

**T.R.**  
**GEBZE TECHNICAL UNIVERSITY**  
**GRADUATE SCHOOL**

**THE DYNAMIC IMPACT OF CLAMPING ON PIPE SYSTEMS: A  
COMPREHENSIVE ANALYSIS THROUGH SIMULATION AND  
EXPERIMENTAL STUDIES**

**ALİ GEZER**

**A THESIS OF MASTER OF SCIENCE  
DEPARTMENT OF ENERGY TECHNOLOGIES  
APPLIED PROPULSION SYSTEM DESIGN AND  
ENGINEERING FOR AEROSPACE TECHNOLOGIES  
PROGRAM  
ADVISOR: PROF. MEHMET ALİ ARSLAN**

**JULY 2025**

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T.C.  
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LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

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## **MASTER of SCIENCE JURY APPROVAL FORM**

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*To my  
Family*

## **ABSTRACT**

In the engineering and design of hydraulic pipeline systems, minimizing the risk of vibration-induced failures is a fundamental requirement, particularly in aerospace applications where reliability and safety are paramount. Structural resonance, if not properly addressed, can lead to high-cycle fatigue and eventual system failure. A primary factor influencing vibrational behavior is the rotational frequencies of aero engines, which may excite resonant modes in pipeline structures if they coincide with the natural frequencies. Thus, accurately predicting and controlling these dynamic characteristics is crucial to prevent resonance-induced damage. This study presents a comprehensive analysis of a double-bent pipe configuration subjected to dynamic excitation under two different support conditions: one fully unsupported and the other supported at its midpoint using a cushioned elastic clamp. Both numerical and experimental methods were employed to evaluate the system's modal response. Finite element analyses were conducted using both three-dimensional solid and two-dimensional shell elements. The resulting modal data were validated through experimental shaker tests. The first three natural frequencies and corresponding mode shapes were compared in detail across all approaches, showing strong agreement. Furthermore, damping ratios for both configurations were determined using the half-power bandwidth (3 dB) method, offering valuable insight into the system's dynamic behavior. The findings highlight the significance of passive vibration control through mechanical support, particularly in modifying modal properties and reducing resonance risk. This research contributes to a broader understanding of vibrational dynamics in pipeline systems and offers practical design strategies for engineers developing robust fluid transport systems in vibration-sensitive environments such as aerospace platforms.

**Keywords: Vibration Test, Pipeline System, Damping Ratio, Excitation Load, Modal Analysis, Modal Behavior**

## ÖZET

Hidrolik boru sistemlerinin mühendisliği ve tasarımında, titreşim kaynaklı arızaların riskinin en aza indirilmesi temel bir gerekliliktir; bu durum, özellikle güvenilirlik ve emniyetin kritik olduğu havacılık ve uzay uygulamalarında önem arz etmektedir. Yapısal rezonans, uygun şekilde ele alınmadığında yüksek çevrimli yorulmaya ve nihai sistem arızasına yol açabilir. Titreşim davranışını etkileyen başlıca faktörlerden biri, havacılık motorlarının dönüş frekanslarıdır; bu frekanslar boru yapılarının doğal frekanslarıyla örtüştüğünde rezonans modlarını uyarabilir. Bu nedenle, rezonans kaynaklı hasarların önlenmesi için dinamik özelliklerin doğru şekilde tahmin edilmesi ve kontrol edilmesi hayati önem taşımaktadır. Bu çalışma, iki farklı destek koşulu altında dinamik uyarıma maruz kalan çift bükümlü bir boru konfigürasyonunun kapsamlı analizini sunmaktadır: biri tamamen desteksiz, diğeri ise orta noktasından yumuşak elastik bir kelepçe ile desteklenmiştir. Sistemin modal tepkisini değerlendirmek için sayısal ve deneysel yöntemler kullanılmıştır. Sonlu eleman analizleri hem üç boyutlu katı eleman modeli hem de iki boyutlu kabuk eleman modeliyle gerçekleştirilmiş ve elde edilen modal veriler deneysel sarsıcı testleri ile doğrulanmıştır. İlk üç doğal frekans ve karşılık gelen mod şekilleri tüm yöntemlerde detaylı olarak karşılaştırılmış ve yüksek uyum gözlenmiştir. Ayrıca, her iki konfigürasyon için sönüm oranları yarı güç bant genişliği (3 dB) yöntemiyle hesaplanmış ve sistemin dinamik davranışına ilişkin önemli bilgiler elde edilmiştir. Bulgular, mekanik destekle sağlanan pasif titreşim kontrolünün modal özelliklerin değiştirilmesinde ve rezonans riskinin azaltılmasında kritik bir rol oynadığını ortaya koymaktadır. Bu araştırma, boru sistemlerinde titreşim dinamiklerine ilişkin daha geniş bir anlayış kazandırmakta ve özellikle havacılık ve uzay platformları gibi titreşim hassasiyeti yüksek ortamlarda dayanıklı akışkan taşıma sistemleri geliştiren mühendisler için uygulanabilir tasarım stratejileri sunmaktadır.

**Anahtar Kelimeler:** Titreşim Testi, Boru Sistemi, Sönümleme Oranı, Tahrik Yüğü, Modal Analiz, Modal Davranış

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## LIST OF SYMBOLS AND ABBREVIATIONS

|                       |                                                                    |
|-----------------------|--------------------------------------------------------------------|
| $c$                   | : Actual Damping Coefficient of the System                         |
| $\{\ddot{u}\}$        | : Acceleration Matrix                                              |
| $\omega$              | : Angular Velocity                                                 |
| $F_o$                 | : Amplitude of the Applied Force                                   |
| $U$                   | : Amplitude of the Steady State Displacement                       |
| $\Delta f_{adjusted}$ | : Bandwidth at the Chosen Offset                                   |
| $c_o$                 | : Critical Damping Coefficient                                     |
| $c$                   | : Damping coefficient                                              |
| $[C]$                 | : Damping Matrix                                                   |
| $\zeta$               | : Damping Ratio                                                    |
| $u(t)$                | : Displacement Response                                            |
| $\{u\}$               | : Displacement Vector                                              |
| $\Delta E$            | : Energy Dissipated per Cycle                                      |
| $f_0$                 | : Frequency of Resonant Peak in Hertz,                             |
| $f_1$                 | : Frequency Value, in Hertz 3 dB Down from Peak Value, $f_1 < f_0$ |
| $f_2$                 | : Frequency Value, in Hertz 3 dB Up from Peak Value, $f_2 > f_0$   |
| $[K_g]$               | : Geometrical Stiffness Matrix                                     |
| $a(t)$                | : Instantaneous Acceleration as a Function of Time                 |
| $k$                   | : Linear Stiffness Coefficient                                     |
| $m$                   | : Mass of the Oscillator                                           |
| $[M]$                 | : Mass Matrix                                                      |
| $\phi$                | : Mode Shape                                                       |
| $\omega_n$            | : Natural Frequency                                                |
| $\alpha$              | : Nonlinear Stiffness Coefficient                                  |
| $A_{offset}$          | : Offset Quantity of the Amplitude                                 |
| $P_{offset}$          | : Offset Quantity of the Power                                     |
| $A_o$                 | : Peak Acceleration as a Function of Time                          |
| $A_{peak}$            | : Peak Value of the Amplitude                                      |
| $P_{peak}$            | : Peak Value of the Power                                          |
| $\varphi$             | : Phase Lag Between Acceleration Load                              |
| $\lambda_i$           | : Principal Stretches                                              |
| $W$                   | : Strain Energy                                                    |
| $[K]$                 | : Stiffness Matrix                                                 |
| $t$                   | : Time                                                             |
| $F(t)$                | : Time-dependent External Force Vector                             |
| $\{\dot{u}\}$         | : Velocity Vector                                                  |
| 1D                    | : 1 Dimensional                                                    |
| 2D                    | : 2 Dimensional                                                    |
| 3D                    | : 3 Dimensional                                                    |
| A.D.                  | : Anno Domini                                                      |
| Al6061                | : Aluminum 6061                                                    |
| AMS:                  | : Automated Multilevel Substructuring                              |
| AS                    | : Aerospace Standard                                               |
| B.C.                  | : Before Christ                                                    |
| CLD                   | : Constrained Layer Damping                                        |

|             |                                             |
|-------------|---------------------------------------------|
| DAQ:        | : Data Acquisition System                   |
| dB          | : Decibel                                   |
| DOF         | : Degree of Freedom                         |
| FEA         | : Finite Element Analysis                   |
| FFT         | : Fast Fourier Transform                    |
| FRF         | : Frequency Response Function               |
| GTÜ         | : Gebze Teknik Üniversitesi                 |
| GVS         | : Ground Vibration Surveys                  |
| GVT         | : Ground Vibration Tests                    |
| HCF         | : High Cycle Fatigue                        |
| HP          | : High Pressure                             |
| Hz          | : Hertz                                     |
| IN625       | : Alloy (Inconel) 625                       |
| IN718       | : Alloy (Inconel) 718                       |
| JSSG        | : Joint Service Specification Guides        |
| LP          | : Low Pressure                              |
| MPC         | : Multi Point Constraint                    |
| PCG Lanczos | : Preconditioned Conjugate Gradient Lanczos |
| SNR         | : Signal to Noise Ratio                     |
| TEI         | : TUSAŞ Engine Industries Inc.              |
| VSD         | : Vibration Schedule Development            |

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# 1. INTRODUCTION

In the design of hydraulic pipeline systems, preventing vibration-related failures—especially those caused by structural resonance—is critically important. In engineering practice, it has been observed that even small changes in clamp positions can significantly affect the damping behavior and vibration stresses in pipelines. These changes may either reduce or increase stress levels; if increased significantly, stress concentrations may occur, potentially leading to fatigue damage.

It is important to consider the rotating frequencies of aero engines with Campbell Diagram to mitigate the risk of high-cycle fatigue failures and to ensure the reliability and longevity of the system. The Campbell Diagram examines the interaction between the obtained mode frequencies and the rotor's rotational frequencies. This graphical tool helps analyze the dynamic behaviour of rotating systems, such as turbines, compressors, and rotors. It identifies critical speeds and potential resonance conditions by exploring the relationship between the system's natural frequencies and rotational speed. In the diagram, straight lines represent excitation frequencies, typically multiples of the rotational speed (e.g.,  $1x$ ,  $2x$ ,  $3x$ , etc., where  $x$  denotes the rotational speed). The points where natural frequency lines intersect with harmonic order lines indicate potential resonance conditions, also called critical speeds. In a twin-spool gas turbine engine, the harmonic order lines in the Campbell Diagram are generated by various excitation sources that create periodic forces within the engine. These forces are typically associated with the rotational speeds of the engine's two spools and other periodic phenomena. The fundamental frequency ( $1x$ ) for either the low-pressure (LP) or HP spool corresponds to one rotation per second. Higher harmonics ( $2x$ ,  $3x$ ,  $4x$ , etc.) occur due to rotor imbalances or periodic loading within the engine. Imbalances in the rotor or misalignments of the LP and HP systems shafts produce harmonics at multiples of the rotational speeds. If the engine includes a reduction gearbox, the interaction of the gears can introduce additional harmonic frequencies. Moreover, aerodynamic disturbances arising from the interaction between the rotating and stationary components generate periodic forces, which can create more harmonics or excite specific frequencies. The blade passing frequency is produced in the airflow path when the rotating blades pass stationary elements.

According to JSSG-2007, analyzing and establishing the rigidity of equipment and subsystems and verifying their design is essential. This analysis aims to minimize potential issues related to vibration and dynamic response. Additionally, testing is required to assess the engine's vibration and dynamic response characteristics. The hydraulic system must withstand operational stresses and vibrations without failing, and there should be no permanent deformation, collapse, or seam cracking when exposed to these conditions. All components must undergo a vibration test by MIL-STD-810 [1].

Modal tests and analyses determine the structural dynamic characteristics of materials and test fixtures. These tests, often called ground vibration tests (GVT) or ground vibration surveys (GVS), involve applying a known dynamic input to the test item while measuring and recording the resulting responses. Modal analysis methods are then utilized to extract modal parameters such as resonant frequencies, mode shapes, and modal damping. Modal parameters can confirm or develop analytical models, troubleshoot issues, identify suitable instrumentation locations, evaluate measured vibration data, and design test fixtures. Modal analysis methods vary, encompassing both frequency domain and time domain techniques as well as single-degree and multi-degree-of-freedom methods. Experimental modal tests typically involve exciting a structure with a measured force and measuring the acceleration response. This allows for the computation of frequency response functions (FRF) at key locations for further modal analysis. The excitation of the structure can be accomplished through different methods. One straightforward technique is the modal impact test, which employs a modally tuned impact hammer with a force gauge. This method generates a low-force impact on the structure that approximates an impulse function, allowing for a quick assessment of resonant frequencies for fixtures and installed components. Alternatively, a more advanced approach involves using burst random excitation with small vibration exciters affixed to the structure. This method is instrumented with a network of accelerometers, making it especially useful for high channel count modal tests on complex structures that require precise measurements to develop mode shapes and validate analytical models accurately. Other options for checking resonant frequencies include sinusoidal and broadband random vibration excitation of a test fixture or item mounted on large vibration exciters, particularly useful in laboratory vibration test setups. It is essential to select a methodology that generates precise,

usable data and provides the detailed information necessary to meet the specific goals of the test. Sinusoidal – Vibration at a single frequency, typically of constant amplitude. Depending on the vibration source, the frequency might remain constant (dwell) or change (sweep) over a pre-defined bandwidth. Broadband Random – Vibration is simultaneously preset at all frequencies over a wide bandwidth. The amplitude may vary significantly over the full bandwidth. Narrowband Random – Vibration is simultaneously present over a narrow bandwidth of frequencies. A centre frequency, bandwidth, and amplitude typically define narrowband vibration. As with sinusoidal vibration, the frequency of the narrowbands is sometimes constant and sometimes swept. Narrowband Random – Vibration is simultaneously present over a narrow bandwidth of frequencies. NA centre frequency, bandwidth, and amplitude typically define narrowband vibrations with sinusoidal vibration; the frequency of the narrowbands is sometimes constant and sometimes swept. **Figure 1.1** illustrates the VSD (Vibration Schedule Development) procedure in an abbreviated block diagram format [2].

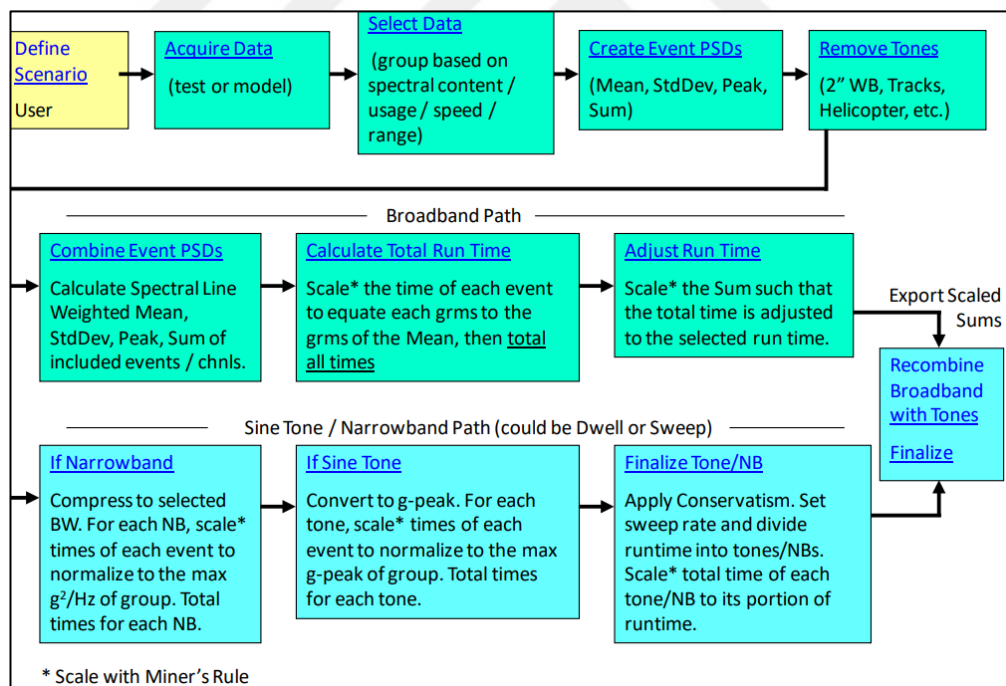


**Figure 1.1:** VSD flowchart from MIL-STD-810G

## 1.1. Theoretical Background

Whenever a machine or structure's natural frequency of vibration coincides with the frequency of the external excitation, a phenomenon known as resonance occurs,

leading to excessive deflections and failure. The literature contains accounts of system failures brought about by resonance and excessive vibration of components and systems. One of the most devastating examples of vibration failure is the Tacoma Narrows bridge, shared in **Figure 1.2**. Because of the devastating effects that vibrations can have on machines and structures, vibration testing has become a standard procedure in the design and development of most engineering systems [3].

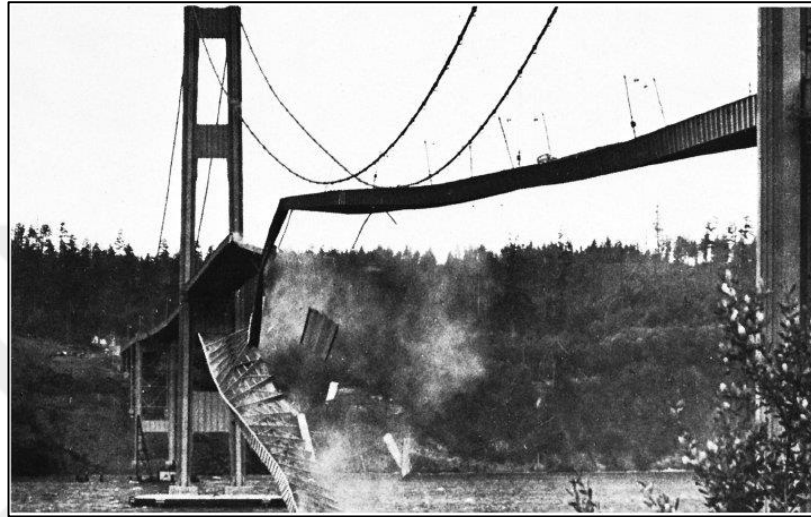


**Figure 1.2:** Tacoma Narrows bridge during wind-induced vibration

#### **1.1.1. Brief History of Vibration**

Interest in vibration began with the creation of early musical instruments like whistles and drums. Pythagoras (582–507 B.C.), a Greek philosopher and mathematician, was the first to investigate musical sounds scientifically. He used a monochord to study vibrating strings and found that shorter strings produce higher notes. Specifically, if a shorter string is half the length of a longer one, it emits an octave higher note. The relationship between pitch and frequency was not understood until Galileo's time in the sixteenth century. In ancient China, Zhang Heng, a historian and astronomer, recognized the need for an earthquake measurement tool and invented the world's first seismograph in A.D. 132. This device resembled a wine jar and contained pendulums and levers. When an earthquake occurred, the pendulum would move and trigger a dragon to release a bronze ball into the mouth of a toad, producing a clanging sound. Galileo Galilei (1564–1642) is known as the founder of modern experimental science. Inspired by a swinging lamp in a church, he discovered that the period of a pendulum's swings is independent of their amplitude. This observation led to further experiments

on the simple pendulum. Until about 40 years ago, vibration studies were conducted using simple models with limited degrees of freedom. The advent of high-speed digital computers in the 1950s enabled the analysis of moderately complex systems, allowing for approximate solutions in semidefinite form through numerical evaluations. The development of the finite element method further enhanced vibration analysis of intricate mechanical, vehicular, and structural systems with thousands of degrees of freedom. Most prime movers face vibrational issues due to imbalances from design flaws or poor manufacturing. For instance, diesel engine imbalances can create disruptive ground vibrations in urban areas, and high-speed locomotives, wheels can lift off the track. Additionally, vibrations can cause severe mechanical failures in turbines [3].

### 1.1.2. Basic Concepts of Vibration

Any motion that repeats itself over time is called vibration or oscillation. Typical examples are a pendulum swinging as shown in **Figure 1.3** and a plucked string's motion. The study of vibration focuses on analyzing the oscillatory motions of bodies and the forces involved. A vibratory system typically consists of a means for storing potential energy (such as a spring or elastic material), a means for storing kinetic energy (such as mass or inertia), and a mechanism for the gradual transfer of energy [3].

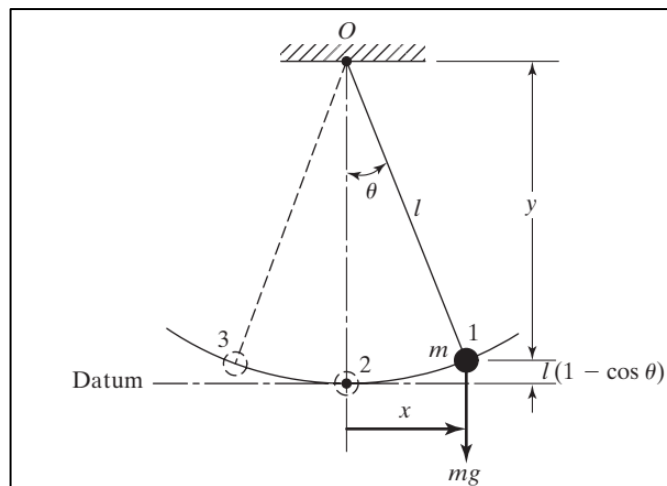


**Figure 1.3:** Swinging pendulum

### 1.1.3. Vibration Analysis Procedure

Vibration analysis is an essential process that extends into four fundamental steps.

Step 1: Mathematical Modeling - Mathematical modelling performs a crucial role in effectively representing the essential features of a system, enabling us to derive the mathematical (or analytical) equations that dictate its behaviour. As shown in **Figure 1.4**, the mathematical model of a structure comprising an anvil and a foundation block highlights the significance of this approach in understanding and predicting system dynamics [3].

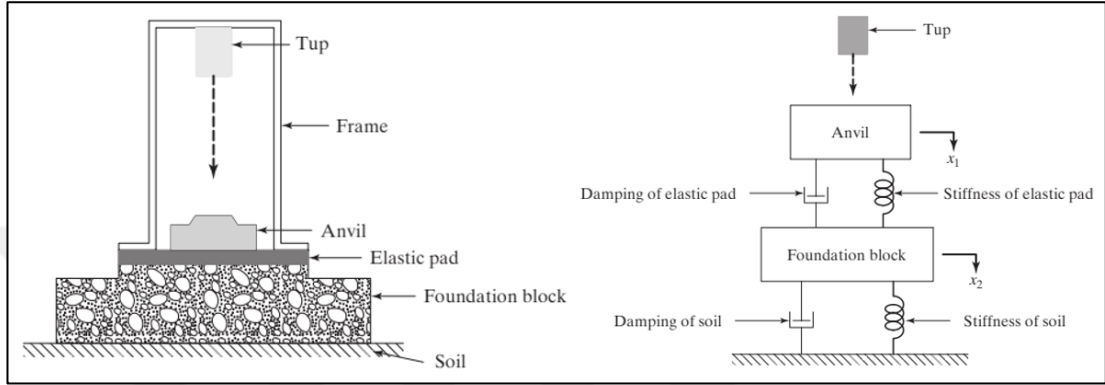


**Figure 1.4:** Derivation to the mathematical model

Step 2: Deriving the Governing Equation - After establishing the mathematical model, we apply the principles of dynamics to derive the equations that describe the system's vibrations. (Equation 1.1)

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \quad (1.1)$$

$[M]$  is the mass matrix,  $[C]$  is damping matrix,  $[K]$  is the stiffness matrix,  $\{u\}$  displacement vector,  $\{\dot{u}\}$  velocity vector,  $\{\ddot{u}\}$  acceleration matrix, and  $F(t)$  time-dependent external force vector.

Step 3: Solving the Governing Equations - To determine the response of the vibrating system, we must solve the equations of motion. Depending on the specific characteristics of the problem, we can employ one of several techniques to find the solution. These techniques include standard methods for solving differential equations, Laplace transform methods, matrix methods, and numerical methods.

Step 4: Interpreting the Results - The solutions to the governing equations provide the displacements, velocities, and accelerations of the various masses within the system. It is essential to interpret these results with a clear understanding of the analysis's objectives and the potential design implications that may arise from these findings.

## 1.2. Literature Review

GAO offers a comprehensive review of general approaches to both passive and active control technologies. These include optimal layout techniques of pipelines and clamps, constrained layer damping, vibration absorbers, hydraulic hoses, optimal pump structures, and active vibration control systems for pipelines [4].

According to statistics from the United States, faults in fuel, air, and hydraulic pipeline systems account for 50% to 60% of all aircraft component failures. Similarly, data from Russian aircraft indicates that issues with fuel, lubricating oil, and hydraulic systems also represent more than 50% of total faults. One primary cause of excessive vibration in these systems is pipeline resonance. When the frequencies of fluid pulsations or base excitations approach the natural frequencies of the pipeline, it induces structural resonance. This resonance can lead to significant displacement at the joint surfaces between the pipeline connections and fixed components. As a result, this can cause surface friction, damage, or collisions with adjacent pipelines. Additionally, excessive vibration may result in the loosening of pipe fittings [4].

Panda investigated an aircraft that experienced an incident during a routine flight operation. The pilot noticed a thin trail of smoke coming out from the tail end of the aircraft. Results showed a fatigue crack in one of the weld joints of the afterburner fuel manifold, resulting in fuel leakage in the form of streaks, as shown in Figure 1.5. The cracks in the hydraulic pipeline system were located at the root of the flat nozzle, 12 mm away from the pipe fitting, as shown in **Figure 1.5** [5].

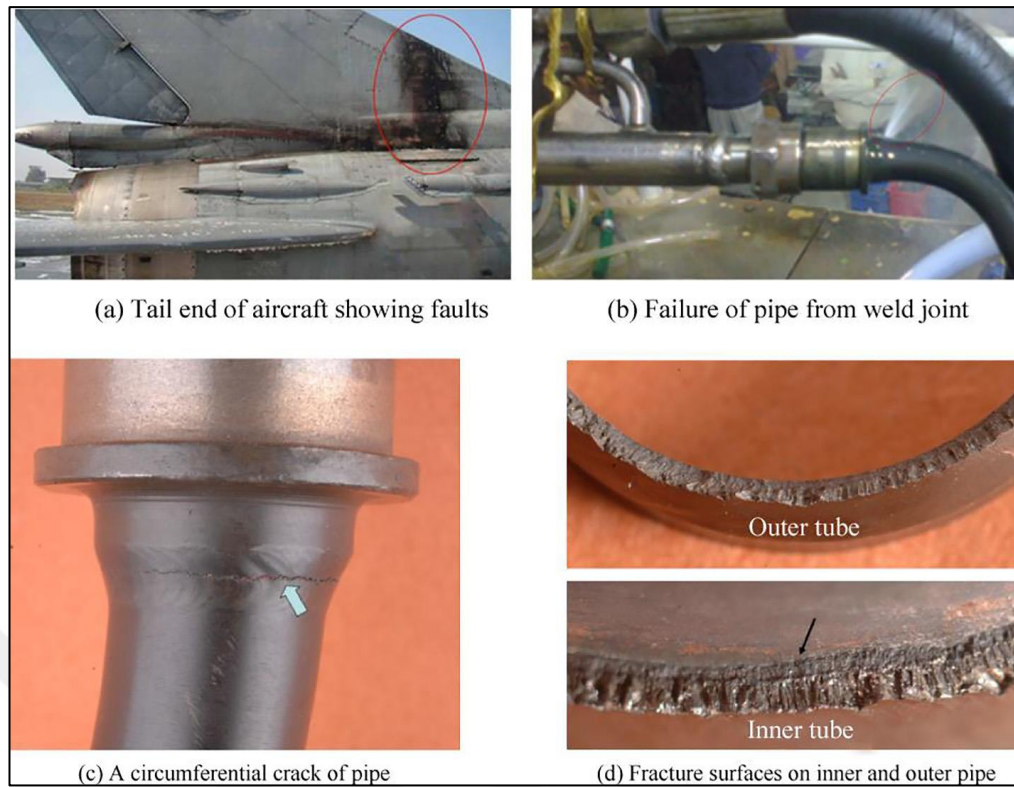


**Figure 1.5:** Vibration failure of a pipeline

Ouyang conducted a vibration analysis of complex hydraulic pipelines used in aircraft. The numerical results indicated that the length of the pipeline between two supports is the most significant factor influencing the natural frequency of the pipeline. Additionally, an increase in the radius of the pipeline can elevate the resonant frequency, as illustrated in **Figure 1.6**. Furthermore, a three-dimensional finite element model of an aircraft hydraulic pipeline was developed, and the results from both experimental tests and simulations closely matched in terms of stress analysis [6].

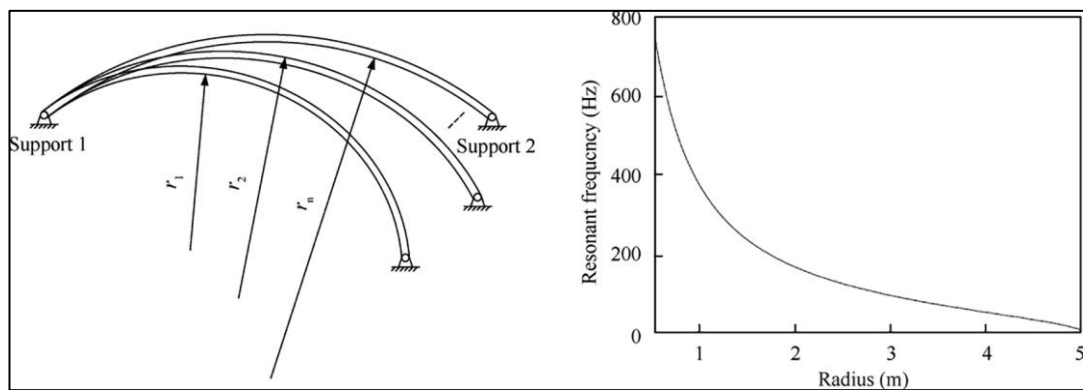


**Figure 1.6:** Effect of bend radius

Mehmood conducted an investigation into two failure cases of the same hydraulic pipeline in an aircraft, as illustrated in **Figure 1.7**. The pipeline experienced midair fractures and cracks, leading to a loss of hydraulic pressure and impairing the movement of the aircraft's control surfaces. The investigation into the pipeline failures involved a visual examination. The results revealed that fluctuations in fluid pressure and stress concentrations in circumferential grooves were the primary factors contributing to fatigue failure in hydraulic pipelines within aircraft [7].

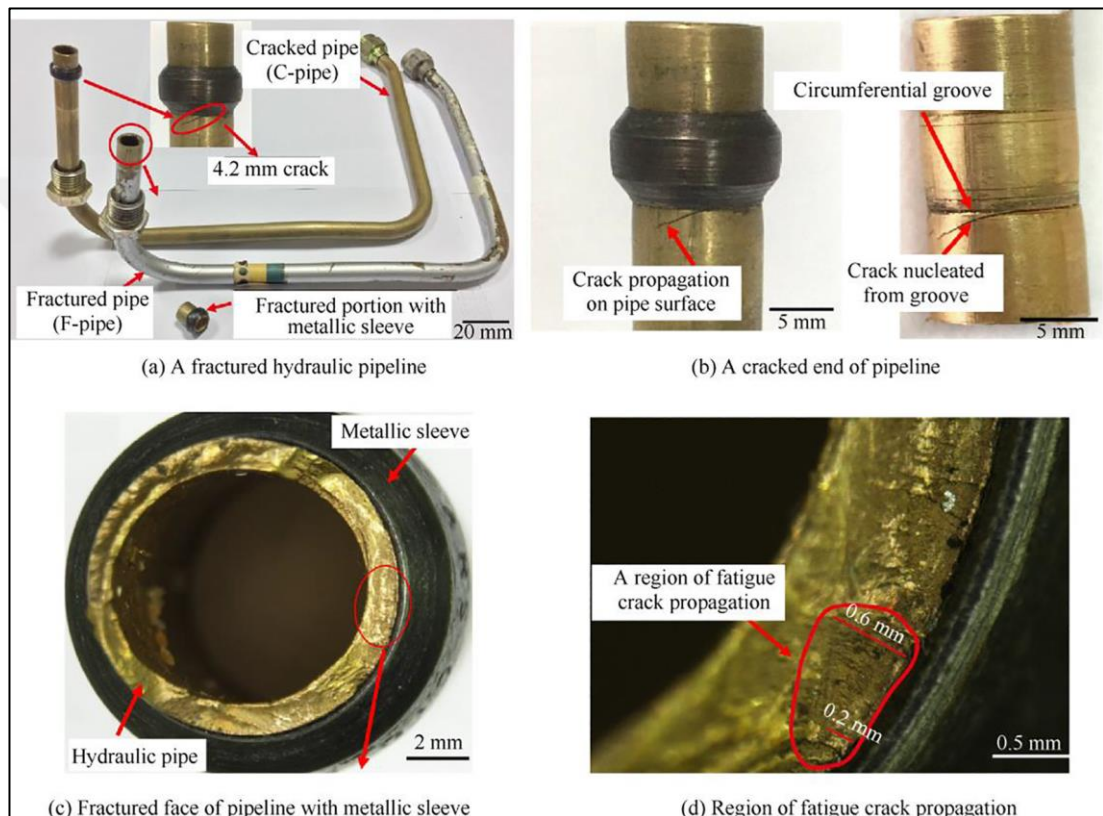


**Figure 1.7:** Vibration failure of a pipeline

Lu summarized the vibration failure of an aircraft hydraulic pipeline. The failure occurred because the natural frequency of the pipeline was nearly equal to the pulsation frequency of the hydraulic pump. The natural frequency coincides, resulting in structural resonance and increased vibration stress on the pipeline. The hydraulic pipeline experienced significant vibration due to fluid pressure fluctuations from the pump and environmental excitations from the aero-engine. These factors combined could lead to excessive vibration stress on the hydraulic pipeline [8].

Liu conducted a fracture fault analysis of pipeline clamps in an aero-engine. The fracture locations of the clamps, which had multiple fractures, were near the installation holes, and the macro morphology of the fracture is depicted in **Figure 1.8** [9].

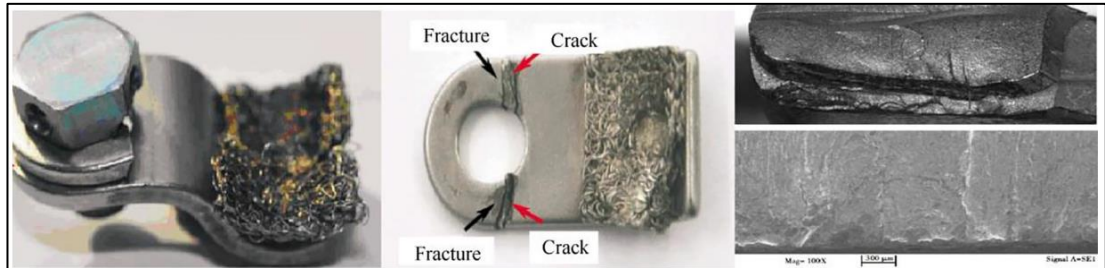


**Figure 1.8:** Vibration failure of a clamp

Liu conducted a test in which the pipe bracket support experienced multiple instances of fracture failure due to high cycle fatigue, as illustrated in **Figure 1.9**. The numerical results indicated that the natural frequencies of the pipeline were close to the engine's rotating frequencies, which directly led to structural resonance in the pipeline. This resonance resulted in relatively high-stress levels on the support bracket and the pipeline. The combination of high-vibration stress led to cumulative damage, which ultimately caused crack propagation and fracture after a certain number of cycles. Finally, increasing the support stiffness can solve the bracket support fracture failure. This adjustment would help shift the natural frequency of the pipeline outside of the aircraft's resonance frequency range [10].

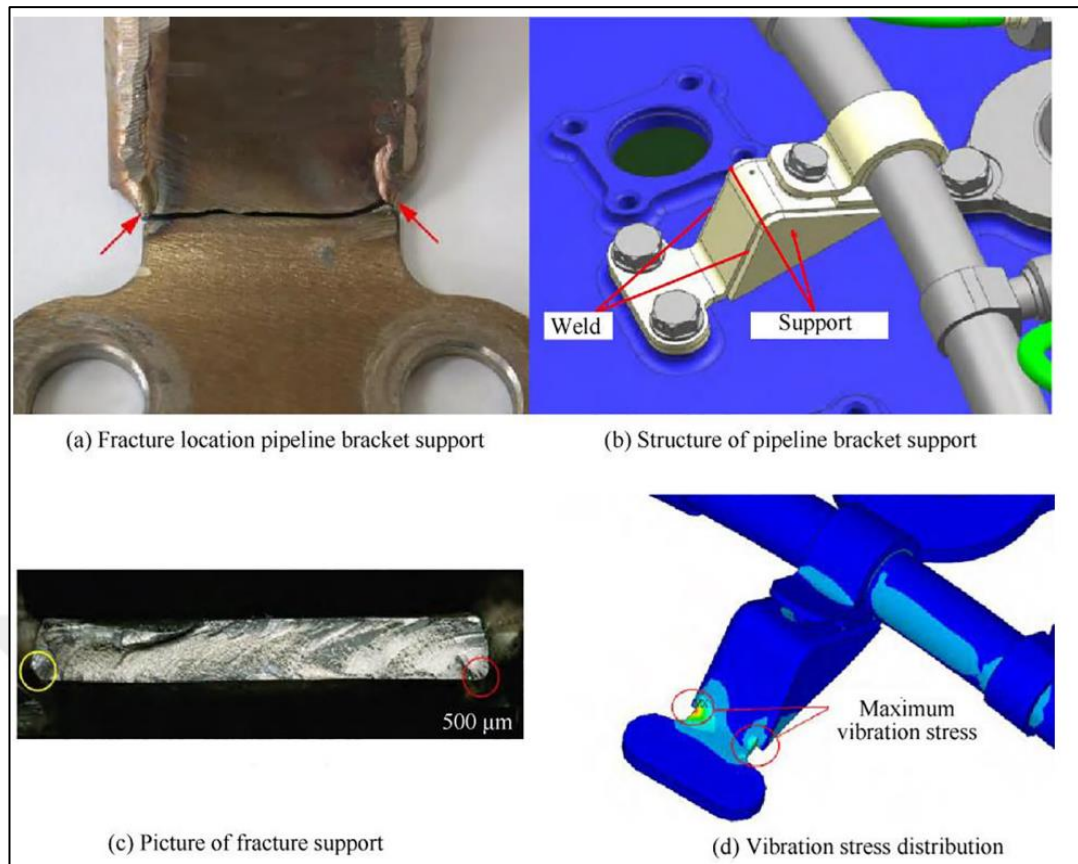


**Figure 1.9:** Vibration failure of a bracket

In recent years, advancements in aircraft development have led to continual improvements in the design of pipeline systems. This progress has increased the reliability requirements for pipeline clamps and supports. To minimize clamp failures, one approach is to enhance the stress corrosion resistance of the support structure. This can be achieved by selecting the appropriate type of clamp and designing a suitable structure to improve the stress levels on the clamp. Additionally, upgrading the materials used for the clamps and supports and improving their environmental adaptability can be beneficial. Another key strategy involves improving manufacturing processes and assembly quality to reduce assembly stress on the clamps and supports.

Clamps can be categorized into two types based on their structure: rigid and elastic. Elastic clamps utilize rubber materials to enhance wear resistance and minimize vibrations in aircraft pipelines, as illustrated in **Figure 1.10**. These clamps consist of a rubber pad, a metal sheet, and a bolt. The bolt and metal sheet work together to pre-tighten and secure the pipeline. On the other hand, rigid clamps, which are typically used in aero-engine pipelines, are made from a combination of metal and rubber, as shown in **Figure 1.10**. A rigid clamp includes metal rubber, a metal sheet, and a bolt.

The metal rubber not only helps to reduce vibrations in the pipeline but also meets high-temperature resistance and wear resistance requirements [4].



**Figure 1.10:** Elastic clamp and rigid clamp

Li conducted an in-depth investigation into the vibration characteristics of a pipeline system using flexible clamp supports. The clamps exhibited piecewise linear stiffness, as demonstrated in **Figure 1.11**, highlighting the innovative design approach [11].

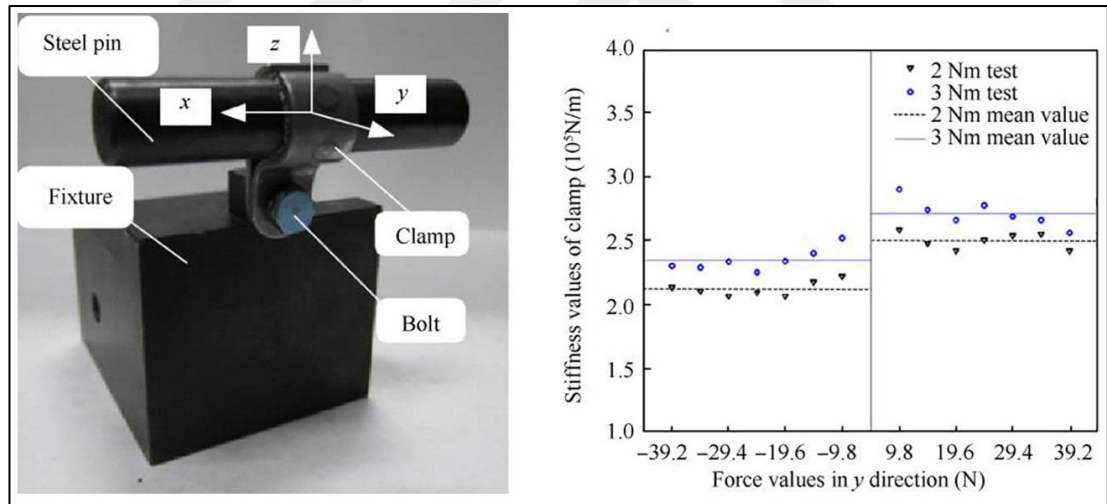


**Figure 1.11:** Experimental set-up and translational stiffness of clamp in y direction

The experiment conducted by Li aimed to analyze the vibrations of clamps under base excitation. The base excitation was generated by a shaker, controlled by an amplified signal from a vibration controller. The study compared two pipelines made from the same material and with identical geometrical configurations. A bare clamp supported one pipeline, while a viscoelastic pad clamp supported the other. An accelerometer was placed on the surface of the pipeline structure, as illustrated in **Figure 1.12**. The test results indicated that the pipeline with the viscoelastic pad clamp significantly

reduced vibration, as shown in **Figure 1.12**. Additionally, compared to the bare clamp, the pipeline with the viscoelastic pad clamp lowered the first-order resonance frequency of the pipeline system [12].

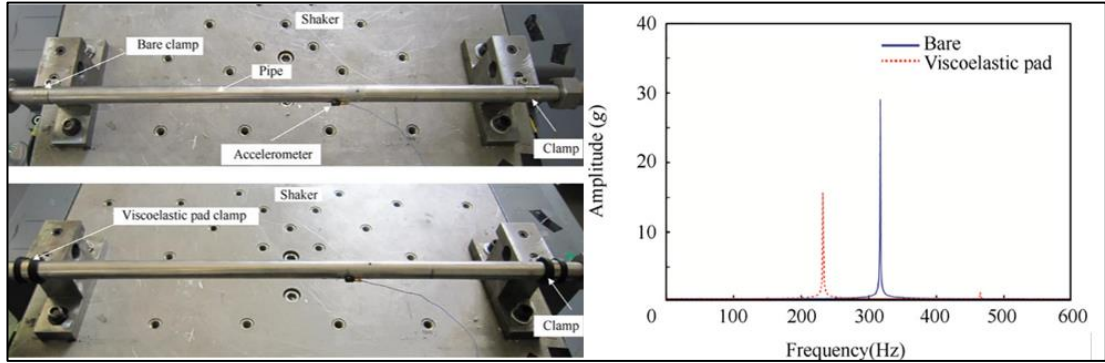


**Figure 1.12:** Vibration response of pipeline with various clamps

Tang proposed an optimization model to reduce vibration by designing the positions of clamps in the hydraulic pipeline system, as shown in **Figure 1.13**. The results showed that the optimization procedure distinctly improved the hydraulic pipeline system's vibration performance. The maximum displacement was reduced by 67.5%, and the maximum axial stress was decreased by 61.5%, which could provide reasonable guidance for the design of a hydraulic pipeline system [13].

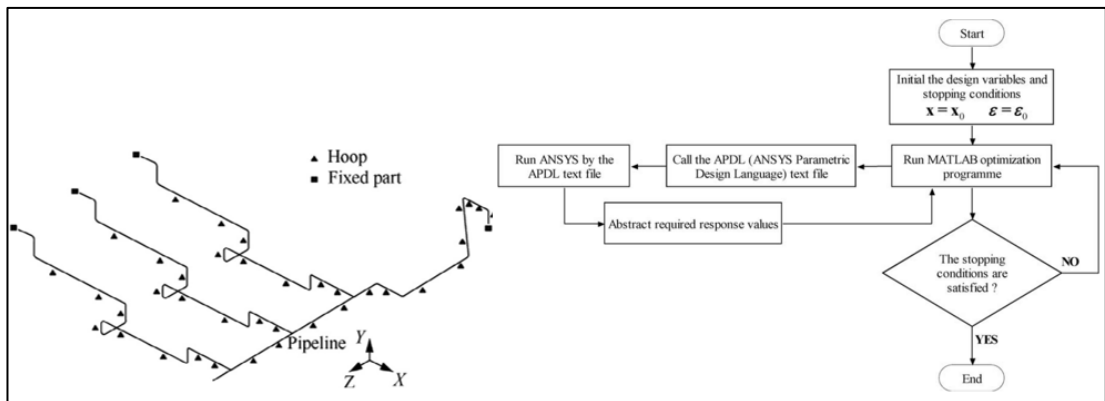


**Figure 1.13:** A hydraulic pipelines system with clamps

Gao proposed a new optimization method for the layout of clamps in hydraulic pipeline systems. A global sensitivity analysis was conducted to evaluate how the position of the clamps affects the difference in natural frequency and the vibration response, as illustrated in **Figure 1.14**. A typical hydraulic pipeline was used to verify the effectiveness of the proposed method. The results demonstrated that optimizing the

clamp layout reduced the maximum vibration stress in the hydraulic pipeline by 43.58% [14].

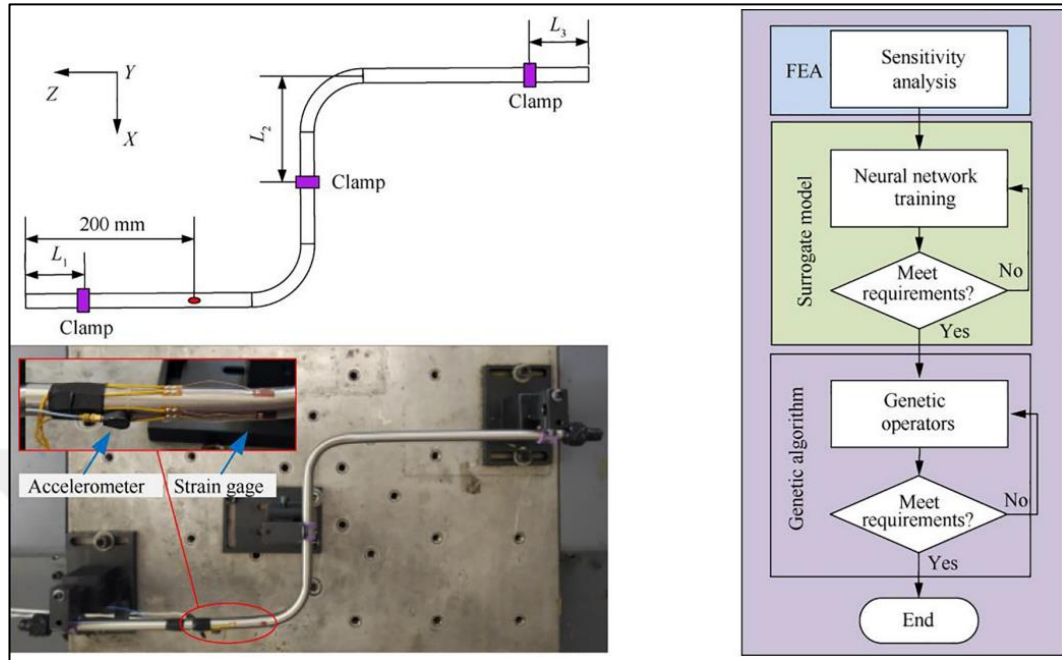


**Figure 1.14:** Optimization layout method for clamps

Gao examined the thickness and elasticity modulus of the viscoelastic and constraining layer parameters using a test setup illustrated in **Figure 1.15** [15].

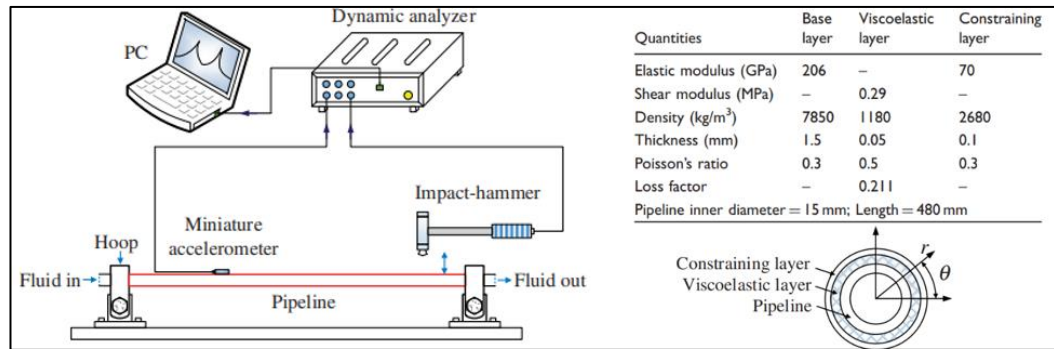


**Figure 1.15:** Schematic diagram of experimental setup and viscoelastic layer

Increasing the fluid velocity and pressure parameters can enhance the damping properties of the pipeline while simultaneously lowering its natural frequencies, as illustrated in **Figure 1.16**. From an engineering perspective, it is crucial to consider the fluid parameters of the constrained layer damping (CLD) pipeline to prevent mechanical resonance. The optimal damping capability is achieved by selecting a

thicker viscoelastic layer combined with a thinner constraining layer, as demonstrated in **Figure 1.17**. An ideal elasticity modulus for both the viscoelastic and constraining layers can maximize damping performance. For modal frequencies, thinner viscoelastic and constraining layers will yield higher natural frequencies. Furthermore, increasing the elasticity modulus of the constraining layer can significantly enhance the natural frequencies, as depicted in **Figure 1.18** [15].

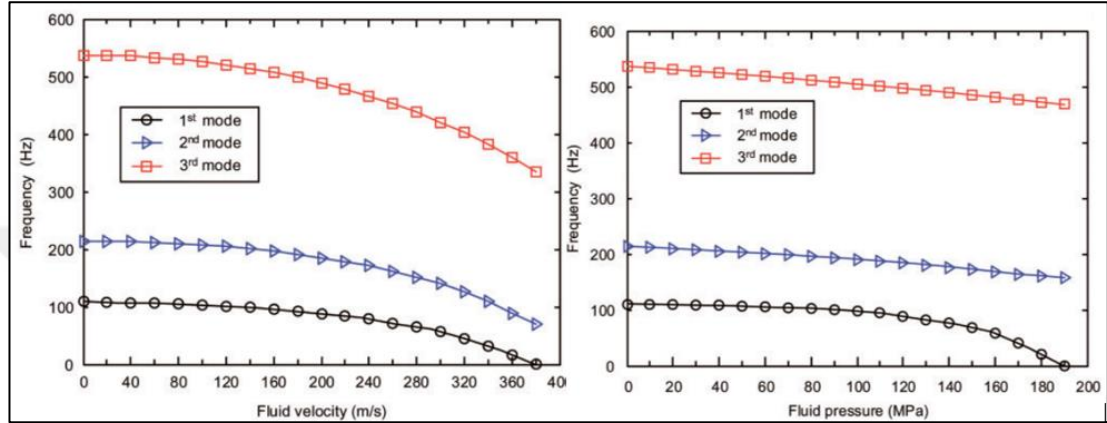


**Figure 1.16:** Influence of the fluid velocity and pressure on the first three modal frequencies

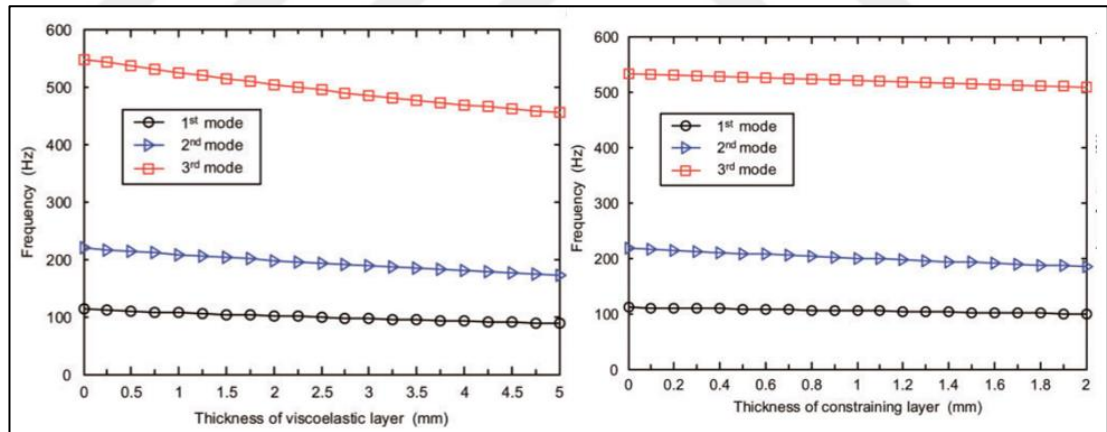


**Figure 1.17:** Influence of viscoelastic layer thickness and containing layer thickness on the first three modal frequencies

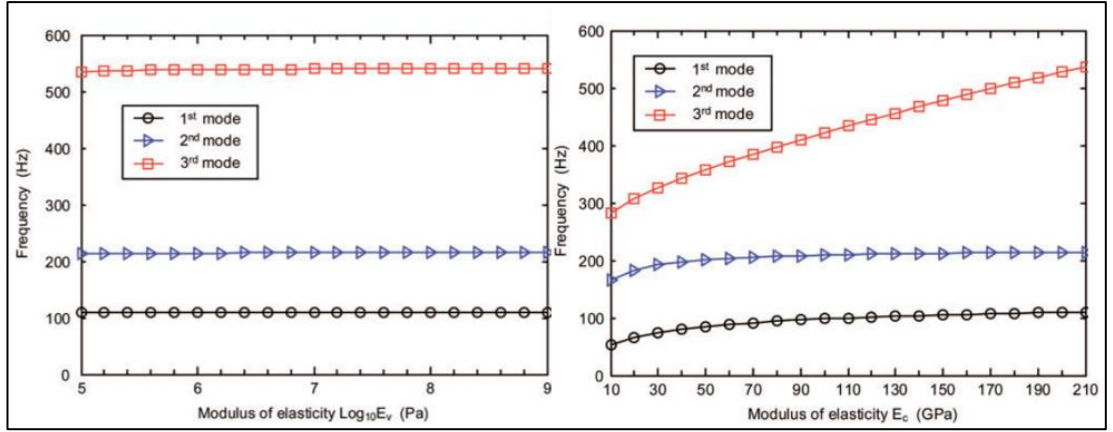


**Figure 1.18:** Influence of the elasticity modulus of viscoelastic layer (left) and containing layer (right) on the first three modal frequencies

Liu introduces a new dynamic modelling method that utilizes pipe-solid element coupling for a spatial parallel pipeline supported by multiple clamps. In this modelling process, the clamp constraints and bending regions should be represented using solid elements (SOLID186). Meanwhile, the other sections of the pipe body can be modelled with pipe elements (PIPE289), as shown in **Figure 1.19** [16].

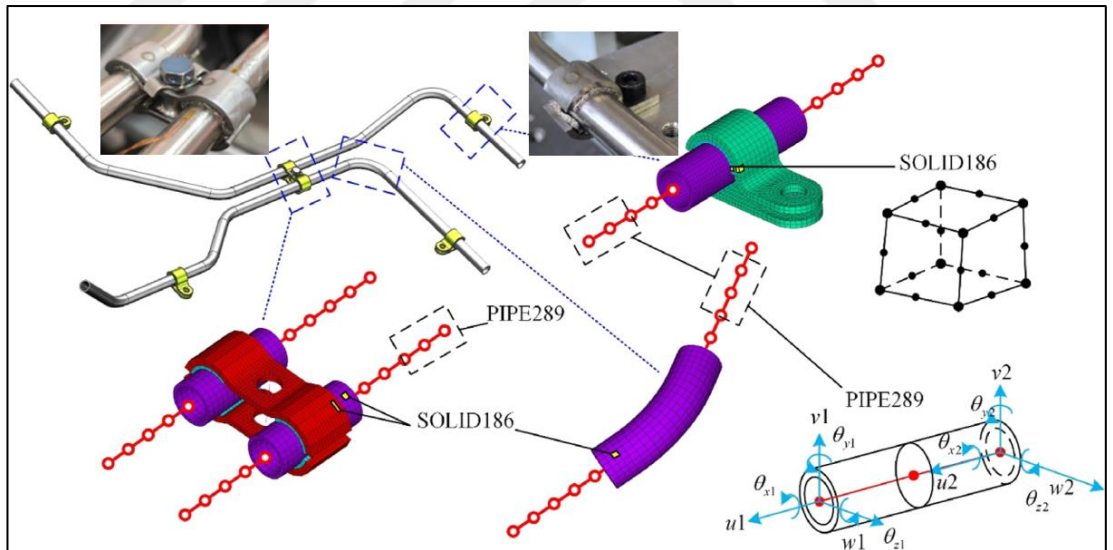


**Figure 1.19:** The modeling idea of pipe-solid element coupling for parallel pipelines

The results reveal that the maximum deviation between the first five natural frequencies obtained through the proposed modelling method and those measured in experiments is 4.98%. Moreover, at three distinct measurement points, the maximum variation in fundamental resonant stress is recorded at -6.27%. In addition, a thorough comparison was conducted between the natural frequencies and vibration stresses

derived from the proposed method and those calculated using the complete solid element method. The analysis demonstrates that while the results from both approaches are broadly consistent, the proposed pipe-solid coupling modelling method offers significantly greater computational efficiency. For example, using identical computational resources, the pipe-solid coupling model accomplished the entire process from modal solving to stress analysis in just 21.53 seconds. This impressive performance represents an increase in efficiency of 688.18 times compared to the traditional full solid model. A parallel pipeline optimization model was developed further to mitigate the fundamental resonant stress within the pipeline, focusing on the adjustment of clamp layouts. The procedure for solving this optimization model was meticulously detailed, incorporating a genetic algorithm. Remarkably, within the specified constraints of clamp mobility in this scenario, the optimization substantially reduced the maximum fundamental resonance stress, achieving a decrease of 45.91% from the initial conditions, as illustrated in **Figure 1.20** [16].

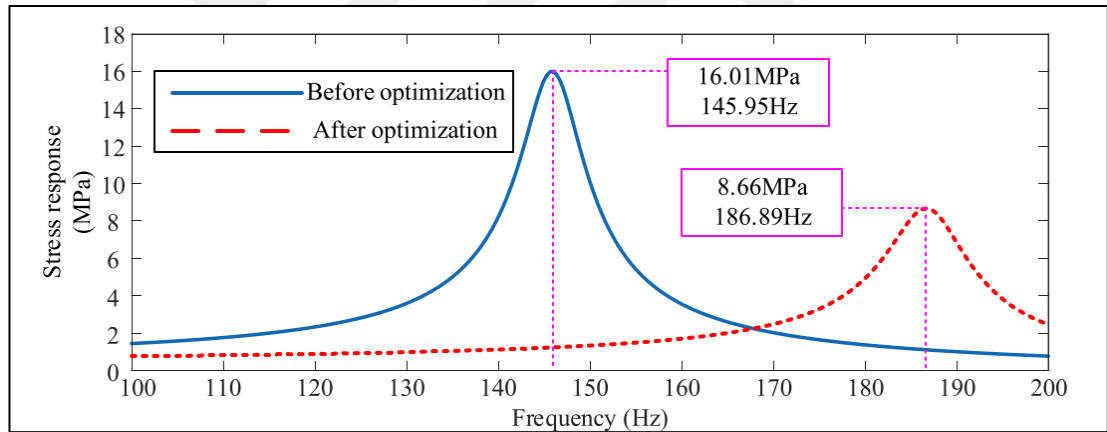


**Figure 1.20:** Comparison of stress response curves before and after optimization

Shang Lv developed an improved dynamic substructure method, known as a reduced-order method, by integrating the dynamic substructure approach with a clamp position parametric model. This innovation enhances the efficiency of subsequent analyses of the influence of clamp position parameters. The effectiveness and rationality of this reduced-order modeling approach are validated through numerical and experimental studies, as illustrated in **Figure 1.21**. Furthermore, the study examines the impact of clamp position and key parameters of various components—such as the pipe body, clamps, and fittings—on pipeline vibration stress. The findings indicate that when the main vibration area of the pipeline system overlaps with the clamp installation region,

significant reductions in vibration stress can be achieved by placing the clamp in the area of maximum modal displacement for the corresponding pipe segment [17].

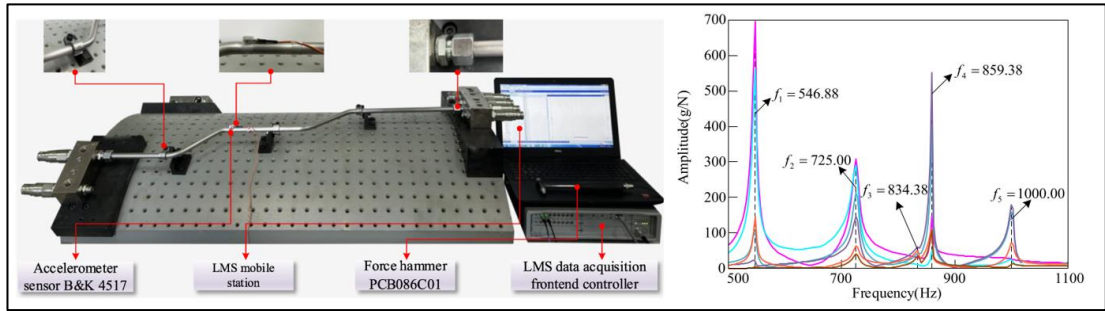


**Figure 1.21:** Test setup and FRF data of the test

The damping property is essential for energy dissipation in structures, enhancing their performance. Wei He conducted an experimental study that examines a lightly damped aluminium bridge without additional damping (**Figure 1.22**) and explores three measures to improve damping: rubber sheet pavements, rubber bearings, and an external passive damper. The analysis shows that the external damper (**Figure 1.25**) significantly outperforms the other methods, expanding damping capacity to 1.5%, a notable increase over rubber bearings (**Figure 1.24**) and much more significant than rubber sheet pavement (**Figure 1.23**). The rubber sheet's low effectiveness stems from the significant difference in elastic modulus between aluminium and rubber, resulting in minimal internal friction during vibrations. Additionally, the external damper's performance is sensitive to its installation location, increasing with modal displacement while showing limited sensitivity to structural frequency. Understanding these dynamics is crucial for selecting the right damping solutions in structural engineering [18].

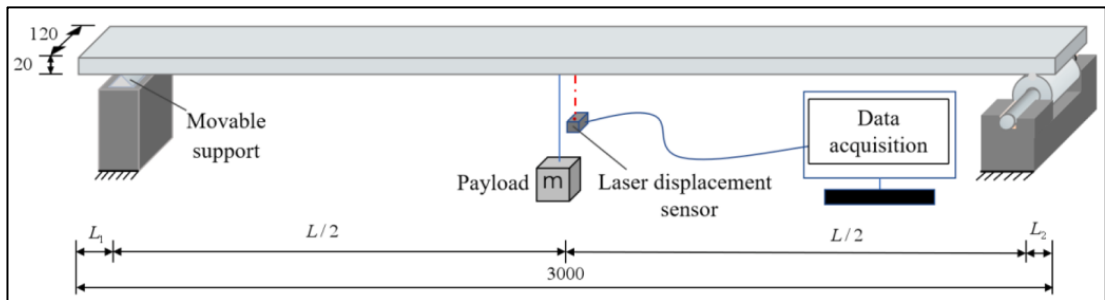


**Figure 1.22:** Case 1, Original structure without additional damping

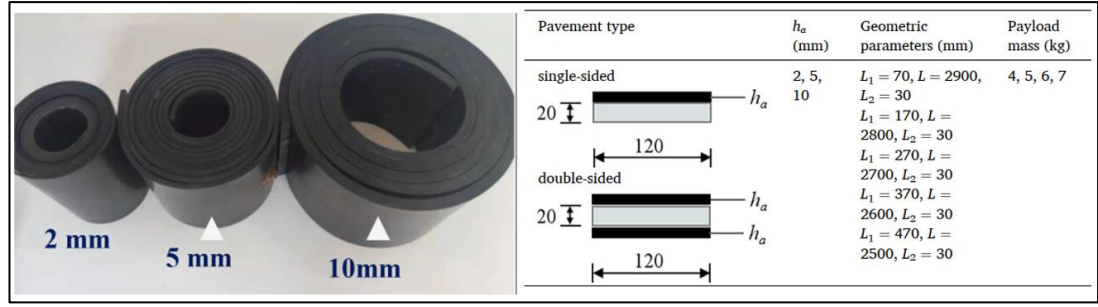


**Figure 1.23:** Case 2, Original structure with additional material damping by rubber sheet pavements

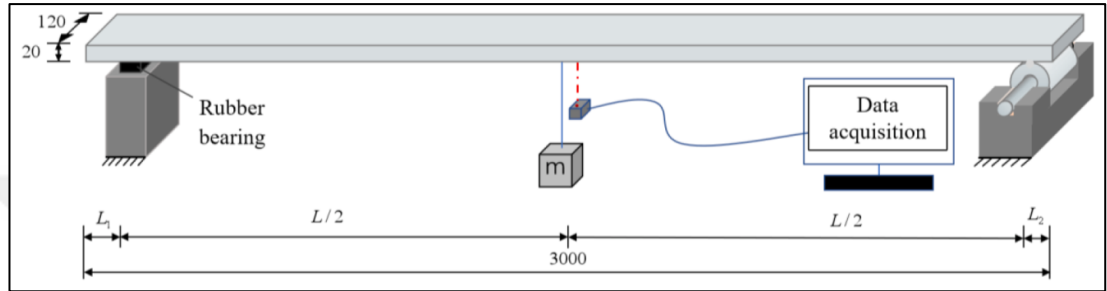


**Figure 1.24:** Case 3, Original structure with additional support damping by rubber bearings

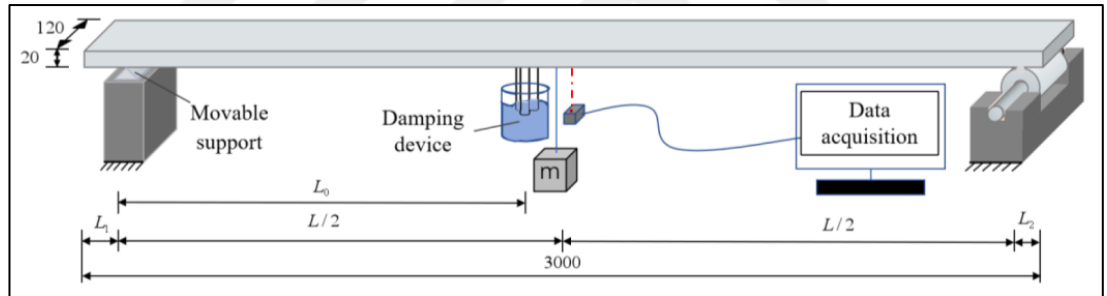


**Figure 1.25:** Case 4, Original structure with additional external damping device

Papagiannopoulos conducted numerical experiments focusing on both single and multi-degree-of-freedom structures. The findings indicate that using the classical form of the half-power bandwidth method can result in significant errors. However, implementing a third-order correction to this method yields more conservative and reliable outcomes. The corrected equation, alongside the classical equation, is illustrated in **Figure 1.26**. Additionally, it was concluded that the half-power bandwidth method should be applied on a mode-to-mode basis to the acceleration frequency response transfer function of the structure. Caution is advised regarding the damping ratio estimates for higher modes, as these should be interpreted carefully [19].

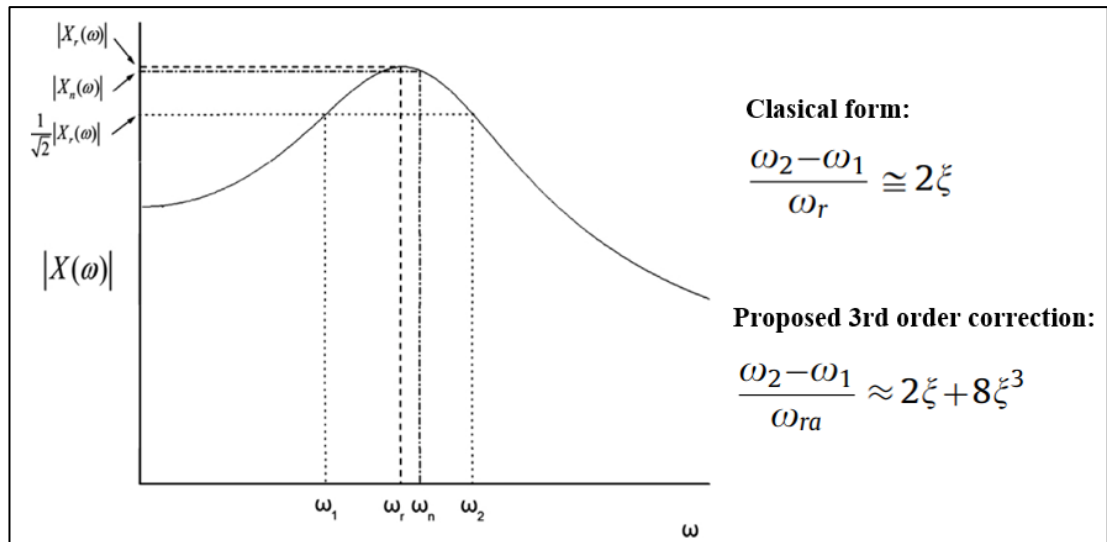


**Figure 1.26:** Classical form and 3rd order corrected form of the hald bandwidth equation

## 2. PURPOSE OF THE STUDY

In our work, the dynamic behavior of a double-bent pipe is investigated under various loading conditions in two configurations: one without support which named configuration-1 and the other supported by a p-clamp at the midpoint. The test specimen consisted of an Inconel 625 AMS 5581 pipe with an outer diameter of 0.188 inches, positioned between two oil jets. The configuration-1, where pipe remained unsupported, is illustrated in **Figure 2.1**.

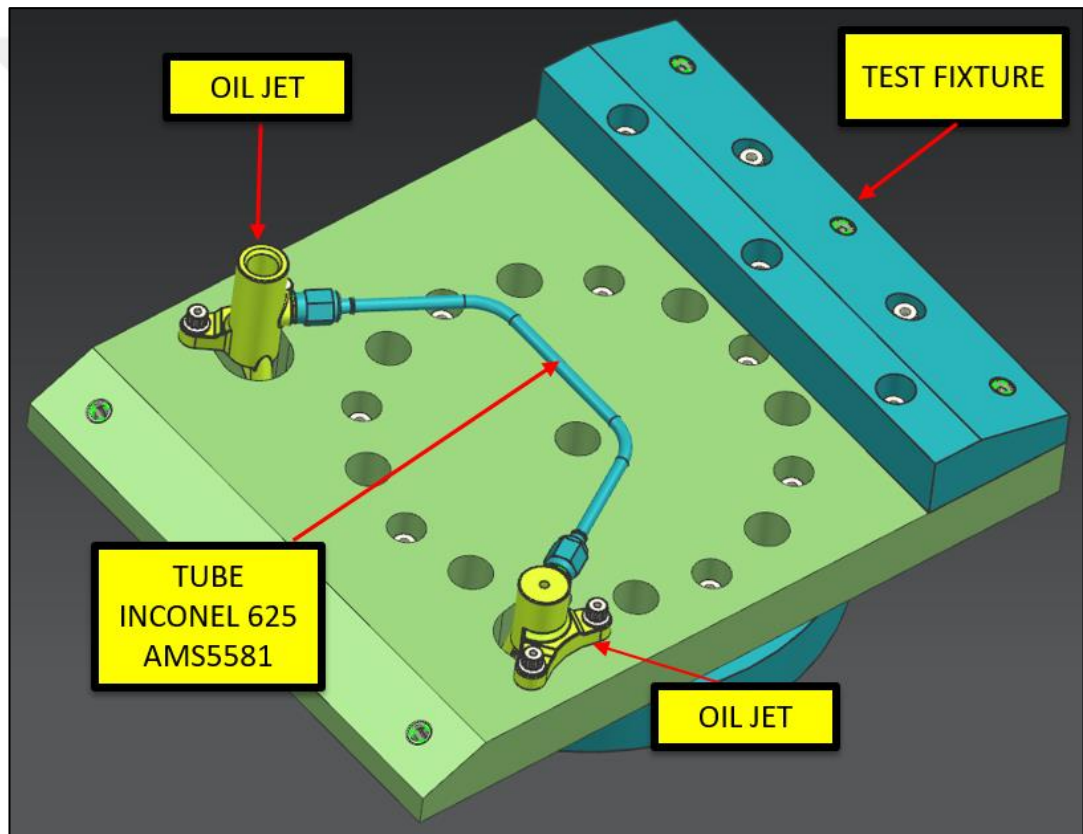


**Figure 2.1:** Test setup for first configuration

The connection between the pipe and the oil jets is established by fastening the AS6230 fitting, which is welded to the pipe with an AS5175 nut. The inseparable assembly is securely tightened to the oil jets with specified torque to ensure structural integrity and prevent leakage, as illustrated in **Figure 2.2**.

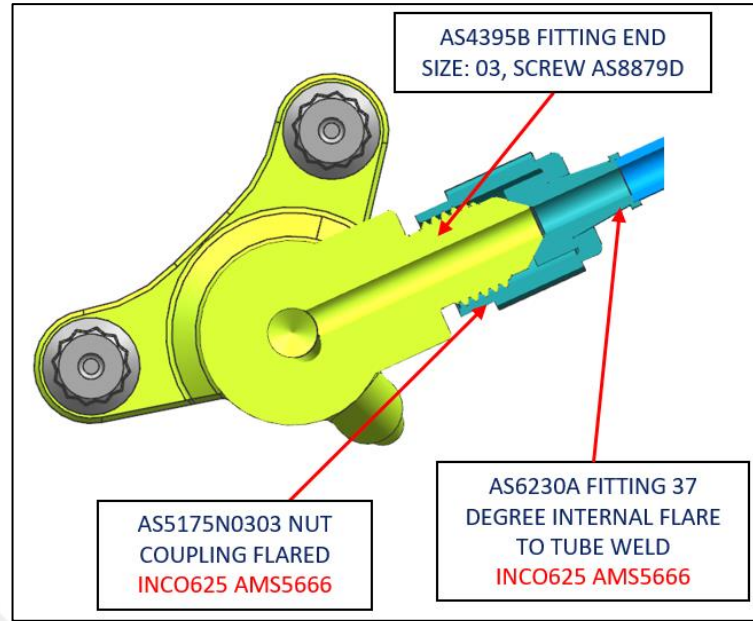


**Figure 2.2:** Tube connections

## 2.1. Validation of the Modal Analysis

In the design of hydraulic pipelines, it is paramount to prevent failures caused by vibrations, focusing on avoiding structural resonance. This necessitates a thorough understanding of the rotating frequencies of aero engines, as these frequencies can significantly heighten the risk of high-cycle fatigue failures. By addressing these concerns, we can enhance the reliability and longevity of the entire hydraulic system. To achieve this, modal analysis is conducted using advanced finite element software.

The modal analysis is a critical technique within finite element analysis (FEA), allowing engineers to uncover essential characteristics such as natural frequencies, mode shapes, and other dynamic properties of the studied structure or mechanical system. This analysis provides invaluable insights into the dynamic behavior of systems, laying the groundwork for a comprehensive understanding of resonance and its associated vibration characteristics. The foundational equation of motion governing a dynamic system is expressed in equation (2.1), offering a mathematical framework to analyze further and validate the system's response to vibrational forces.

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F(t)\} \quad (2.1)$$

For modal analysis, the focus is on the free vibration of the system without external forces ( $F(t) = 0$ ) and typically without damping ( $[C] = 0$ ). The simplified equation becomes equation (2.2).

$$[M]\{\ddot{u}\} + [K]\{u\} = 0 \quad (2.2)$$

The displacement response in free vibration is assumed to have a harmonic form:  $\{u(t)\} = \{\phi\}e^{i\omega t}$ .  $\phi$  is the mode shape,  $\omega$  angular velocity and  $t$  is the time. Substituting this into the governing equation (2.3).

$$-\omega^2[M]\{\phi\} + [K]\{\phi\} = 0 \quad (2.3)$$

Solving the eigenvalue problem, eigenvalues ( $\omega^2$ ) correspond to the square of the natural frequencies of the system. The natural frequencies of the system can be obtained from equation (2.4).

$$f = \omega/2\pi \quad (2.4)$$

Eigenvectors ( $\phi$ ) represent the mode shapes, which describe the deformation patterns of the structure at the corresponding natural frequencies.

**Figure 2.3** illustrates the first and the second configuration of the design, where the pipe is clamped at the midpoint to increase the first mode frequency.

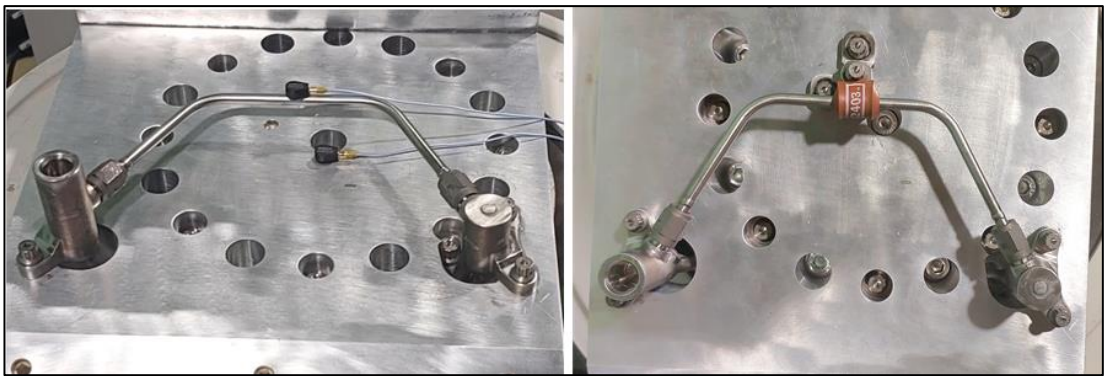


**Figure 2.3:** First (left) and second (right) design configuration

Validating a structure's vibration characteristics is a crucial step in the design process. It ensures that the structure performs as intended and remains safe and functional throughout its lifecycle. The primary aim of this study is to validate the modal analysis

for two different configurations by determining the natural frequencies of the structures and comparing the numerical results with experimental data.

## 2.2. Gathering the Damping Ratio

Uncontrolled vibrations can lead to structural damage in machinery; however, when correctly managed, vibrations may pose no significant threat. Consequently, it is imperative to analyze the effects of vibrations to ensure components' high-cycle fatigue (HCF) life. The Goodman criterion is a guiding framework for managing the HCF life of parts under resonant conditions. Suppose the combined mean stress—resulting from temperature and pressure during operation—and the alternating stress induced by oscillation at resonance remain beneath the Goodman line. In that case, the HCF life of the component is anticipated to exceed  $10^7$  cycles. Mean stress is evaluated through static FEA applied to the structure while alternating stress is obtained from harmonic FEA. Harmonic analysis determines the steady-state response of a structure or system when subjected to sinusoidal loading. This analytical approach is commonly employed to examine the behaviour of systems exposed to periodic forces or vibrations, such as those occurring in rotating machinery, acoustic waves, or oscillating forces. The equation governing harmonic analysis is derived from the general Equation (2.1) of motion for a dynamic system. For harmonic analysis, the applied load is assumed to vary sinusoidally with time. (Equation (2.5)) Similarly, the displacement response is assumed to be sinusoidal. (Equation (2.6))

$$F(t) = F_o \cdot e^{i\omega t} \quad (2.5)$$

$$u(t) = U \cdot e^{i\omega t} \quad (2.6)$$

$F_o$  is the amplitude of the applied force. In harmonic analysis, the system is assumed to behave linearly, meaning the relationship between force and response is proportional. Transient effects are neglected, focusing only on the steady-state behavior. Substituting the harmonic load and response into the equation of motion and considering only steady-state behavior, we get linear algebraic Equation (2.7) solved in the frequency domain to obtain the complex displacement  $U$ , which contains both the magnitude and phase information of the response.

$$(-\omega^2[M] + i\omega[C] + [K])\{U\} = F_o \quad (2.7)$$

$-\omega^2[M]$  represents inertial forces,  $i\omega[C]$  represents damping forces, and  $[K]$  represents stiffness forces.

As illustrated in Equation (2.7), both damping and external force excitation are essential parameters in the harmonic analysis, which requires specific values pertinent to the system. The external load is unique to each system, and the magnitude of this external load must be determined through engine-level testing.

In this study, the values for the damping ratio were acquired by varying the vibration load using the half-bandwidth or 3 dB method. Consequently, the damping ratio obtained through testing was input into the analysis, allowing for observation of the alternating stress distribution on the component through finite element analysis (FEA) software. The 3 dB bandwidth or half-power bandwidth method is used in signal processing, vibration analysis, and acoustics to determine a system's bandwidth or quality factor. This method is predicated on the principle that a reduction of 3 decibels (dB) in amplitude corresponds to a halving of the amplitude level on a logarithmic scale, frequently employed as a criterion to outline the effective range of a system's response.

### 2.3. Effect of the External Load

A sinusoidal acceleration load in vibration testing denotes a harmonic excitation applied to a structure or component characterized by a time-dependent sinusoidal acceleration input. This loading method effectively simulates dynamic conditions, including vibrations induced by engines, rotating machinery, and external forces such as seismic activity or vibrations resulting from transportation. In vibration testing, the dynamic response to the acceleration load is evaluated using the equation of motion (refer to Equation 2.8).

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -M \cdot a(t) \quad (2.8)$$

$-M \cdot a(t)$  is the force induced by the sinusoidal acceleration (inertia force). The applied acceleration is expressed in Equation (2.9).

$$a(t) = A_o \cdot \sin(\omega t) \quad (2.9)$$

$a(t)$  is the instantaneous acceleration as a function of time,  $A_o$  is the peak acceleration as a function of time (e.g, in  $\text{m/s}^2$  or G, where  $1\text{G}=9,81 \text{ m/s}^2$ )

The resulting displacement can be expressed as equation (2.10)

$$u(t) = U \cdot \sin(\omega t - \varphi) \quad (2.10)$$

$U$  is the amplitude of the steady state displacement,  $\varphi$  phase lag between acceleration load and  $u(t)$  is the displacement response.

The response of a mechanical system is contingent upon the relationship between the excitation frequency and the system's natural frequency. When operating below the natural frequency, the amplitude of the response increases with rising frequency, characterized by small displacement responses due to the dominance of stiffness. At the natural frequency, resonance occurs, producing the maximum response amplitude determined by the damping ratio. In this scenario, the system experiences peak displacement and stress. Conversely, when the frequency exceeds the natural frequency, the response amplitude decreases rapidly as the system's mass becomes more dominant. The excitation load significantly impacts the resonance frequency during vibration testing by altering the system's effective stiffness, mass, and damping characteristics. In some instances, elevated excitation levels may induce nonlinear behaviours in stiffness or damping, causing resonance frequency shifts. One may observe softening nonlinearity, in which the resonance frequency decreases with an increase in load, or hardening nonlinearity, where the resonance frequency increases as the load rises. Under low load conditions, the system typically exhibits linear behaviour with negligible frequency variations. However, under high excitation loads, introducing nonlinearities, changes in boundary conditions, and dynamic effects can result in significant shifts in the resonance frequency. Although damping influences the resonance amplitude, excessive excitation loads can affect the energy dissipation mechanisms, indirectly leading to the preload that can significantly modify the effective stiffness, shifting the resonance frequency and minor adjustments in resonance frequency. In systems that introduce frictional effects, the resonance frequency can be modified.

Additionally, in systems subjected to preloads, such as tensioned pipes or clamped beams, the preload can substantially alter the effective stiffness, resulting in a shift of the resonance frequency. This study investigates the effects of changes in excitation load on the resonance frequency. A comprehensive understanding of these factors ensures accurate testing and effective system design.



### 3. EXPERIMENT

As highlighted earlier, the initial validation step to mitigate potential vibration issues involves conducting a thorough structural analysis of the component. This analysis begins with pre-stress modal analysis, where the structure is evaluated to determine its natural frequencies. It is crucial to assess these frequencies because if they align with the operational frequencies of the aero-engine, a resonance phenomenon may arise, potentially leading to detrimental vibrations. To investigate this further, harmonic analysis is performed once resonance is detected. This analysis helps ascertain whether the structural components are at risk of fatigue due to High Cycle Fatigue (HCF) mechanisms, which can significantly impact the longevity and reliability of the structure. ANSYS software is employed for this modal analysis, providing advanced computational capabilities to accurately model the structural behaviour under various conditions. The second step in the part's design validation process involves calibrating the structural analysis findings with empirical test data. It is important to note that discrepancies in natural frequency predictions can arise from various factors, such as inaccuracies in the mesh configurations or inadequacies in the contact definitions used within the Finite Element Analysis (FEA) software. Two distinct design configurations are subjected to experimental testing in a shaker test setup to address these potential inaccuracies. This hands-on evaluation provides critical insights and validates the theoretical predictions, ensuring the design is robust and aligned with real-world performance expectations. The methodology and setup of these shaker tests are explained in detail in the subsequent sections.

#### 3.1. Test Setup Description

A shaker test setup, often used for vibration testing, typically includes the following components: Shaker (vibration generator), power amplifier, controller, test fixture, test specimen, sensors, data acquisition system (DAQ), cooling system, software, isolation base. Components of the shaker test setup is illustrated in **Figure 3.1**. The components interact in a coordinated manner to produce, control, measure, and analyze vibrations. Shaker is the core equipment that produces controlled vibrations. Power amplifier supplies power to the shaker to generate the required vibrations. Controller provides

precise control over the vibration profile. Also it allows user to program and monitor test parameters like frequency, amplitude, and duration. In closed-loop control, the controller ensures the shaker's output matches the test requirement by continuously adjusting inputs. Test fixture is a mechanical structure used securely mount specimen to the shaker. It ensures proper vibration transfer and avoids introducing unwanted modes. Accelerometer measures vibration levels at different points on the test specimen or fixture. It sends analog signals representing vibration data to the DAQ. Data acquisition system captures data from sensors during test. It processes and stores information like acceleration, displacement, and force. For high powered shakers, cooling is a necessary to manage heat dissipation. Software used for test setup, control, and data analysis. It enables plotting, logging, and post-processing of vibration data. Isolation base prevents vibration transmission to the surroundings and ensures external vibrations don't interfere with the test results.

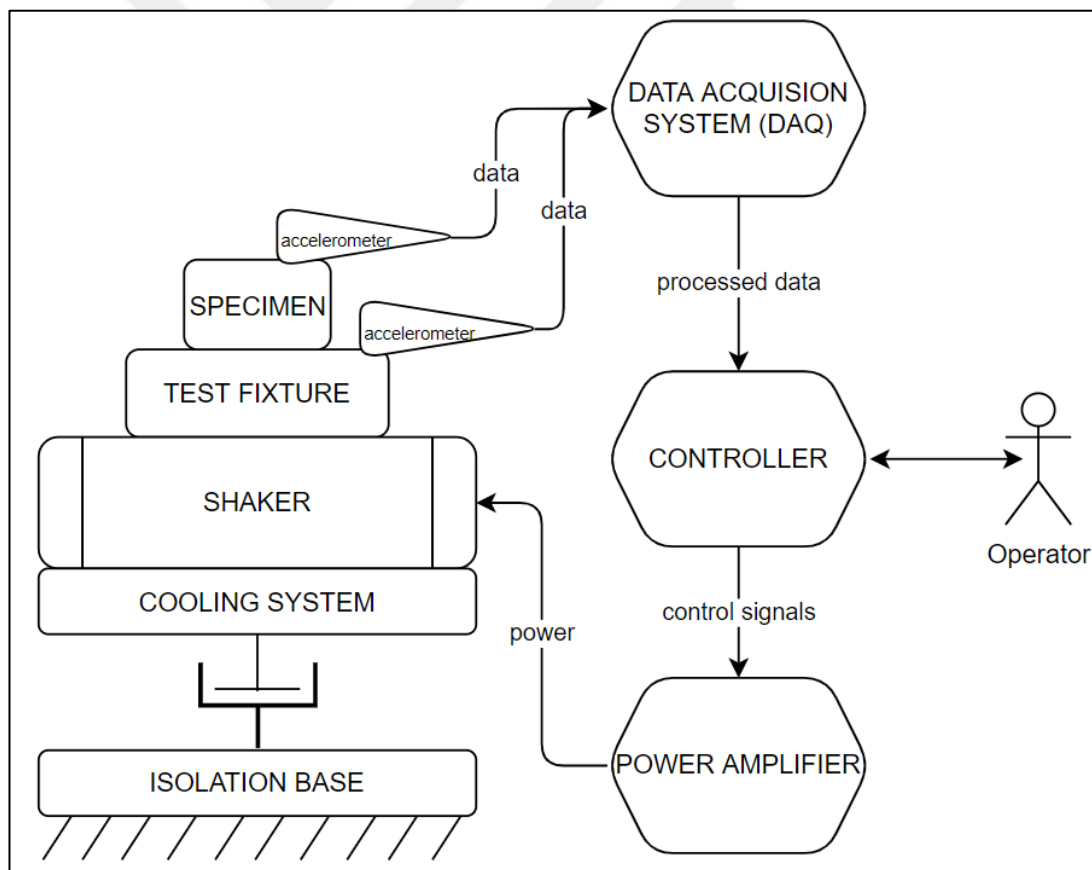


**Figure 3.1:** Shaker test setup

### 3.2. Analysis and Test of the First Configuration

Modal analysis in Ansys Mechanical is a powerful technique used to determine the natural frequencies and mode shapes of a structure or mechanical system. These natural frequencies are critical as they are the points at which the system can resonate, leading to amplified vibrations. Conducting modal analysis early in the design process enables engineers to anticipate and mitigate potential resonance issues, optimizing the system's performance and longevity. It also aids in ensuring compliance with safety standards by identifying at-risk frequency ranges. Consequently, understanding and applying modal analysis not only enhances the reliability and durability of the design but also contributes to better overall engineering practices by harnessing insights into the dynamic behavior of structures and mechanical systems.

Modal analysis is based on solving the eigenvalue problem derived from the equation (2.1) of motion for a vibrating system. For modal analysis, the focus is on the free vibration of the system without external forces ( $F(t) = 0$ ) and typically without damping ( $[C] = 0$ ). The simplified equation becomes Equation (2.2). The displacement response in free vibration is assumed to have a harmonic form:  $\{u(t)\} = \{\phi\}e^{i\omega t}$ .  $\phi$  is the mode shape,  $\omega$  angular velocity and  $t$  is the time. Substituting this into the governing Equation (3.1).

$$-\omega^2[M]\{\phi\} + [K]\{\phi\} = 0 \quad (3.1)$$

Solving the eigenvalue problem, eigenvalues ( $\omega^2$ ) correspond to the square of the natural frequencies of the system. The natural frequencies of the system can be obtained from Equation (3.2)

$$f = \omega/2\pi \quad (3.2)$$

It is assumed material properties and deformations are within the elastic range. Pure modal analysis assumes undamped vibrations. Only free vibrations are analyzed, assumed no external forces. Constraints remain constant throughout analysis.

### 3.2.1. Geometry and Material Properties

The experimental setup and corresponding finite element analysis (FEA) model of the test assembly are illustrated in **Figure 3.2**. The FEA model is based on an aluminum 6061 (Al6061) mounting table rigidly attached to a shaker system to facilitate dynamic excitation. Two IN718 oil jets are mounted onto this table using AS54511 bolts, while a connecting IN625 pipe forms the primary conduit for oil transport between the jets. An AS6230 end fitting is positioned at the terminal end of the pipe to ensure mechanical integrity, and the assembly is stabilized using an AS5175 nut. The analysis utilizes material properties such as Young's modulus, Poisson's ratio, and density derived from the TEI material database. This comprehensive configuration facilitates an in-depth examination and analysis of the mechanical behavior of the components under defined conditions.

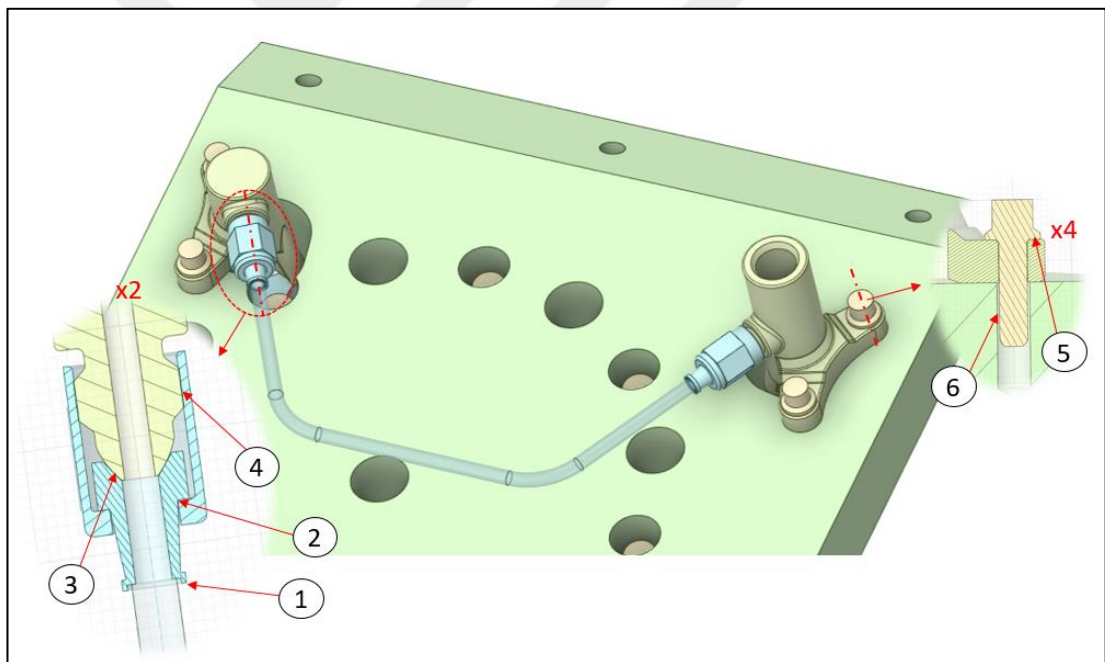


**Figure 3.2:** Contact definitions of the FEA model for configuration 1

### 3.2.2. Boundary Conditions

Ansys Mechanical, contacts are categorized into several types, including bonded, no separation, frictionless, rough, and frictional. The bonded contact type assumes no relative motion between the contacting surfaces, allowing the contact pairs to function as a single continuous entity. This approach is ideal for permanently adhered or welded

components, as it simplifies eigenvalue extraction while enhancing computational performance. Generally, bonded or frictionless contact is preferred to maximize computational efficiency.

During operational conditions, the oil jets are torqued onto the table, and the end fittings are similarly secured. The pipe end fittings are permanently joined by welding. To simplify the FEA model, the weld ring-to-pipe interface (denoted as location 1 in **Figure 3.2**) is idealized using bonded contact conditions. Likewise, the clamped regions (locations 2 and 3) are represented with bonded contacts to incorporate preload effects. The threaded engagement region (location 4), along with the contact between the bolt head surface and the oil jet flange (location 6), is also modeled using bonded contact definitions. These regions encompass the four bolts responsible for securing the oil jets in place.

Ansys Mechanical employs various mathematical theories to address contact mechanics, primarily utilizing the penalty, multi-point constraint (MPC), and augmented Lagrange methods. In this analysis, the MPC method was utilized as the contact formulation. The MPC formulation in Ansys Mechanical is a specialized approach for modelling bonded contacts. It utilizes constraint equations to govern the relative motion between contact and target surfaces, ensuring perfect bonding with minimal computational cost. This efficiency makes it particularly advantageous for scenarios involving bonded contacts, as it does not necessitate iterative adjustments, unlike the Augmented Lagrange or Penalty methods. This study neglects stress-stiffening effects, solving the analysis in a non-prestress environment. The stress stiffening effect accounts for the influence of initial stresses on a structure's stiffness and natural frequencies. For stress-stiffened systems, the total stiffness matrix includes an additional geometric stiffness matrix ( $[K_g]$ ) due to initial stresses given in the Equation 3.3.

$$([K] + K_g)\{u\} = [M]\{u\} \quad (3.3)$$

At the contact regions, compressive stress is present, which contributes to a decrease in natural frequencies by reducing the structure's effective stiffness. Conversely, considering the pipe's tolerances, tensile stresses may arise, leading to an increase in natural frequencies as a result of the system's enhanced stiffness.

### 3.2.3. Mesh

This analysis encompasses modal assessments conducted on both 2D shell element pipe and 3D solid element pipe. The mesh utilized in Ansys Mechanical is a fundamental aspect of finite element analysis (FEA). It partitions the model's geometry into more minor, manageable elements, thereby enabling the approximation of structural behaviour under specified conditions. Collectively, these elements represent the entire geometry, allowing the software to solve complex equations across the defined domain. In Ansys Mechanical, there are three distinct types of mesh based on geometric form: 1D elements, which are linear and typically employed for beams and trusses; 2D elements, consisting of triangular or quadrilateral shapes, commonly used for shells and thin plates; and 3D elements, which include tetrahedral, hexahedral, pyramidal, or wedge forms, primarily used for solid structures. This analysis encompasses modal assessments conducted on both 2D shell pipes and 3D solid pipes. The mesh configurations for both shell and solid elements are illustrated in **Figure 3.3** and **Figure 3.4**.

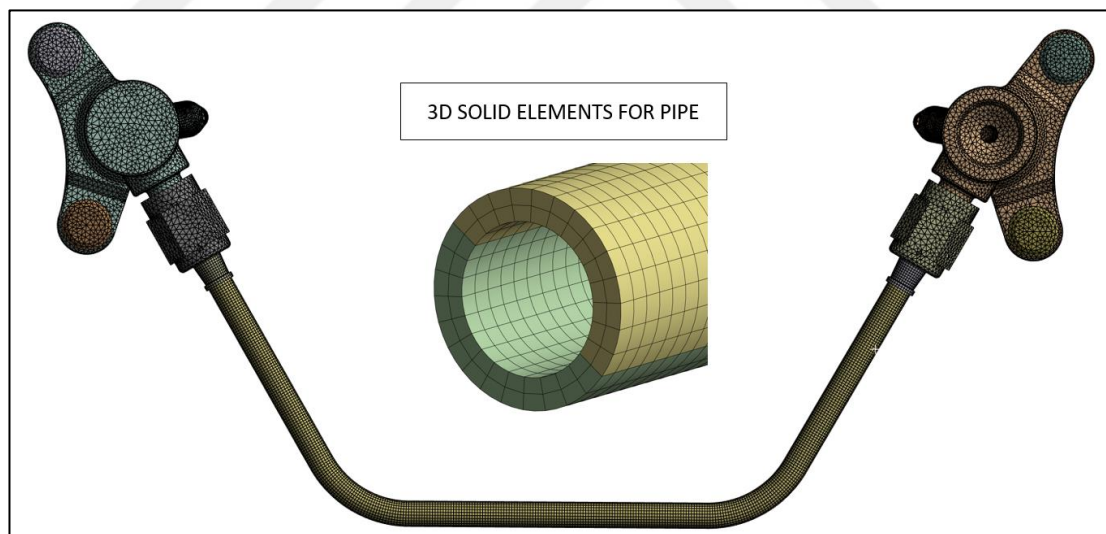


**Figure 3.3:** Solid element mesh structure for configuration 1

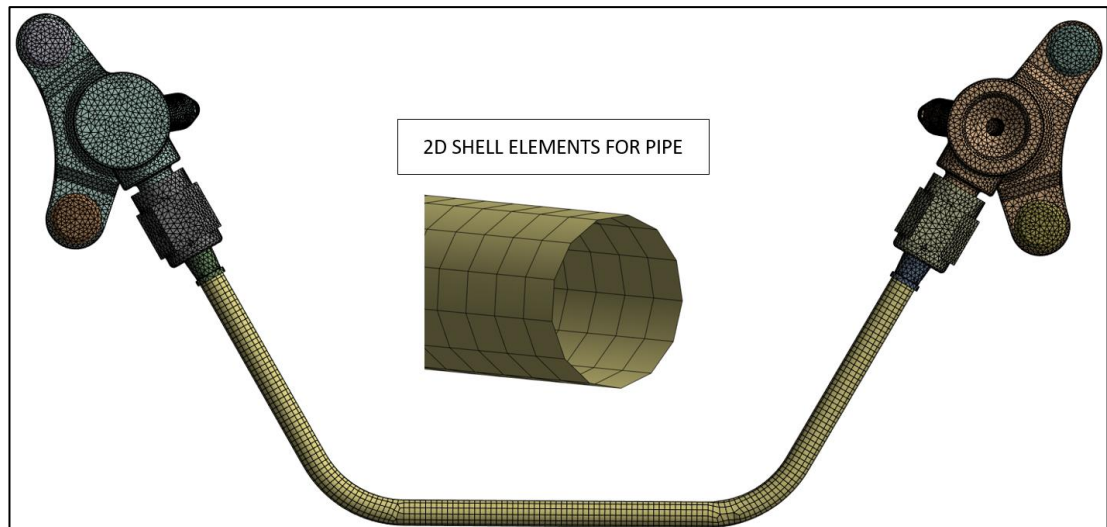


**Figure 3.4:** Shell element mesh structure for configuration 1

A total of 358498 elements and 649527 nodes have been employed in the solid model. In the shell model, there are 343862 elements and 534350 nodes. A comprehensive analysis of the mesh metrics for both models was conducted, and the findings were considered acceptable.

### 3.2.4. Solution and Post-processing

Ansys Mechanical provides a range of solvers designed for the execution of modal analysis. The Block Lanczos method is the default iterative solver, renowned for its efficient handling of large models with a high degree of freedom. This solver employs the Lanczos iteration method to effectively address sizeable sparse eigenvalue problems. It applies to extensive assemblies, intricate geometries, and prestressed modal analyses. The subspace solver operates by projecting the original problem into a reduced subspace, facilitating the determination of eigenvalues and eigenvectors. This approach demonstrates superior speed to Block Lanczos when addressing simpler systems with fewer modes. The PCG Lanczos (Preconditioned Conjugate Gradient Lanczos) represents a variant of the Lanczos method optimized for parallel computation. It incorporates preconditioners to enhance convergence rates. The QR solver utilizes the QR decomposition technique to resolve eigenvalue problems; while offering greater accuracy, it is considerably more computationally intensive than iterative solvers. Furthermore, the AMS (Automated Multilevel Substructuring) solver, another iterative method, partitions the model into smaller substructures, computes the eigenvalues for each segment, and subsequently integrates these results.

This solver is particularly suited for exceedingly large models with millions of degrees of freedom. Lastly, the Unsymmetric Solver is expressly designed for systems characterized by non-symmetric stiffness or mass matrices. It adeptly addresses gyroscopic effects and non-symmetric boundary conditions, making it applicable in scenarios involving rotating machinery, systems exhibiting damping, or coupled-field problems.

**Table 3.1** presents a comparative analysis of the modal frequencies of two structures: the solid-modeled tube and the shell-modeled tube. The findings indicate that the solid-modeled tube demonstrates more stiff behaviour when contrasted with the shell-modeled tube.

**Table 3.1:** Solid and shell model mode frequencies for configuration 1

| Mode | Solid modelled tube | Shell modelled tube | Difference (%) |
|------|---------------------|---------------------|----------------|
| 1    | 593.92              | 567.47              | 4.45%          |
| 2    | 1259.1              | 1216.1              | 3.42%          |
| 3    | 1479.1              | 1424.3              | 3.70%          |

The mode shapes identified throughout the analysis demonstrate a remarkable alignment with the expected outcomes, thereby lending strong support to the initial hypotheses proposed at the outset of the study. This consistency is effectively captured in **Figure 3.5**, which provides a visual representation of the distinct and intricate patterns of vibration associated with the mode of the structure. For instance, the first mode shape typically reflects the fundamental frequency, where the structure undergoes the most significant deformation, while higher modes exhibit increasingly complex patterns of vibration. These shapes not only highlight the various dynamic characteristics of the structure but also offer essential insights that can inform design decisions and predictive maintenance strategies. Analyzing these mode shapes allows engineers and researchers to better comprehend the performance and stability of the structure under operational conditions, thereby enhancing safety and reliability in potential applications.

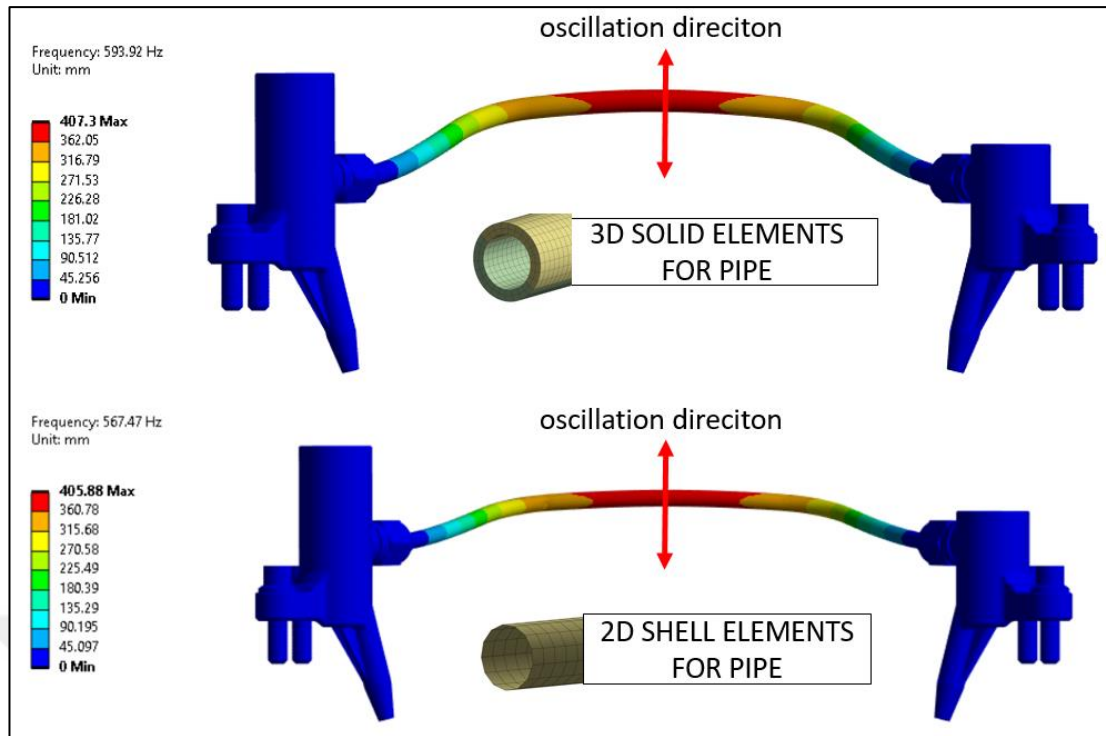


**Figure 3.5:** First mode shape of the configuration 1

### 3.2.5. Shaker Test

A shaker test involves systematically evaluating a test specimen's response to controlled vibrations. The procedure begins with defining test objectives such as identifying resonance frequencies, assessing structural integrity, or validating compliance with vibration standards. Test profile should be specified such as sine sweep, random vibration or shock pulse. The sine sweep test profile is specified which sweeping through a range of frequencies to identify resonances. The test specimen is securely mounted to the shaker's vibration platform using fixtures designed to minimize interference. Sensors, like accelerometers, strain gauges, displacement sensors and force transducers, are attached to critical locations on the specimen to monitor its response. The shaker test setup for Configuration 1 is illustrated in **Figure 3.6**.

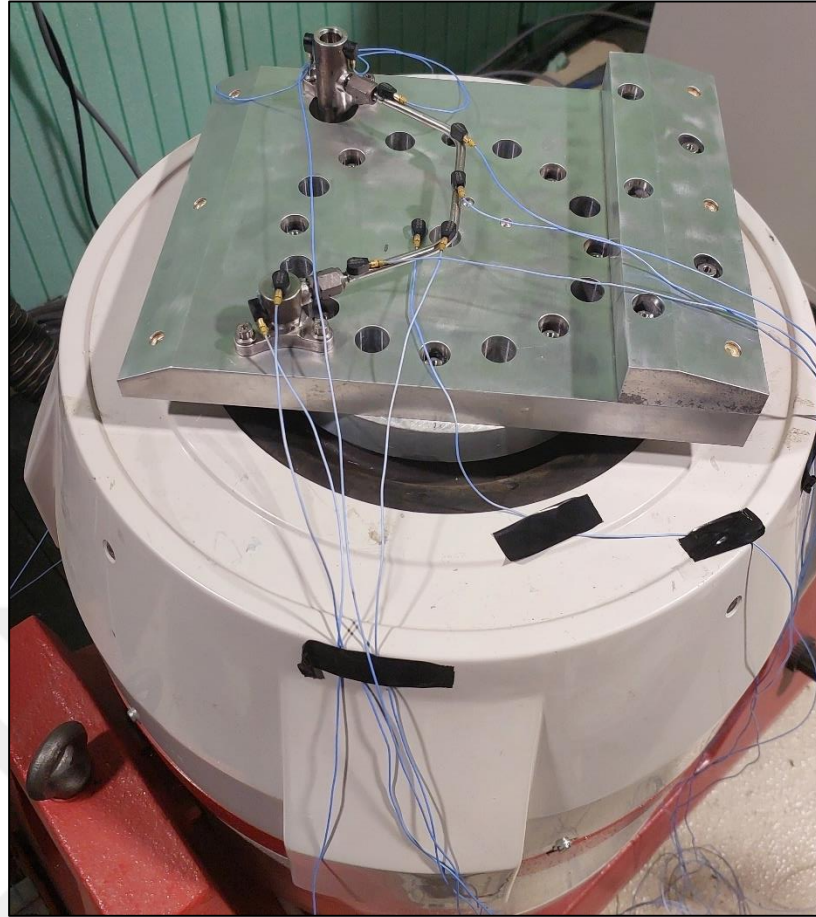


**Figure 3.6:** Shaker test setup of the configuration 1

The shaker system is calibrated, ensuring accurate force, displacement, and frequency generation. The controller is programmed to send excitation signals as sine sweeps through the power amplifier, which drives the shaker. Real-time feedback from sensors helps adjust the shaker's output to maintain the desired vibration profile. A data acquisition system collects vibration response data for analysis during the test. Post-test, the frequency response function (FRF) is calculated to determine resonances, damping, and mode shapes. FRF is a linear-frequency-based transfer function that consists of real and imaginary numbers to calculate magnitudes and phases. Each function is a spectral function of a Fourier transform to compute the relationship between a response of the structure and an applied excitation. The main advantage of FRF is its capability to analyse complex structures that demonstrate noisy vibration responses with high SNRs (signal-to-noise ratio). Besides, FRF data can provide more practical information about the vibration response than limited information captured by modal data, such as coordinates and several modes. FRF is a linear-frequency-based transfer function that consists of real and imaginary numbers to calculate magnitudes

and phases. Each function is a spectral function of a Fourier transform to compute the relationship between a response of the structure and an applied excitation [20].

The Frequency Response Function (FRF) is a fundamental concept in vibration testing and structural dynamics. It illustrates the relationship between an applied input force and the corresponding output response of a system within the frequency domain. The FRF encapsulates the vibrational characteristics by transforming time-domain vibration data into the frequency domain utilizing the Fast Fourier Transform (FFT), as illustrated in **Figure 3.7**. This methodology is essential for comprehensively understanding system behaviour under dynamic conditions.

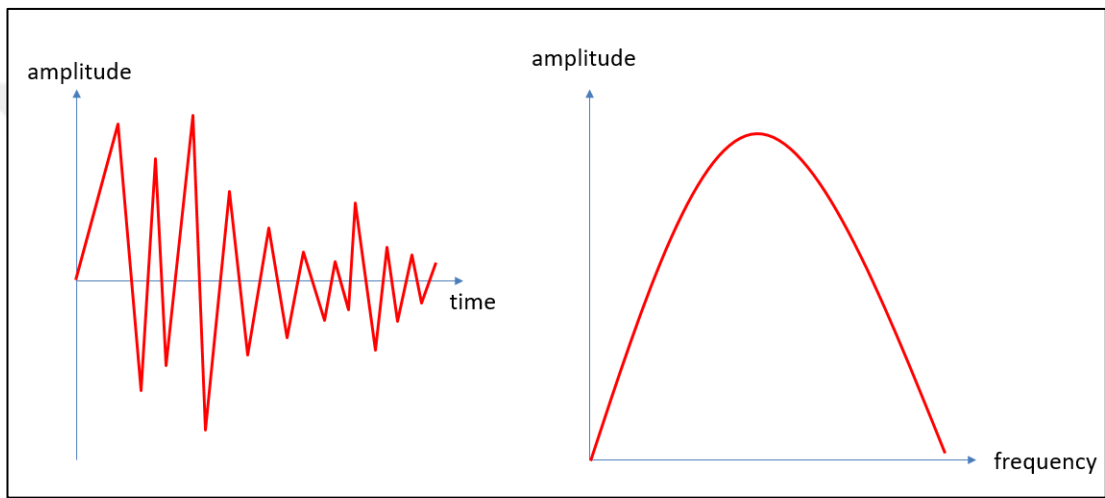


**Figure 3.7:** FRF transformation

The FRF is mathematically expressed as the ratio of the output response  $X(f)$  (e.g., displacement, velocity, or acceleration) to the input excitation  $F(f)$  (force), both as functions of frequency ( $f$ ) (Equation 3.6)

$$FRF(f) = \frac{X(f)}{F(f)} \quad (3.6)$$

The specimen and shaker system are then inspected for damage or wear. Finally, the results, including test conditions, observed responses, and conclusions, are compiled into a detailed report for validation or further analysis. The test was conducted three times using a 1G excitation force. The system's response is illustrated in **Figure 3.8**.

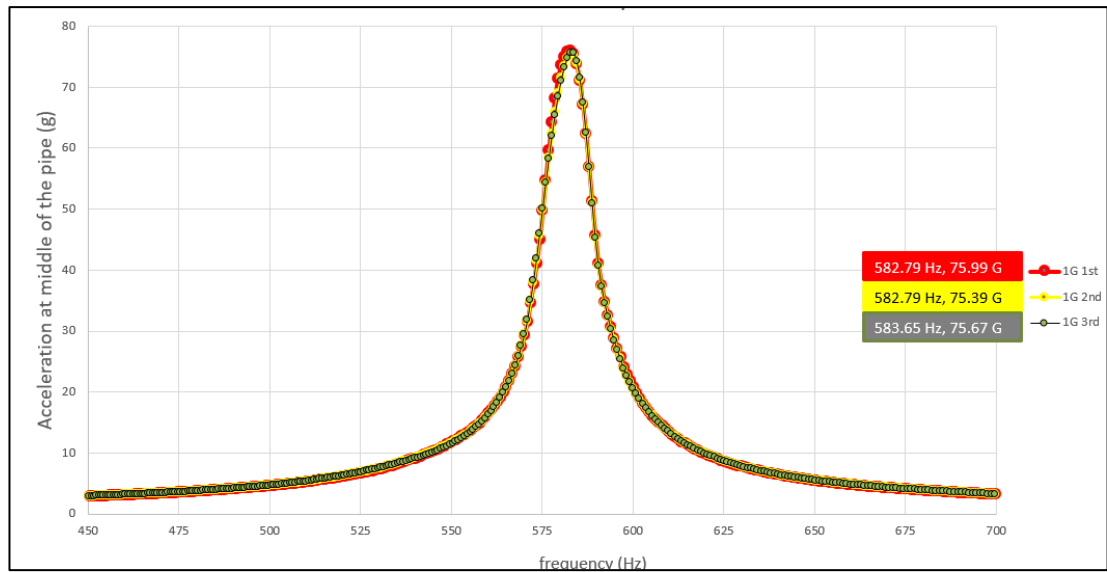


**Figure 3.8:** FRF plot of the configuration 1

### 3.3. Analysis and Test of the Second Configuration

In the second concept, a cushioned clamp (**Figure 3.9**) is integrated into the midpoint of the structure, along with an adapter component that facilitates the integration of this clamp with the fixture. The clamps utilized within gas turbine engine piping systems are essential for mitigating vibration-induced challenges that may result in fatigue, leaks, or damage to the system. These clamps are specifically engineered to secure and support the piping while isolating it from excessive vibrations. Cushioned clamps feature an elastomeric lining, typically composed of rubber or silicone, which dampens vibrations and diminishes the transmission of mechanical energy between the piping and the supporting structure. They are predominantly employed in hydraulics, fuel, and airlines, where the mitigation of vibration and noise reduction is paramount. The typical materials used for cushioned clamps include stainless steel or aluminium combined with high-temperature-resistant elastomers. Commonly utilized cushioned clamps in the aviation sector include p-clamps and saddle clamps. For general applications in aircraft, loop-style clamps should be limited to a diameter of 1 inch for liquid-filled lines and 2 inches for air duct piping. Conversely, saddle-style or line blocks (rigid clamps) are recommended for larger-diameter applications.

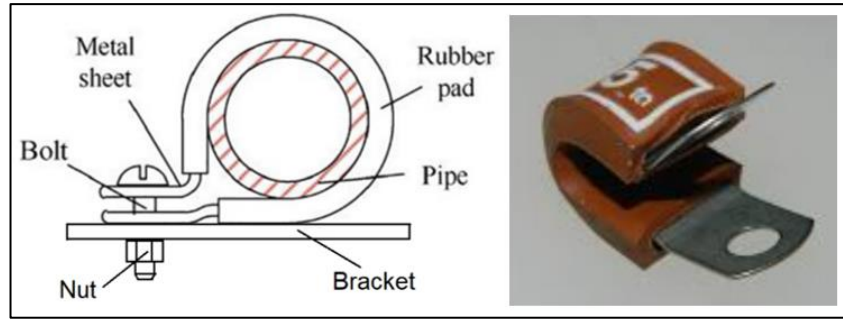


**Figure 3.9:** P-clamp application with a bracket

### 3.3.1. Geometry and Material Properties

In the second configuration, an AS62403 p-clamp is employed for case 2 to secure a 0.188-inch Inconel 625 AMS5581 tube. In the AS62400 series of p-clamps, stainless steel is utilized as the clamp material, while silicone reinforced with glass fibre fabric is the cushioning material. The operational temperature range for this assembly extends from -65 to 300 degrees Celsius. An Al6061 adapter with two bolts is used to secure the pipe to the fixture, as illustrated in **Figure 3.10**.

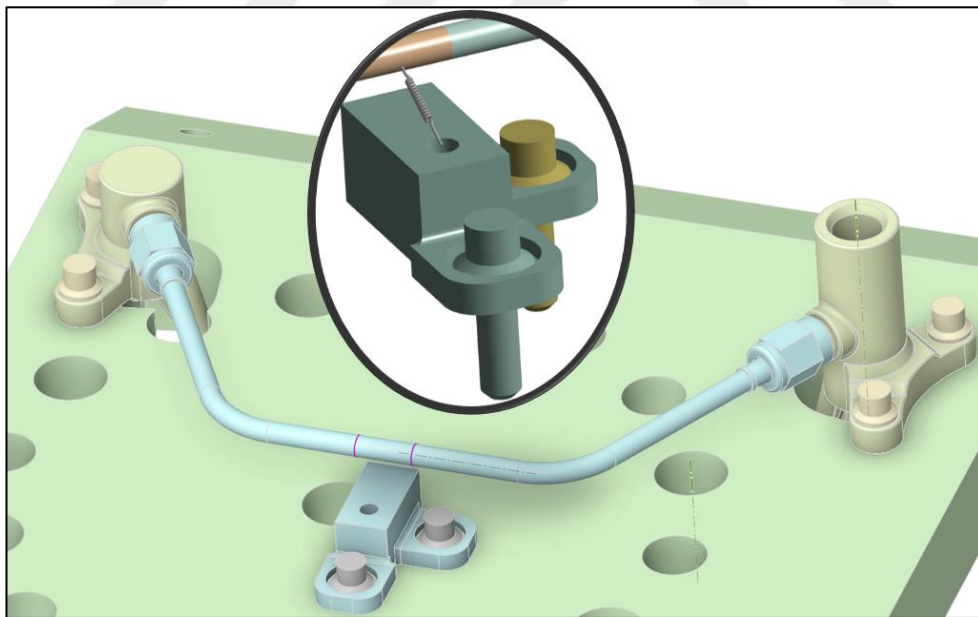


**Figure 3.10:** P-clamp application with a bracket

### 3.3.2. Boundary Conditions

The structure is securely fixed to the bottom surface of the table using bolt holes. Under actual operating conditions, the oil jets are connected to the table with a torque, while

the end fitting of the oil jets is secured with a torque. Additionally, the pipe end fitting is attached through welding. The weld ring-pipe interface, denoted as number 1 in **Figure 3.2**, is represented using bonded contact to simplify the model. Likewise, the clamped areas labelled as 2 and 3, which incorporate the specified preload effect, are also modelled with bonded contact. The thread engagement area, indicated as number 4, is treated similarly. The contact interface between the surface of the bolt head and the oil jet flange for the four bolts securing the oil jets, along with the thread engagement area labelled as number 6, are modelled using bonded contact as well.

Instead of the P-clamp, the spring element available within ANSYS Mechanical was utilized. In ANSYS, spring elements represent linear or nonlinear elastic connections between two points, adhering to Hooke's Law, which outlines the relationship between force and displacement in elastic materials. These elements can simulate various behaviours, including tension, compression, and torsion. The most commonly utilized spring element in ANSYS is COMBIN14, which accommodates linear and nonlinear stiffness. It connects two nodes and facilitates translational or rotational spring-damper behaviour. Spring elements present several advantages: they are simpler to define and implement compared to nonlinear material models, which necessitate extensive material property specifications. Furthermore, they decrease the complexity associated with meshing and geometry creation, as they solely interconnect nodes instead of necessitating a volumetric mesh. Spring elements considerably minimize computation time by involving fewer degrees of freedom and employing more straightforward mathematical formulations. In contrast, nonlinear material models typically require iterative solutions, which can incur significant computational costs. However, it is important to note that spring elements are limited to modelling connections and cannot adequately capture volumetric or intricate three-dimensional behaviours, such as substantial plastic deformations or material damage, which nonlinear materials can simulate. Given that this study emphasizes system-level behaviour rather than detailed stress-strain analysis, spring elements are considered most appropriate. As illustrated in **Figure 3.11**, a spring element is inserted between two remote points that link the pipe clamping area and the bolt hole.

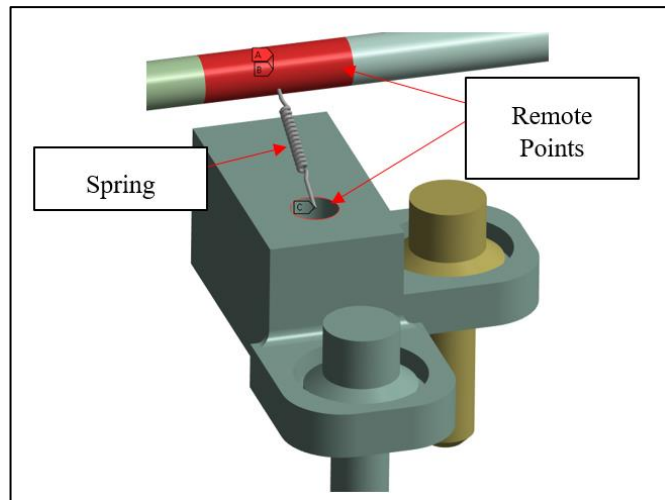


**Figure 3.11:** Remote points and spring definition

A remote point in ANSYS serves as a virtual node or reference point, facilitating the application of boundary conditions, loads, or constraints at a designated location that may not physically exist within the geometry. These remote points are linked to the geometry through constraint equations or multi-point constraints (MPCs), which allow for accurate load transfer or motion control. The utilization of remote points helps to prevent local stress concentrations that could result from the direct application of loads.

### 3.3.3. Mesh

**Figure 3.12** and **Figure 3.13** illustrate the mesh structures, which include both shell and solid elements, in detail. These figures depict the various interpretations of the mesh, showcasing how the shell elements represent thin-walled structures while the solid elements capture the three-dimensional characteristics of the material. This distinction in mesh types is crucial for understanding the structural analysis and performance of the design under various conditions.

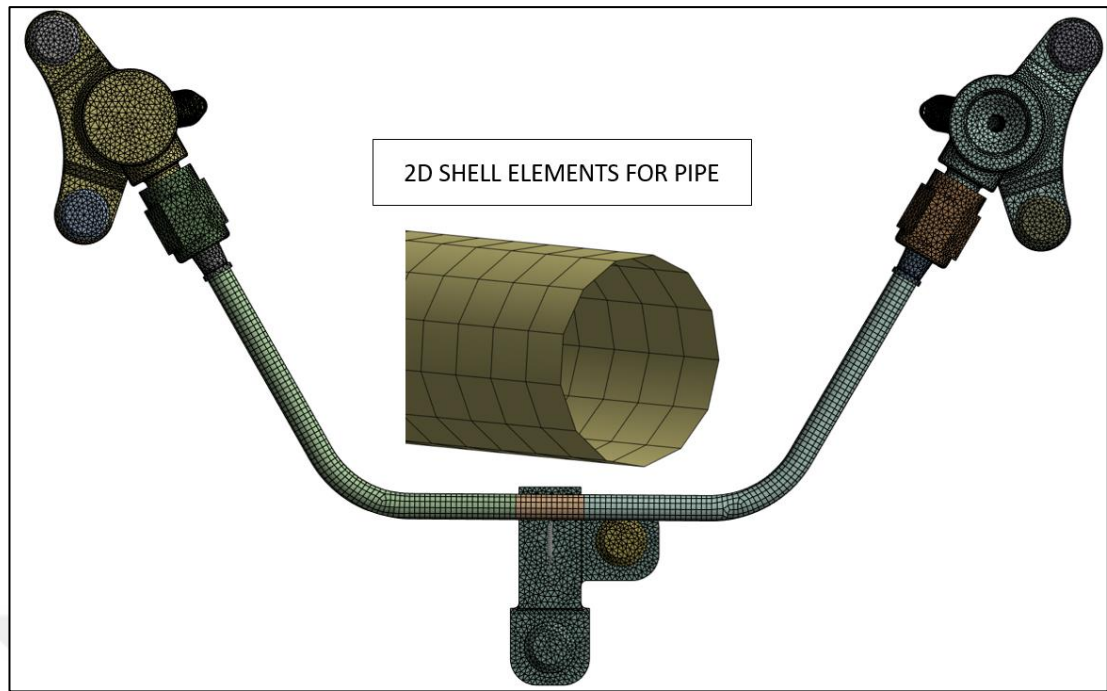


**Figure 3.12:** Shell element mesh structure for configuration 2

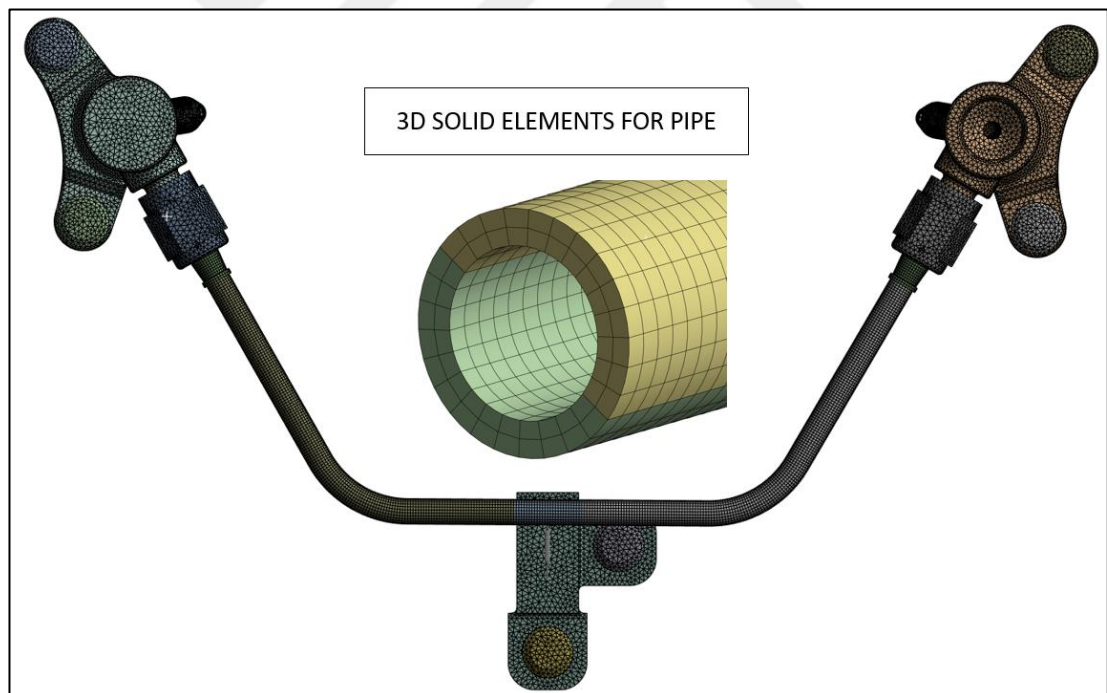


**Figure 3.13:** Shell element mesh structure for configuration 2

### 3.3.4. Solution and Post-processing

The structure's bending mode frequency was found to be elevated to a range between 950 and 1000 Hz when clamping was applied at the midpoint of the pipe. **Table 3.2**

presents a comparative analysis of the mode frequencies for both the solid-modeled tube and the shell-modeled tube. The solid-modeled tube exhibits greater stiffness compared to the shell-modeled tube.

**Table 3.2:** Solid and shell model mode frequencies for configuration 2

| Mode | Solid modelled tube | Shell modelled tube | Difference (%) |
|------|---------------------|---------------------|----------------|
| 1    | 977.24              | 959.32              | 1.83%          |
| 2    | 1311.8              | 1262.2              | 3.78%          |
| 3    | 1496.9              | 1435.1              | 4.13%          |

The first bending mode shape remained consistent with the original configuration as the frequency increased to 977 Hz. The first mode shape is illustrated in **Figure 3.14**.

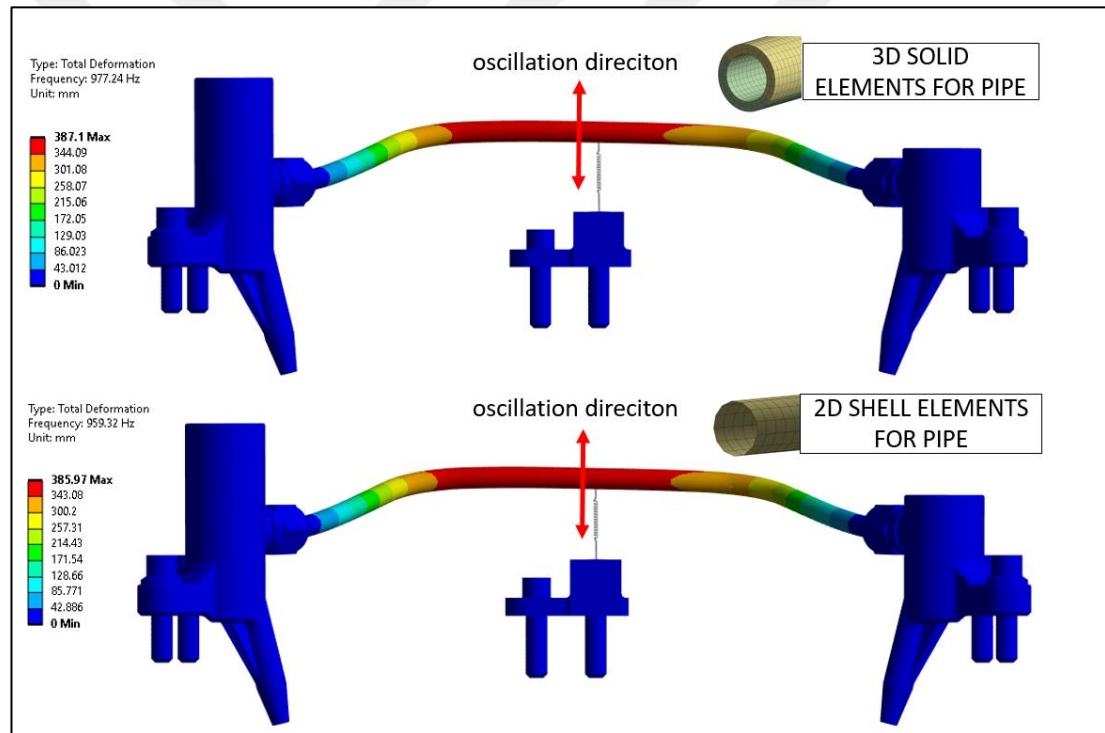


**Figure 3.14:** First mode shape of the configuration 2

The second mode shape is oriented in the tangential direction. This mode shape is illustrated in **Figure 3.15**.

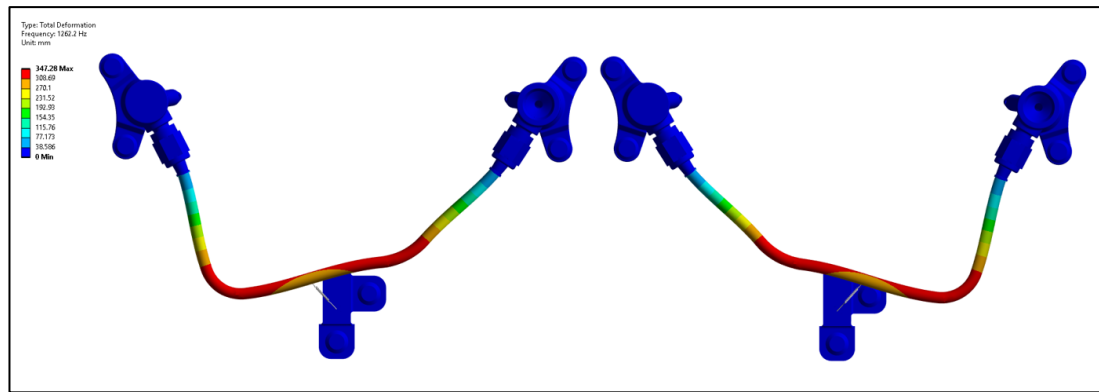


**Figure 3.15:** Second mode shape of the configuration 2

The third mode shape is oriented in the axial direction, as illustrated in **Figure 3.16**.

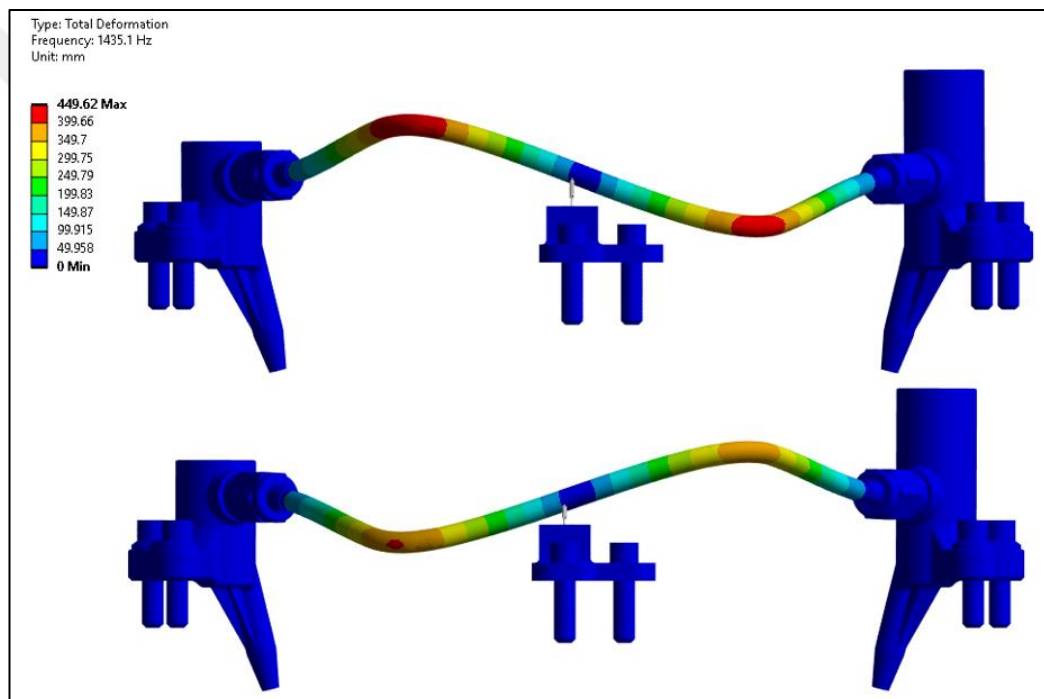


**Figure 3.16:** Third mode shape of the configuration 2

The increase in the frequency of the first, second, and third modes is presented in the table for solid tube models. The first mode exhibits an increase of 64.54%, while the second and third modes show increases of 4.19% and 1.2%, respectively. In the second mode, oscillation occurs in the axial direction of the tube. Due to the lower stiffness in this direction, the increase in mode frequency is minimal. In the third mode, the clamped point acts as an anti-node. Consequently, clamping the pipe at this point does not influence the mode frequency, as demonstrated in the **Table 3.3** and **Table 3.4**.

**Table 3.3:** Mode frequency increase between configurations for solid model first 3 modes

| Mode | Configuration-1 | Configuration-2 | % increase frequency |
|------|-----------------|-----------------|----------------------|
| 1    | 593.92          | 977.24          | 64.54%               |
| 2    | 1259.1          | 1311.8          | 4.19%                |
| 3    | 1479.1          | 1496.9          | 1.20%                |

**Table 3.4:** Mode frequency increase between configurations for shell model first 3 modes

| Mode | Configuration-1 | Configuration-2 | % increase frequency |
|------|-----------------|-----------------|----------------------|
| 1    | 567.47          | 959.32          | 69.05%               |
| 2    | 1216.1          | 1262.2          | 3.79%                |
| 3    | 1424.3          | 1435.1          | 0.76%                |

### 3.3.5. Shaker Test

The procedure for the shaker test is identical to that detailed in section 3.2.5. A block is secured to the fixture using a torque to facilitate the installation of the p-clamp. Subsequently, the p-clamp is attached to the pipe and bolted to the block, also utilizing a torque, as illustrated in **Figure 3.17**.



**Figure 3.17:** Shaker test setup of the configuration 2

The shaker system has been calibrated to guarantee precise force, displacement, and frequency generation. The controller is programmed to transmit excitation signals through sine sweeps conveyed through the power amplifier to the shaker. Real-time feedback from sensors enables adjustment of the shaker's output to align with the desired vibration profile consistently. A data acquisition system systematically collects vibration response data for subsequent analysis during the testing procedure. Following the test, the frequency response function (FRF) is computed to ascertain the specimen's resonances, damping characteristics, and mode shapes. The shaker system and the specimen are then subject to thorough inspection for any signs of damage or wear. Ultimately, the findings, encompassing test conditions, observed responses, and conclusive insights, are compiled into a comprehensive report for validation or further examination. The testing is repeated for varying excitation forces, with system responses presented in **Figure 3.18** for 0.5 G excitation force, **Figure 3.19** for 1 G excitation force, **Figure 3.20** for 5 G excitation force, and **Figure 3.21** for 10 G excitation force.

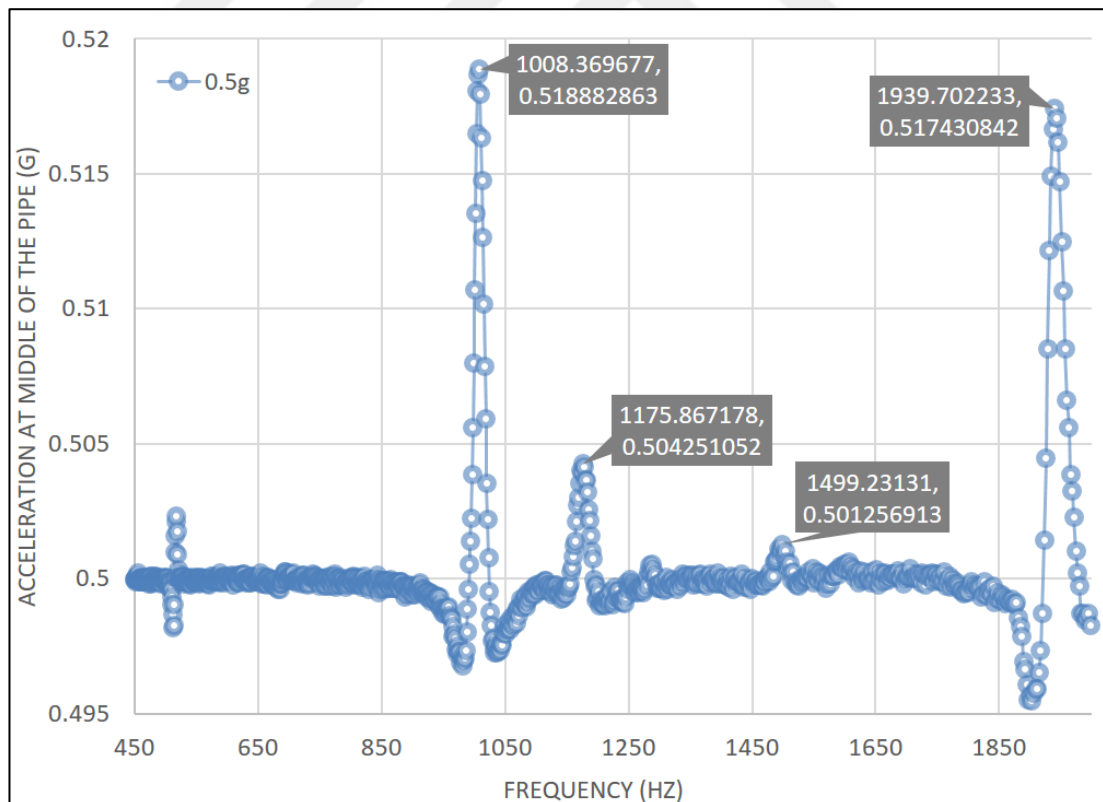


**Figure 3.18:** FRF plot of the configuration 2 with 0.5 G excitation

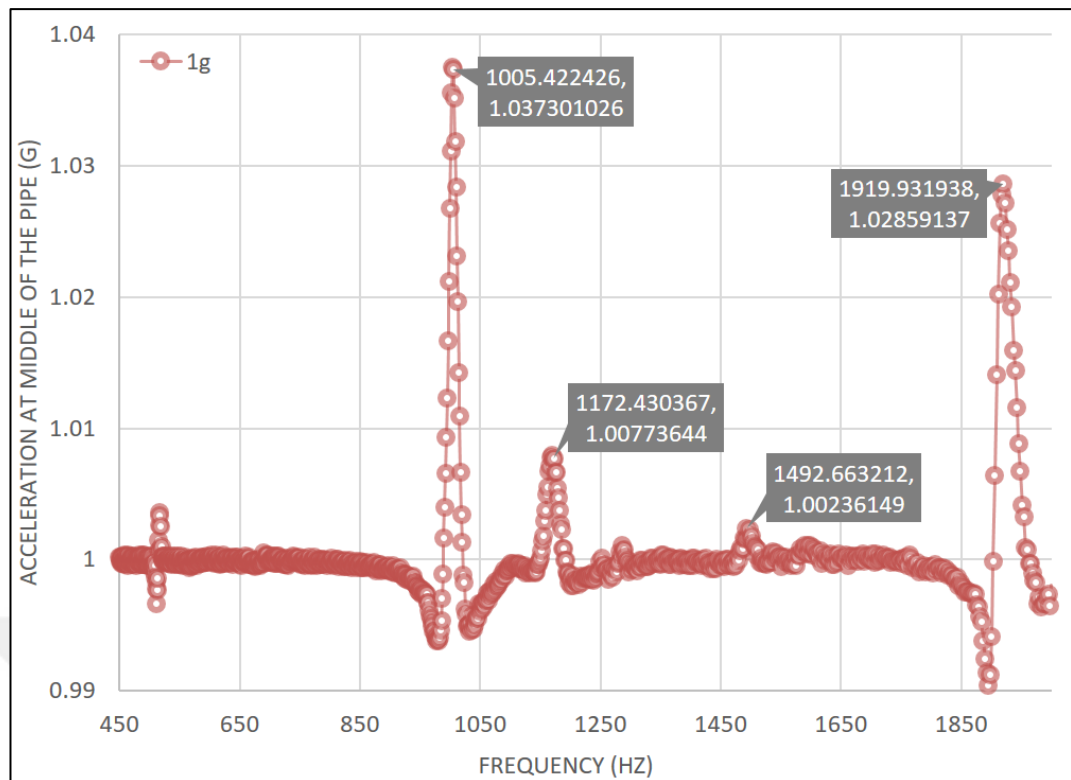


**Figure 3.19:** FRF plot of the configuration 2 with 1 G excitation

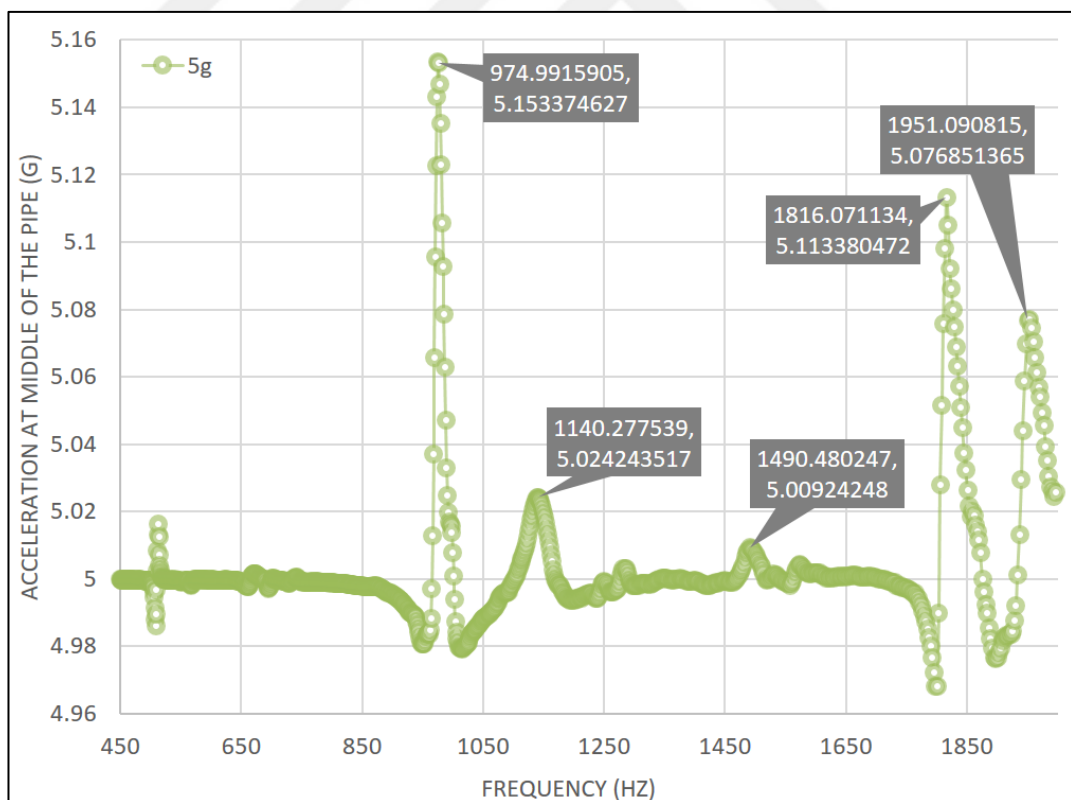


**Figure 3.20:** FRF plot of the configuration 2 with 5 G excitation

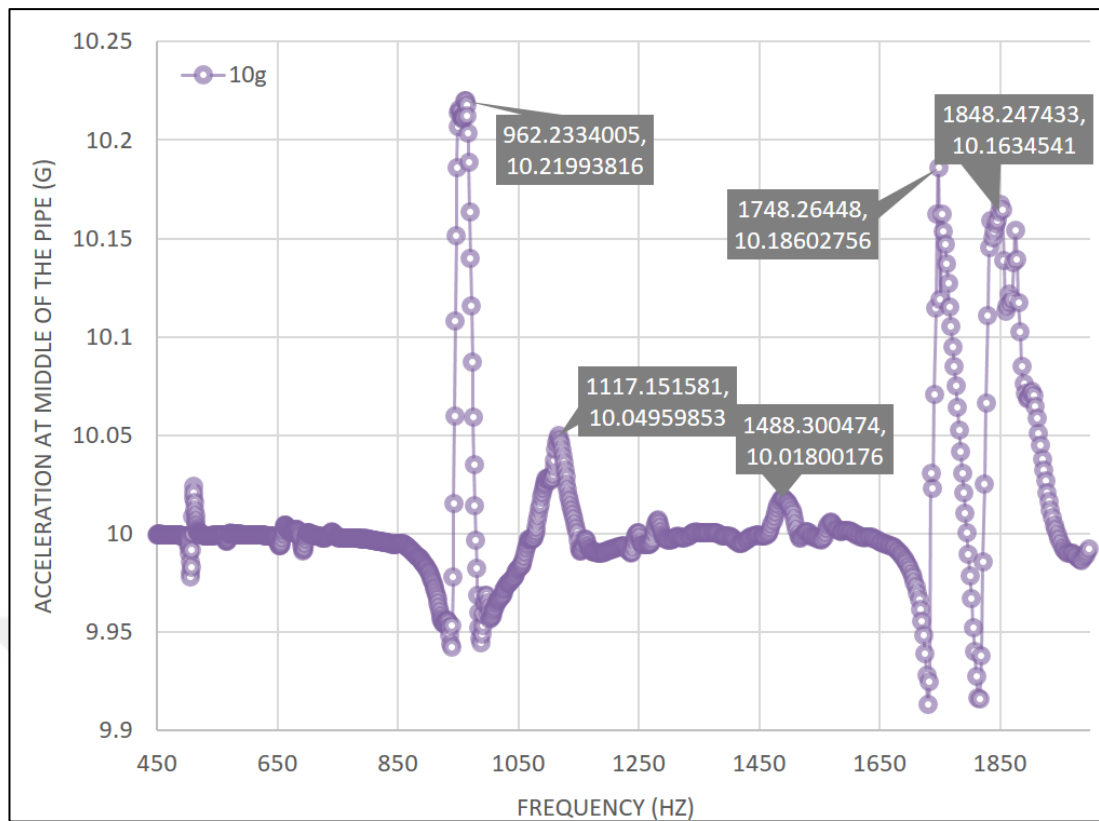


**Figure 3.21:** FRF plot of the configuration 2 with 10 G excitation

## 4. RESULTS

In this study, modal analysis was conducted on two configurations of piping systems. The first configuration, a double-twisted pipe, experienced analysis utilizing two different mesh elements, accompanied by vibration testing at an excitation level of 1 G. The second configuration, a pipe secured with a p-clamp, was similarly analyzed using two different mesh elements and subjected to vibration testing at excitation levels of 0.5 G, 1 G, 5 G, and 10 G. The findings from the analysis and testing are presented herein.

### 4.1. Validation of Mode Frequency

In analyzing the first mode frequency within the initial configuration, three distinct modelling approaches were employed for the pipe: shaker testing, a three-dimensional solid element model and a two-dimensional shell element model. The first mode frequency recorded for the three-dimensional solid element model was 593.92 Hz, while the two-dimensional shell element model yielded a frequency of 567.47 Hz. Additionally, the shaker test produced a frequency of 582.8 Hz. The deviation of the frequency obtained from the three-dimensional solid element model compared to the shaker test was 1.91%, whereas the two-dimensional shell element model exhibited a deviation of 2.63% (as presented in **Table 4.1**). The shaker test was conducted using a high-speed camera, and it was observed that the mode shape confirmed the findings of the modal analysis.

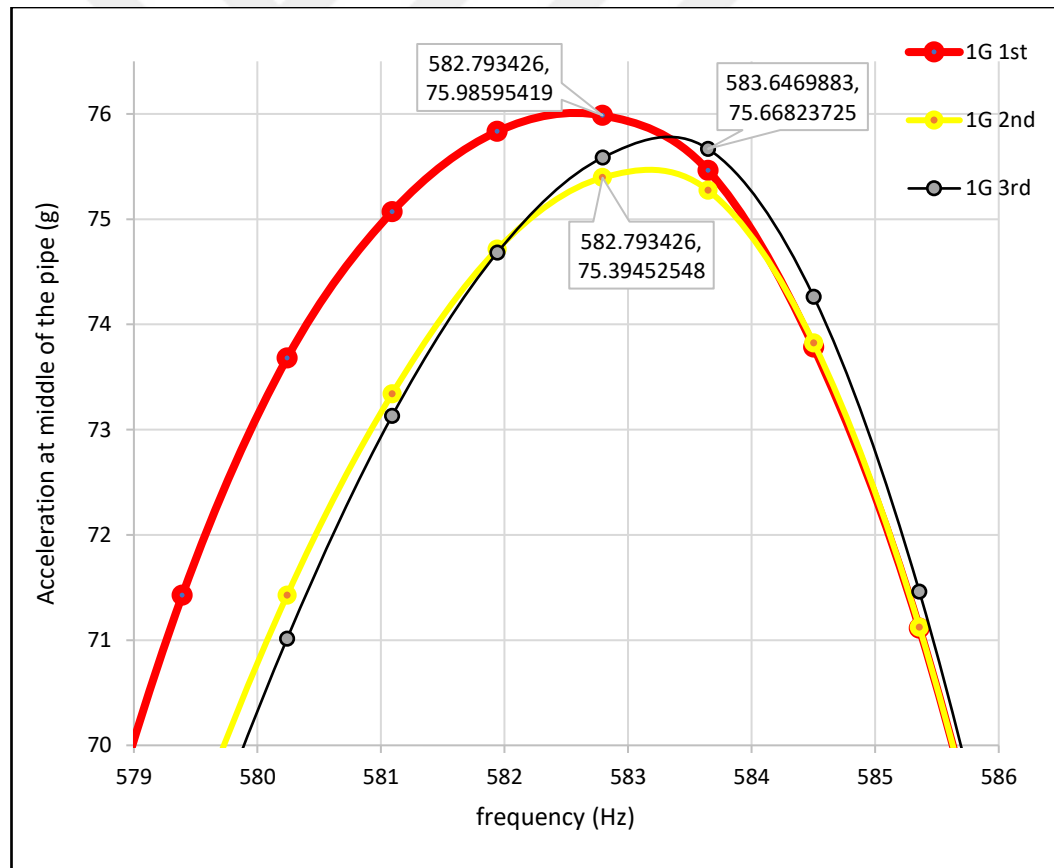
The results of the three shaker tests further demonstrated strong compatibility. Considering the simplifications applied to the contact conditions and the assumptions made throughout the analysis, the results obtained are reasonable. Notably, the three-dimensional solid model provides a more realistic illustration, albeit requiring more extensive computational time.

**Table 4.1:** First mode frequencies for configuration 1 and deviations from test

| Excitation        | 1st MODE<br>FEA Modal solid<br>tube | 1st MODE<br>FEA Modal shell<br>tube | 1st MODE<br>Shaker test |
|-------------------|-------------------------------------|-------------------------------------|-------------------------|
| 1 G               | 593.92                              | 567.47                              | <b>582.8</b>            |
| Difference<br>(%) | 1.91%                               | 2.63%                               | 0                       |

When 1 G of excitation (9806.65 mm/s<sup>2</sup>) is applied during the shaker test, an acceleration of 76 G (745305.4 mm/s<sup>2</sup>) is recorded at the midpoint of the pipe, as illustrated in **Figure 4.1**. The amplitude ratio is subsequently calculated as 76, utilizing Equation 4.1.

$$a_{ratio} = \frac{a_{output}}{a_{input}} = 76 \quad (4.1)$$

**Figure 4.1:** System response for configuration 1

The first mode frequency in the second configuration was assessed through modal analysis using 3D solid and 2D shell elements. The results indicate that the first mode frequency in the 3D solid element model is 977.24 Hz, while the 2D shell element

model yields a frequency of 959.32 Hz. Furthermore, during shaker testing, the frequencies observed were 1008.4 Hz at 0.5 G, 1005.4 Hz at 1 G, 975 Hz at 5 G, and 962.2 Hz at 10 G. The analysis revealed an increase in the first bending mode of 69.05% in the finite element analysis (FEA) shell model, 64.54% in the FEA solid model, and 72.52% during the shaker test, as detailed in **Table 4.2**. The results are normalized with the shaker test frequencies, and the deviations from the test are shared in **Table 4.3**.

**Table 4.2:** First mode frequency increase between configurations with test results

|             | Configuration-1 | Configuration-2 | % increase in frequency |
|-------------|-----------------|-----------------|-------------------------|
| FEA Shell   | 567.47          | 959.32          | 69.05%                  |
| FEA Solid   | 593.92          | 977.24          | 64.54%                  |
| Shaker Test | 582.79          | 1005.42         | 72.52%                  |

**Table 4.3:** First mode frequencies for configuration 2 and deviations from test

| Excitation        | 1st MODE<br>FEA Modal solid<br>tube | 1st MODE<br>FEA Modal shell<br>tube | 1st MODE<br>Shaker test |
|-------------------|-------------------------------------|-------------------------------------|-------------------------|
| 0.5 G             | 977.24                              | 959.32                              | <b>1008.4</b>           |
| Difference<br>(%) | 3.09%                               | 4.87%                               | -                       |
| 1 G               | 977.24                              | 959.32                              | <b>1005.4</b>           |
| Difference<br>(%) | 2.80%                               | 4.58%                               | -                       |
| 5 G               | 977.24                              | 959.32                              | <b>975</b>              |
| Difference<br>(%) | -0.23%                              | 1.61%                               | -                       |
| 10 G              | 977.24                              | 959.32                              | <b>962.2</b>            |
| Difference<br>(%) | -1.56%                              | 0.30%                               | -                       |

The second mode frequency in the second configuration was assessed through modal analysis using 3D solid and 2D shell elements. The results indicate that the first mode frequency in the 3D solid element model is 1311.8 Hz, while the 2D shell element model yields a frequency of 1262.2 Hz. Furthermore, during shaker testing, the frequencies observed were 1175.9 Hz at 0.5 G, 1172.4 Hz at 1 G, 1140.3 Hz at 5G and

1117.2 Hz for 10G. The results are normalized with the shaker test frequencies, and the deviations from the test are shared in **Table 4.4**.

**Table 4.4:** Second mode frequencies for configuration 2 and deviations from test

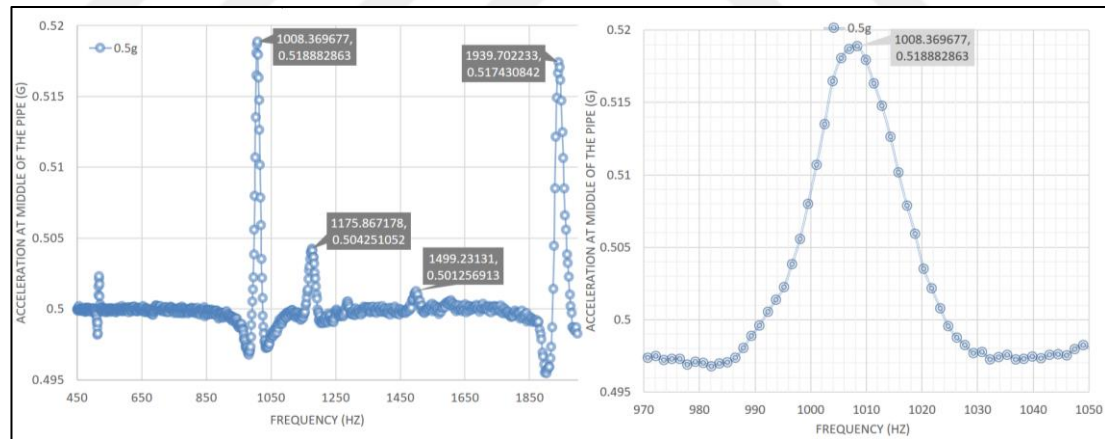
| Excitation        | 2nd MODE<br>FEA Modal solid<br>tube | 2nd MODE<br>FEA Modal shell tube | 2nd MODE<br>Shaker test |
|-------------------|-------------------------------------|----------------------------------|-------------------------|
| 0.5 G             | 1311.8                              | 1262.2                           | <b>1175.9</b>           |
| Difference<br>(%) | 11.56%                              | 7.34%                            | 0                       |
| 1 G               | 1311.8                              | 1262.2                           | <b>1172.4</b>           |
| Difference<br>(%) | 11.89%                              | 7.66%                            | 0                       |
| 5 G               | 1311.8                              | 1262.2                           | <b>1140.3</b>           |
| Difference<br>(%) | 15.04%                              | 10.69%                           | 0                       |
| 10 G              | 1311.8                              | 1262.2                           | <b>1117.2</b>           |
| Difference<br>(%) | 17.42%                              | 12.98%                           | 0                       |

The third mode frequency in the second configuration was assessed through modal analysis using 3D solid and 2D shell elements. The results indicate that the first mode frequency in the 3D solid element model is 1496.9 Hz, while the 2D shell element model yields a frequency of 1452.4 Hz. Furthermore, during shaker testing, the frequencies observed were 1499.2 Hz at 0.5 G, 1492.7 Hz at 1 G, 1490.5 Hz at 5G and 1488.3 Hz at 10G. The results are normalized with the shaker test frequencies, and the deviations from the test are shared in **Table 4.5**.

**Table 4.5:** Third mode frequencies for configuration 2 and deviations from test

| Excitation     | 3rd MODE<br>FEA solid tube | 3rd MODE<br>FEA shell tube | 3rd MODE<br>Shaker test |
|----------------|----------------------------|----------------------------|-------------------------|
| 0.5 G          | 1496.9                     | 1452.4                     | <b>1499.2</b>           |
| Difference (%) | 0.15%                      | 3.12%                      | 0                       |
| 1 G            | 1496.9                     | 1452.4                     | <b>1492.7</b>           |
| Difference (%) | 0.28%                      | 2.70%                      | 0                       |
| 5 G            | 1496.9                     | 1452.4                     | <b>1490.5</b>           |
| Difference (%) | 0.43%                      | 2.56%                      | 0                       |
| 10 G           | 1496.9                     | 1452.4                     | <b>1488.3</b>           |
| Difference (%) | 0.58%                      | 2.41%                      | 0                       |

During the shaker test, an excitation of 0.5 G (4903.325 mm/s<sup>2</sup>) was applied, resulting in an observed acceleration of 0.519 G (5089.651 mm/s<sup>2</sup>) at the midpoint of the pipe, as illustrated in **Figure 4.2**. Consequently, the amplitude ratio is calculated to be 1.038.

**Figure 4.2:** System response for configuration 2 at 0.5 G excitation

During the shaker test, an excitation of 1 G, equivalent to 9806.65 mm/s<sup>2</sup>, is applied. At the midpoint of the pipe, an acceleration reading of 1.037 G (10169.5 mm/s<sup>2</sup>) is observed. As illustrated in **Figure 4.3**, the amplitude ratio is determined to be 1.037.

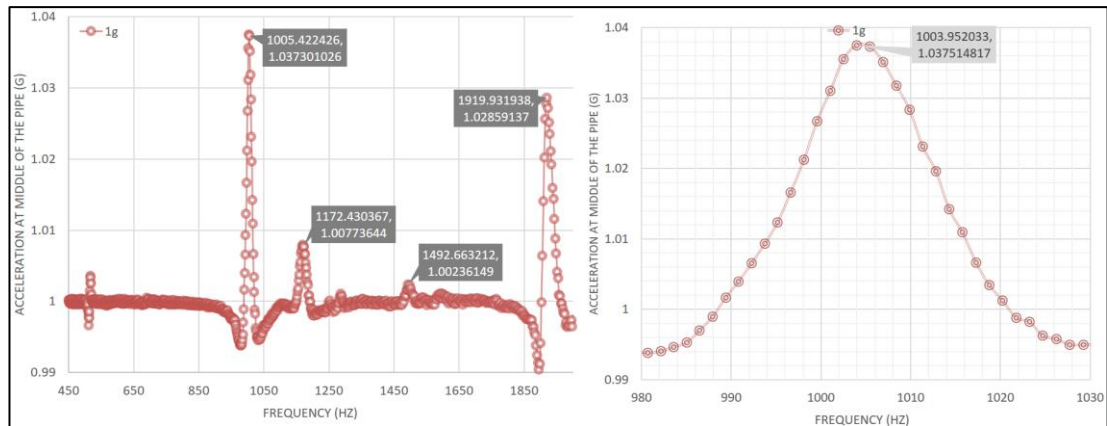


**Figure 4.3:** System response for configuration 2 at 1 G excitation

During the shaker test, an excitation of 5 G (49033.25 mm/s<sup>2</sup>) was applied, resulting in a recorded acceleration of 5.153 G (50533.66 mm/s<sup>2</sup>) at the midpoint of the pipe, as illustrated in **Figure 4.4**. The amplitude ratio calculated from this data is 1.031.

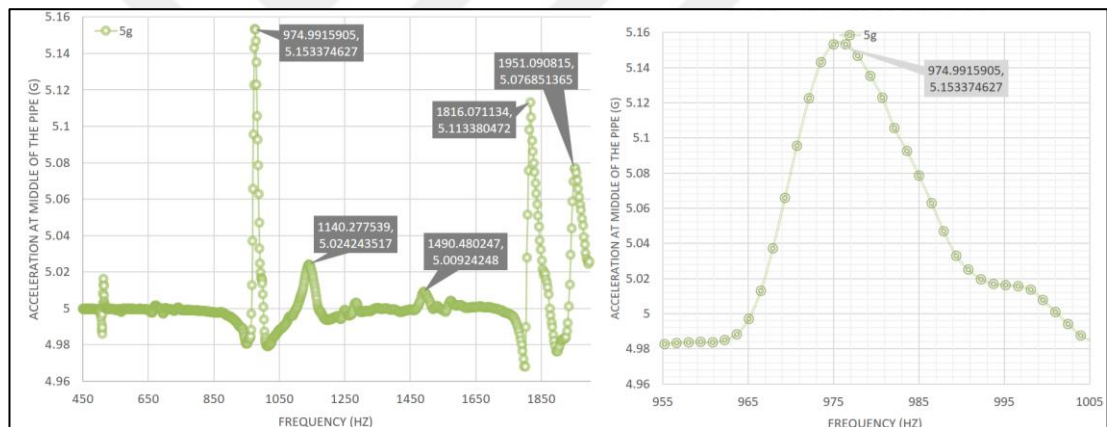


**Figure 4.4:** System response for configuration 2 at 5 G excitation

During the shaker test, an excitation of 10 G (49033.25 mm/s<sup>2</sup>) was applied, resulting in an observed acceleration of 10.22 G (100223.963 mm/s<sup>2</sup>) at the midpoint of the pipe, as illustrated in **Figure 4.5**. This yields an amplitude ratio of 1.022.

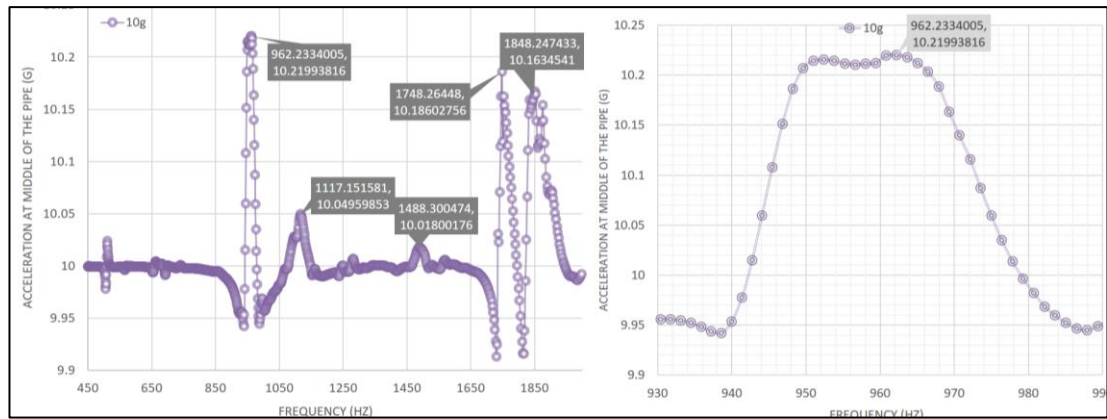


**Figure 4.5:** System response for configuration 2 at 10 G excitation

## 4.2. Damping Ratio

Damping refers to the process by which energy is dissipated from a vibrating system, reducing the amplitude of oscillations over time. It is a mechanism that resists motion and converts kinetic energy into other forms, such as heat or sound. **(Figure 4.6 [21])** Damping can be classified as viscous damping, where resistance is proportional to velocity, commonly seen in systems like shock absorbers; coulomb (dry friction) damping, which involves constant resistance due to friction between surfaces; structural (material) damping, where energy is lost due to internal material friction; aerodynamic damping caused by air resistance and hysteretic damping, which results from energy dissipation due to material deformation.

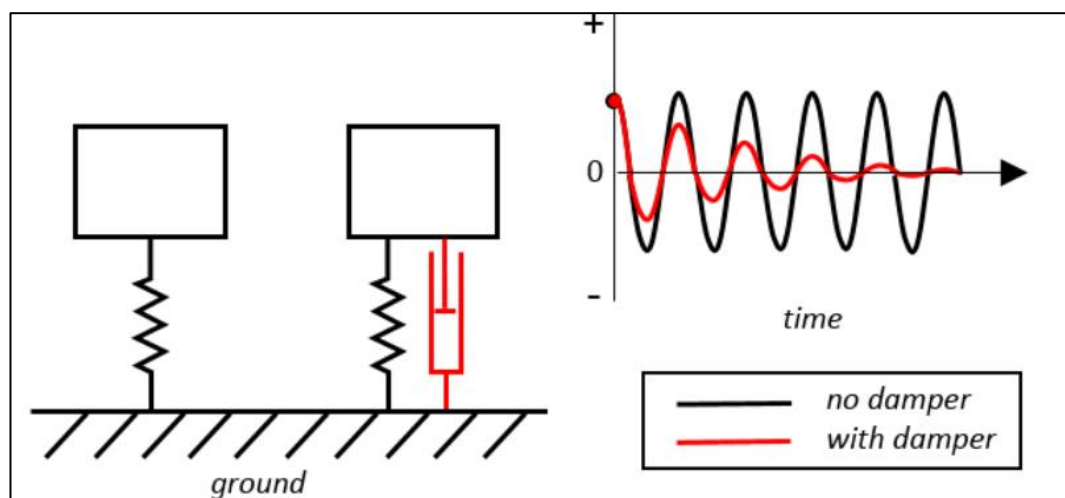


**Figure 4.6:** Damping mechanism of a structure

The damping ratio ( $\zeta$ ) is a dimensionless measure that measure that describes how much damping is present in a vibrating system relative to the systems's critical damping. It indicates the degree of resistance to motion in relation to the system's natural oscillation. (Equation 4.2)

$$\zeta = \frac{c}{c_o} \quad (4.2)$$

$c$  is the actual damping coefficient of the system and  $c_o$  is the critical damping coefficient, the minimum damping required to prevent oscillations. In an underdamped system ( $\zeta < 1$ ), the system oscillates with gradually decreasing amplitude. A critically damped system ( $\zeta = 1$ ) returns to equilibrium as quickly as possible without oscillating. In an overdamped system ( $\zeta > 1$ ), the system returns to equilibrium slowly without oscillating. An undamped system ( $\zeta = 0$ ) oscillates indefinitely without energy loss. As mentioned in section 2.2 damping ratio is critical input for harmonic analysis.

#### 4.2.1. Damping Calculation Methods

There are wide variety of damping calculation methods such as half-power bandwidth method, logarithmic decrement method, energy dissipation method, numerical methods, random decrement technique, time frequency analysis. The logarithmic decrement method analyzes the decay of successive amplitudes in the time domain, suitable for lightly damped systems with observable oscillation in time domain. It assumes constant damping over time. Energy dissipation method calculates damping based on energy loss in each vibration cycle, suitable for systems with non-linear damping. It is challenging to measure energy dissipated per cycle ( $\Delta E$ ) accurately and not practical for small energy losses. Random Decrement Technique estimates damping using random vibration data, ideal for systems under ambient excitation. It relies on statistical assumptions. Time-frequency analysis uses advanced techniques like wavelet transforms to estimate damping. It used for systems with time varying damping, applicable in machinery diagnostics and earthquake engineering. The choice of method depends on the type of system, available data, and the precision required. In this study, the half-bandwidth method is used to calculate damping ratio. The half-bandwidth method is commonly used in vibration analysis and signal processing to

determine the bandwidth or identify the damping characteristics of a system. The term “3 dB” refers to the point where the power of the system’s response drops to half its maximum value (Equation 4.3), a significant and meaningful threshold in vibration. 3 dB points are the frequencies on either side of the resonant frequency ( $f_1$  and  $f_2$ ) where the amplitude decreases by  $\sqrt{2}/2$  (approximately 0.707) of the peak amplitude. (Equation 4.4) The 3 dB point is widely used because halving the power is a practical and recognizable measure of energy dissipation or signal attenuation.

$$P_{offset} = 0.5 \times P_{peak} \quad (4.3)$$

$$A_{offset} = 0.707 \times A_{peak} \quad (4.4)$$

In a FRF, the damping is proportional to the width of the resonant peak about the peak’s center frequency. (Equation 4.5) By looking at three dB down from the peak level, one can determine the associated damping.

$$\zeta = \frac{f_1 - f_2}{2 \times f_0} \quad (4.5)$$

$f_0$  is the frequency of resonant peak in Hertz,  $f_2$  is the frequency value, in Hertz, 3 dB down from peak value, higher than  $f_0$ ,  $f_1$  frequency value, in Hertz, 3 dB down from peak value, lower than  $f_0$ . (**Figure 4.7**)

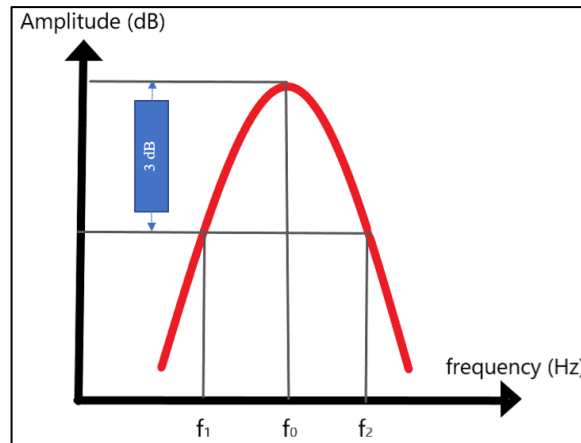


**Figure 4.7:** Half-bandwidth method

#### 4.2.2. Damping of the First Configuration

Initially, the amplitudes are converted to decibels utilizing Equation 4.6.

$$dB_a = 20 \log_{10} \left( \frac{a}{a_{ref}} \right), a_{ref} = 10^{-6} m/s^2 \quad (4.6)$$

Subsequently, the peak amplitude is reduced by 3 dB, resulting in a halving of the power, as demonstrated in **Figure 4.8**.

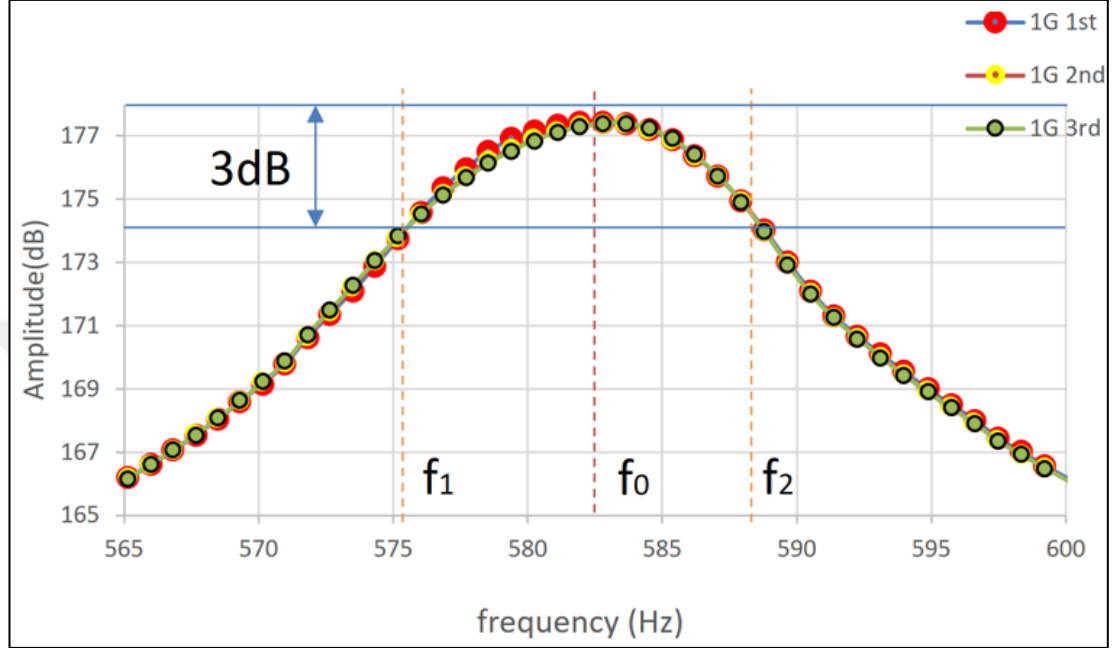


**Figure 4.8:** Half-bandwidth method for configuration 1

The corresponding frequencies,  $f_0$  which is the frequency of resonant peak in Hertz,  $f_2$  which is the frequency value, in Hertz, 3 dB down from peak value, higher than  $f_0$ ,  $f_1$  which is frequency value, in Hertz, 3 dB down from peak value, lower than  $f_0$  gathered for all 3 tests and shared in **Table 4.6**. Damping ratios are determined using Equation 4.5. The calculated damping ratios for the conducted tests are as follows: 1.08% for the first test, 1.08% for the second test, and 1.07% for the third test.

**Table 4.6:** 3 dB points for configuration 1

|       | 1G 1st test |         | 1G 2nd test |         | 1G 3rd test |         |
|-------|-------------|---------|-------------|---------|-------------|---------|
|       | f (Hz)      | A (dB)  | f (Hz)      | A (dB)  | f (Hz)      | A (dB)  |
| $f_1$ | 575.854     | 174.445 | 575.845     | 174.377 | 575.882     | 174.409 |
| $f_0$ | 582.793     | 177.445 | 582.793     | 177.377 | 583.647     | 177.409 |
| $f_2$ | 588.396     | 174.445 | 588.428     | 174.377 | 588.398     | 174.409 |

The 3 dB method demonstrates the highest precision for lightly damped systems, typically characterized by a damping ratio within the  $0 < \zeta < 0.2$  range. A damping

ratio of 1.08% was accurately determined for the first configuration. This value will be subsequently refined in the harmonic analysis, which will allow for assessing the alternating stress experienced by the component.

### 4.2.3. Damping of the Second Configuration

Similar to the 4.2.2, the amplitudes converted to the dB with Equation (4.6) for second configuration vibration data and shared in **Table 4.7**.

**Table 4.7:** 3 dB points for configuration 1

|                | 0.5 G excitation |        | 1 G excitation |        | 5 G excitation |        | 10 G excitation |        |
|----------------|------------------|--------|----------------|--------|----------------|--------|-----------------|--------|
|                | f (Hz)           | A (dB) | f (Hz)         | A (dB) | f (Hz)         | A (dB) | f (Hz)          | A (dB) |
| f <sub>1</sub> | 100              | 133.81 | 100            | 139.82 | 100            | 153.81 | 100             | 159.83 |
| f <sub>0</sub> | 1008.37          | 134.12 | 1003.95        | 140.15 | 974.99         | 154.07 | 962.23          | 160.02 |
| f <sub>2</sub> | 2000.24          | 133.78 | 2000.24        | 139.81 | 2000.24        | 153.85 | 2000.24         | 159.82 |

When the data in **Table 4.7** are examined, it is seen that an amplitude offset of 3 dB does not occur. This is seen in systems with high damping ratios and where the system is approaching critically damped. Although the 3 dB method does not give a precise result, the approximate damping ratio of the system will be obtained by an approach. In this approach offset level will be adjusted for a different offset level. The adjusted bandwidth can be calculated by using Equation (4.7), damping ratio with adjusted bandwidth can be calculated by using Equation (4.8) with third order correction proposed by Papagiannopoulos [19]. The stiffness is assumed constant with increasing excitation load. ( $\Delta f_{adjusted}$  bandwidth at the chosen offset)

$$\Delta f_{adjusted} = \Delta f_{offset} \cdot \sqrt{\frac{-3dB}{-Offset \text{ dB}}} \quad (4.7)$$

$$\frac{\Delta f_{adjusted}}{w_n} = \frac{2\zeta + 8\zeta^3}{\sqrt{1 - 2\zeta^2}} \quad (4.8)$$

For the test with 0.5 G excitation force, peak amplitude is 134.132 dB at 1008.37 hZ, 0.329 dB offset is selected as shown in **Figure 4.9**. Corresponding frequencies are shared in **Table 4.8**.

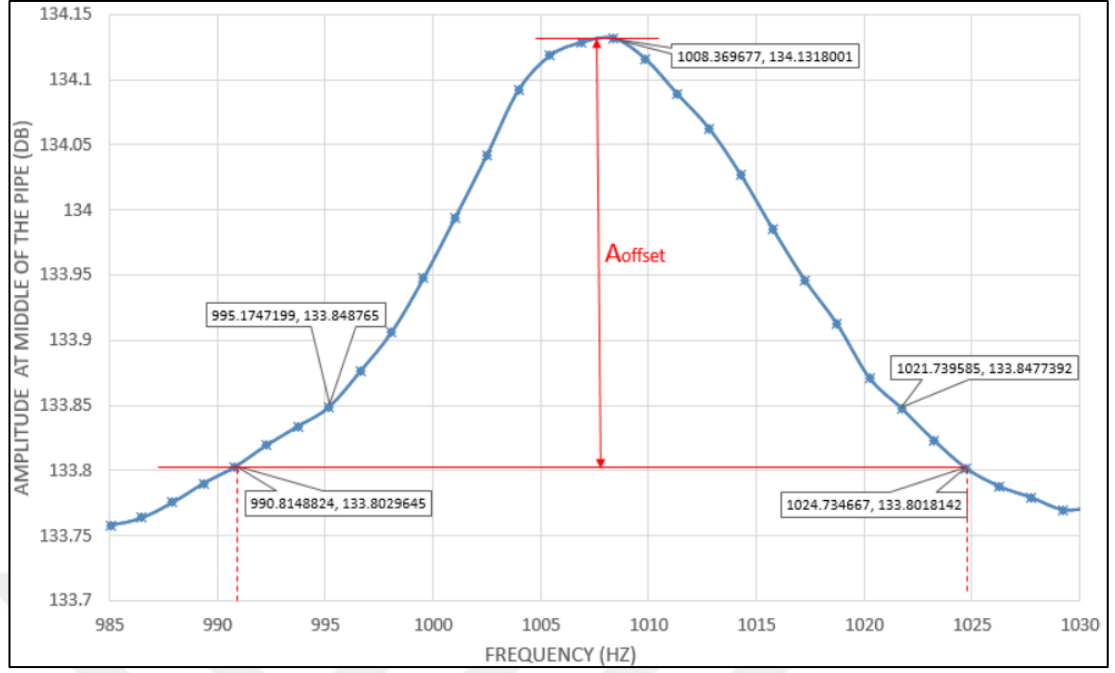


**Figure 4.9:** Scaled half-bandwidth method for configuration 2 at 0.5 G

**Table 4.8:** Scaled 3 dB points for configuration 2 at 0.5 G

|                | f (Hz)   | A (dB)  |
|----------------|----------|---------|
| f <sub>1</sub> | 990.815  | 133.803 |
| f <sub>0</sub> | 1008.367 | 134.132 |
| f <sub>2</sub> | 1024.734 | 133.802 |

$\Delta f_{adjusted}$  is calculated as 102.453 Hz by using Equation (4.7). By substituting the adjusted frequency and natural frequency Equation (4.9) is obtained. Multiplying both side with  $\sqrt{1 + 2\zeta^2}$ , Equation (4.10) is obtained. Simplifying Equation (4.10), Equation (4.11) is obtained. Removing the square roots and solving the obtained six order differential equation, the solution roots obtained as Equation (4.12). The positive root is the damping ratio of the system as 5.016% in Equation (4.13).

$$\frac{2\zeta + 8\zeta^3}{\sqrt{1 - 2\zeta^2}} - 0.101603 = 0 \quad (4.9)$$

$$\frac{2\zeta + 8\zeta^3}{\sqrt{1 - 2\zeta^2}} \times \sqrt{1 + 2\zeta^2} - 0.101603 \times \sqrt{1 + 2\zeta^2} = 0 \quad (4.10)$$

$$2\zeta + 8\zeta^3 - 0.101603 \times \sqrt{1 + 2\zeta^2} = 0 \quad (4.11)$$

$$\zeta_1 = \sqrt{\frac{2.52687 \times 10^{15}}{1 \times 10^{18}}}, \zeta_2 = -\sqrt{\frac{2.52687 \times 10^{15}}{1 \times 10^{18}}} \quad (4.12)$$

$$\zeta = 5.016\% \quad (4.13)$$

For the test with 1 G excitation force, peak amplitude is 140.15 dB at 1003.95 hZ, 0.346 dB offset is selected as shown in **Figure 4.10**. Corresponding frequencies are shared in **Table 4.9**.  $\Delta f_{adjusted}$  is calculated as 112.679 Hz by using Equation (4.7), damping ratio calculated as 5.527% by using same methodology with Equation (4.8).

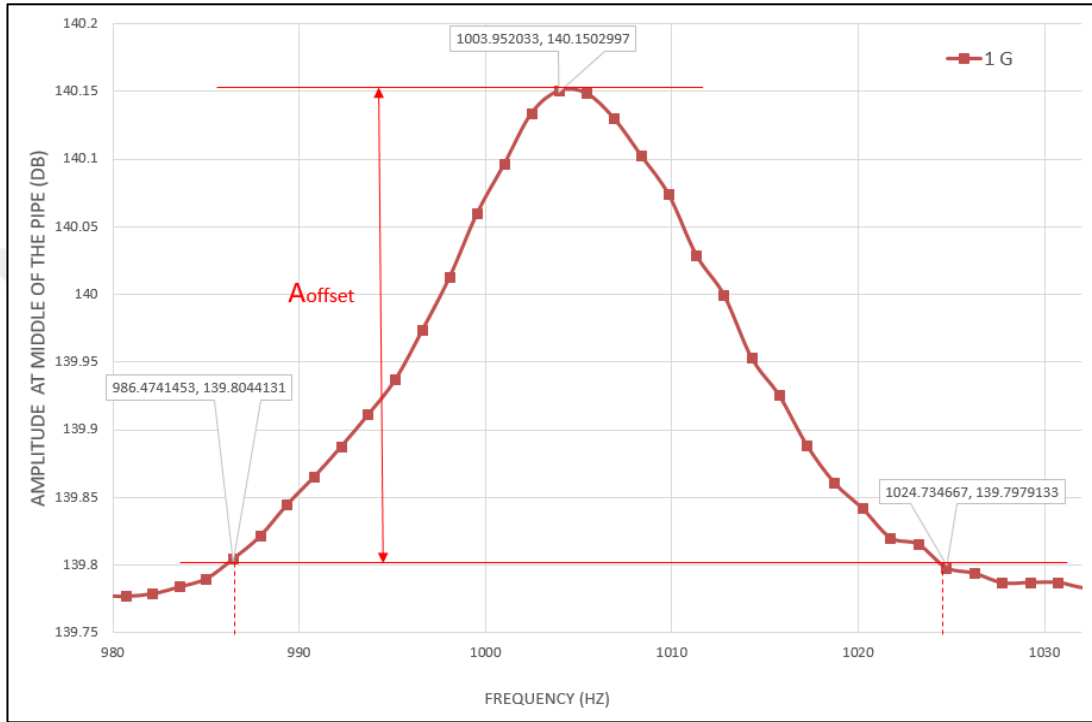


**Figure 4.10:** Scaled half-bandwidth method for configuration 2 at 1 G

**Table 4.9:** Scaled 3 dB points for configuration 2 at 1 G

|       | <b>f (Hz)</b> | <b>A (dB)</b> |
|-------|---------------|---------------|
| $f_1$ | 986.474       | 139.804       |
| $f_0$ | 1003.952      | 140.150       |
| $f_2$ | 1024.735      | 139.798       |

For the test with 5 G excitation force, peak amplitude is 154.072 dB at 974.992 hZ, 0.268 dB offset is selected as shown in **Figure 4.11**. Corresponding frequencies are shared in **Table 4.10**.  $\Delta f_{adjusted}$  is calculated as 125.296 Hz by using Equation (4.7), damping ratio calculated as 6.299% by using same methodology with Equation (4.8).

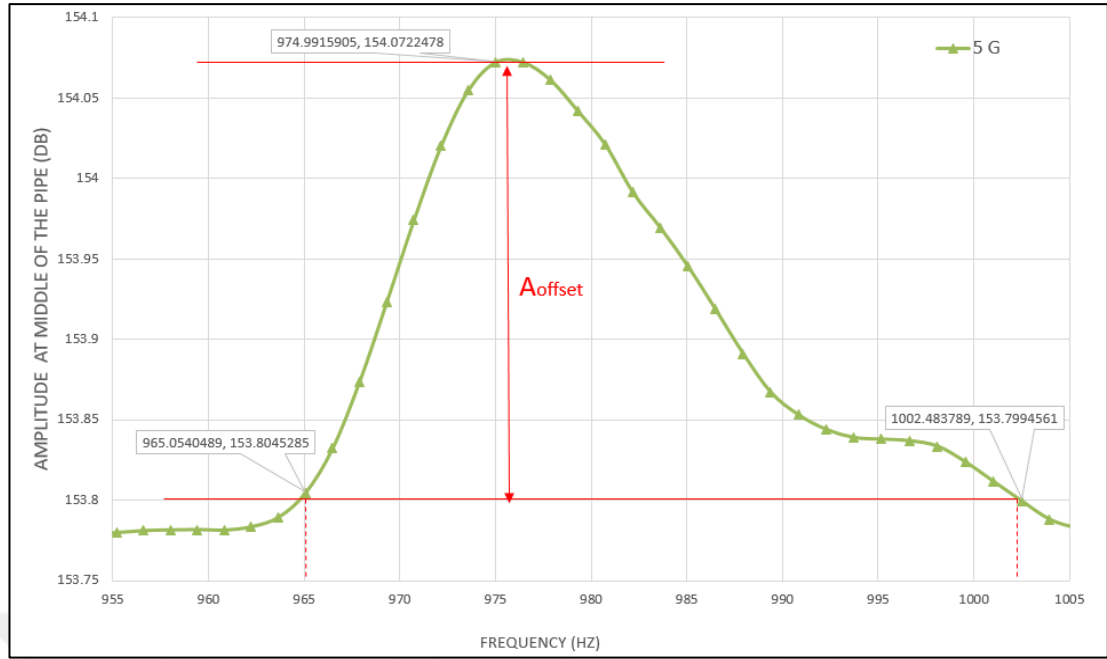


**Figure 4.11:** Scaled half-bandwidth method for configuration 2 at 5 G

**Table 4.10:** Scaled 3 dB points for configuration 2 at 5 G

|       | <b>f (Hz)</b> | <b>A (dB)</b> |
|-------|---------------|---------------|
| $f_1$ | 965.05405     | 153.8045      |
| $f_0$ | 974.99159     | 154.07225     |
| $f_2$ | 1002.4838     | 153.7995      |

For the test with 10 G excitation force, peak amplitude is 160.019 dB at 962.233 hZ, 0.208 dB offset is selected as shown in **Figure 4.12**. Corresponding frequencies are shared in **Table 4.11**.  $\Delta f_{adjusted}$  is calculated as 149.436 Hz by using Equation (4.7), damping ratio calculated as 7.548% by using same methodology with Equation (4.8).

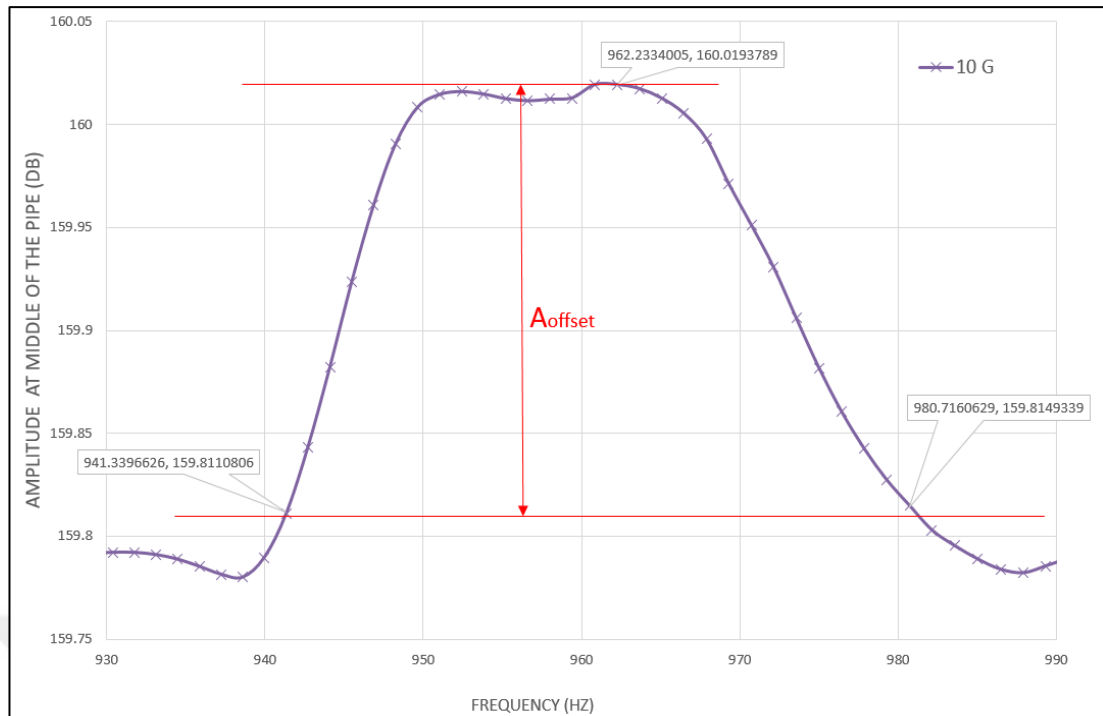


**Figure 4.12:** Scaled half-bandwidth method for configuration 2 at 10 G

**Table 4.11:** Scaled 3 dB points for configuration 2 at 10 G

|       | <b>f (Hz)</b> | <b>A (dB)</b> |
|-------|---------------|---------------|
| $f_1$ | 941.33966     | 159.8111      |
| $f_0$ | 962.2334      | 160.01938     |
| $f_2$ | 980.71606     | 159.8149      |

By assuming constant stiffness with increasing excitation, the damping ratio change for configuration-2 is plotted in **Figure 4.13** for both conventional Equation (4.5), and 3rd order corrected Equation (4.8). The damping ratio values are shared for both configuration in **Table 4.12**.

**Table 4.12:** Damping ratio values for all configurations

| Excitation amplitude | 0.5 G | 1G    | 5G    | 10    |
|----------------------|-------|-------|-------|-------|
| Configuration-1      | -     | 1.08% | -     | -     |
| Configuration-2      | 5.02% | 5.53% | 6.30% | 7.55% |

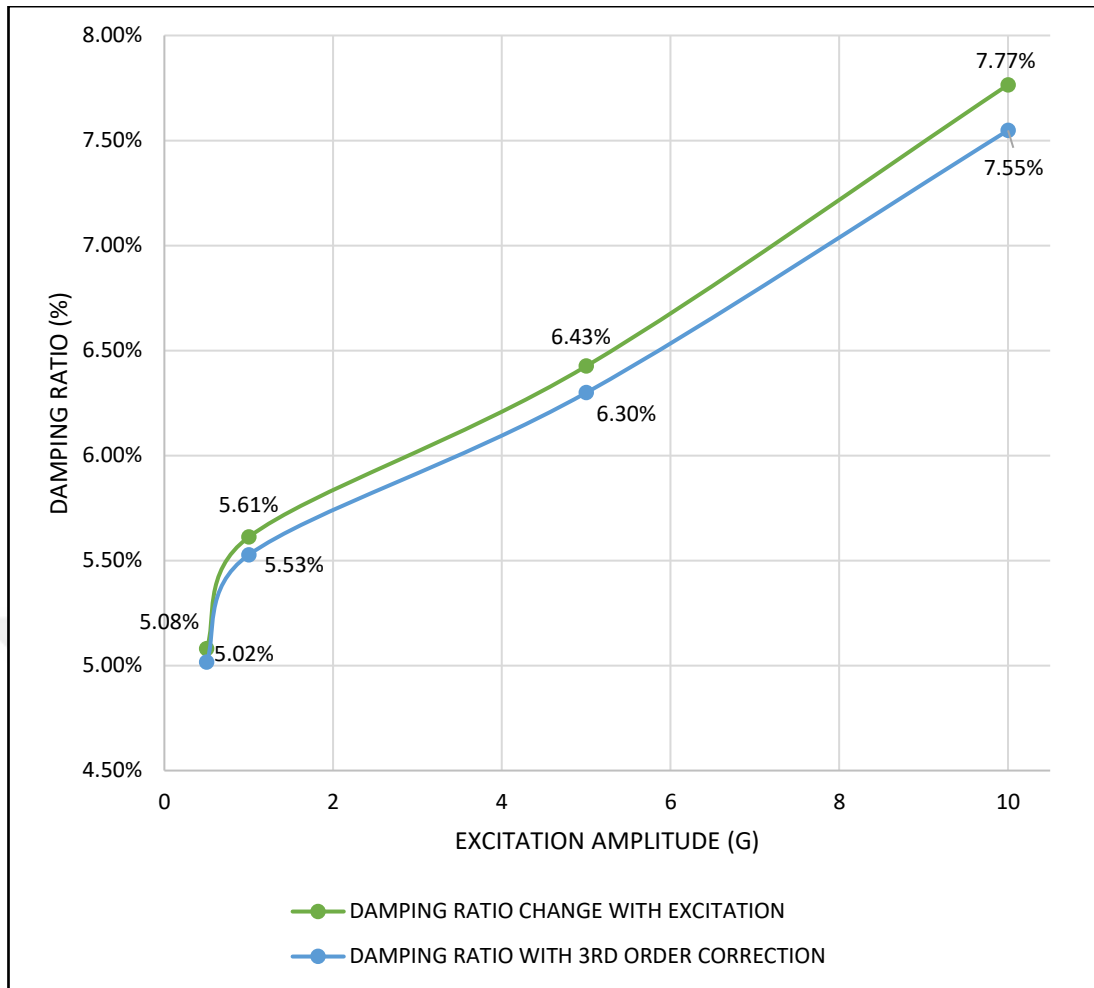


**Figure 4.13:** Damping ratio change with increasing excitation load

### 4.3. Excitation and Frequency Shift

In a linear system, the resonant frequency is just influenced by the characteristics of the system's stiffness and mass. There is no change in frequency when the amplitude of excitation increases, since both stiffness and mass are unchanged. In real-world situations, however, various non-linear factors—like material stiffening, geometric influences, thermal effects, or damping—may cause stiffness or damping to vary based on displacement or velocity. The system can portray softening nonlinearity, where the resonant frequency decreases with a rising load, or hardening nonlinearity, where the resonant frequency increases with a higher load. Initially, the system generally shows linear behavior at low excitation levels, with insignificant frequency shifts. Nonetheless, increased excitation loads can trigger non-linear behavior, modifications in boundary conditions, and dynamic effects that may lead to changes in the resonance frequency. While damping primarily affects the amplitude of resonance, considerable

excitation loads could also impact the mechanisms of energy dissipation, as shown in **Figure 4.13**, resulting in slight changes in the resonance frequency, as represented in the scaled Frequency Response Function (FRF) plot in **Figure 4.14**.

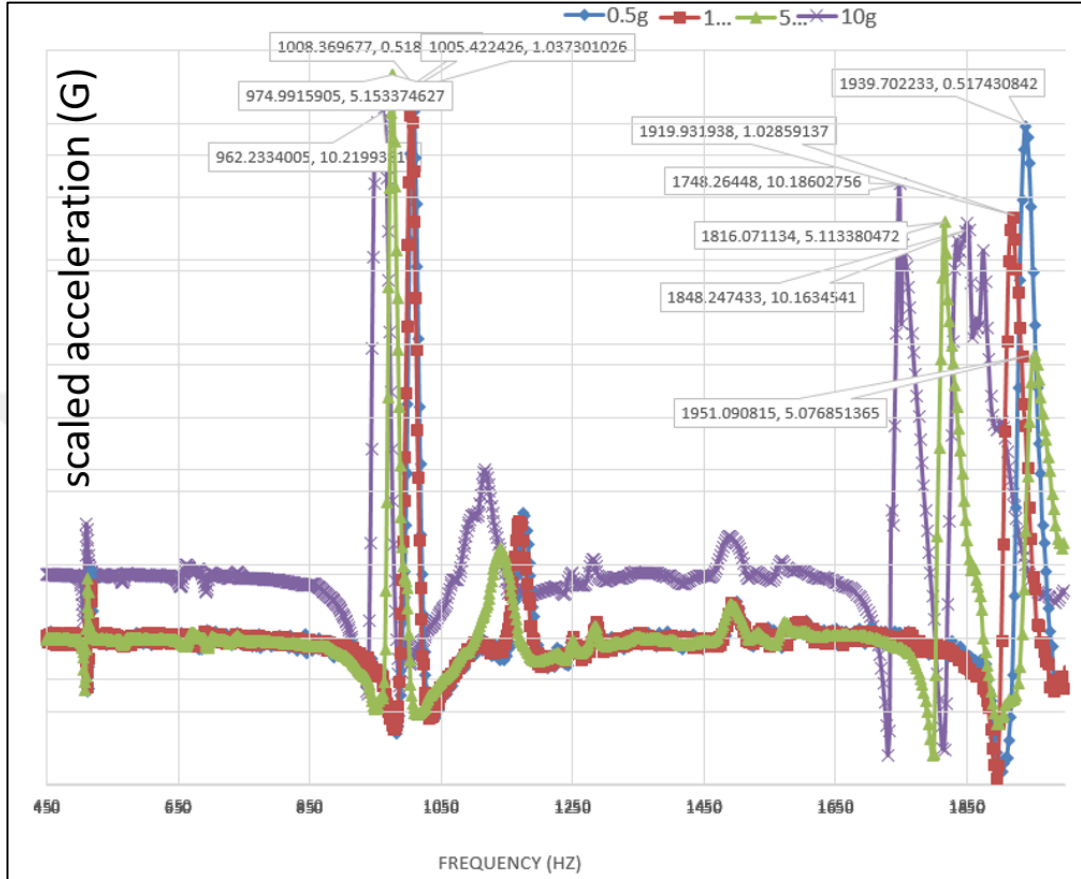


**Figure 4.14:** The FRF plot for all excitation levels with scaling

In systems involving joints or contact interfaces, the excitation loads may influence the contact stiffness or induce friction effects, thereby altering the resonance frequency. For systems subjected to preloads, such as tensioned pipes or clamped beams, the preload can substantially modify the effective stiffness, resulting in a shift in the resonance frequency. Furthermore, high excitation levels can increase the material's temperature, consequently reducing stiffness and leading to a decrease in the natural frequency. The Duffing equation serves as a model for a classic nonlinear system, with the Duffing oscillator representing a paradigm within nonlinear dynamics. Researchers employ analytical approximations, such as perturbation methods, alongside numerical tools to investigate its behaviour. The Duffing oscillator is described by the following second-order nonlinear differential equation (Equation (4.17)) [22].

$$m\ddot{x} + c\dot{x} + kx + \alpha x^3 = F_o \cos(wt) \quad (4.17)$$

$m$  is the mass of the oscillator,  $c$  is the damping coefficient,  $k$  is the linear stiffness coefficient,  $\alpha$  is the nonlinear stiffness coefficient. The term  $\alpha x^3$  introduces nonlinearity. With the hardening spring ( $\alpha > 0$ ) stiffness increases with displacement, with the softening spring ( $\alpha < 0$ ) stiffness decreases with displacement.

For configuration 2, the frequency shifts for first 3 modes plotted in **Figure 4.15**.

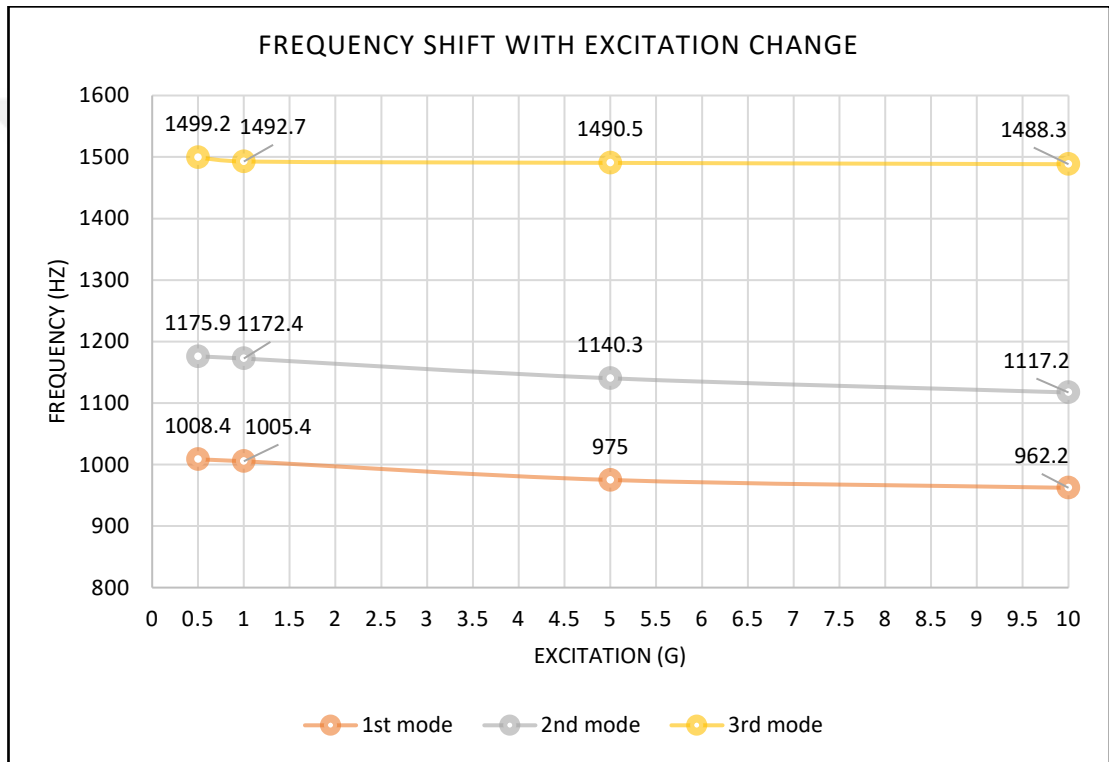


**Figure 4.15:** Frequency shift for all excitation levels

The first mode exhibited a frequency shift of 46.2 Hz, reflecting a percentage change of 4.58% from 0.5 G to 10 G. The second mode demonstrated a frequency shift of 58.7 Hz, corresponding to a percentage change of 4.99% from 0.5 G to 10 G. In contrast, the third mode experienced a frequency shift of 10.9 Hz, indicating a percentage change of 0.73% from 0.5 G to 10 G. Notably, the third mode illustrates a more linear response than the first and second modes. This linear behaviour can be attributed to the positioning of the support point of the cushioned clamp, which is situated in the anti-node region, thereby minimizing its impact on the system.

The observed nonlinearity in the first and second modes is primarily associated with the p-clamp. Examples of softening nonlinearities include slackened cables or strings and rubber or elastomer materials. This phenomenon can be observed in guitar strings that lose tension when plucked with excessive force and exhibit a "floppy" response. Furthermore, when a cable or string is displaced beyond its tensioned state, it becomes slack, reducing its effective stiffness. Similar behaviour is observed in automotive bushings or shoe soles that soften under repeated compression. Under cyclic loading conditions, rubber materials demonstrate the Mullins effect, characterized by a decrease in stiffness following substantial strain.

Hyperelastic models, such as the Ogden or Mooney-Rivlin models, effectively capture the softening phenomenon, as described by Equation (4.18), where  $W$  represents the strain energy and the terms denote the principal stretches.

$$W = \sum_{i=1}^N \frac{\mu_i}{\alpha_i} (\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3) \quad (4.18)$$

After analyzing the data gathered from the tests alongside the various field examples, it becomes evident that the observed nonlinearity within the system is primarily attributed to the properties of the cushion material used in the cushioned clamp. This material consists of silicone that is reinforced with glass fibre fabric. This combination significantly influences the system's performance, leading to the nonlinearity observed in the test results. Further investigation into the material's behaviour under different conditions may provide additional insights into its impact on system functionality.

## 5. CONCLUSIONS

This study presents a fully comprehensive experimental investigation into the dynamic behavior of a double-bent pipe system under varying excitation levels and support conditions. Modal validation was performed using both finite element models—3D solid and 2D shell—and experimental shaker testing. The first three mode frequencies were compared in detail across these methods under multiple excitation levels (0.5 G, 1 G, 5 G, and 10 G), revealing strong correlation and highlighting the accuracy of the numerical models. Among them, the 3D solid model demonstrated the closest agreement with the experimental results, with a frequency deviation of 1.91% in the first mode. In addition, the mode shapes obtained from the high-speed camera recordings during shaker testing were found to be consistent with those predicted by both FEA models, confirming the validity of the numerical approach.

The introduction of a cushioned p-clamp at the midpoint of the pipe (Configuration 2) significantly increased the first mode frequency by more than 70%, resulting a considerable rise in structural stiffness. In contrast, the unsupported configuration (Configuration 1) exhibited a much higher amplitude ratio. For instance, under 1 G input excitation, an output acceleration of 76 G was recorded at the midpoint, yielding an amplitude ratio of 76. This ratio—defined as the output acceleration amplitude divided by the input acceleration amplitude—emphasizes the pipe's susceptibility to vibration amplification in the absence of support.

In the supported configuration, amplitude ratios were close to unity, indicating a more stable response under dynamic loads. However, nonlinear frequency shifts were observed in the first and second modes as excitation levels increased. These nonlinearities are attributed to the material properties of the clamp padding, made of silicone reinforced with glass fiber, which exhibits softening behavior under cyclic loading—a phenomenon similar to the Mullins effect. The third mode showed minimal frequency variation, likely due to the clamp's location near an anti-node, reducing its influence on stiffness.

The results demonstrate significant differences in the damping characteristics between the two configurations. In the unsupported configuration (configuration-1), the

damping ratio was determined to be 1.08%, indicating minimal energy dissipation. In contrast, in the supported configuration (configuration-2), the damping ratio was evaluated under different excitation levels (0.5G, 1G, 5G, and 10G), yielding values of 5.02%, 5.53%, 6.3%, and 7.55%, respectively. These findings indicate a nonlinear increase in damping with increasing excitation, suggesting that higher excitation levels enhance energy dissipation due to the structural interaction between the pipe and the support. Furthermore, the observed nonlinearity in the damping response highlights the complex interplay between excitation magnitude and structural constraints, which must be carefully considered in the design and analysis of piping systems, subjected to dynamic loads.

Overall, this study demonstrates the significant role of clamping design in the dynamic response of pipe systems, emphasizes the role of external supports, potentially influencing their fatigue life and structural integrity. The validated mode shapes and accurate frequency predictions across different modeling techniques and excitation levels provide a solid foundation for future design improvements in vibration-critical applications. Further, these insights contribute to the broader understanding of damping mechanisms in bent piping systems and provide a foundation for future research aimed at optimizing support configurations to enhance stability and mitigate excessive vibrations.

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## BIOGRAPHY

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