feature of the SPM effect, underground hiding technology can also be studied. The effects of the SPM parameters, the depth of the SPM soils, the different antenna configurations and the different source signals can also be considered the other future studies. Another difficult soil type with the IP effect together the SPM soils is also a good future research topic.



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BIOGRAPHY

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PUBLICATIONS AND PRESENTATIONS FROM THE THESIS

- [1] Saydam T., Aksoy S., (2025), “QS-FDTD modeling of dispersive superparamagnetic soils for time-domain electromagnetic method”, IEEE Geoscience and Remote Sensing Letters, 22, 7505405.
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