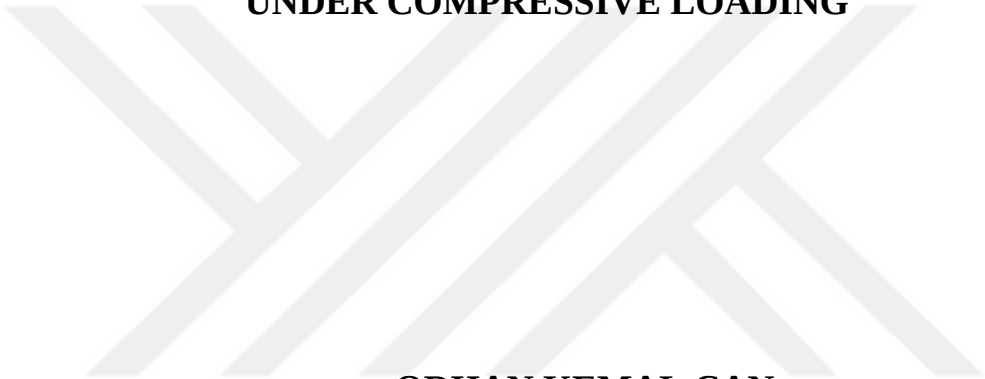


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ORHAN KEMAL CAN
A THESIS SUBMITTED FOR THE DEGREE OF
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**THESIS SUPERVISOR
PROF. DR. OSMAN SAİM DİNÇ**

GEBZE

2022

GEBZE TEKNİK ÜNİVERSİTESİ	YÜKSEK LİSANS JÜRİ ONAY FORMU
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İMZA/MÜHÜR

SUMMARY

In this work, buckling capacity of isogrid-stiffened cylinders is examined with changing parameters as R/t , isogrid triangle number on the circumference and manufacturing tolerances. Firstly, hand calculations are performed based on NASA Isogrid Design Handbook (NASA CR 124075) and NASA/SP-8007 (Buckling of Thin-Walled Circular Cylinders). Secondly, a shell-beam FEA model is created using ANSYS Workbench 2020 R1 where the skin and stiffeners are represented as shell and beams respectively. This FEA model is employed to perform linear Eigenvalue buckling analysis and nonlinear buckling analysis. Nonlinear analysis is run as large deflection activated for with geometrical nonlinearity, nonlinear material properties activated and with an initial geometrical imperfection applied to the FEA model.

Results of hand calculation and FEA are compared and interpreted considering different R/t ratios and circumferentially isogrid triangle number.

Buckling capacity is determined and compared for isogrid-stiffened cylinder and unstiffened cylinder which have same diameter and weight.

Knock down factors (KDF's) which scale down the theoretical buckling load to actual critical buckling are compared and interpreted for hand calculation and FEA methodologies. Additionally, the effect of the geometrical imperfections caused by manufacturing tolerances is showed on the KDF level.

Key Words: Isogrid Stiffened Structure, Non-Linear Buckling Analysis, NASA Isogrid Design Handbook, Knock Down Factor (KDF) for buckling.

ÖZET

Bu çalışmada; R/t, çevredeki güçlendirici izo-kafes üçgen sayısı ve imalata toleransları gibi parametrelerin değişimi ile izo-kafes güçlendirilmiş silindir yapılarının burkulma kapasiteleri incelenmiştir. İlk olarak, NASA Isogrid Design Handbook (NASA CR 124075) ve NASA/SP-8007 (Buckling of Thin-Walled Circular Cylinders) dokümanlarına dayanarak el hesapları yapılmıştır. İkinci olarak, ANSYS Workbench 2020 R1 programı kullanılarak, yapının ana-kabuk kısmının kabuki eleman olarak, güçlendirici kafes yapısının ise kiriş eleman olarak modellendiği bir kabuk-kiriş SEA(sonlu elemanlar analiz) modeli oluşturulmuştur. Bu SEA modeli ile lineer özdeğer burkulma ve lineer olmayan burkulma analizleri gerçekleştirilmiştir. Lineer olmayan analizde, malzeme ve geometrik lineer olmama durumları ile imalat kaynaklı geometri hataları da hesaba katılmıştır.

Farklı R/t oranları ve çevredeki güçlendirici izo-kafes üçgen sayılarına göre yapılan el hesaplamaları ve SEA sonuçları karşılaştırılmış ve yorumlanmıştır.

Aynı çapa ve ağırlığa sahip, izo-kafes güçlendirilmiş silindir ile güçlendirilmemiş tek kabuk silindirin burkulma kapasitesi belirlenmiş ve karşılaştırılmıştır.

Teorik burkulma kuvvetini, gerçek burkulma kuvvetine dönüştürmek için kullanılan düşürme katsayısı (DK) değerleri, el hesapları ve SEA için karşılaştırılıp, yorumlanmıştır. Ek olarak, imalat toleransları kaynaklı geometrik kusurların DK üzerine olan etkisi incelenmiştir.

Anahtar Kelimeler: İzogrid Güçlendirilmiş Yapı, Doğrusal olmayan burkulma Analizi, NASA İzogrid Tasarım Elkitabı, Burkulma Düşürme Katsayıları.

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LIST of ABBREVIATIONS and ACRONYMS

<u>Abbreviations</u>	<u>Explanations</u>
<u>and Acronyms</u>	
k_x	: Buckling Coefficient
N_x	: Critical Buckling Load
m	: Axial Half-Waves of the buckling mode shape
γ	: Knockdown factor
E	: Young Modulus
t	: Thickness of the shell
n	: Circumferential full waves of the buckling mode shape
ν	: Poisson's Ratio
b	: Rib thickness
d_{iso}	: Rib depth
h_{iso}	: Height of isogrid triangle
t_{iso}	: Thickness of skin
E^*	: Equivalent Young Modulus
t^*	: Equivalent Thickness of the Shell

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1. INTRODUCTION

1.1 Grid Structures

Grid structures means a thin outer skin and above, repetitive pattern of stiffening ribs to support the whole structure as shown in Figure 1. In that way, structures increases their stiffness while the weight addition is limited.

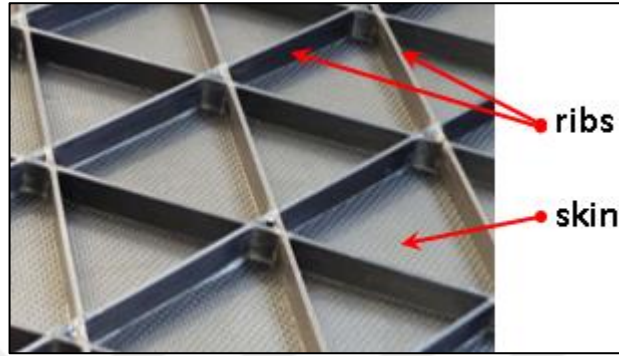


Figure 1-1 Grid Structure Example.

Structural efficiency and environmental robustness are major advantages of grid structures. [1] By changing the direction of ribs, required stiffness is achieved in any direction, which is needed per the loading condition of the structure. Other directions that are not on a load path might be used for weight reduction opportunities. Additionally, cellular structure provides higher damage tolerance capability.

According to the pattern of the repetitive ribs, grid structures are subdivided as orthogrid, diamond grid, triangle rectangle grid and isogrid shown in Figure 1-2. [2] The pattern affects the directional material behavior of the whole structure.

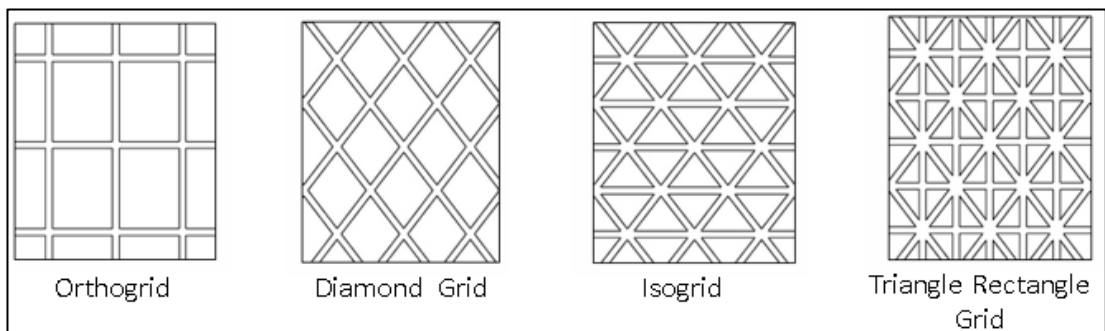


Figure 1-2 Grid Types.

1.2 Isogrid Structures and Usage in Aerospace

Isogrid structures has equilateral triangular pattern of stiffening ribs. Thus, tri-directional grid provides isotropic material properties in all these directions. Isotropic

material properties ensure good compressive and bending combined load capabilities. Isogrid structure has high stiffness to weight ratio, thus it is used in aerospace and aviation applications. As an example, Eurojet EJ200 [3] which is a low-bypass turbofan engine has isogrid stiffened casings as shown in Figure 1-3 and powers Eurofighter Typhoon aircraft.

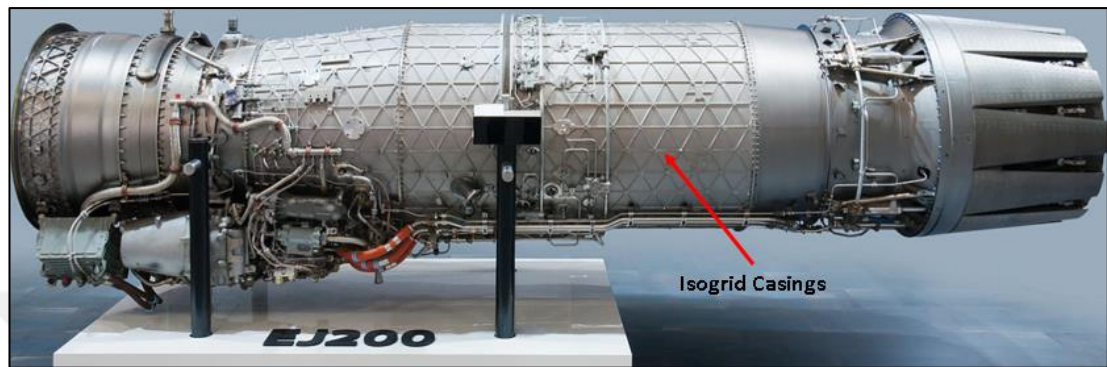


Figure 1-3 EJ200 Engine.

In turbojet engines of aircrafts, cold section casings which works under compression loads are very sensitive to buckling failure. Due to weight restrictions, design engineers aim to keep the casing thickness as thin as possible while the hardware has still enough buckling margin.

Also, aerospace launch vehicle program are interested in grid stiffened cylindrical structures. Hilburger Mark W. et al worked on the buckling of large-scale , integrally-stiffened metallic cylinder. [4] In the scope of this study, eight different 8 ft diameter cylinders were fabricated and tested under various loading conditions and test results were compared with finite element analysis results. Figure 1-4 is taken from the article and shows the fabrication process of integrally stiffened, 2195-T3 Aluminum alloy cylinder with 96 inch diameter and 78 inch length. 2195 Al-Li flat plates were machines to stiffened panels and bump-formed to curved ones. Finally, curved stiffened panels were friction-stir welded together.

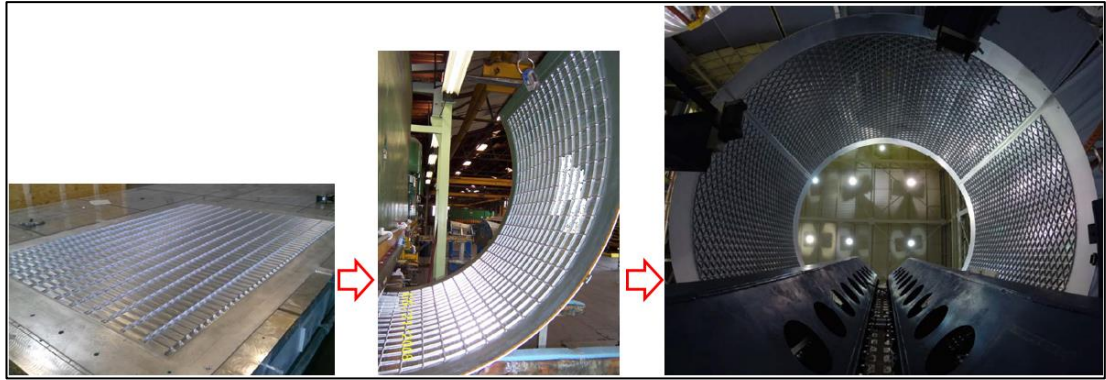


Figure 1-4 Fabrication Process of Stiffened Test Structure.

1.3 Buckling

Buckling is an instability issue for structures under compressive loading condition. Especially, it is very critical failure mode for long and slender structures [5]. The shape of the structure changes suddenly at a critical level if a gradually increasing load is applied on it. For thin and slender structure as shell cylinders, the critical level is well below the yielding stress, so their strength is limited by buckling. Figure 1-5 is taken from reference and shows buckling test of a cylindrical shell under axial compression loading



Figure 1-5 Buckling Test of a Cylindrical Shell.

1.4 Scope of the Thesis

The aim of this study is to investigate the effect of added isogrid stiffeners on the buckling capacity and the total weight of the cylindrical structure. While the critical buckling load of the structure increases, also the total weight, which is one of the most important key parameters for aircrafts, is increasing. Thus, the correlation between the interval of the stiffeners and stiffness to weight ratio is worth to investigate.

Firstly, analytical calculations are performed per NASA Isogrid Design Handbook. Theoretical, critical buckling loads are calculated and these values are multiplied with KDF to get the real critical loads. Following, finite element analysis is used to study the theoretical critical buckling loads and the nonlinear results considering geometrical and material nonlinearities. This method is implemented by the software ANSYS Workbench 2020 R1.

Titanium alloy is chosen as material because of its often usage in aero engine cold section casings.

2. ANALYTICAL CALCULATIONS

2.1 Buckling of Thin Walled Cylindrical Structure

Based on Donnell's shell theory, critical buckling load for a isotropic, moderately long thin walled cylinder under axial compression is given by [7]:

$$N_x = k_x \frac{\pi^2 D}{L^2} \quad (2-1)$$

Where

$$k_x = m^2 (1 + \beta^2)^2 + \frac{12}{\pi^4} \frac{(\gamma Z)^2}{m^2 (1 + \beta^2)^2} \quad (2-2)$$

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (2-3)$$

$$Z = \frac{L^2}{Rt} \sqrt{1 - \nu^2} \quad (2-4)$$

$$\beta = \frac{nL}{m\pi R} \quad (2-5)$$

For moderately long cylinders, which have $\gamma Z > 2,85$, equation (2-2) is minimized to:

$$k_x = \frac{4\sqrt{3}}{\pi^2} \gamma Z \quad (2-6)$$

Using equations (2-1)-(2-6), theoretical critical buckling load is obtained as:

$$N_{critical,theoretical} = \frac{2\pi Et^2}{R\sqrt{1-\nu^2}} \quad (2-7)$$

As seen in equation (2-7), critical buckling load is dependent on the Young's Modulus and Poisson Ratio of the material and thickness and radius of the geometry.

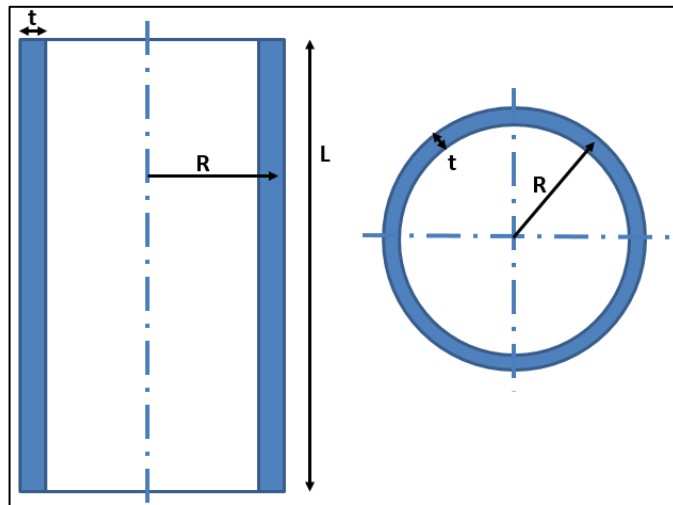


Figure 2-1 Dimensions for a thin walled cylinder.

In reality, actual critical buckling load is less than calculated theoretical due to non-linearities and geometrical imperfections. To consider this reality, a knock down factor (KDF) γ is applied

$$\gamma = 1 - 0,901(1 - e^{-\varphi}) \quad (2-8)$$

$$\varphi = \frac{1}{16} \sqrt{\frac{R}{t}} \left(\text{for } \frac{R}{t} < 1500 \right) \quad (2-9)$$

2.2 Buckling of Isogrid Stiffened Thin Walled Cylindrical Structure

As described in previous section, those formulas (2-1)-(2-9) are applicable for isotropic unstiffened thin walled cylinders. However, isogrid stiffened cylinders have ribs on the skin structure. Based on NASA Isogrid Design Handbook [8], an equivalent Young's Modulus E^* and equivalent thickness t^* can be calculated for isogrid stiffened cylinders using formula (2-10) and (2-11) in order to be treated as a monocoque, isotropic unstiffened cylinder. With those equivalent Young's Modulus E^* and equivalent thickness t^* values, equations (2-1)-(2-7) are applicable as stated in Chapter 2.1.

$$t^* = t_{iso} \frac{\beta_{iso}}{1 + \alpha_{iso}} \quad (2-10)$$

$$E^* = E \frac{(1 + \alpha_{iso})^2}{\beta_{iso}} \quad (2-11)$$

β_{iso} , α_{iso} and δ_{iso} are non-dimensional parameters and formulated as equations (2-12)-(2-14) below using the values of the skin thickness, rib thickness, rib height and the triangle height shown in Figure 7.

$$\beta_{iso} = \sqrt{[3\alpha_{iso}(1 + \delta_{iso})^2 + (1 + \alpha_{iso})(1 + \alpha_{iso}\delta_{iso}^2)]} \quad (2-12)$$

$$\alpha_{iso} = \frac{b_{iso}d_{iso}}{t_{iso}h_{iso}} \quad (2-13)$$

$$\delta_{iso} = \frac{d_{iso}}{t_{iso}} \quad (2-14)$$

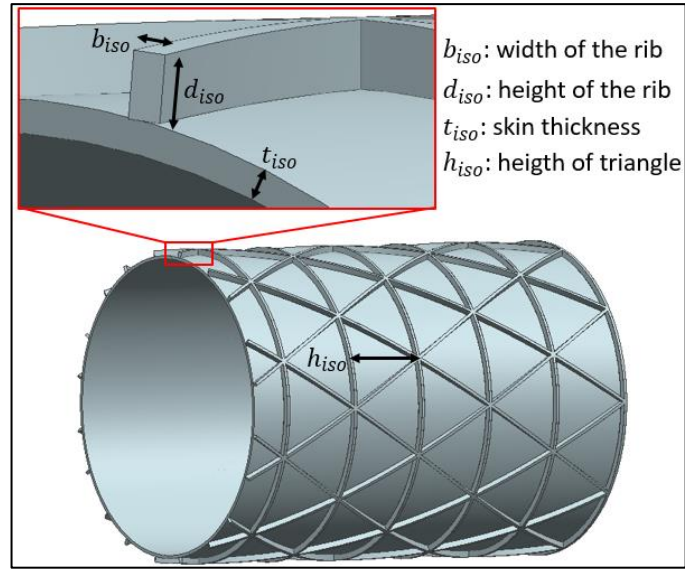


Figure 2-2 Isogrid Parameters.

Using the equivalent Young's Modulus E^* and equivalent thickness t^* , theoretical critical buckling load for the isogrid stiffened cylinder is calculated per the formula (2-7).

NASA-SP8007 suggest KDF per formula (2-8) for lightly stiffened cylinders and KDF of 0.65 for moderate or heavy stiffened cylinders based on NASA-TN-D-5561[9].

As a initial design, an unstiffened, isotropic cylinder from Titanium Alloy is taken with a radius of 150 mm, length of 450 mm and a shell thickness of 1 mm. Following, six design iterations are studied by increasing the circumferential triangle numbers with the same rib cross section of 1x2 mm as in Table 2-1.

Table 2-1 Design Iterations.

Skin Radius(mm)	150						
Skin Thickness(mm)	1						
Skin Length(mm)	450						
Rib Height(mm)	0	2					
Rib Width(mm)	0	1					
Circumferential Triangle Number	0	12	16	24	36	48	72

Table 2-2 Material Properties.

Young's Modulus(MPa)	9,60E+04
Poisson's Ratio	0,36
Density (kg/mm ³)	4,62E-06

Material properties of titanium alloy is taken from ANSYS Library [2] and listed in Table 2-2.

Although the limits are not clear for a moderate or heavy stiffening definition, KDF is taken as 0,65 for 6 iterations.

Table 2-1 Calculation Results.

Skin Radius(mm)	150						
Skin Thickness(mm)	1						
Skin Length(mm)	450						
Rib Height(mm)	0	2					
Rib Width(mm)	0	1					
Circumferential Triangle Number	0	12	16	24	36	48	72
Mass(kg)	1,966	2,142	2,198	2,319	2,486	2,663	3,007
t*(Eq. Thickness) mm	N/A	1,355	1,447	1,607	1,804	1,964	2,212
E*(Eq. Youngs Modulus)	N/A	7,295E+04	6,895E+04	6,325E+04	5,791E+04	5,462E+04	5,106E+04
Theoretical Critical Buckling Load (N)	3,733E+05	5,205E+05	5,612E+05	6,352E+05	7,328E+05	8,194E+05	9,712E+05
KDF	0,518	0,65	0,65	0,65	0,65	0,65	0,65
Actual Critical Buckling Load(N)	1,934E+05	3,383E+05	3,648E+05	4,129E+05	4,763E+05	5,326E+05	6,313E+05
Buckling Load to Weight Ratio (N/kg)	9,836E+04	1,579E+05	1,660E+05	1,780E+05	1,916E+05	2,000E+05	2,099E+05

Calculated equivalent thickness and Young's Modulus, theoretical and actual buckling load, buckling load to weight ratio are listed in Table 2-3.

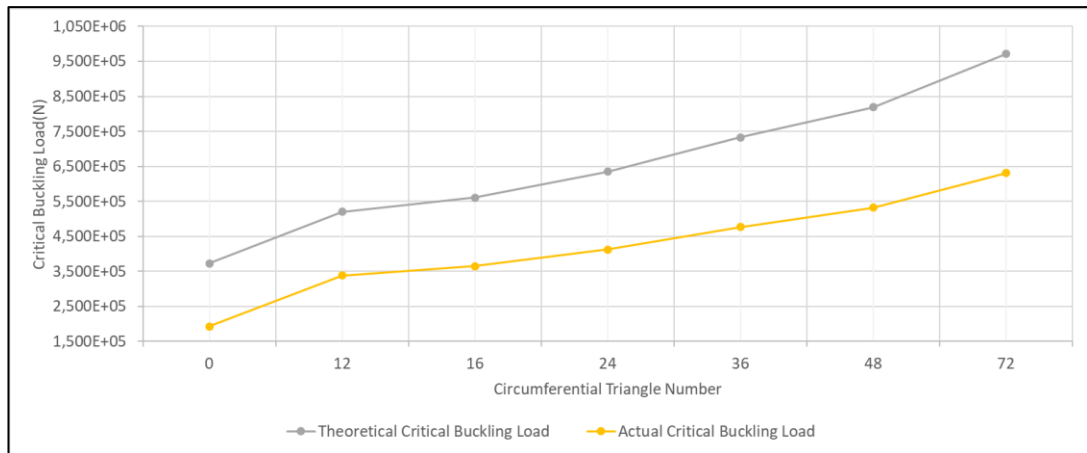


Figure 2-3 Critical Buckling Loads per Triangle Number.

As expected, increasing the number of triangles per circumference makes the structure more capable to buckling. Changes in theoretical and actual critical buckling load is seen in Figure 2-3. In addition, buckling capability to weight ratio is plotted in Figure 2-4.

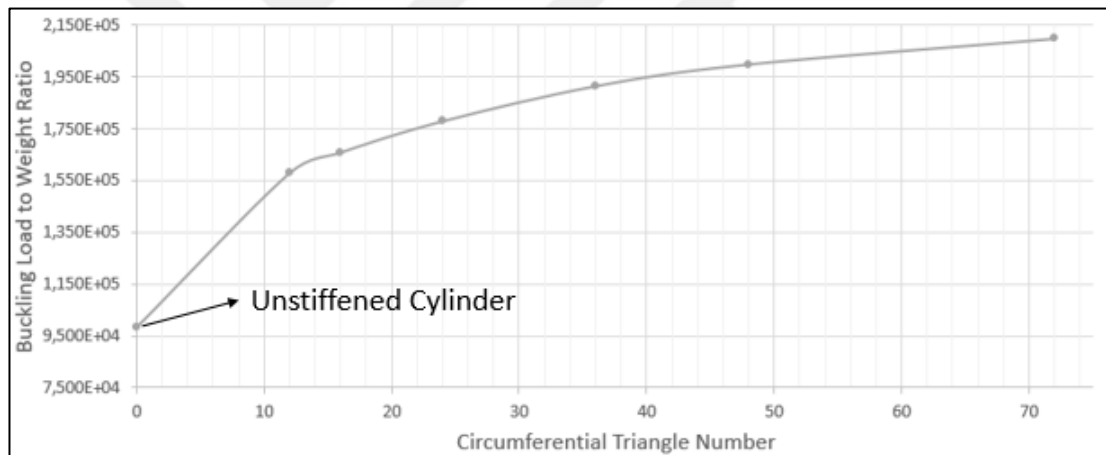


Figure 2-4 Buckling Load to Weight Ratio per Triangle Number.

3. FINITE ELEMENT ANALYSIS

3.1 Introduction

In this section, a finite element model of isogrid stiffened cylinder is developed using the software ANSYS Workbench 2020 R1. Geometrical model, meshing details, material properties and analysis types are described in following sub-sections.

Because the cylinder thickness is too much smaller compared to the diameter and length dimension, shell model is used for the skin [10]. If the R/t ratio is above 25, using solid elements become increasingly uneconomical considering the accuracy of the results and computational time.

Stiffeners have a constant cross-section and the dimension of this cross-section is too much smaller compared to the length dimension, thus they are modelled as beam elements.

As described in section 2, one unstiffened cylinder and six different design iterations as shown in Table 1 are studied using FEA. Six iterations differ in the stiffener density as shown in Figure 3-1.

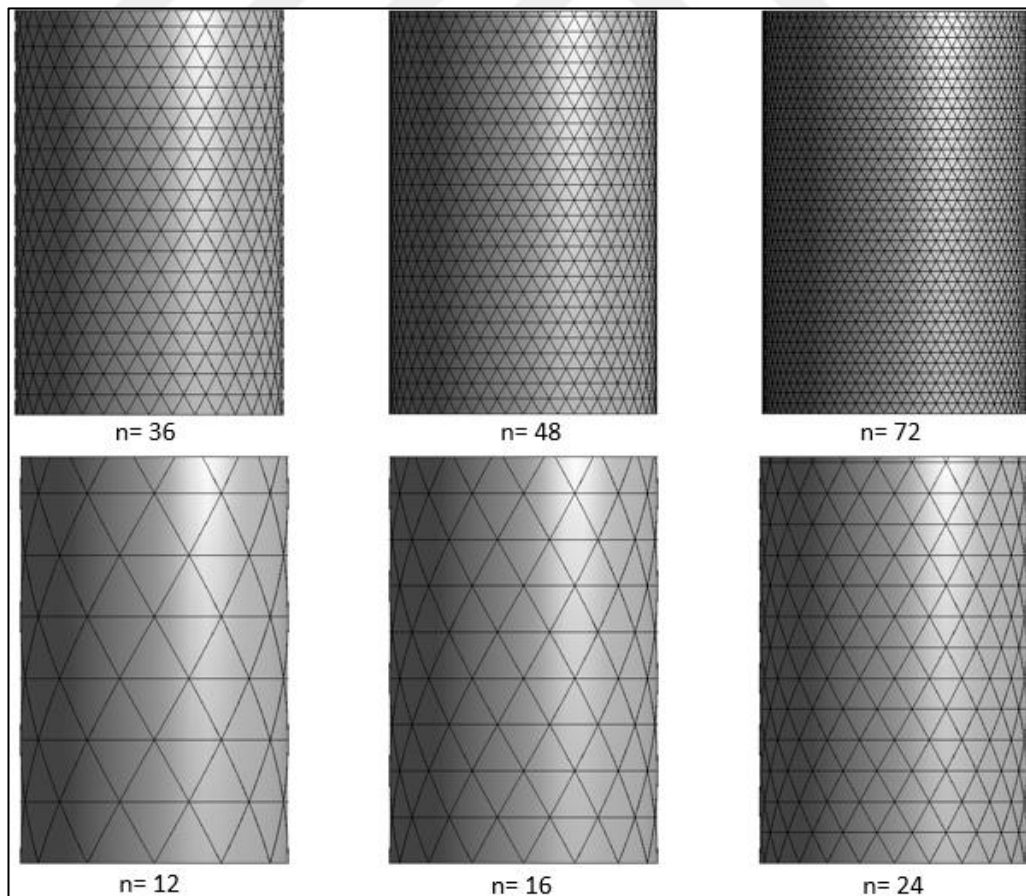


Figure 3-1 Design Iterations per the circumferentially triangle number.

3.2 Geometry

Using Geometry Modeler module available in ANSYS Workbench, a parametric model consisting of a surface body and line bodies is developed as shown in Figure 3-2.

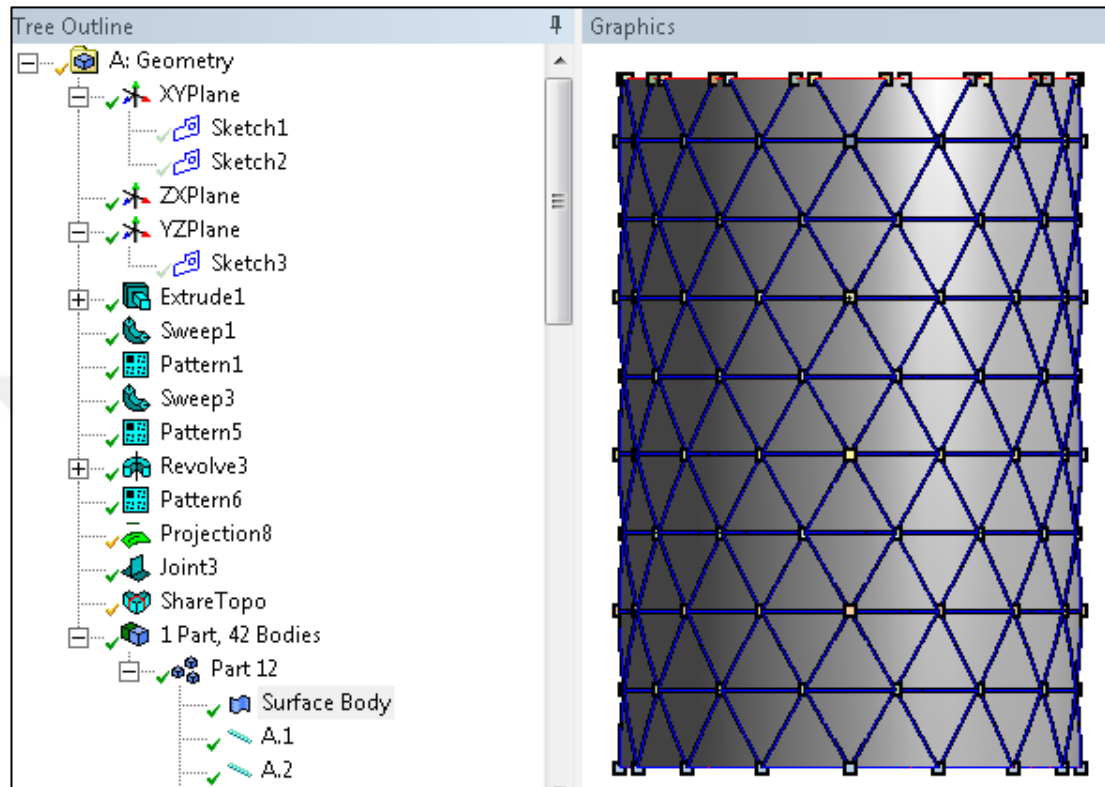


Figure 3-2 Geometry Modeler.

The related parameters are the skin outer diameter, the skin length and the grid number as shown in Figure 3-3. With the first two parameters creates the top surface of the cylinder skin. Grid number parameter defines the number of the equilateral triangle on the circumference. Therefore, it changes the density of the stiffeners. The more the grid number is, the more the isogrid structure is stiffened. However, stiffeners adds also weight to the structure.

	Name	Value	Type
✓	Skin_Outer_Dia	301 mm	Length
✓	skin_length	450 mm	Length
✓	Grid_Number	16	Dimensionless

Figure 3-3 Geometry Model Parameters.

3.3 Material

Titanium alloys are suitable materials for casings of aero-engine cold sections modules as fan and compressor due to their low weight to stiffness ratio [11]. Due to this fact, titanium alloy is chosen as material to investigate in this study.

ANSYS Workbench material library provides physical and material properties as shown in Table 3-1.

Table 3-1 Material Properties.

Young's Modulus(MPa)	9,60E+04
Poisson's Ratio	0,36
Density (kg/mm ³)	4,62E-06
Bulk Modulus(MPa)	1,1429E+05
Shear Modulus(MPa)	35294

In order to take the material non-linearity's, bilinear isotropic hardening curve of titanium alloy is also provided by ANSYS Material library as shown in Figure 3-4.

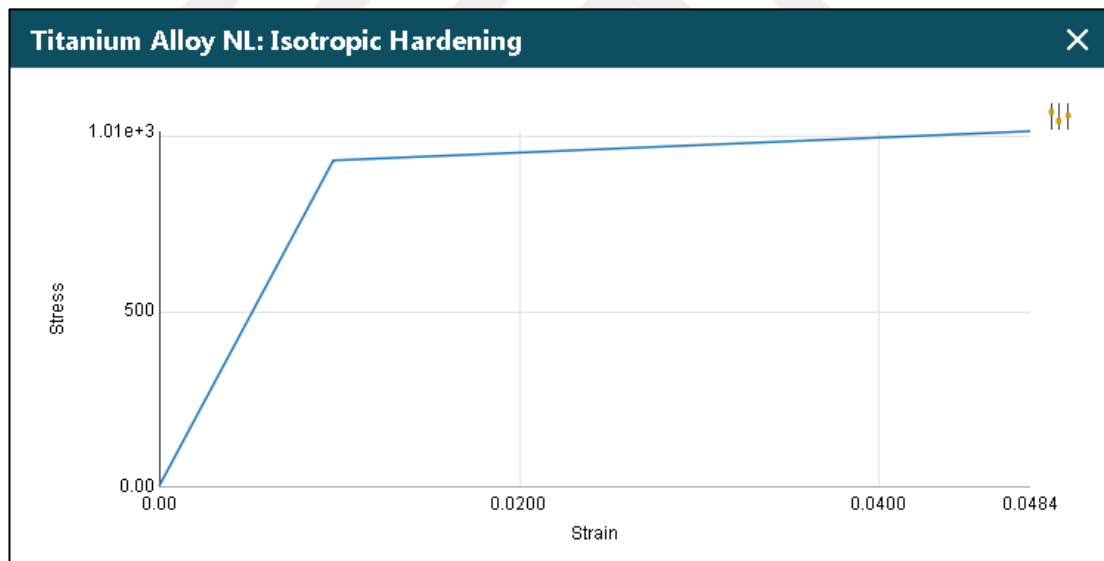


Figure 3-4 Titanium Alloy Bilinear Isotropic Hardening Curve.

3.4 Loading and Boundary Conditions

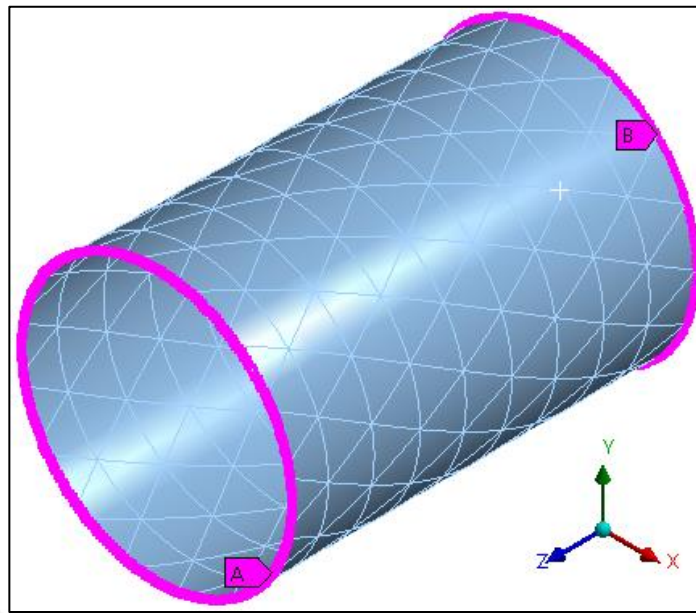


Figure 3-5 Boundary Conditions.

A remote unit force of 1 N is applied on the edge shown as A in Figure 3-5 in negative Z- direction. The rotations of nodes on the same edge are also fixed.

The edge shown as B is fixed in Z-direction and free in radial direction. Finite elements are SHELL281 and BEAM189 for surface body and line bodies respectively.

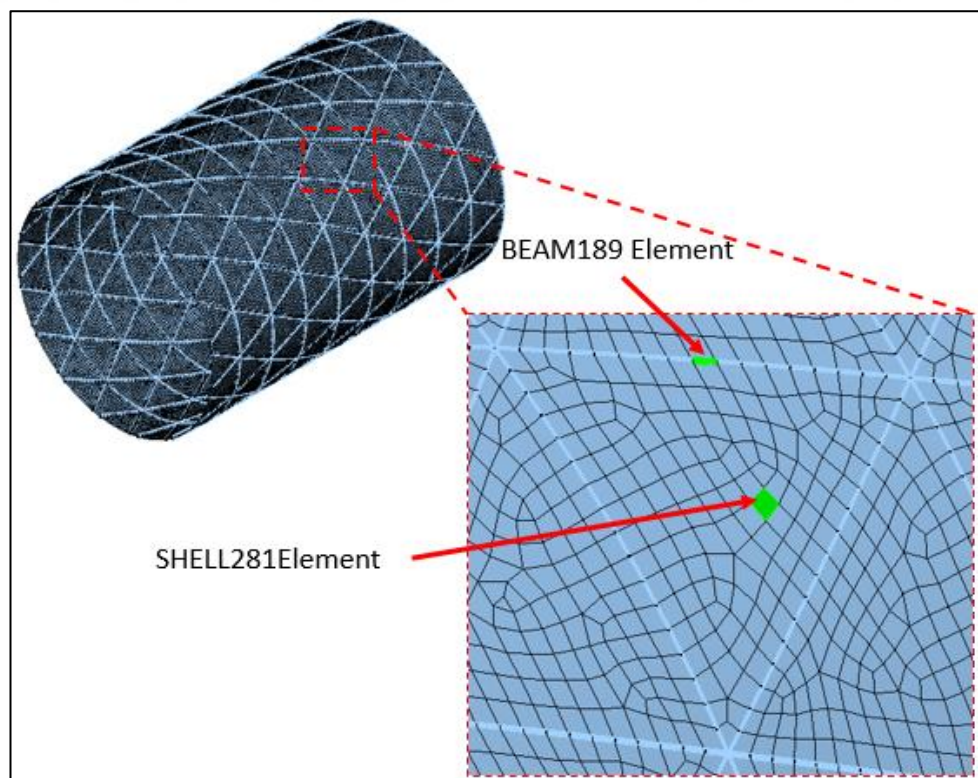


Figure 3-6 Finite Element Types.

SHELL181 is suitable element with four nodes with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes for analyzing thin to moderately thick structures [12].

BEAM188 is proper element with two nodes, which have six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. It is suitable for analyzing slender to moderately stubby/thick beam structures [12].

3.5 Meshing

FEA method is based on the concept of discretization of the interested system in smaller simpler elements. This process is called as meshing [13]. Mesh density that is determined by the element size is very critical for the accuracy of the analysis results. Decreasing the mesh size may result in results that are more accurate; however, it means also increase in element and node numbers. The more the element number is, the more the computing time is. Optimum point should be determined between the result accuracy and solution efficiency. A mesh sensitivity analysis is performed for the design option with circumferential triangle number of 36. The resulted eigenvalue buckling load, the created node number and their changes for each iteration is listed in Table 5.

Table 3-2 Mesh Sensitivity Results.

Circumferential Triangle Number	Mesh Element Size(mm)	Critical Eigenvalue Buckling Load(N)	Change in Buckling Load	Node Number	Node Number Increase
36	15	6,93E+05	REF	17064	REF
	12	6,808E+05	-1,80%	17640	3,38%
	9	6,811E+05	0,04%	30780	74,49%
	8	6,746E+05	-0,95%	35620	15,72%
	7	6,741E+05	-0,07%	58754	64,95%
	5	6,729E+05	-0,18%	79325	35,01%

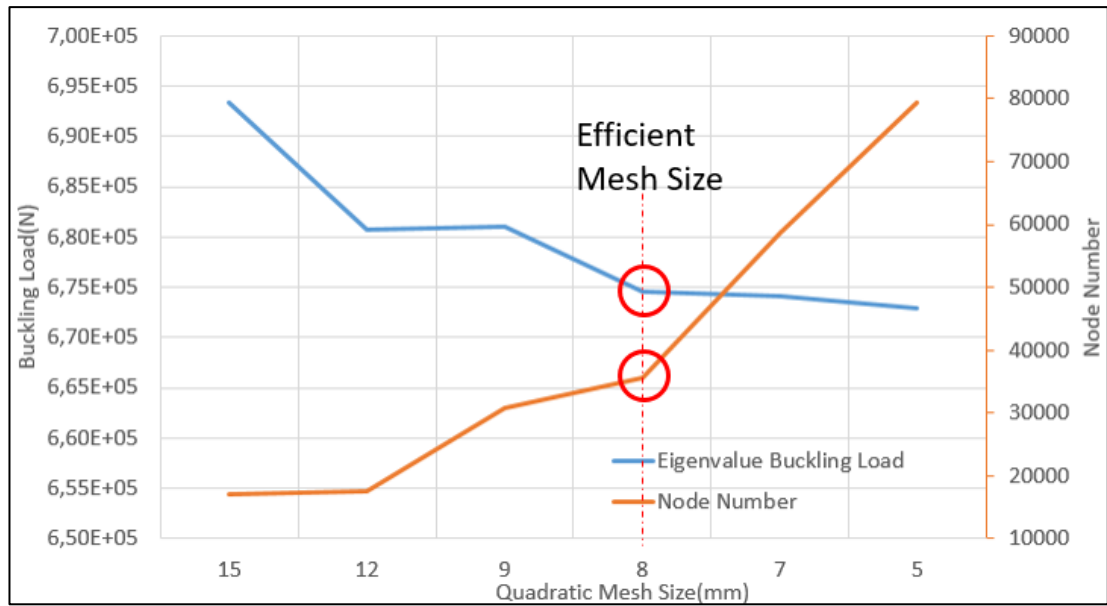


Figure 3-7 Buckling Load-Node Number-Mesh Size Plot.

Element size of 8 mm is chosen as the accuracy-efficiency point where the critical load result is 6,746E5 N and the node number is 35620 by viewing Figure 3-7.

3.6 Eigenvalue Buckling Analysis

The eigenvalue buckling analysis is performed using the static structural analysis as pre-stressed model for isogrid stiffened model. Project schematic is seen in Figure 3-8.

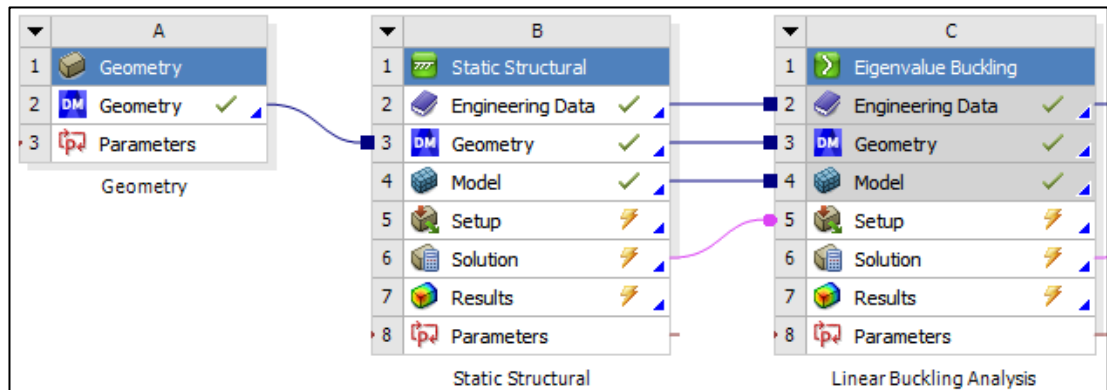


Figure 3-8 Project Schematic for Eigenvalue Buckling Analysis.

Per the solution of the model, the first critical eigenvalue buckling load of the isogrid cylinder with 36 triangles in the circumference is 6,746 E+5 N and the mode shape which has 16x deformation scale is seen in Figure 3-9.

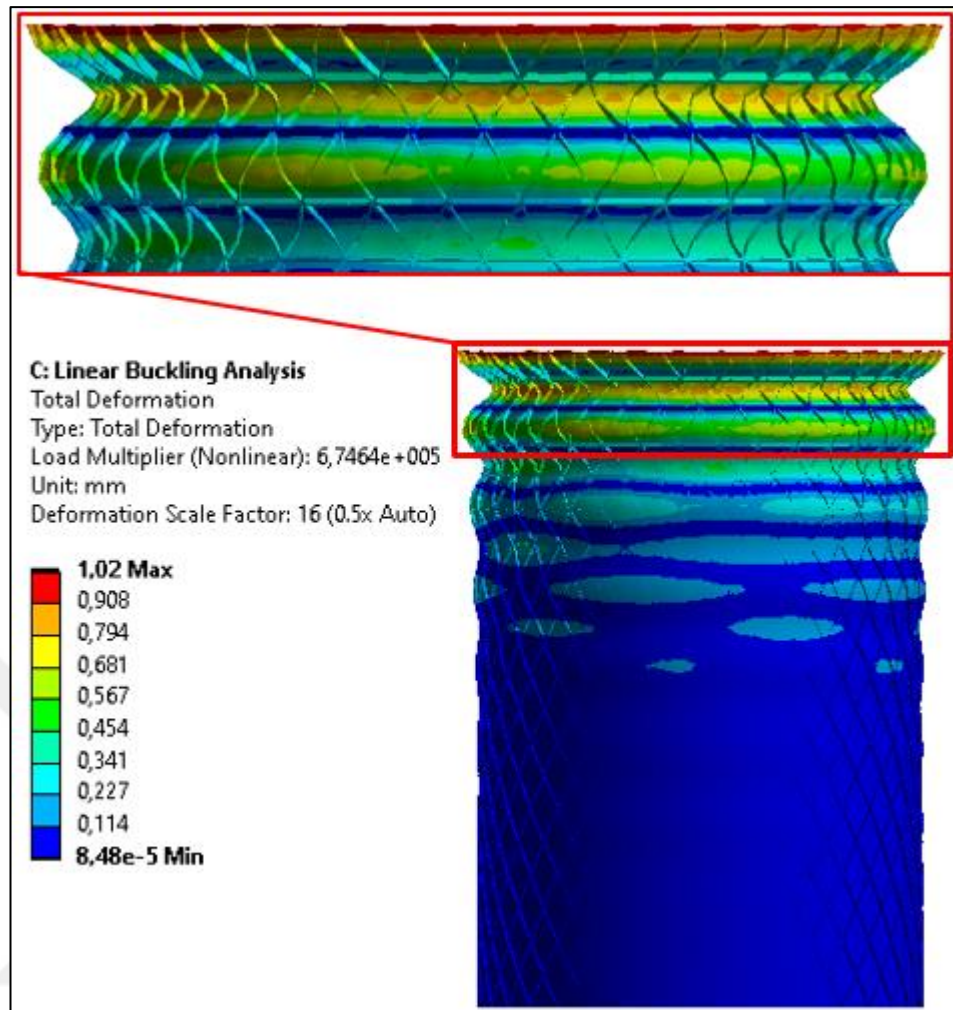


Figure 3-9 Buckling Mode Shape for the option w/36 circumferentially triangles.

3.7 Non-Linear Buckling Analysis

Using the same geometry with the same mesh structure, a non-linear static analysis is performed to take the geometrical and material nonlinearities and geometrical imperfections into account. For geometrical nonlinearity effect, large deflection is enabled in the ANSYS solver control options. (Figure 3-10)

Solver Controls	
Solver Type	Program Controlled
Weak Springs	Off
Solver Pivot Checking	Program Controlled
Large Deflection	On
Inertia Relief	Off

Figure 3-10 Ansys Solver Controls.

By linear elastic analysis, stiffness matrix is kept same. However in reality, stiffness of the structure changes due to change in the shape of the structure [14].

In order to take the material non-linearities, bilinear isotropic hardening curve of titanium alloy is also provided by ANSYS Material library as shown in Figure 3-4.

Titanium alloy has a yield strength of 930 MPa. It has a strain of 0.00969 at yield stress 930 MPa. Up to this point, material behaves linear elastic. After that stress level of 930 MPa, plasticity begins until the strain limit of 0.0484 at the stress level of 1010 MPa.

The critical buckling load of $6,746 \text{ E}+5 \text{ N}$ determined by the eigenvalue buckling analysis is applied in Z-direction by pre-defined substeps in Analysis settings as shown in Figure 3-11.

Step Controls	
Number Of Steps	1,
Current Step Number	1,
Step End Time	1, s
Auto Time Stepping	On
Define By	Substeps
Initial Substeps	100,
Minimum Substeps	50,
Maximum Substeps	1000,

Figure 3-11 Ansys Step Controls.

Geometry used in the FEA model is perfect, however in reality; there are imperfections due to manufacturing tolerances. To simulate this condition, the buckling mode shape and deformations are transferred with a scale of 0,025 to the nonlinear static analysis as shown in Figure 3-12.

Maximum total deformation for the first buckling mode is 1,02 mm as seen in Figure 18. Considering inward and outward deformations, total relative deflection is 2 mm. So, scale factor of 0,025 means 0.05 mm profile tolerance, which is a reasonable machining tolerance for a diameter 300 mm in aviation industry.

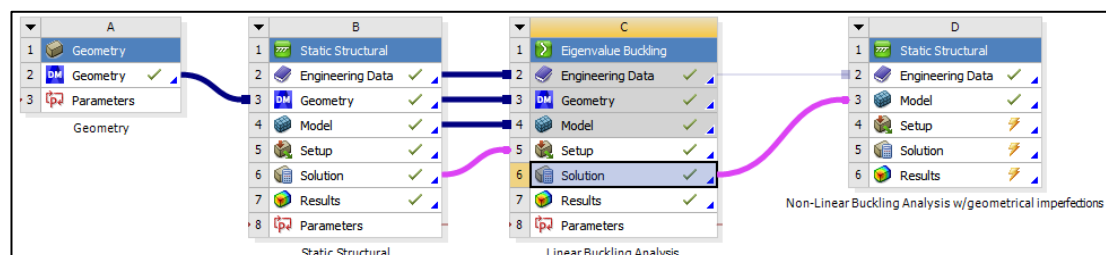


Figure 3-12 Project Schematic for Non-Linear Buckling Analysis.

At the load level where the structure becomes unstable, solution will be unconverged and the time step at that point is the knock down factor (KDF).

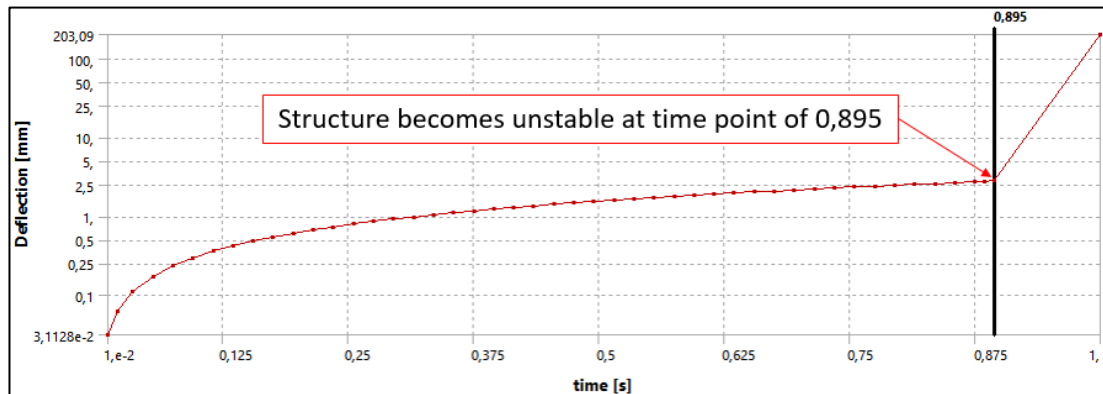


Figure 3-13 KDF for Actual Buckling Load.

As seen in Figure 3-13, KDF due to geometrical and material non-linearities and geometrical imperfections is found as 0,895. This means structure buckles under 6,038 E+5 N compression load which is the 0,895 times of the theoretical critical buckling load of 6,746 E+5 N.

Before buckling, maximum equivalent von-Mises stress is 874,5 MPa at the upper edge and stress level is around 500 MPa overall per Figure 3-14. Both are well below the yield stress of 930 MPa. Thus, cylinder buckles in elastic region. Therefore, failure mechanism is buckling.

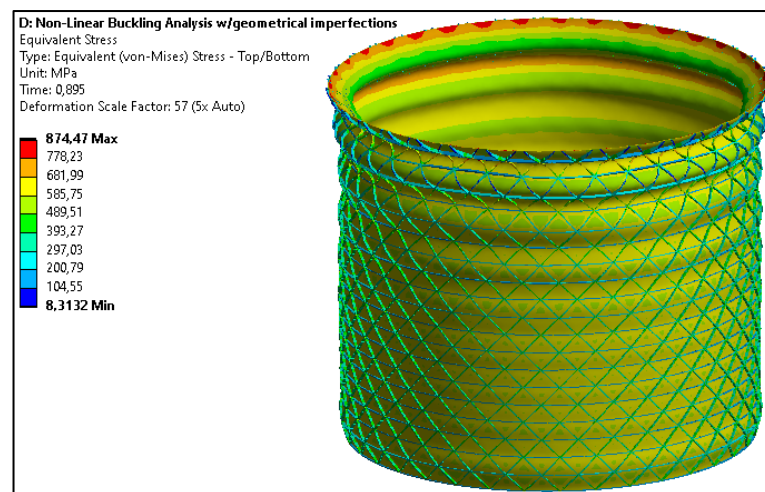


Figure 3-14 Stress Levels before Buckling Failure.

Total deformation before buckling is maximum 2,8 mm at the upper edge of the cylinder. As seen in Figure 3-15, total deformation shape before buckling failure matches well with the linear buckling mode shape.

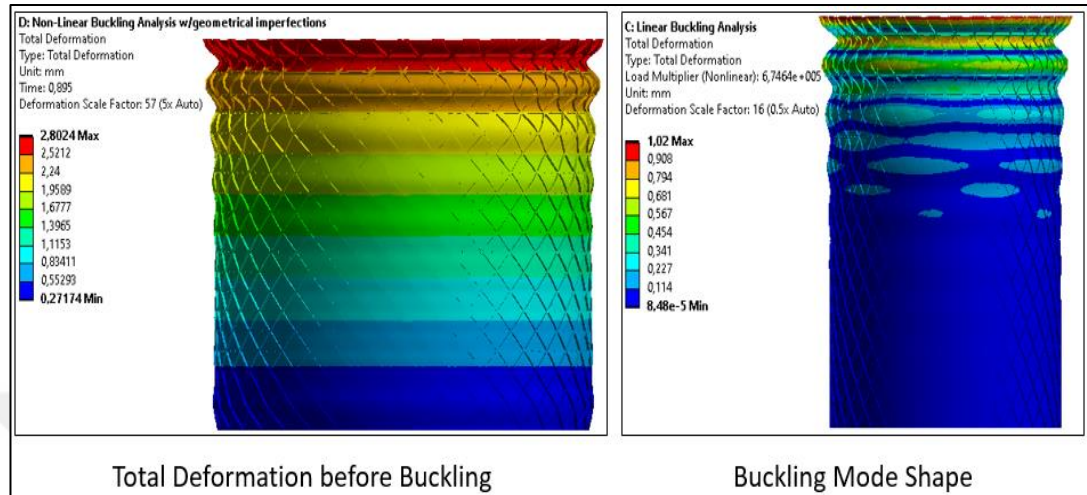


Figure 3-15 Non-linear Deformation – Buckling Mode Shape Comparison for isogrid option w/36 circumferentially triangle number.

Unstiffened cylinder has a theoretical critical buckling load of 3,675 E+5 N and KDF of 0,881. Thus, the actual critical buckling load is 3,238 E+5 N and total deformation shape before buckling point matches well with the linear buckling mode shape as seen in Figure 3-16.

The isogrid-stiffened cylinder is 26 % heavier than the unstiffened cylinder, however it has 87 % higher actual critical buckling load.

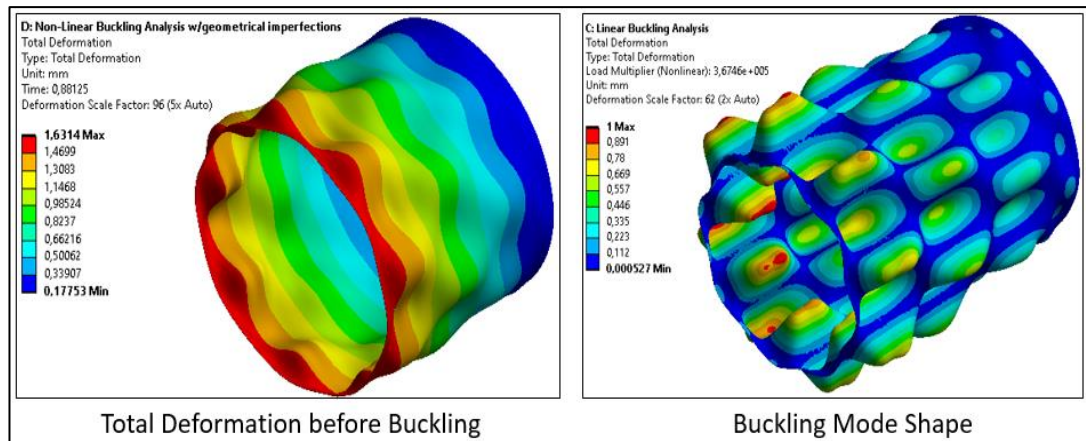


Figure 3-16 Non-linear Deformation – Buckling Mode Shape Comparison for Unstiffened Cylinder.

The FEA results for the other design iterations with different circumferential triangle number are listed in Table 3-3.

Table 3-3 FEA Results.

Skin Radius(mm)	150						
Skin Thickness(mm)	1						
Skin Length(mm)	450						
Rib Height(mm)	0	2					
Rib Width(mm)	0	1					
Circumferential Triangle Number	0	12	16	24	36	48	72
Mass(kg)	1,966	2,142	2,198	2,319	2,486	2,663	3,007
Linear Critical Buckling Load (N)	3,675 E+05	3,962 E+05	4,335 E+05	5,497 E+05	6,746 E+05	7,665 E+05	9,068 E+05
KDF for geometrical and material non-linearity w/geometrical imperfections	0,881	0,808	0,875	0,896	0,895	0,896	0,923
Nonlinear Actual Critical Buckling Load	3,238 E+05	3,201 E+05	3,793 E+05	4,925 E+05	6,038 E+05	6,868 E+05	8,370 E+05
Actual Buckling Load to Weight Ratio (N/kg)	1,647 E+05	1,495 E+05	1,726 E+05	2,124 E+05	2,429 E+05	2,579 E+05	2,783 E+05

Linear and actual critical buckling loads are graphed per circumferential triangle number in Figure 3-17. As expected, increasing triangle number lead to a higher buckling load.

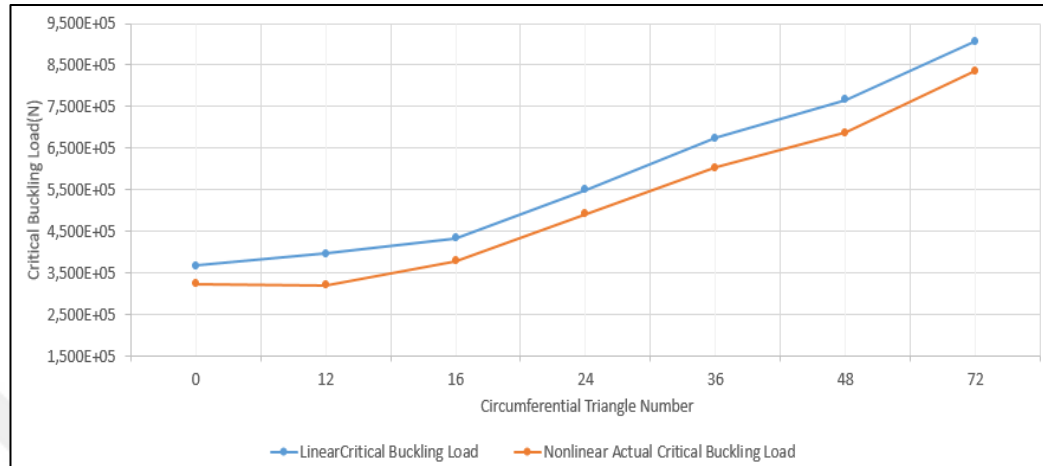


Figure 3-17 Critical Buckling Loads per Triangle Number.

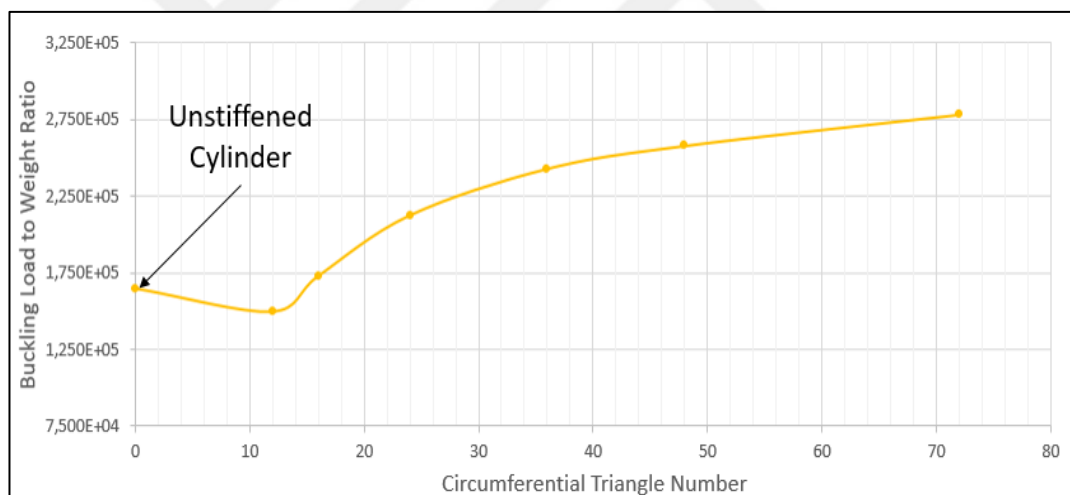


Figure 3-18 Buckling Load to Weight Ratio per Triangle Number.

4. CONCLUSIONS AND DISCUSSIONS

4.1 Comparison of FEA and Hand Calculation

This section compares the FEA results from shell-beam model with the hand calculation results per NASA design manual, concerning theoretical buckling load. Figure 4-1 shows the FEA and calculation results of unstiffened cylinder and six isogrid-stiffened cylinder with different triangle numbers. Regarding unstiffened cylinder, FEA and hand calculations result in $3,675 \text{ E}+5 \text{ N}$ and $3,733 \text{ E}+5$ respectively. The difference is only 1,6 % and it is acceptable for early design phase activities.

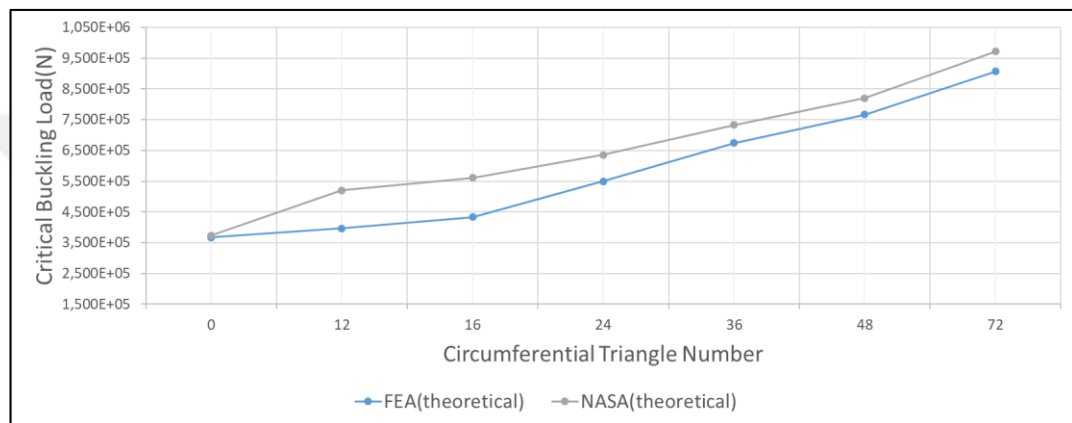


Figure 4-1 Theoretical Buckling Loads per FEA and NASA Manual.

As seen in the plot (Figure 4-1), hand calculation overestimate the theoretical buckling load for isogrid designs. Difference between FEA and calculations reduces with increasing number of triangles in the circumference. If the structure is lightly stiffened, difference is up to 31%. However, it decreases to 7-8% if the cylinder is heavily stiffened as minimum 36 triangle on the circumference.

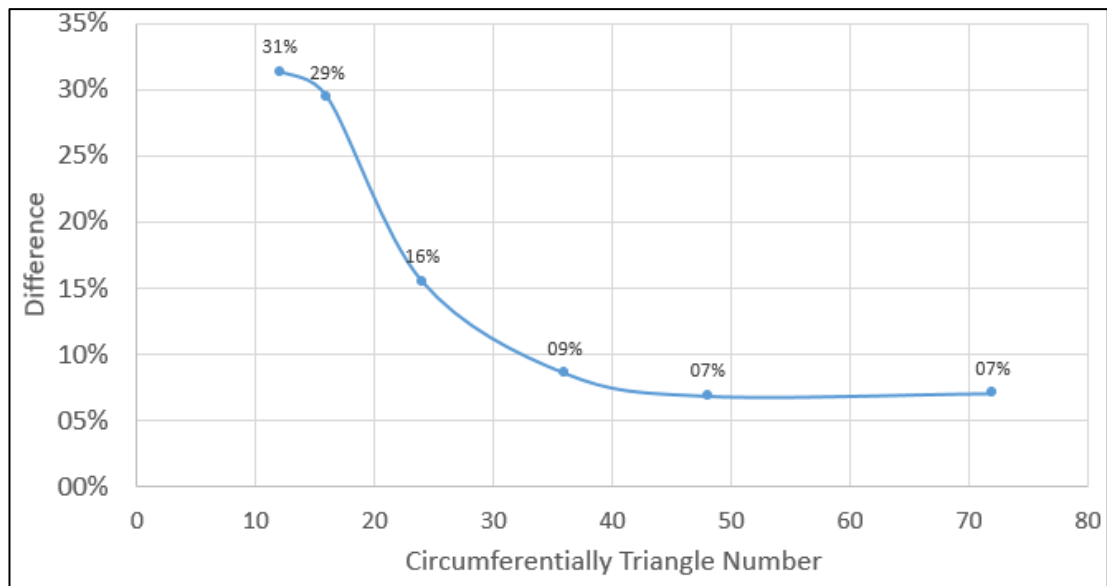


Figure 4-2 FEA-Hand Calculation Difference per Stiffener Density.

Above results are for the specific R/t ratio of three. Similar assessment is performed for different R/t ratios from 50 to 750. Cylinders are moderate/heavily stiffened as 36 triangle on the circumference. Figure 4-2 and Figure 4-3 shows the results and difference between them again for FEA and hand calculation.

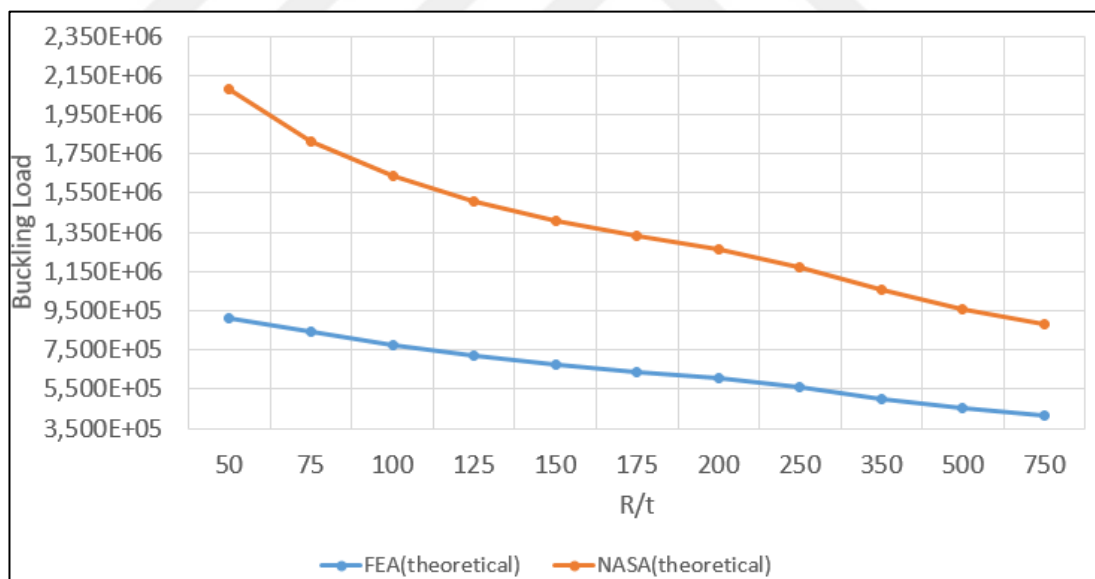


Figure 4-3 Theoretical Buckling Loads per FEA&NASA Manual for different R/t ratios.

As seen in Figure 4-4, hand calculation is not suitable for R/t below 100. Difference is 28% and 15% for R/t ratio of 50 and 15 respectively. Hand calculation approach does not work properly if the slenderness is low.

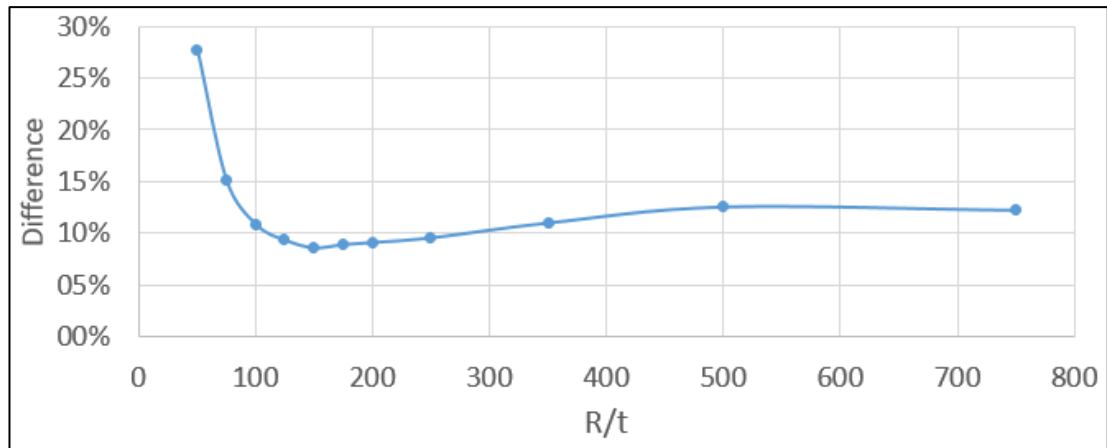


Figure 4-4 FEA-Hand Calculation Difference per R/t.

To sum up, hand calculation and FEA match very well if the cylinder is not stiffened. For isogrid-stiffened cylinders, hand calculation approach overestimate the buckling capability of the structure. The overestimate level is around 10% for slender, moderate/heavily stiffened cylinders, which have minimum 36 stiffener triangle number on the circumference and minimum slenderness level with R/t ratio of 100 to 800. If the slenderness is below R/t ratio of 100 or the structure is just lightly stiffened, using hand calculation might mislead the design.

4.2 Isogrid Effect on Buckling Capacity and Weight

Design option with circumferential triangle number 36 has a weight of 2,486 kg. Considering the same radius of 150 mm and the same weight and same material, 2,486 kg corresponds to an unstiffened cylinder with shell thickness of 1,269 mm. Nonlinear buckling analysis of this shell results in a critical buckling value of 5,322E5 N while the same weight isogrid stiffened cylinder has 6,038 E5 N. Isogrid design provides 13 % improvement for the same weight as listed in Table 4-1.

Table 4-1 Buckling Capacity of Unstiffened and Isogrid Stiffened Cylinder.

	Isogrid Stiffened Cylinder	Unstiffened Cylinder w/same Weight	
Skin Radius(mm)	150	150	
Skin Thickness	1	1,269	
Skin Length	450	450	
Circumferential Rib Number	36	N/A	
Rib Height(mm)	2	N/A	
Rib Width(mm)	1	N/A	
Mass(kg)	2,486	2,486	Improvement
Linear Critical Buckling Load(N)	6,746E+05	5,842E+05	15%
Nonlinear Critical Buckling Load(N)	6,038E+05	5,322E+05	13%

4.3 KDF by NASA Manual and FEA

In reality, structures buckle earlier than the expected, theoretical critical load due to the geometrical and material nonlinearities and geometrical imperfection caused by manufacturing tolerances. In order to consider this phenomenon, theoretical buckling load is multiplied with KDF and actual critical load is obtained. As discussed in 2.2, NASA suggests a KDF of 0,65 for moderate/heavily stiffened cylinder. This number is based on the test data obtained at NASA from the 1930s through the 1960s [15]. Considering the high accuracy of today's modern manufacturing processes and technology; this KDF looks conservative. Less conservative factors will help to gain weight saving for aerospace industry.

On the other hand, non-linear FEA buckling analysis result in an average KDF of around 0,85 depending on the stiffening level as discussed in 3.7.

Due to this fact, hand calculation ends with a lower actual critical buckling load compared to FEA result, although it overestimate the theoretical, critical load by 7-16 % for moderate/heavily stiffened ones.

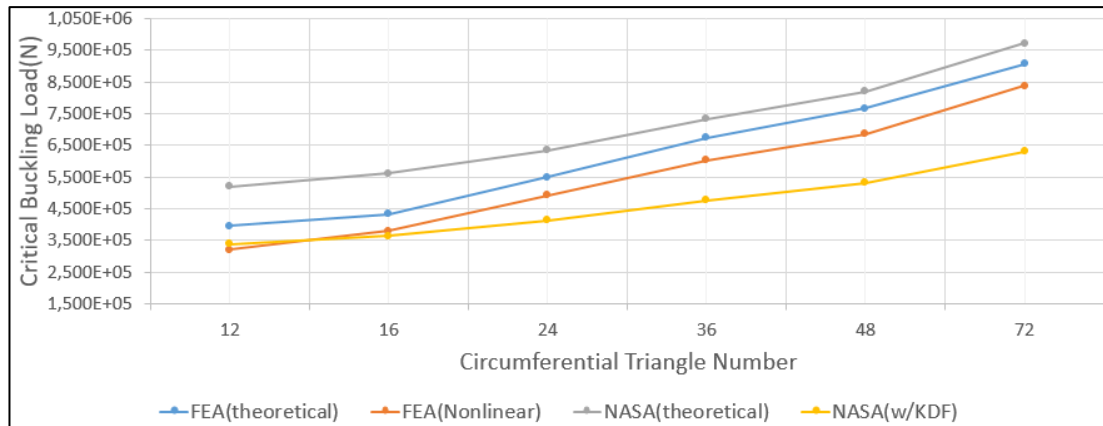


Figure 4-5 Theoretical and Actual Buckling Loads per FEA and Hand Calculations.

4.4 KDF and Geometrical Imperfection Amplitude

There are always initial imperfections due to nature of manufacturing processes and this fact reduces the buckling strength of the structure. Obviously, imperfections are random and could not predicted until we measure the actual, manufactured part. Real 3D geometry of the part can be modelled by modern scanning technology and can be transferred to FEA software. However, this methodology is piece part unique and very costly process considering mass production. Instead of this technique, a pre-deformed 3d model of the structure can be built up for initial geometry with imperfections. As discussed xxx, first buckling mode shape from eigenvalue buckling analysis is suitable for this technique. A scale factor is used to regulate the amplitude of the imperfections. To remember, the structure that is studied in 3 is 300 mm diameter cylinder with a shell thickness of 1 mm and it has 36 isogrid triangles on the circumference with rib cross section 2x1 mm. If one checks the first eigenvalue buckling mode shape, 300 mm diameter varies between 302 and 298,2 mm as seen in Figure 4-6. Total deviation is 1,9 mm for 300 mm if the scale factor is assumed as 1.

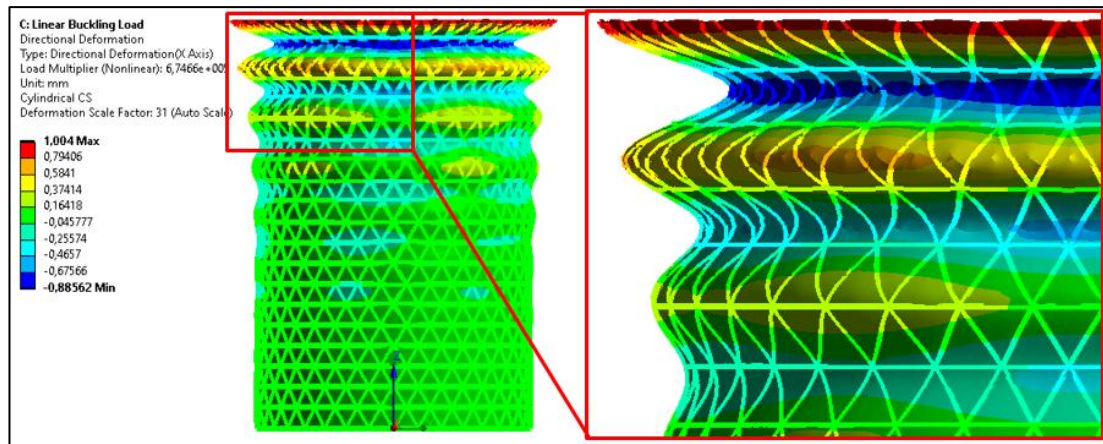


Figure 4-6 Radial Deflection of First Eigenvalue Buckling Mode Shape.

Scale factors and the corresponding manufacturing tolerances are listed in Table 4-2 and Table 4-3.

Table 4-2 KDF's per Manufacturing Tolerances-1.

Scale Factor	0,01	0,02	0,03	0,04	0,05	0,06	0,07
Corresponding Manufacturing Tolerance($\pm$)(mm)	0,02	0,04	0,06	0,08	0,1	0,12	0,14
KDF	0,96	0,91	0,88	0,86	0,84	0,84	0,81

Table 4-3 KDF's per Manufacturing Tolerances-2.

Scale Factor	0,08	0,09	0,1	0,2	0,5	0,75	1
Corresponding Manufacturing Tolerance($\pm$)(mm)	0,16	0,18	0,2	0,4	1	1,5	2
KDF	0,79	0,78	0,77	0,67	0,5	0,43	0,38

As expected, the more the structure have manufacturing tolerance, the lower KDF is for actual critical buckling load. Figure 4-7 shows the change of KFD per manufacturing tolerance. In section, $\pm 0,05$ mm tolerance is assumed for the isogrid structure with a diameter of 300 mm. Considering the modern manufacturing techniques used in aero industry, this tolerance level looks quite achievable.

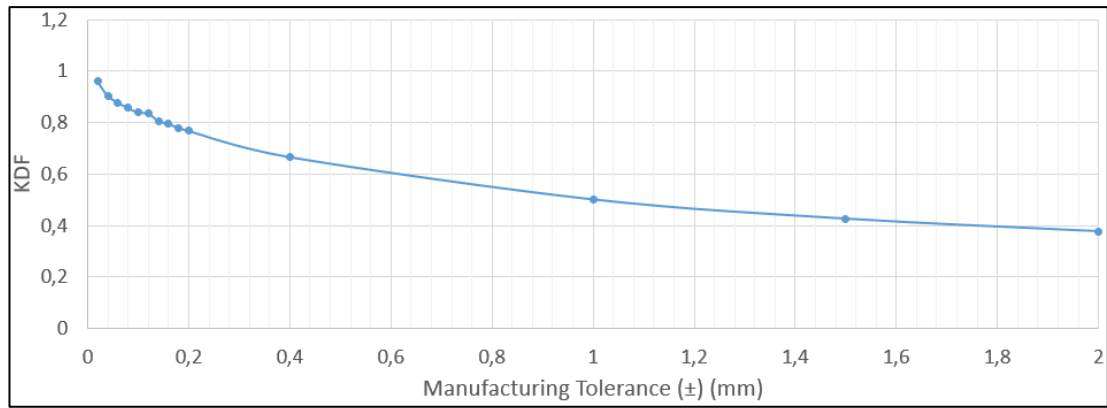


Figure 4-7 KDF's per Manufacturing Tolerances.

This correlation (Figure 4-7) shows the importance of the manufacturing tolerances for aero/aerospace industries considering thin walled cylinder structures. Buckling capacity of those type structures is very sensitive to geometrical imperfections. For instance, manufacturing tolerance of $\pm 0,1$ mm results in a KDF of 0,84, while KDF is 7 % lower for $\pm 0,2$ mm tolerance level. Just 0,1 mm change in manufacturing accuracy reduces the buckling capacity by 7 %.

4.5 Conclusion

Regarding thin walled cylinders, results of hand calculation per Donnell's shell theory and FEA method are very close as 1,6 % for critical buckling load. However; hand calculation per NASA's manual overestimates theoretical critical buckling load by 7-8 % compared to FEA results for moderately isogrid stiffened cylinders with R/t ratio of 150. The difference increases up to 31 % if the structure is lightly stiffened. In addition, if the R/t ratio is below 100, 7-8 % difference increases up to 28 %. Thus, hand calculation is not proper for R/t ratios below 100.

One of the most important design criterions in aerospace technologies is the weight. Using isogrid stiffeners increases buckling capability of the structure. This study shows that isogrid-stiffened cylinder has 13% more buckling capability than the monocoque cylinder with the same weight.

Manufacturing tolerance is very critical factor to determine the buckling capacity of the structure, because it effects KDF, which allows calculating the critical buckling load from theoretical buckling load. This study shows that the change in manufacturing tolerance from $\pm 0,1$ to $\pm 0,2$ mm decreases the buckling capacity by 7 %.

REFERENCES

- [1] Steven M. Huybrechts, Grid Stiffened Structures: A Survey of Fabrication, Analysis and Design Methods, 1999
- [2] Web 1, (2021), <https://www.satorsrl.it/grid-structures/>, (Eriřim Tarihi: 20/06/2021).
- [3] Web 2, (2021), https://en.wikipedia.org/wiki/Eurojet_EJ200, (Eriřim Tarihi: 20/06/2021).
- [4] Hilburger Mark W., Test and Analysis of Buckling-Critical Stiffened Metallic Launch Vehicle Cylinders,
- [5] Hafiz Pouria and Baru A. Maikanti, An Investigation of Buckling Phenomenon in Steel Elements, 2018
- [6] Web 3, (2021), https://www.dlr.de/fa/en/desktopdefault.aspx/tabid-10589/18840_read-43672/, (Eriřim Tarihi: 20/06/2021).
- [7] NASA/SP-8007, Buckling of Thin-Walled Circular Cylinders, 2019 Revision
- [8] NASA CR-124075, Isogrid Design Handbook, 1973
- [9] NASA TN D-5561, Buckling of Stiffened Cylinders in Axial Compression and Bending- A Review of Test Data, 1969
- [10] Sadowski Adam J. and Rotter J. Michael, Solid or shell finite elements to model thick cylindrical tubes and shells under global bending, 2013
- [11] Okura Takehiro, Materials for Aircraft Engines – ASEN 5063 Aircraft Propulsion Final Report, 2015
- [12] ANSYS Workbench Help Document
- [13] Hassan Ahmed and Kurgan Naci, Modeling and Buckling Analysis of Rectangular Plates in ANSYS, 2019
- [14] Web 4, (2021), <https://www.padtinc.com/blog/to-use-large-deflection-or-not-that-is-the-question/>, (Eriřim Tarihi: 20/06/2021).
- [15] Lovejoy Andrew E. and Hilburger Mark W., SBKF Modeling and Analysis Plan: Buckling Analysis of Compression-Loaded Orthogrid and Isogrid Cylinders, 2013

BIOGRAPHY

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