

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

TIME SERIES MODELLING AND CRYPTOCURRENCIES



ABDÜLSAMET AÇA
A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
DEPARTMENT OF INDUSTRIAL ENGINEERING

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**THESIS SUPERVISOR
ASSIST. PROF. DR. KEMAL DİNÇER DİNGEÇ**

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2023**

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SUMMARY

Cryptocurrencies have compelled the attention of people after Nakamoto introduced Bitcoin. Cryptocurrencies, whose effectiveness has increased rapidly in the economy and finance, has also started to be studied in academia and has increased in popularity in recent years. The Non-linear Autoregressive Distributed Lags (NARDL) method is the focus of this thesis as it can examine the asymmetric relationships between cryptocurrencies and economic-financial variables. Previously, Monte Carlo simulation experiments were carried out to investigate finite sample properties of the estimators under normal distribution for the model. In this thesis, the reliability of NARDL model has been studied when non-normal distributions are used. This is an important research problem, as the distributions of returns on cryptocurrencies are non-normal and heavy-tailed. In this thesis, the NARDL model was subjected to Monte Carlo simulation experiments for normal distribution and two heavy-tailed distributions called Student-t and Skew-t distributions. To our knowledge, there is no study in the literature on finite sample properties of NARDL estimators for time series with non-normality. In addition, estimations were carried out using the packages both in R and STATA, which are open and closed source software, respectively, in order to evaluate their reliabilities by comparison.

The results of thesis indicate that the NARDL model is unable to reveal the short-run asymmetric relationships of variables following non-normal distributions with certain parameters although the inferences made about the effects of cryptocurrencies on the economy and finance are mostly accurate. Another important finding is that the package developed in R is not utterly reliable and accurate.

Key Words: Monte Carlo Simulation, Non-linear Autoregressive Distributed Lags (NARDL), Cryptocurrencies, Non-Normal Distributions.

ÖZET

Nakatomo'nun Bitcoin'i piyasaya sürmesinden sonra kripto paralar insanların dikkatini çekmiştir. Dünya ekonomi ve finans ekosisteminde etkinliği hızlı bir şekilde artan kripto paralar, akademi tarafında da çalışılmaya başlanmış ve son yıllarda popülaritesi giderek artmıştır. Doğrusal Olmayan Otoregresif Dağılım Gecikmeleri (NARDL) yöntemi kripto paralar ile ekonomik ve finansal değişkenler arasındaki asimetric ilişkileri inceleyebildiği için bu tezin odak noktası olmuştur. Bu modelin normal dağılım varsayımı altında tahmin edicilerin sonlu örnek özelliklerini araştırmak için Monte Carlo simülasyon deneyleri yapılmıştır. Bu tezde NARDL metodunun normal olmayan dağılımlar kullanıldığında güvenilir olup olmadığı incelenmektedir. Kripto para birimlerinin getirilerinin sahip olduğu dağılımların normal olmadığı ve ağır kuyruklu olduğu için bu önemli bir araştırma problemidir. Bu tezde, NARDL modeli normal dağılım ve ağır kuyruklu dağılımlardan olan Student-t ve Skew-t dağılımları için Monte Carlo Simülasyon deneylerine tabi tutulmuştur. Bildiğimiz kadarıyla, literatürde normal dağılıma sahip olmayan zaman serileri için doğrusal olmayan ARDL modelinin tahmin edicilerinin sonlu örnek özellikleri üzerine bir çalışma mevcut değildir. Ayrıca tezimizde tahmin işlemleri biri sırasıyla açık kaynaklı diğeri ise açık kaynaklı olmayan iki farklı yazılımda (R ve STATA) oluşturulan NARDL paketi kullanılarak gerçekleştirilmiştir. Açık kaynaklı yazılım programında oluşturulan modelin güvenilirliği açık kaynaklı olmayan yazılım programında oluşturulan model ile karşılaştırılarak değerlendirilmiştir.

Tezin sonuçları, kripto para birimlerinin ekonomi ve finans üzerindeki etkileri hakkında yapılan değerlendirmelerin çoğunlukla doğru olmasına rağmen NARDL modelinin belirli parametrelere sahip normal olmayan dağılımlı değişkenlerin kısa dönemli asimetric ilişkilerini göstermede yetersiz kaldığını göstermiştir. Tezin bir önemli bulgusu da açık kaynaklı program (R)'da oluşturulan NARDL paketinin tam olarak güvenilir ve doğru sonuçlar vermediğinin saptanmasıdır.

Anahtar Kelimeler: Monte Carlo Simülasyonu, Doğrusal Olmayan Otoregresif Dağılım Gecikmeleri (NARDL), Kripto paralar, Normal Olmayan Dağılımlar.

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TABLE of CONTENTS

	<u>Page</u>
SUMMARY	v
ÖZET	vi
ACKNOWLEDMENTS	vii
TABLE of CONTENTS	viii
LIST of ABBREVIATIONS and ACRONYMS	x
LIST of FIGURES	xi
LIST of TABLES	xii
1. INTRODUCTION	1
2. LITERATURE REVIEW	4
3. MONTE CARLO SIMULATION & DATA GENERATION PROCESS	7
3.1. Model	7
3.2. Distributions	9
3.2.1. Normal Distribution	9
3.2.2. Student-t Distribution	9
3.2.3. Skew-t Distribution	11
3.3. Hypotheses	13
3.4. Algorithm	14
4. MONTE CARLO SIMULATION RESULTS	15
4.1. Normal Distribution	15
4.2. Student-t Distribution	19
4.3. Skew-t Distribution	22
4.4. Normal Distribution and Student-t Distribution	25
5. COMPARISON OF STATA AND R	28
6. AN EMPRICIAL STUDY: GOLD AND CRYPTOCURRENCIES	31
7. CONCLUSIONS AND FUTURE RESEARCH	40
REFERENCES	42

BIOGRAPHY	46
APPENDICES	47



LIST OF ABBREVIATIONS AND ACRONYMS

<u>Abbreviations</u>	<u>Explanations</u>
<u>and Acronyms</u>	
\$	: Dollar
ADF	: Augmented Dickey–Fuller Test
ARDL	: Autoregressive Distributive Lags
BSV	: Bitcoin SV Price
BTC	: Bitcoin Price
ECM	: Error Correction Model
GARCH	: Generalized Autoregressive Conditional Heteroskedasticity
GOLD	: Gold Price
KPSS	: Kwiatkowski-Phillips-Schmidt-Shin Test
LSK	: Lisk Price
NARDL	: Nonlinear Autoregressive Distributive Lags
OLS	: Ordinary Least Square
PP	: Phillips–Perron Test
RMSE	: Root Mean Square Error
VECM	: Vector Error Correction Model
XLM	: Stellar Price

LIST OF FIGURES

<u>Figure No:</u>	<u>Page</u>
3.1: Comparison of Student-t and Standard Normal Distributions.	10
3.2: Comparison of Skew-t Distributions when $df=3$.	12
3.3: Comparison of Skew-t Distributions when $a_{skew} = -2$.	13
6.1: The Correlation Matrix.	32



LIST OF TABLES

<u>Table No:</u>	<u>Page</u>
2.1: The summary of literature review.	5
3.1: Parameter Definition for NARDL ECM.	7
3.2: Assumption Table for Simulation.	8
3.3: Assumption Table for Student-t Distribution.	10
3.4: Assumption Table for Skew-t Distribution.	12
3.5: Pseudocode for Monte Carlo Simulation of NARDL.	14
4.1: Monte Carlo Simulation Results for Normal Distribution: Size and Power of Wald and PSS Tests.	17
4.2: Monte Carlo Simulation Results for Normal Distribution: Bias, Standard Error and RMSE of the OLS Estimators.	18
4.3: Monte Carlo Simulation Results for Student-t Distribution: Size and Power of Wald and PSS Tests.	20
4.4: Monte Carlo Simulation Results for Student-t Distribution: Bias, Standard Error and RMSE of the OLS Estimators.	21
4.5: Monte Carlo Simulation Results for Skew-t Distribution: Size and Power of Wald and PSS Tests ($a_{skew} = 2$).	23
4.6: Monte Carlo Simulation Results for Skew-t Distribution: Bias, Standard Error and RMSE of the OLS Estimators ($a_{skew}=2$ and $\omega=-0.5$).	24
4.7: Monte Carlo Simulation Results for Normal and Student-t Distribution: Size and Power of Wald and PSS Tests.	26
4.8: Monte Carlo Simulation Results for Normal and Student-t Distribution: Bias, Standard Error and RMSE of the OLS Estimators.	27
5.1: Monte Carlo Simulation Results for Normal Distribution: Size and Power of Wald and PSS Tests (for <i>nardl</i> package in R).	30
5.2: Monte Carlo Simulation Results for Normal Distribution: Bias and Standard Error of the OLS Estimators (for <i>nardl</i> package in R).	30
6.1: Descriptive Statistics.	32

6.2:	The Standard Unit Root Tests (ADF, PP, and KPSS).	33
6.3:	Cointegration Test Statistics from STATA.	35
6.4:	The Results of NARDL Model.	37
6.5:	The Model Diagnostics.	38



1. INTRODUCTION

Cryptocurrencies are a monetary system that utilizes blockchain technological infrastructure and is used as an internet-based medium of exchange. The acceptance and popularity of cryptocurrencies by people is based on the following three features: Decentralization, transparency, and immutability [Lin and An, 2021]. The most well-known cryptocurrency is Bitcoin, created by Nakamoto [2008]. As of August 2022, Bitcoin's market capitalization on coinmarketcap is worth approximately \$456 billion [Web 1, 2022]. It is inevitable that a cryptocurrency of this volume have an effect on economy, finance, and people's daily lives. With the rapid increase in the volume of Bitcoin and the spread of blockchain technology around the world, studies on the relationship between cryptocurrencies and economic-financial variables has accelerated by academics and researchers. The linear methods, for instance linear regression model [Leirvik, 2022], correlation analysis [Baur et al., 2018] and different cointegration methods such as vector error correction model (VECM) [Li and Wang, 2017], Generalized Autoregressive Conditional Heteroskedasticity (GARCH) [Dyhrberg, 2016], ARDL [Ciaian et al., 2018], were used to analyze these relationships. Although linear causality or co-integration tests are widely used in the studies, especially with the development of econometric methods, the issue of whether there is asymmetry or nonlinearity in the relations between cryptocurrencies and economic-financial variables has recently started to be investigated.

The nonlinear ARDL model is an improved version of the ARDL model. The ARDL model, developed by [Pesaran et al., 2001], is one of the co-integration analyses used to identify long- and short-run causality relationships between non-stationary series. An important feature of this cointegration analysis is that it does not require the series to be of the same order. The aim of the nonlinear ARDL model is to develop a simple and flexible nonlinear dynamic framework that can simultaneously and consistently model both the long-term relationships between variables and the asymmetries in dynamic adjustment models [Shin et al., 2014]. This framework provides a theoretical foundation that allows us to track asymmetric effects following positive and negative shocks to variables, and the flexibility of this theoretical framework can accommodate different general combinations of long-run and short-run asymmetries. [Shin et al., 2014] conducted a series of Monte Carlo

simulation experiments based on a simple data generation process to investigate finite sample properties of estimators when constructing the NARDL model. The data were generated based on normal distribution and finite sample properties of estimators under normal distribution were investigated.

[Shanaev and Ghimire, 2021] applied Cramer-von Mises, Anderson-Darling, Kuiper, Kolmogorov-Smirnov, and Chi-squared goodness-of-fit tests for 22 different theoretical distributions to determine the distributions of 772 different cryptocurrencies. They found that cryptocurrencies have non-normal distributions such as student-t, skew-t, Johnson SU, and asymmetric power function distributions. The research problem that motivates this thesis emerges at this point. The main research question we address is whether the NARDL model is reliable when analyzing the relationship between variables with non-normal distributions.

The aim of this thesis is to test the reliability of NARDL model, which shows the long and short-run asymmetric relationships between cryptocurrencies and other variables that do not have a normal distribution, under different heavy-tailed distributions (Student t-distribution and Skew-t distribution).

Within the scope of this thesis, Monte Carlo Simulation experiments are conducted using the NARDL packages developed in R and STATA applications. The main reason for using two different software applications is to raise a question mark on the reliability of the results of the analyses and studies conducted using R since the NARDL package created in the R programming language has not undergone the validation and evaluation processes. In order to remove this question mark, it is aimed to compare the NARDL packages developed in R with STATA. In this way, the reliability of NARDL package developed in R will be tested.

In this context, the difference of our proposed thesis from the existing studies and its contribution to the existing literature and society can be summarized as follows:

- It is shown whether the NARDL model is reliable when non-normal variables are analyzed. In this way, the accuracy of financial and economic inferences obtained using the NARDL model is demonstrated.
- Monte Carlo simulation experiments and comparison methods reveal the reliability of open-coded software used in the NARDL analysis.

- The relationships between cryptocurrencies, which have a large monetary volume, and their relationships with other variables (gold, silver, etc.) are examined and the accuracy of these relationships is determined.

In the second part of this thesis, a literature review is conducted on studies that examine the relationships between cryptocurrencies and other variables (gold, gasoline, natural gas prices, etc.) using the NARDL model. Chapter 3 describes the simple data generation process for Monte Carlo simulation experiments and how the simulation experiments are performed. Chapter 4 evaluates the simulation results. Chapter 5 presents a comparison and analysis of the NARDL packages developed in STATA and R applications. Experimental studies using real data are presented in chapter 6. Finally, chapter 7 presents the conclusion of this thesis and some recommendations for future work.

2. LITERATURE REVIEW

There are abounding quantitative and qualitative studies on cryptocurrencies in the literature. An important research area has also emerged in academia after the rapid development of cryptocurrency markets and the rapid popularity of cryptocurrencies on the basis of society. [Corbet et al., 2019] have prepared a detailed literature study by systematically analyzing the academic studies conducted after the proposal of Bitcoin to the world. In this literature study of the authors, almost all studies on cryptocurrencies until 2019 have been examined under two categories, quantitative and qualitative. For the literature study conducted within the scope of this thesis, the literature study of [Corbet et al., 2019] has been taken as a basis in the quantitative studies on cryptocurrencies until 2019. The literature review conducted by [Corbet et al., 2019] give the following information: the dependent and control variables of the quantitative studies in the literature, the selected methodologies, the reasons for the studies, the frequency of data (daily, monthly, etc.), data sources, and the size of data. In the literature review in Table 2.1, the variables used, the methodologies used, especially NARDL and ARDL models, and the software or program types used in the studies conducted from 2017 to present are shown. In most of the studies, no information could be found about the name or version of the program used for the analysis.

It has been seen in the literature study that the studies in the literature are aimed at determining the relationships between stock price, market volume, cryptocurrencies, and other variables. In the studies we examined in the literature, there is no research on the reliability of the selected methodologies and models. Only [Shanaev and Ghimire, 2021] conducted a good-of-fit analysis using 22 different theoretical distribution functions and the Cramer-von Mises, Anderson-Darling, Kuiper, Kolmogorov-Smirnov, and Chi-squared goodness-of-fit tests to determine the distributions of 772 different cryptocurrencies. After [Shanaev and Ghimire, 2021] indicated that cryptocurrencies have non-normal distributions, this thesis, which investigates the reliability assumption of the NARDL model developed under normal distribution, makes an important contribution to the literature.

Table 2.1: The summary of literature review.

Authors	Variables	Methods/Models	Software
Sayed et al. (2022)	Green Bond, Economic Policy uncertainty, Bitcoin, S&P Global Clean Energy Index and Oil Prices	NARDL	Unspecified
Zhang et al. (2022)	Daily data of NFTs, Stocks, Bonds, US dollar, Gold, Crude oil and Bitcoin	NARDL	Unspecified
Zhu et al. (2022)	COVID-19 sentiment indicators, Bitcoin and gold	NARDL	Unspecified
Ghorbel et al. (2022)	6 cryptocurrencies (Bitcoin, Litecoin, Bitcoin gold, Dash, Maker, ve Ethereum) and 7 stock markets	NARDL	STATA
Jareño et al. (2022)	Bitcoin, Ethereum, Cardano, Tether and commodity returns	NARDL	Unspecified
Lin and An (2021)	Bitcoin price and resource commodity futures price	NARDL	Unspecified
Moussa et al. (2021)	Bitcoin, Oil, Natural Gas, Gold and Coal	Smooth Transition Error Correction Model (STECM)	Unspecified
Jareño et al. (2021)	Oil prices and the cryptocurrency	NARDL	Unspecified
Gaies et al. (2021)	Bitcoin	ARDL and NARDL	Unspecified

Table 2.1: Continued.			
Long et al. (2021)	Bitcoin, gold and global economic policy uncertainty, US stock market volatility index and the CBOE crude oil ETF volatility index	NARDL	Unspecified
Çaşkurlu and Arslan (2021)	Financial Depth, Cryptocurrency Volume, Precious Metals Account, Stock	ARDL and NARDL	Unspecified
Contuk (2021)	Bitcoin, Gold, Brent Oil	NARDL	Unspecified
Jeribi et al. (2021)	The top five cryptocurrencies (Bitcoin, Ethereum, Dash, Monero, and Ripple) and gold for BRICS stock markets	NARDL	Unspecified
Demir et al. (2020)	Bitcoin and altcoins, namely Ethereum, Ripple and Litecoin	NARDL	Unspecified
Jareño et al. (2020)	Bitcoin Returns and Gold Price returns	NARDL and Quantile Regression Approach	EViews
De la O González et al. (2020)	Bitcoin returns and returns of ten additional cryptocurrencies	NARDL	Unspecified
Bouri et al. (2018)	Aggregate commodity index and gold prices on the price of Bitcoin	NARDL and Quantile ARDL	Unspecified
Ciaian et al. (2018)	Bitcoins and six altcoins	ARDL	Unspecified
Canoz and Dirican (2017)	Bitcoin, BIST100, FTSE100 and NIKKEI225	ARDL	EViews 9.0

3. MONTE CARLO SIMULATION & DATA GENERATION PROCESS

3.1. Model

In order to examine whether the non-linear ARDL model is reliable when used for variables with non-normal distribution, Monte Carlo simulation experiments were conducted using the NARDL error verification model designed by [Shin et al., 2014] when the NARDL model was first created. The definitions of the parameters in the model are given in table 3.1.

Table 3.1: Parameter Definitions for NARDL ECM.

Parameters	Parameter Definition
y_t	Dependent Variable
x_t	Independent Variable
α	Constant Term
ρ	Error Correction Parameter
β^+	Positive long-run asymmetric parameter
β^-	Negative long-run asymmetric parameter
φ^+	Positive short-run asymmetric parameter
φ^-	Negative short-run asymmetric parameter
x_t^+	Positive changes of x_t
x_t^-	Negative changes of x_t
u_t	Error term
ω	Correlation Parameter

In Monte Carlo simulation experiments, the following NARDL error correction model (ECM) has been used to generate data for the dependent and independent variables.

$$\Delta y_t = \alpha + \rho(y_{t-1} - \beta^+ x_{t-1}^+ - \beta^- x_{t-1}^-) + \varphi^+ \Delta x_t^+ + \varphi^- \Delta x_t^- + u_t \quad (3.1)$$

where $\Delta x_t = \varepsilon_t$, and (u_t, ε_t) are serially uncorrelated.

x_t^+ and x_t^- denote the partial sum process of positive and negative changes in x_t

$$x_t^+ = \sum_{i=1}^t \Delta x_i^+ = \sum_{i=1}^t \max(\Delta x_i, 0) \quad (3.2)$$

$$x_t^- = \sum_{i=1}^t \Delta x_i^- = \sum_{i=1}^t \min(\Delta x_i, 0) \quad (3.3)$$

When $\omega \neq 0$, formula (3.1) can be estimated by:

$$\Delta y_t = \alpha + \rho y_{t-1} + \theta^+ x_{t-1}^+ + \theta^- x_{t-1}^- + \pi^+ \Delta x_t^+ + \pi^- \Delta x_t^- + e_t \quad (3.4)$$

where $\pi^+ = \varphi^+ + \omega$ and $\pi^- = \varphi^- + \omega$, and the parameters shown to the long run

relationship is denoted as $\beta^+ = \frac{-\theta^+}{\rho}$ ve $\beta^- = \frac{-\theta^-}{\rho}$.

After performing the simple data generation process for the dependent and independent variables, Monte Carlo Simulation procedures were performed for equation (3.1) using the default parameter values by [Shin et al., 2014] as shown in Table 3.2.

Table 3.2: Parameter Values for Simulation.

Parameter Definition	Parameter	Value
Constant Term	α	0
Error Correction Parameter	ρ	-0.2
Positive long-run asymmetric parameter	β^+	0.5
Negative long-run asymmetric parameter	β^-	$\beta^+ + \delta_\beta$
Negative long-run asymmetric delta parameter	δ_β	0.5
Positive short-run asymmetric parameter	φ^+	0.5

Table 3.2: Continued.		
Negative short-run asymmetric parameter	φ^-	$\varphi^+ + \delta_\varphi$
Negative short-run asymmetric delta parameter	δ_φ	0.5
Sample Size	T	100,200,400
Number of Simulation Replication	N	3000
Correlation Parameter	ω	-0.5,0,0.5
Degrees of Freedom	v or df	2.1,3,10

3.2. Distributions

The student-t distribution, Skew-t distribution, which are non-normal distributions, and Normal Distribution have been used in the scope of this thesis.

3.2.1. Normal Distribution

The parameters of u_t and $\Delta x_t = \varepsilon_t$ are generated in the Monte Carlo Simulation under Normal Distribution.

$$u_t = \omega z_1 + \sqrt{1 - \omega^2} z_2 \quad (3.5)$$

$$\Delta x_t = \varepsilon_t = z_1 \quad (3.6)$$

In the standard model based on the normal distribution, z_1 and z_2 are independent random values with zero mean and unit standard deviation derived under the standard normal distribution.

3.2.2 Student-t Distribution

The parameters of u_t and $\Delta x_t = \varepsilon_t$ are generated in the Monte Carlo Simulation under Student-t Distribution by using equation (3.5) and (3.6), respectively. z_1 and z_2 are independent random variable under student-t distribution.

$$z_1, z_2 \sim \sqrt{\frac{v-2}{v}} rt \quad (3.7)$$

rt are random values generated under Student-t distribution. The reason these values are multiplied by $\sqrt{\frac{v-2}{v}}$ is to compensate for the fact that z_1 and z_2 have unit variance. Table 3.3 shows the values of the degrees of freedom parameter of Student-t Distribution, which are used in the experiments.

Table 3.3: Parameter Values for Student-t Distribution.

Parameter Definition	Parameter	Value
Degrees of Freedom	v or df	2.1,3,10

In the statistics, the normal distribution seen in Figure 3.1 is the most widely used continuous probability distribution. In the context of this thesis, the student-t distribution's degrees of freedom of 2.1, 3 and 10 were chosen because, while bell-shaped and symmetrical, the student-t distribution has heavier tails than the normal distribution, which means that more values in the distribution are located in the tail ends than the center when compared to the normal distribution. Also, [Shanaev and Ghimire, 2021] illustrated that the distributions of cryptocurrencies are heavy tailed and non-normal. The lowest degrees of freedom was selected 2.1. During the simple random data generation, values less than 2 could not be included due to the equation (3.7) added to ensure that the z_1 and z_2 values have unit variance.

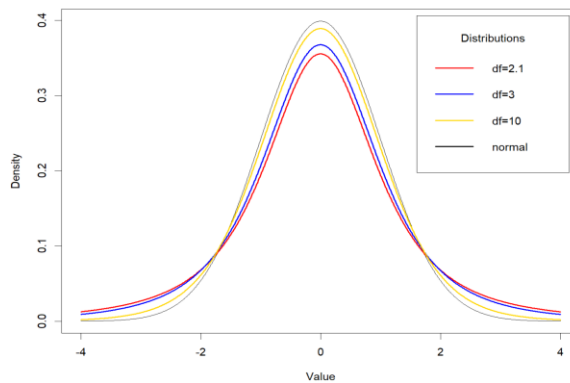


Figure 3.1: Comparison of Student-t and Standard Normal Distributions.

3.2.3. Skew-t Distribution

Another non-normal distribution to use is the Skew-t Distribution. In Monte Carlo Simulation experiments under this distribution, the parameters of u_t and $\Delta x_t = \varepsilon_t$ are generated by using equation (3.5) and (3.6), respectively. z_1 and z_2 are independent random variable under skew-t distribution.

$$z_1, z_2 \sim \frac{rst - \mu}{\sqrt{\sigma^2}} \quad (3.8)$$

rst are random values generated under the skew-t distribution. The generated random values are standardized to ensure that z_1 and z_2 have zero mean and unit standard deviation. The formulas suggested by [Azzalini and Capitanio, 2014] were used to calculate the mean μ and variance σ^2 values of the skew-t distribution. The authors stated that assuming the random variable Y has a Skew-t distribution, it would be written as follows:

$$Y \sim ST(\xi, \omega_{skew}^2, \alpha_{skew}, v) \quad (3.9)$$

ξ , ω_{skew} , α_{skew} and v denote respectively position parameter, scale parameter, skewness parameter and degrees of freedom. The mean and variance formula of Y variable is as follows.

$$\mu = \xi + \omega_{skew} b_v \delta, \quad \text{if } v > 1 \quad (3.10)$$

$$\sigma^2 = \omega_{skew}^2 \left[\frac{v}{v-2} - (b_v * \delta) \right], \quad \text{if } v > 2 \quad (3.11)$$

$$b_v = \frac{\sqrt{v} \Gamma(\frac{1}{2}(v-1))}{\sqrt{\pi} \Gamma(\frac{1}{2}v)}, \quad \text{if } v > 1 \quad (3.12)$$

$$\delta = \frac{\alpha_{skew}}{\sqrt{1 + \alpha_{skew}^2}} \quad (3.13)$$

Table 3.4 shows the parameter values to generate simple data for Skew-t Distribution.

Table 3.4: Parameter Values for Skew-t Distribution.

Parameter Definition	Parameter	Value
Degrees of freedom	ν or df	2.1,3,10
Location parameter	ξ	0
Scale parameter	ω_{skew}	1
Skewness parameter	a_{skew}	-2, -1,1,2

The Skew-t distribution is deemed to be a highly adaptable family of distributions capable of approximating a vast spectrum of empirical return distributions. Such distributions can range from being close to Gaussian (esp. when the degree of freedom parameter is high) to those exhibiting heavy-tailed or even infinite-variance (in cases where the degrees of freedom parameter is less than two) [Shanaev and Ghimire, 2021]. During the Monte Carlo simulation trials concerning the skew-t distribution, the skewness parameter values were ascertained utilizing the exploration performed by [Shanaev and Ghimire, 2021]. Moreover, -1 and -2 values were included in the experiments.

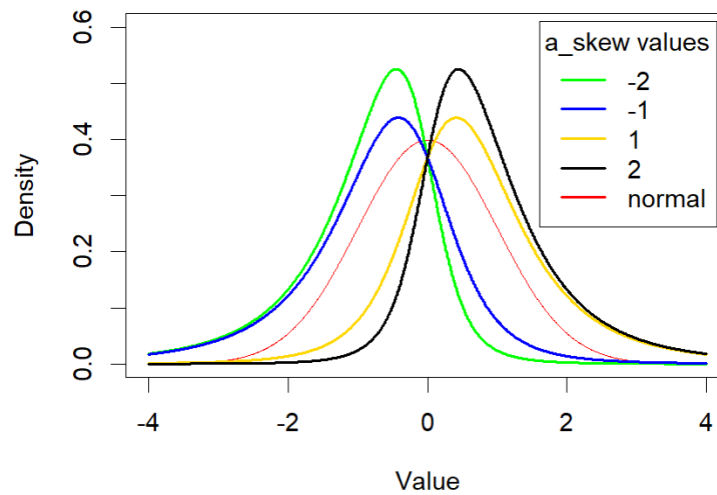


Figure 3.2: Comparison of Skew-t Distributions when df=3.

In Figure 3.2, the comparison of probability density distribution graphs with different skewness parameter values in cases where the degree of freedom value is 3 is given.

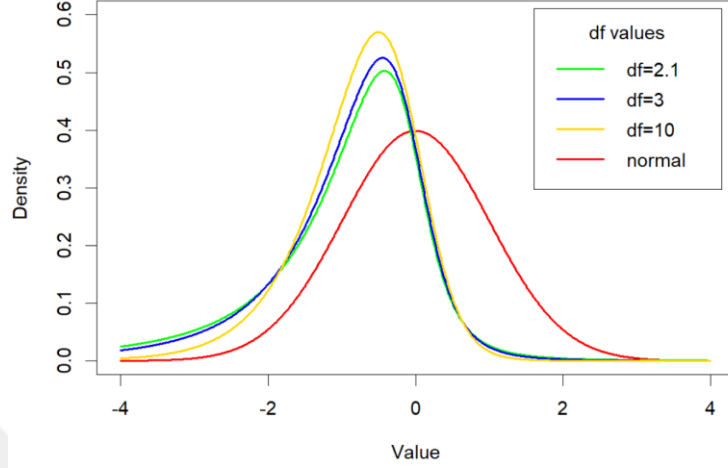


Figure 3.3: Comparison of Skew-t Distributions when $a_{\text{skew}} = -2$.

In Figure 3.3, the comparison of the probability density distribution graphs with different degree of freedoms parameter values in cases where the skewness parameter value is -2 is given.

3.3. Hypotheses

The finite sample properties of the estimators and the sizes (type-I error probabilities) and the powers of Wald Statistics and PSS Boundary tests for the following hypotheses have been investigated by performing Monte Carlo simulation experiments for each distribution mentioned above.

$$H_{LR}^S: \beta^+ = \beta^- \text{ (Null hypothesis for long – run symmetry)} \quad (3.14)$$

$$\Delta y_t = \alpha + \rho (y_{t-1} - \beta x_{t-1}^+ + \varphi^+ \Delta x_t^+ + \varphi^- \Delta x_t^- + u_t \text{ (Reduced model under } H_{LR}) \quad (3.15)$$

$$H_{SR}^S: \varphi^+ = \varphi^- \text{ (Null hypothesis for short – run symmetry)} \quad (3.16)$$

$$\Delta y_t = \alpha + \rho (y_{t-1} - \beta^+ x_{t-1}^+ - \beta^- x_{t-1}^-) + \varphi \Delta x_t + u_t \text{ (Reduced model under } H_{SR}) \quad (3.17)$$

$$H_{PSS}: \rho = \beta^+ = \beta^- = 0 \text{ (Null Hypothesis for no asymmetric cointegration)} \quad (3.18)$$

$$\Delta y_t = \alpha + \varphi^+ \Delta x_t^+ + \varphi^- \Delta x_t^- + u_t \text{ (Reduced model under } H_{PSS}) \quad (3.19)$$

3.4. Algorithm

During the Monte Carlo simulation experiments, the simple data generation process was carried out using R version 4.1.3 [R Core Team, 2022]. The random values of rt are generated from the student-t distribution by using the built-in function in R. The random values of rst following the Skew-t distribution, on the other hand, are generated by using the R package *sn* [Azzalini, 2022] with reference to [Azzalini and Capitanio, 2014]. After the simple data generation process, the NARDL package developed by Marco Sunder [Web 2, n.d.] for STATA was used with *RStata* package [Braglia, 2016]. The results obtained in STATA were analyzed in R.

The algorithm for the simulation experiment is shown in Table 3.5.

Table 3.5: Pseudocode for Monte Carlo Simulation of NARDL.

```

 $y_0, x_0 = 0$  [default parameter]
Create empty data frames for coefficient values, Wald Tests and PSS Tests
Create empty data frames for  $\alpha, \rho, \varphi^+, \varphi^-, \pi^+, \pi^-, \beta^+$  and  $\beta^-$ 
for  $i$  from 1 to N
  Create empty data frames for  $y, x, \Delta x, \Delta x^+, \Delta x^-, x^+, x^-$ 
  for  $t$  from 1 to T
    Generate  $z_1$  and  $z_2$  from the selected probability distribution
    Compute error term ( $u_t$ ) by using equation (3.5)
    Set  $\Delta x_t = z_1$ 
    Create positive and negative changes in  $x_t$ 
    Compute NARDL Error correction model by using equation (3.1)
    Compute  $y_t, x_t, x_t^-$  and  $x_t^+$ 
  end
  Create a data table for  $y_1 \dots y_T$  and  $x_1 \dots x_T$ 
  Connect to STATA with RStata to run NARDL model or run the model by using nardl library in R.
  Extract the coefficient values to empty data frames
  Extract WSR, WLR and FPSS values to empty data frames by comparing critical values.
end
Compute bias, standard deviation, and Root Mean Square Error (RMSE) for the coefficient values
Compute estimates of size and power of Wald Tests and PSS Tests
Create tables for the coefficient values, Wald Tests and PSS Tests

```

4. MONTE CARLO SIMULATION RESULTS

In this section, the NARDL model's simulation results from Monte Carlo experiments using variables with various distributions are displayed. During the experiments, the normal distribution, student-t distribution, and skew-t distribution were all used. The first three sections provide the results of the assumption that the random variables z_1 and z_2 have the same type of distribution, while the fourth section provides the results of the assumption that z_1 has a normal distribution and z_2 has a student-t distribution. The results of the experiments are presented in the tables that are placed in the four sections that are deemed to be important, and the remaining tables are placed in the appendix section.

The parameter estimates based on the simulations utilizing 3,000 replications of the decided baseline case are summarized in Tables 4.2, 4.4, 4.6, and 4.8 along with a variety of summary statistics. As can be observed, bias and error in estimating each parameter are virtually minimal (this is true for the other parameterizations of the data generating process that we take into consideration as well). The error correction parameter, which exhibits a little downward bias, especially when $T < 100$, is the lone exception to this generality. However, considering the well-established downward bias connected with the estimate of AR(1) coefficients in time series models, this discovery is not at all surprising [Shin et al., 2014]

4.1. Normal Distribution

In Table 4.1, W_{LR} , W_{SR} and F_{PSS} refer to the long-term Wald Statistical Test, the short-term Wald Statistical Test and the PSS Boundary Test for Asymmetric Cointegration, respectively. The finite sample properties for the normal distribution and the estimates of the size (type-I error probability) and power of the Wald Statistics and PSS Boundary tests for the hypotheses in equations (3.14), (3.16) and (3.18) are shown. These values summarize the results of the simulation experiments performed assuming a 5% significance level. For $T = 100$, the long-term Wald test has a very high power, the short-term Wald test has a medium-low, and the PSS tests have a medium power, but these powers seem to improve rapidly as the sample size increases. When the sample size is equal to 400, all tests for W_{LR} and F_{PSS} reach close to 100% power.

The size values show the type-I error probabilities and the size values close to 5% are desired. As a result of the simulations, it is seen that the size value mostly decreases as the T value increases in all ω values. The largest size value has emerged in the simulation experiment where T is 100 and ω is 0 for W_{LR} . In this experiment, it was observed that the type 1 error value decreased when the number of samples was increased. The findings of the simulation show that when T rises, outcomes almost match the desired value.

The simulation results performed under the normal distribution within the scope of the thesis largely overlap with the results of the simulation experiments conducted by [Shin et al., 2014] to examine the finite sample properties of the estimators. This overlap shows that the algorithm we have established for the simulation experiments works correctly and can be used for different distributions.

Table 4.1: Monte Carlo Simulation Results for Normal Distribution: Size and Power of Wald and PSS Tests.

Normal Distribution									
	T = 100			T = 200			T = 400		
	Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$	W_{LR}	0.995	0.128	W_{LR}	1.000	0.098	W_{LR}	1.000	0.075
	W_{SR}	0.196	0.054	W_{SR}	0.363	0.050	W_{SR}	0.661	0.053
	F_{PSS}	0.604	0.079	F_{PSS}	0.995	0.075	F_{PSS}	1.000	0.070
$\omega = 0$	W_{LR}	0.989	0.132	W_{LR}	1.000	0.098	W_{LR}	1.000	0.072
	W_{SR}	0.163	0.048	W_{SR}	0.299	0.054	W_{SR}	0.521	0.049
	F_{PSS}	0.430	0.075	F_{PSS}	0.959	0.067	F_{PSS}	1.000	0.066
$\omega = 0.5$	W_{LR}	0.993	0.120	W_{LR}	1.000	0.083	W_{LR}	1.000	0.061
	W_{SR}	0.196	0.050	W_{SR}	0.375	0.048	W_{SR}	0.655	0.042
	F_{PSS}	0.585	0.076	F_{PSS}	0.995	0.071	F_{PSS}	1.000	0.075

Table 4.2: Monte Carlo Simulation Results for Normal Distribution: Bias, Standard Error and RMSE of the OLS Estimators.

Normal Distribution												
	T = 100				T = 200				T = 400			
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = -0.5$	α	0.000	0.001	0.001	α	0.004	0.000	0.000	α	0.002	4.989	5.065
	ρ	-0.074	0.078	0.107	ρ	-0.034	0.048	0.058	ρ	-0.016	0.031	0.035
	$\theta+$	-0.037	0.059	0.380	$\theta+$	-0.038	0.031	0.388	$\theta+$	-0.039	0.017	0.393
	$\theta-$	-0.073	0.082	0.743	$\theta-$	-0.077	0.049	0.771	$\theta-$	-0.078	0.031	0.786
	$\pi+$	-0.050	0.177	0.534	$\pi+$	-0.049	0.125	0.514	$\pi+$	-0.050	0.087	0.507
	$\pi-$	-0.050	0.186	0.533	$\pi-$	-0.050	0.122	0.517	$\pi-$	-0.050	0.087	0.508
	$\beta+$	-0.030	0.213	0.215	$\beta+$	-0.011	0.104	0.105	$\beta+$	-0.002	0.052	0.052
	$\beta-$	-0.530	0.213	0.571	$\beta-$	-0.511	0.104	0.521	$\beta-$	-0.502	0.052	0.505
$\omega = 0$	α	-0.047	0.456	0.459	α	-0.051	0.313	0.317	α	-0.012	0.075	0.076
	ρ	-0.089	0.089	0.126	ρ	-0.041	0.053	0.067	ρ	-0.021	0.034	0.039
	$\theta+$	-0.035	0.078	0.365	$\theta+$	-0.037	0.038	0.381	$\theta+$	-0.039	0.021	0.390
	$\theta-$	-0.071	0.109	0.720	$\theta-$	-0.075	0.060	0.761	$\theta-$	-0.077	0.036	0.780
	$\pi+$	-0.003	0.211	0.211	$\pi+$	0.002	0.144	0.144	$\pi+$	0.000	0.099	0.099
	$\pi-$	-0.002	0.211	0.211	$\pi-$	-0.002	0.143	0.143	$\pi-$	-0.003	0.099	0.099
	$\beta+$	-0.004	0.237	0.237	$\beta+$	0.001	0.114	0.114	$\beta+$	0.001	0.057	0.057
	$\beta-$	-0.504	0.237	0.557	$\beta-$	-0.499	0.114	0.512	$\beta-$	-0.499	0.057	0.503
$\omega = 0.5$	α	-0.004	0.031	0.032	α	-0.001	0.009	0.009	α	0.000	0.002	0.002
	ρ	-0.073	0.081	0.109	ρ	-0.035	0.047	0.058	ρ	-0.017	0.032	0.036
	$\theta+$	-0.035	0.081	0.361	$\theta+$	-0.037	0.037	0.381	$\theta+$	-0.039	0.021	0.391
	$\theta-$	-0.071	0.114	0.724	$\theta-$	-0.076	0.057	0.764	$\theta-$	-0.078	0.035	0.783
	$\pi+$	0.050	0.183	0.532	$\pi+$	0.050	0.124	0.516	$\pi+$	0.050	0.087	0.508
	$\pi-$	0.049	0.184	0.526	$\pi-$	0.050	0.122	0.517	$\pi-$	0.050	0.087	0.507
	$\beta+$	0.026	0.217	0.219	$\beta+$	0.010	0.103	0.104	$\beta+$	0.003	0.051	0.051
	$\beta-$	-0.474	0.217	0.521	$\beta-$	-0.490	0.103	0.501	$\beta-$	-0.497	0.051	0.500

4.2. Student-t Distribution

Table 4.3 displays the results of simulation experiments on the long-term and short-term Wald Statistical Tests (W_{LR} and W_{SR}) and the PSS Boundary Test for Asymmetric Cointegration (F_{PSS}). The experiments aimed to investigate the finite sample properties for the student-t distribution and the outcomes of the size and power of the hypotheses in equations (3.14), (3.16) and (3.18). All values in this table were obtained using the 5% significance level. For $T = 100$, the long-term Wald test demonstrated a high power, while the short-term Wald test and PSS tests had medium-low and medium power, respectively. However, the power of all tests improved considerably as the sample size increased, with all tests W_{LR} and F_{PSS} reaching almost 100% power for a sample size of 400. When the degree of freedom is 2.1, the test power for W_{SR} increases to nearly 80%. When the degree of freedom increases, the power for W_{LR} decreases slightly.

As a result of the simulation experiments performed under different omega and degrees of freedoms in the student-t distribution, it has been noticed that the size of the long-term Wald Statistical Test has a value close to 10% when T is 100. This size was shown to decline and approach 5% as T increased. The type-I probability remained unchanged despite variations in the degrees of freedom. In the size values of the short-term Wald Statistics, changes in T , omega or degrees of freedoms did not cause a large change, and values close to 5% emerged in all cases.

Table 4.3: Monte Carlo Simulation Results for Student-t Distribution: Size and Power of Wald and PSS Tests.

T-Distribution										
		T = 100			T = 200			T = 400		
		Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$	df= 2.1	W_{LR}	0.950	0.127	W_{LR}	0.997	0.095	W_{LR}	1.000	0.084
	df= 3		0.980	0.117		1.000	0.100		1.000	0.073
	df=10		0.991	0.133		1.000	0.110		1.000	0.075
	df= 2.1	W_{SR}	0.304	0.054	W_{SR}	0.512	0.056	W_{SR}	0.789	0.054
	df= 3		0.261	0.061		0.496	0.043		0.794	0.052
	df=10		0.198	0.049		0.410	0.060		0.695	0.060
	df= 2.1	F_{PSS}	0.604	0.088	F_{PSS}	0.993	0.079	F_{PSS}	1.000	0.073
	df= 3		0.596	0.082		0.989	0.065		1.000	0.073
	df=10		0.595	0.083		0.992	0.070		1.000	0.072
$\omega = 0$	df= 2.1	W_{LR}	0.932	0.128	W_{LR}	0.996	0.087	W_{LR}	1.000	0.067
	df= 3		0.976	0.132		0.999	0.096		1.000	0.077
	df=10		0.990	0.134		1.000	0.104		1.000	0.066
	df= 2.1	W_{SR}	0.226	0.053	W_{SR}	0.425	0.062	W_{SR}	0.688	0.058
	df= 3		0.208	0.058		0.389	0.047		0.682	0.049
	df=10		0.171	0.053		0.316	0.051		0.573	0.054
	df= 2.1	F_{PSS}	0.400	0.080	F_{PSS}	0.964	0.080	F_{PSS}	1.000	0.075
	df= 3		0.414	0.081		0.963	0.073		1.000	0.069
	df=10		0.421	0.071		0.958	0.074		1.000	0.078
$\omega = 0.5$	df= 2.1	W_{LR}	0.945	0.104	W_{LR}	0.999	0.079	W_{LR}	1.000	0.062
	df= 3		0.978	0.110		1.000	0.087		1.000	0.073
	df=10		0.990	0.121		1.000	0.086		1.000	0.061
	df= 2.1	W_{SR}	0.298	0.055	W_{SR}	0.525	0.052	W_{SR}	0.796	0.055
	df= 3		0.259	0.056		0.485	0.051		0.790	0.053
	df=10		0.209	0.052		0.403	0.050		0.689	0.051
	df= 2.1	F_{PSS}	0.628	0.085	F_{PSS}	0.993	0.077	F_{PSS}	1.000	0.075
	df= 3		0.602	0.078		0.992	0.073		1.000	0.075
	df=10		0.576	0.077		0.995	0.071		1.000	0.070

Table 4.4: Monte Carlo Simulation Results for Student-t Distribution: Bias, Standard Error and RMSE of the OLS Estimators.

		T-Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = 0$	df = 2.1	α	-0.026	0.287	0.288	α	-0.020	0.208	0.209	α	-0.017	0.257	0.258
		ρ	-0.083	0.085	0.119	ρ	-0.040	0.050	0.065	ρ	-0.020	0.033	0.039
		$\theta+$	-0.036	0.121	0.380	$\theta+$	-0.038	0.047	0.382	$\theta+$	-0.039	0.025	0.390
		$\theta-$	-0.072	0.140	0.733	$\theta-$	-0.076	0.063	0.761	$\theta-$	-0.078	0.038	0.780
		$\pi+$	-0.003	0.317	0.317	$\pi+$	-0.003	0.156	0.156	$\pi+$	0.000	0.116	0.116
		$\pi-$	-0.003	0.291	0.291	$\pi-$	0.003	0.166	0.166	$\pi-$	-0.003	0.131	0.131
		$\beta+$	-0.008	0.427	0.427	$\beta+$	0.005	0.169	0.169	$\beta+$	0.001	0.082	0.082
		$\beta-$	-0.508	0.427	0.663	$\beta-$	-0.495	0.169	0.524	$\beta-$	-0.499	0.082	0.506
	df = 3	α	-0.046	0.461	0.464	α	-0.043	0.295	0.298	α	-0.037	0.217	0.220
		ρ	-0.088	0.087	0.123	ρ	-0.043	0.053	0.068	ρ	-0.020	0.034	0.039
		$\theta+$	-0.035	0.082	0.364	$\theta+$	-0.038	0.041	0.381	$\theta+$	-0.039	0.022	0.391
		$\theta-$	-0.071	0.113	0.719	$\theta-$	-0.076	0.062	0.760	$\theta-$	-0.078	0.037	0.781
		$\pi+$	0.002	0.202	0.202	$\pi+$	-0.003	0.136	0.136	$\pi+$	0.001	0.092	0.092
		$\pi-$	0.000	0.209	0.209	$\pi-$	0.005	0.137	0.137	$\pi-$	0.000	0.115	0.115
		$\beta+$	0.006	0.254	0.254	$\beta+$	0.001	0.126	0.126	$\beta+$	0.002	0.061	0.061
		$\beta-$	-0.494	0.254	0.556	$\beta-$	-0.499	0.126	0.515	$\beta-$	-0.498	0.061	0.502
	df = 10	α	-0.050	0.471	0.474	α	-0.040	0.315	0.318	α	-0.038	0.235	0.238
		ρ	-0.087	0.089	0.124	ρ	-0.042	0.053	0.068	ρ	-0.018	0.034	0.039
		$\theta+$	-0.036	0.083	0.367	$\theta+$	-0.038	0.039	0.381	$\theta+$	-0.039	0.021	0.391
		$\theta-$	-0.071	0.115	0.722	$\theta-$	-0.076	0.061	0.760	$\theta-$	-0.078	0.036	0.782
		$\pi+$	0.007	0.206	0.206	$\pi+$	0.001	0.138	0.138	$\pi+$	0.003	0.095	0.095
		$\pi-$	-0.001	0.204	0.204	$\pi-$	-0.001	0.138	0.138	$\pi-$	0.000	0.096	0.096
		$\beta+$	-0.004	0.244	0.244	$\beta+$	0.001	0.119	0.119	$\beta+$	0.000	0.059	0.059
		$\beta-$	-0.504	0.244	0.560	$\beta-$	-0.499	0.119	0.513	$\beta-$	-0.500	0.059	0.503

4.3. Skew-t Distribution

In this section, the results obtained with the Monte Carlo simulation for the skew-t distribution are shown. The PSS Boundary Test for Asymmetric Cointegration (F_{PSS}) and the long- and short-term Wald Statistical Tests (W_{LR} and W_{SR}) simulation experiment findings are shown in Table 4.5. The studies sought to learn more about the skew-t distribution's finite sample qualities as well as the results of the size and power of the hypothesis tests in equations (3.14), (3.16), and (3.18). The 5% significance level was used to evaluate all of the values in this table. The PSS tests had a medium power, whereas the long-term Wald test showed a high power for $T = 100$. However, as the sample size rose, all tests' power increased significantly, with all W_{LR} and F_{PSS} tests reaching about 100% power with a sample size of 400. It is expected that the values increase when the degrees of freedom approach 10 for each a_{skew} value, but there is no significant difference between low degrees of freedom and high degrees of freedom. It was observed that the power of Wald and PSS tests increased as the number of samples increased for each degree of freedom. As in the normal and student-t distributions, it is observed that the short-term improves as the sample size increases in the skew-t distributions. Finally, the most important conclusion noticed in Table 4.5 is that when $\omega = -0.5$, $a_{skew} = 2$, and $T=400$, the short-term Wald Test has a very low power compared to other situations. This means that the NARDL model may not accurately show the short-run asymmetric relationship when it is used for a variable that fits the above-mentioned distribution.

Table 4.5: Monte Carlo Simulation Results for Skew-t Distribution: Size and Power of Wald and PSS Tests ($a_{\text{skew}} = 2$).

Skew-t Distribution										
		T = 100			T = 200			T = 400		
		Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$ $a_{\text{skew}}=2$	df= 2.1	W_{LR}	0.931	0.123	W_{LR}	0.996	0.084	W_{LR}	0.997	0.072
	df= 3		0.978	0.111		0.999	0.098		1.000	0.062
	df=10		0.988	0.119		1.000	0.093		1.000	0.084
	df= 2.1	W_{SR}	0.192	0.054	W_{SR}	0.338	0.045	W_{SR}	0.380	0.045
	df= 3		0.214	0.056		0.374	0.060		0.622	0.053
	df=10		0.202	0.054		0.367	0.048		0.669	0.051
	df= 2.1	F_{PSS}	0.610	0.082	F_{PSS}	0.989	0.071	F_{PSS}	0.993	0.075
	df= 3		0.602	0.081		0.989	0.073		1.000	0.070
	df=10		0.590	0.078		0.992	0.079		1.000	0.079
$\omega = 0$ $a_{\text{skew}}=2$	df= 2.1	W_{LR}	0.924	0.125	W_{LR}	0.993	0.093	W_{LR}	0.999	0.087
	df= 3		0.954	0.128		1.000	0.094		1.000	0.075
	df=10		0.985	0.128		1.000	0.096		1.000	0.077
	df= 2.1	W_{SR}	0.153	0.053	W_{SR}	0.272	0.052	W_{SR}	0.461	0.046
	df= 3		0.167	0.063		0.296	0.056		0.509	0.056
	df=10		0.160	0.049		0.295	0.042		0.546	0.054
	df= 2.1	F_{PSS}	0.414	0.071	F_{PSS}	0.962	0.063	F_{PSS}	1.000	0.070
	df= 3		0.420	0.077		0.961	0.079		1.000	0.071
	df=10		0.423	0.083		0.953	0.082		1.000	0.067
$\omega = 0.5$ $a_{\text{skew}}=2$	df= 2.1	W_{LR}	0.940	0.110	W_{LR}	0.994	0.088	W_{LR}	0.999	0.067
	df= 3		0.975	0.113		1.000	0.083		1.000	0.075
	df=10		0.976	0.126		1.000	0.087		1.000	0.064
	df= 2.1	W_{SR}	0.182	0.059	W_{SR}	0.336	0.055	W_{SR}	0.553	0.049
	df= 3		0.200	0.051		0.372	0.051		0.617	0.051
	df=10		0.201	0.054		0.373	0.055		0.620	0.049
	df= 2.1	F_{PSS}	0.620	0.076	F_{PSS}	0.991	0.069	F_{PSS}	1.000	0.077
	df= 3		0.616	0.082		0.993	0.073		1.000	0.075
	df=10		0.618	0.075		0.992	0.073		1.000	0.066

Table 4.6: Monte Carlo Simulation Results for Skew-t Distribution: Bias, Standard Error and RMSE of the OLS Estimators ($a_{\text{skew}} = 2$ and $\omega = -0.5$).

		Skew-t Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = -0.5$ $a_{\text{skew}} = 2$	df = 2.1	α	-0.021	0.227	0.228	α	-0.022	0.143	0.145	α	-0.023	0.137	0.139
		ρ	-0.067	0.073	0.099	ρ	-0.030	0.044	0.053	ρ	-0.028	0.041	0.050
		$\theta+$	-0.381	0.096	0.393	$\theta+$	-0.387	0.039	0.389	$\theta+$	-0.388	0.036	0.390
		$\theta-$	-0.745	0.103	0.753	$\theta-$	-0.772	0.049	0.774	$\theta-$	-0.774	0.046	0.776
		$\pi+$	-0.500	0.213	0.543	$\pi+$	-0.502	0.124	0.517	$\pi+$	-0.500	0.122	0.514
		$\pi-$	-0.506	0.425	0.661	$\pi-$	-0.508	0.298	0.589	$\pi-$	-0.509	0.274	0.578
		$\beta+$	-0.040	0.352	0.354	$\beta+$	-0.006	0.153	0.153	$\beta+$	-0.007	0.141	0.141
		$\beta-$	-0.540	0.352	0.644	$\beta-$	-0.506	0.153	0.529	$\beta-$	-0.507	0.141	0.526
	df = 3	α	-0.043	0.362	0.364	α	-0.037	0.243	0.246	α	-0.035	0.199	0.202
		ρ	-0.069	0.077	0.104	ρ	-0.031	0.046	0.056	ρ	-0.015	0.029	0.032
		$\theta+$	-0.376	0.067	0.382	$\theta+$	-0.388	0.031	0.389	$\theta+$	-0.393	0.017	0.394
		$\theta-$	-0.741	0.085	0.746	$\theta-$	-0.772	0.048	0.774	$\theta-$	-0.786	0.029	0.786
		$\pi+$	-0.498	0.173	0.527	$\pi+$	-0.499	0.100	0.509	$\pi+$	-0.497	0.064	0.501
		$\pi-$	-0.501	0.268	0.568	$\pi-$	-0.507	0.185	0.540	$\pi-$	-0.505	0.128	0.521
		$\beta+$	-0.024	0.250	0.251	$\beta+$	-0.010	0.112	0.112	$\beta+$	-0.004	0.054	0.054
		$\beta-$	-0.524	0.250	0.581	$\beta-$	-0.510	0.112	0.523	$\beta-$	-0.504	0.054	0.507
	df = 10	α	-0.049	0.389	0.392	α	-0.045	0.274	0.277	α	-0.043	0.208	0.212
		ρ	-0.072	0.079	0.107	ρ	-0.033	0.048	0.058	ρ	-0.016	0.031	0.034
		$\theta+$	-0.376	0.060	0.381	$\theta+$	-0.386	0.030	0.388	$\theta+$	-0.393	0.018	0.393
		$\theta-$	-0.740	0.082	0.745	$\theta-$	-0.770	0.049	0.772	$\theta-$	-0.785	0.031	0.786
		$\pi+$	-0.502	0.161	0.527	$\pi+$	-0.500	0.108	0.512	$\pi+$	-0.501	0.072	0.506
		$\pi-$	-0.498	0.207	0.539	$\pi-$	-0.503	0.143	0.523	$\pi-$	-0.502	0.097	0.511
		$\beta+$	-0.028	0.225	0.226	$\beta+$	-0.009	0.104	0.105	$\beta+$	-0.003	0.051	0.051
		$\beta-$	-0.528	0.225	0.573	$\beta-$	-0.509	0.104	0.520	$\beta-$	-0.503	0.051	0.505

4.4. Normal Distribution and Student-t Distribution

Table 4.7 uses the terms W_{LR} , W_{SR} , and F_{PSS} to denote the long-term, short-term, and PSS Boundary Test for Asymmetric Cointegration, respectively. The results of the size and power of the Wald Statistics and PSS Boundary tests for the hypotheses in equations (3.14), (3.16), and (3.18) are provided, together with the finite sample characteristics for the normal and student-t distributions. These numbers provide a summary of the simulation experiments' findings at the 5% significance level. The PSS tests have a medium power, the short-term Wald test has a medium-low power, and the long-term Wald test has an extremely high power for $T = 100$, however these powers appear to increase quickly as the sample size grows. All W_{LR} and F_{PSS} tests have a power close to 100% when the sample size is 400. There is an intriguing circumstance where the short-term Wald test's power is extremely high when the degrees of freedom is 2.1. The power of the short-term Wald tests abruptly drop to approximately 60% as the degree of freedom rises to 3 and 10.

Table 4.7: Monte Carlo Simulation Results for Normal and Student-t Distribution: Size and Power of Wald and PSS Tests.

Normal and T-Distribution										
		T = 100			T = 200			T = 400		
		Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$	df= 2.1	W_{LR}	0.997	0.120	W_{LR}	1.000	0.079	W_{LR}	1.000	0.067
	df= 3		0.991	0.122		1.000	0.081		1.000	0.068
	df=10		0.993	0.130		1.000	0.088		1.000	0.069
	df= 2.1	W_{SR}	0.597	0.052	W_{SR}	0.825	0.048	W_{SR}	0.955	0.051
	df= 3		0.232	0.061		0.411	0.052		0.685	0.050
	df=10		0.197	0.053		0.381	0.046		0.657	0.052
	df= 2.1	F_{PSS}	0.853	0.071	F_{PSS}	1.000	0.083	F_{PSS}	1.000	0.071
	df= 3		0.615	0.083		0.995	0.082		1.000	0.074
	df=10		0.588	0.092		0.992	0.075		1.000	0.070
$\omega = 0$	df= 2.1	W_{LR}	0.994	0.123	W_{LR}	0.999	0.086	W_{LR}	1.000	0.075
	df= 3		0.990	0.138		1.000	0.088		1.000	0.079
	df=10		0.989	0.139		1.000	0.090		1.000	0.083
	df= 2.1	W_{SR}	0.480	0.046	W_{SR}	0.742	0.053	W_{SR}	0.896	0.050
	df= 3		0.182	0.056		0.337	0.056		0.562	0.053
	df=10		0.157	0.057		0.299	0.053		0.546	0.052
	df= 2.1	F_{PSS}	0.403	0.078	F_{PSS}	0.964	0.078	F_{PSS}	1.000	0.073
	df= 3		0.423	0.079		0.969	0.074		1.000	0.079
	df=10		0.424	0.077		0.951	0.065		1.000	0.076
$\omega = 0.5$	df= 2.1	W_{LR}	0.996	0.100	W_{LR}	1.000	0.078	W_{LR}	1.000	0.064
	df= 3		0.990	0.121		1.000	0.079		1.000	0.067
	df=10		0.994	0.123		1.000	0.083		1.000	0.066
	df= 2.1	W_{SR}	0.615	0.050	W_{SR}	0.831	0.047	W_{SR}	0.950	0.053
	df= 3		0.222	0.055		0.400	0.058		0.687	0.054
	df=10		0.191	0.055		0.365	0.048		0.652	0.052
	df= 2.1	F_{PSS}	0.862	0.081	F_{PSS}	0.999	0.069	F_{PSS}	1.000	0.071
	df= 3		0.607	0.073		0.993	0.078		1.000	0.072
	df=10		0.620	0.079		0.993	0.077		1.000	0.066

Table 4.8: Monte Carlo Simulation Results for Normal and Student-t Distribution: Bias, Standard Error and RMSE of the OLS Estimator.

		Normal and T-Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = 0$	df = 2.1	α	-0.049	0.332	0.335	α	-0.039	0.321	0.323	α	-0.050	0.210	0.216
		ρ	-0.084	0.083	0.118	ρ	-0.040	0.049	0.063	ρ	-0.020	0.033	0.039
		$\theta+$	-0.357	0.062	0.363	$\theta+$	-0.380	0.036	0.381	$\theta+$	-0.390	0.019	0.390
		$\theta-$	-0.715	0.094	0.721	$\theta-$	-0.760	0.057	0.762	$\theta-$	-0.780	0.034	0.781
		$\pi+$	0.000	0.129	0.129	$\pi+$	-0.002	0.105	0.105	$\pi+$	0.002	0.067	0.067
		$\pi-$	0.001	0.131	0.130	$\pi-$	0.002	0.162	0.162	$\pi-$	-0.002	0.065	0.065
		$\beta+$	0.013	0.570	0.570	$\beta+$	0.002	0.116	0.116	$\beta+$	0.001	0.043	0.043
		$\beta-$	-0.487	0.570	0.750	$\beta-$	-0.498	0.116	0.512	$\beta-$	-0.499	0.043	0.501
	df = 3	α	-0.058	0.484	0.487	α	-0.045	0.311	0.315	α	-0.047	0.235	0.239
		ρ	-0.086	0.085	0.121	ρ	-0.043	0.052	0.067	ρ	-0.019	0.033	0.038
		$\theta+$	-0.356	0.081	0.365	$\theta+$	-0.378	0.039	0.380	$\theta+$	-0.391	0.021	0.391
		$\theta-$	-0.714	0.109	0.722	$\theta-$	-0.757	0.059	0.759	$\theta-$	-0.782	0.035	0.782
		$\pi+$	-0.006	0.212	0.212	$\pi+$	0.001	0.138	0.138	$\pi+$	0.002	0.099	0.099
		$\pi-$	0.002	0.204	0.204	$\pi-$	-0.003	0.138	0.138	$\pi-$	-0.003	0.098	0.098
		$\beta+$	0.001	0.254	0.254	$\beta+$	0.000	0.116	0.116	$\beta+$	-0.001	0.057	0.057
		$\beta-$	-0.499	0.254	0.560	$\beta-$	-0.500	0.116	0.513	$\beta-$	-0.501	0.057	0.504
	df = 10	α	-0.050	0.460	0.463	α	-0.041	0.318	0.321	α	-0.044	0.243	0.247
		ρ	-0.088	0.088	0.125	ρ	-0.040	0.053	0.067	ρ	-0.021	0.034	0.040
		$\theta+$	-0.357	0.079	0.365	$\theta+$	-0.381	0.039	0.383	$\theta+$	-0.390	0.021	0.390
		$\theta-$	-0.713	0.110	0.721	$\theta-$	-0.761	0.060	0.763	$\theta-$	-0.779	0.036	0.780
		$\pi+$	-0.007	0.204	0.204	$\pi+$	0.001	0.142	0.142	$\pi+$	-0.002	0.099	0.099
		$\pi-$	0.005	0.209	0.209	$\pi-$	0.003	0.143	0.143	$\pi-$	0.001	0.099	0.099
		$\beta+$	-0.003	0.236	0.236	$\beta+$	-0.004	0.117	0.117	$\beta+$	-0.001	0.058	0.058
		$\beta-$	-0.503	0.236	0.556	$\beta-$	-0.504	0.117	0.517	$\beta-$	-0.501	0.058	0.504

5. COMPARISON OF STATA AND R

In this part of our thesis, the results of the Monte Carlo simulation experiments are demonstrated by comparing the results using two different applications. It has been seen that the values shown by the *nardl* package developed by [Zaghdoudi, 2017] in R, an open-source software, show different results with the values displayed by the *nardl* package in STATA. Due to this difference, the applications used in the studies using the NARDL model are determined and shown in the literature study in Table 2.1. Studies employing the NARDL model to analyze cryptocurrencies have been given priority because there is an immense amount of studies utilized NARDL model. In addition to the literature study in Table 2.1, [Allen and McAleer, 2021] used the *nardl* package developed by Taha Zaghdoudi in their two studies using the NARDL model. Information about the software and packages used in the majority of the studies have not been disclosed. The default maximum value for the number of lags of the variables in the model is 1. If the maximum lag value is greater than 1, the non-significant coefficient values in the model could not be reached from the results and the necessary analyzes could not be made. *Nardl* package developed by Marco Sunder [Web 2, n.d.] for STATA is used in the version shared by the author from his own site. *RStata* package [Braglia, 2016] is used to connect R and STATA. The results from STATA are saved in Excel and transferred to R and analyzed. In the Monte Carlo simulation experiments performed in STATA, the maximum lag values are chosen as 1.

The results in Table 5.1 show that the type I error probability (size) values were quite high as a result of the experiments using the *nardl* package in R. According to [Shin et al., 2014]’s study, the size values for W_{LR} , W_{SR} , and F_{PSS} are all less than 10%.

As can be seen from Table 5.1, the size values, especially for W_{LR} and F_{PSS} , are quite high. This means that although there is a symmetrical relationship in the long run, the results in R show that this relationship is asymmetrical. According to the results of the simulations in R, it seems that there is an asymmetrical relationship in the long run for almost all of the 3000 experiments with the increase in the value of T . A similar situation applies to the size value in F_{PSS} . Although there is no asymmetric cointegration between the variables, it seems that there is an asymmetric

in all experiments when ω are equal to 0 and 0.5. While it is expected that the size value close to 5%, which is a type 1 error probability, the size values close to 100% were found in the simulations made using the *nardl* package in R. Such type I error suggests that the R-created *nardl* package is not operating properly. Because the size values in Table 4.1, which were acquired when the identical study was conducted in STATA, were similar to those in the investigations by [Shin et al., 2014].



Table 5.1: Monte Carlo Simulation Results for Normal Distribution: Size and Power of Wald and PSS Tests (using *nardl* package in R).

Normal Distribution									
	T = 100			T = 200			T = 400		
	Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$	W _{LR}	0.760	0.668	W _{LR}	0.771	0.846	W _{LR}	0.832	0.976
	W _{SR}	0.494	0.204	W _{SR}	0.500	0.209	W _{SR}	0.608	0.288
	F _{PSS}	0.873	0.236	F _{PSS}	1.000	0.398	F _{PSS}	1.000	0.622
$\omega = 0$	W _{LR}	0.688	0.784	W _{LR}	0.925	0.933	W _{LR}	0.991	0.994
	W _{SR}	0.068	0.188	W _{SR}	0.316	0.180	W _{SR}	0.695	0.052
	F _{PSS}	0.999	0.978	F _{PSS}	1.000	1.000	F _{PSS}	1.000	1.000
$\omega = 0.5$	W _{LR}	0.853	0.859	W _{LR}	0.971	0.969	W _{LR}	0.999	0.998
	W _{SR}	0.232	0.020	W _{SR}	0.485	0.014	W _{SR}	0.845	0.020
	F _{PSS}	1.000	1.000	F _{PSS}	1.000	1.000	F _{PSS}	1.000	1.000

Table 5.2: Monte Carlo Simulation Results for Normal Distribution: Bias and Standard Error of the OLS Estimators (using *nardl* package in R).

Normal Distribution									
	T = 100			T = 200			T = 400		
	Coef	Bias	STDE	Coef	Bias	STDE	Coef	Bias	STDE
$\omega = -0.5$	ρ	-0.074	0.072	ρ	-0.040	0.044	ρ	-0.023	0.030
	α	-0.116	0.376	α	-0.115	0.288	α	-0.101	0.236
$\omega = 0$	ρ	-0.095	0.096	ρ	-0.041	0.056	ρ	-0.018	0.032
	α	-0.167	0.456	α	-0.070	0.308	α	-0.046	0.227
$\omega = 0.5$	ρ	-0.062	0.073	ρ	-0.028	0.043	ρ	-0.014	0.028
	α	-0.052	0.381	α	-0.041	0.263	α	-0.040	0.201

6. AN EMPIRICAL STUDY: GOLD AND CRYPTOCURRENCIES

In recent years, crypto money markets have grown in popularity among people, so cryptocurrencies have been accepted by people and investors as an investment tool. The price of Bitcoin, one of the most important cryptocurrencies, has been observed to increase from 327\$ to 27950\$, with an increase of approximately 8447% between November 2015 and April 2023. While this phenomenal growth demonstrates the significance of cryptocurrencies, its consequences on gold and other precious metals, which are now utilized as investment instruments, have been a source of speculation. In recent years, there has been a lot of research on the relationship between cryptocurrencies as well as the relates to between precious metals and cryptocurrencies. [Jareno et al., 2020] claim that they used the non-linear ARDL technique to investigate the long and short-term correlations between Bitcoin and gold price returns and discovered a positive and statistically significant correlation between these variables. [Demir et al., 2020], on the other hand, stated that they utilized the NARDL model to examine the relationship between Bitcoin and altcoins and identified proof of the asymmetrical impact of Bitcoin on altcoins in both the short- and long-term.

The NARDL model is used in this section of the thesis to analyze the impact of bitcoin and some of the cryptocurrencies we have discovered on gold as well as their interactions. Lisk and Stellar were picked as alternative cryptocurrencies since they were found to demonstrate the skew-t distribution in [Shanaev and Ghimire, 2021] 's study. The investigation also takes into account the cryptocurrencies Bitcoin and Bitcoin SV, which have non-normal distributions.

The data was obtained from the investing.com website. The dataset, which has 1033 data, covered the period from January 3, 2020 to October 31, 2022. The data was the logarithmic daily price return. The null data in the gold price data are filled with the moving average method.

Table 6.1: Descriptive Statistics.

	Mean	Median	Max	Min	Standard Deviation	Skewness	Kurtosis
Gold Price Log Return	0.0001	0.0004	0.0580	-0.0476	0.0089	-0.3658	3.0993
Bitcoin Price Log Return	0.0010	0.0013	0.1774	-0.4973	0.0401	-1.8648	27.2425
Bitcoin SV Price Log Return	-0.0006	-0.0002	0.8979	0.8979	0.0678	1.4381	45.6155
Lisk Price Log Return	0.0006	0.0028	1.8963	-1.8480	0.1050	0.1728	198.5599
Stellar Price Log Return	0.0009	0.0021	0.5536	-0.4403	0.0598	0.5091	16.8593

The descriptive statistics for the logarithmic daily price return values of Gold, Bitcoin (BTC), Bitcoin SV (BSV), Lisk (LSK), and Stellar (XLM) cryptocurrencies are provided in Table 6.1. The mean values of the variables are quite tiny and have values that are very near to zero since logarithmic returns are analyzed. The log-return of Lisk price is the most volatile variable with the biggest standard deviation, according to the risk metric known as standard deviation. In contrast, gold has a lower standard deviation than all other variables. Gold and Bitcoin have negative skewness values, whereas the remaining variables have positive skewness values. The variables other than gold have excess kurtosis, as seen by their kurtosis values. The correlation matrix shown in Figure 6.1 shows a statistically significant relationship between bitcoin and the determined cryptocurrencies. The correlation matrix in Figure 6.1 displays the correlation values between the variables.



Figure 6.1: The Correlation Matrix.

The first step in the empirical implementation of the nonlinear ARDL technique is to perform the standard unit root tests (ADF and PP) and stationary test (KPSS) to determine if the variables are integrated of order zero, $I(0)$, or integrated of order one, $I(1)$. If an integrated of order two, $I(2)$, variable is present, the computed F-statistics for testing cointegration are erroneous [Demir et al., 2020].

Table 6.2: The Standard Unit Root Tests (ADF, PP, and KPSS).

	ADF Stat.	PP Stat.	KPSS Stat.
Gold Price Log Return	-11.11***	-1275,6***	0.247
Bitcoin Price Log Return	-9.3936***	-1189,3***	0.494
Bitcoin SV Price Log Return	-10.756***	-1167,1***	0.179
Lisk Price Log Return	-9.5621***	-1294,9***	0.217
Stellar Price Log Return	-9.5434***	-1140,1***	0.310

Table 6.2 shows the standard Unit Root Tests (ADF and PP Tests). Augmented Dickey–Fuller test is one of unit root tests which tests the null hypothesis that the time series is non-stationary. If the p-value of the ADF test is lower than a pre-specified significant level or if the test statistic is less than the critical value, the null hypothesis is rejected. That means the time series is stationary [Dickey and Fuller, 1979]. Another unit root test is Phillips–Perron test which tests the null hypothesis that the time series is integrated of order one, [Phillips and Perron, 1988]. The same procedure is like ADF test. The typical symbols for statistical significance at the 10%, 5%, and 1% levels are *, **, and ***, respectively. Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are used to compare the unit root alternative to the null hypothesis that a detectable time series is stationary around a deterministic trend [Kwiatkowski et al, 1992]. According to these three tests, the log-gold price return and the determined log-cryptocurrencies price return have stationary time series.

To investigate potential asymmetrical and nonlinear interactions, according to [Shin et al., 2014]'s suggestion, the designated cryptocurrencies return series are divided into a partial sum of positive (negative) decompositions as follows:

$$BTC_t^+ = \sum_{i=1}^t \Delta BTC_i^+ = \sum_{i=1}^t \max(\Delta BTC_i, 0), BTC_t^- = \sum_{i=1}^t \Delta BTC_i^- = \sum_{i=1}^t \min(\Delta BTC_i, 0) \quad (6.1)$$

$$BSV_t^+ = \sum_{i=1}^t \Delta BSV_i^+ = \sum_{i=1}^t \max(\Delta BSV_i, 0), BSV_t^- = \sum_{i=1}^t \Delta BSV_i^- = \sum_{i=1}^t \min(\Delta BSV_i, 0) \quad (6.2)$$

$$LSK_t^+ = \sum_{i=1}^t \Delta LSK_i^+ = \sum_{i=1}^t \max(\Delta LSK_i, 0), LSK_t^- = \sum_{i=1}^t \Delta LSK_i^- = \sum_{i=1}^t \min(\Delta LSK_i, 0) \quad (6.3)$$

$$XLM_t^+ = \sum_{i=1}^t \Delta XLM_i^+ = \sum_{i=1}^t \max(\Delta XLM_i, 0), XLM_t^- = \sum_{i=1}^t \Delta XLM_i^- = \sum_{i=1}^t \min(\Delta XLM_i, 0) \quad (6.4)$$

where BTC, BSV, LSK and XLM denote Bitcoin, Bitcoin SV, Lisk and Stellar log-prices, respectively. The NARDL configuration for the log-price of gold (GOLD) is as follows:

$$\begin{aligned} \Delta GOLD_t = & \alpha + \alpha_1 GOLD_{t-1} + \beta_1 BTC_{t-1}^+ + \beta_2 BTC_{t-1}^- + \beta_3 BSV_{t-1}^+ + \beta_4 BSV_{t-1}^- + \beta_5 LSK_{t-1}^+ \\ & + \beta_6 LSK_{t-1}^- + \beta_7 XLM_{t-1}^+ + \beta_8 XLM_{t-1}^- + \sum_{i=1}^p \delta_i \Delta GOLD_{t-i} \\ & + \sum_{i=0}^q (\varphi_{1i}^+ \Delta BTC_{t-i}^+ + \varphi_{2i}^- \Delta BTC_{t-i}^- + \varphi_{3i}^+ \Delta BSV_{t-i}^+ + \varphi_{4i}^- \Delta BSV_{t-i}^- \\ & + \varphi_{5i}^+ \Delta LSK_{t-i}^+ + \varphi_{6i}^- \Delta LSK_{t-i}^- + \varphi_{7i}^+ \Delta XLM_{t-i}^+ + \varphi_{8i}^- \Delta XLM_{t-i}^-) + u_t \end{aligned} \quad (6.5)$$

where p is the number of lags of the dependent variable and q is the number of lags for the independent variables. δ_i is the autoregressive parameter. u_t is the error term. The parameters shown to the long run relationship is denoted as β_i for $i = 1, 2, 3, 4, 5, 6, 7, 8$ and $\varphi_{1i}^+, \varphi_{3i}^+, \varphi_{5i}^+, \varphi_{7i}^+, \varphi_{2i}^-, \varphi_{4i}^-, \varphi_{6i}^-$ or φ_{8i}^- for $i = 0, 1 \dots q$ denote the parameters of the short run relationship.

The following are hypotheses for short-run symmetry, long-run symmetry and no asymmetric cointegration.

- $H_{SR}: \varphi_{1i}^+ = \varphi_{2i}^-$ for BTC, $H_{SR}: \varphi_{3i}^+ = \varphi_{4i}^-$ for BSV, $H_{SR}: \varphi_{5i}^+ = \varphi_{6i}^-$ for LSK, $H_{SR}: \varphi_{7i}^+ = \varphi_{8i}^-$ for XLM when $i = 0, \dots, q$. These hypotheses are null hypothesis for short-run symmetry.
- $H_{LR}: \beta_1 = \beta_2$ for BTC, $H_{LR}: \beta_3 = \beta_4$ for BSV, $H_{LR}: \beta_5 = \beta_6$ for LSK, and $H_{LR}: \beta_7 = \beta_8$ for XLM. These hypotheses are null hypotheses for long-run symmetry.
- $H_{PSS}: \alpha_1 = \beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = \beta_6 = \beta_7 = \beta_8 = 0$ null hypothesis for no asymmetric cointegration.

In this methodology, using BDM t-statistic (t_{BDM}) [Banerjee et al., 1998], the limits testing method (PSS F-statistic) of [Pesaran et al., 2001] and [Shin et al., 2014], the presence of cointegration among the variables are tested based on the estimated NARDL. The hypotheses for the PSS F-statistic can be shown as, against the alternative that the null hypothesis is not true. For these tests, two values are crucial. It can be said there is cointegration if the value derived from the test result is higher than the upper bound. On the other hand, there is no proof of a long-term relationship if the test statistic is less than the lower bound. If the test statistic is between the boundaries, it is inconclusive.

Table 6.3: Cointegration Test Statistics from STATA.

$t_{BDM} =$	-14.654***
$F_{PSS} =$	25.366***

According to Table 6.3, there is cointegration between the gold price and the cryptocurrencies due to the fact that it can be examined the long-run relationship between gold price returns and the designated cryptocurrencies returns. The typical symbols for statistical significance at the 10%, 5%, and 1% levels are *, **, and ***, respectively. The critical values are 5.15 and 6.36 for 1% for FPSS.

When the cointegration criteria is met, the estimated short-run and long-run coefficients, which are shown in Table 6.4, can be analyzed. The Akaike Information Criteria (AIC) is used in the study to choose the optimal lag length. In R, VARselect

command in *vars* library is utilized to find the optimal lag length for the dependent and independent variables. The optimal lag length for the dependent and independent variables is selected as 4 lags. In the model, it is evident that Bitcoin has a statistically significant and negative impact in short-run. In other words, falling Bitcoin prices cause falling gold prices. It has been determined that the prices of BSV and XLM have a statistically significant effect on the gold price in the long run. Likewise, it has been observed that the prices of gold in the previous days are statistically significant to the price of gold today. In the short run, it has been discovered that BSV is statistically related to with the price of current and previous day's gold, and this relationship is symmetrical according to the Wald test (W_{SR}^{BSV}).

[Demir et al., 2020] used [Shin et al., 2009]'s "additive short-run symmetry condition" to evaluate whether the specified cryptocurrencies have an unequal influence on gold in the short term. The symmetry criterion can only be rejected, according to this proposition, if the sum of declines is considerably different from the sum of rises. Therefore, Wald statistical for short-run symmetry (WSR) is constructed according to this proposition. The results in Table 6.4 indicate that the null hypothesis at 10% significance level is rejected for only Bitcoin. There is symmetrical condition in short-run for the rest of cryptocurrencies. It concludes that the impact of falling Bitcoin prices on gold prices is greater than the impact of rising prices. This means that the price of gold is more sensitive to Bitcoin price drops than to increases.

With the logic made for the short-term symmetry situation, there is no asymmetric situation in the determined cryptocurrencies in the long-term symmetry situation. As can be seen in Table 6.4, all long-term Wald statistics (W_{LR}) values were insignificant. Finally, it was found that the current and previous day price of XLM has a statistical effect on gold in the short term.

The results of Table 6.4 show that the coefficient of the lagged ECM term ($GOLD_{t-1}$) has a statistically significant and negative effect in the long-term equilibrium. The values of PSS F-statistic is 25.366 and it is higher than the critical value, meaning that there is a long-term relationship between the return of gold price and the cryptocurrencies' price return.

Table 6.4: The Results of NARDL Model.

	Coef.	Std. Err.	P Value
Constant Term	0.005**	0.002	0.029
$GOLD_{t-1}$	-0.997***	0.068	0.000
BTC_{t-1}^+	0.043	0.026	0.102
BTC_{t-1}^-	0.043*	0.026	0.097
BSV_{t-1}^+	0.072***	0.019	0.000
BSV_{t-1}^-	0.071***	0.018	0.000
LSK_{t-1}^+	-0.013	0.014	0.352
LSK_{t-1}^-	-0.014	0.014	0.340
XLM_{t-1}^+	-0.053***	0.018	0.004
XLM_{t-1}^-	-0.053***	0.018	0.004
$\Delta GOLD_{t-1}$	-0.181***	0.060	0.003
$\Delta GOLD_{t-2}$	-0.137***	0.049	0.005
$\Delta GOLD_{t-3}$	-0.003	0.032	0.927
ΔBTC_t^+	-0.009	0.014	0.510
ΔBTC_{t-1}^+	-0.007	0.023	0.778
ΔBTC_{t-2}^+	0.020	0.020	0.323
ΔBTC_{t-3}^+	0.004	0.015	0.779
ΔBTC_t^-	0.045***	0.015	0.002
ΔBTC_{t-1}^-	0.010	0.022	0.663
ΔBTC_{t-2}^-	-0.009	0.018	0.606
ΔBTC_{t-3}^-	0.014	0.014	0.301
ΔBSV_t^+	0.012*	0.007	0.080
ΔBSV_{t-1}^+	-0.042**	0.016	0.010
ΔBSV_{t-2}^+	-0.018	0.013	0.180
ΔBSV_{t-i}^+	-0.018*	0.010	0.061
ΔBSV_t^-	0.018*	0.009	0.053
ΔBSV_{t-1}^-	-0.040***	0.014	0.004
ΔBSV_{t-2}^-	-0.030***	0.010	0.004
ΔBSV_{t-3}^-	-0.007	0.007	0.285
ΔLSK_t^+	0.001	0.004	0.847
ΔLSK_{t-1}^+	0.009	0.013	0.494
ΔLSK_{t-2}^+	-0.011	0.011	0.321
ΔLSK_{t-3}^+	-0.002	0.004	0.569
ΔLSK_t^-	-0.001	0.006	0.838
ΔLSK_{t-1}^-	0.001	0.011	0.928
ΔLSK_{t-2}^-	0.006	0.007	0.428
ΔLSK_{t-3}^-	-0.004	0.006	0.462
ΔXLM_t^+	-0.004	0.009	0.691
ΔXLM_{t-1}^+	0.024	0.018	0.166

Table 6.4: Continued.			
ΔXLM_{t-2}^+	0.016	0.015	0.294
ΔXLM_{t-3}^+	0.014	0.012	0.235
ΔXLM_t^-	-0.024**	0.011	0.027
ΔXLM_{t-1}^-	0.033**	0.014	0.019
ΔXLM_{t-2}^-	0.014	0.011	0.207
ΔXLM_{t-3}^-	-0.001	0.008	0.949
W_{LR}^{BTC}	-	-	0.622
W_{LR}^{BSC}	-	-	0.224
W_{LR}^{LSK}	-	-	0.583
W_{LR}^{XLM}	-	-	0.243
W_{SR}^{BTC}	-	-	0.077*
W_{SR}^{BSV}	-	-	0.644
W_{SR}^{LSK}	-	-	0.402
W_{SR}^{XLM}	-	-	0.243
R^2	0.6125	-	-
Adjusted R^2	0.5952	-	-
Sum of Squared Residuals	0.1847	-	-
P-Value for F-Statistics	35.35	-	0.00001***

The Portmanteau test, Breusch/Pagan heteroskedasticity test, Ramsey RESET test, and Jarque-Bera test on normality tests were all performed on the established NARDL model. Using STATA's *nardl* command, these tests were conducted.

Table 6.5: Model Diagnostics.

Model Diagnostics Tests	P-Value
Portmanteau test	0.131
Breusch/Pagan heteroskedasticity test	0.854
Ramsey RESET test	0.367
Jarque-Bera test on normality	0.000

The portmanteau test, called Box-Pierce test, developed by Box and Pierce [1970] is a way to check the diagnostic of a time series model. The portmanteau test assesses the null hypothesis that a series of residuals exhibits no autocorrelation for a fixed number of lags. According to Table 6.5, there is no autocorrelation in the residuals of the fitted NARDL model. The Breusch–Pagan test, developed by [Breusch T. S. and Pagan A. R., 1979] is used to test for heteroskedasticity. In the

Breusch–Pagan test, the null hypothesis is that there is a homoscedasticity which means the residuals are distributed with equal variance. Table 6.5 shows that there is no heteroskedasticity in the residuals of the fitted NARDL model. A broad specification test called the Ramsey Regression Equation Specification Error Test (RESET), refined by [Ramsey J. B., 1969], determines whether non-linear combinations of the explanatory variables can adequately describe the response variable. The Ramsey RESET test is performed for the null hypothesis that there is no omitted variable in the model. According to Table 6.5, there is no omitted variable in the NARDL model. The Jarque-Bera test, which is developed by [Jarque and Bera, 1980], is a goodness-of-fit test that determines whether sample data contain skewness and kurtosis that are consistent with a normal distribution. The Jarque-Bera test assesses the null hypothesis that the data is normally distributed. According to Table 6.5, there are sufficient evidence to say that the residuals of the fitted NARDL model is not normally distributed. This finding is an expected phenomenon since the selected independent variables have non-normal distributions.

7. CONCLUSION AND FUTURE RESEARCH

A genuinely remarkable asset class is cryptocurrencies. Cryptocurrencies, which were first created as decentralized digital money encrypted on the blockchain, are now exchanged for speculative purposes. Cryptocurrencies have evolved into a relatively new asset class with unique characteristics as a result of the rising trading volume of private cryptocurrency exchanges and the launch of Bitcoin futures on the commodities market [Borgards, 2021]. A variety of linear models have been used in academic research to examine cryptocurrencies, which are rapidly increasing in value and significance. The time series data of cryptocurrencies, however, have been found to follow non-normal distributions, and also linear models are unable to reveal possible non-linear dependencies between the cryptocurrencies. At this stage, non-linear models were used for the data analysis. This thesis has focused on the NARDL model, which is one of these nonlinear models.

In this thesis, Monte Carlo simulation experiments were used to test the validity of the NARDL model, which depicts the long- and short-term asymmetric relationships between non-normally distributed cryptocurrencies and other variables, under heavy-tailed distributions (Student t-distribution and Skew-t distribution).

Using NARDL packages created in R and STATA programs, simulation experiments were conducted for the purposes of this thesis. Since the NARDL package developed in the R programming language has not through the testing and evaluation process, the major motivation for employing two separate software programs is to cast doubt on the reliability of the findings of analyses and research using R. A comparison between STATA, which verifies packages, and NARDL packages created in R has been done in order to answer this question. In this manner, the dependability of the R-developed NARDL package has been evaluated.

The following are the ways in which the thesis differs from previous research and how it contributes to the literature and society:

- As a result of the simulations made under student-t and skew-t distributions, which are non-normal distributions, it has been shown that the statistical inferences obtained when the NARDL model developed by [Shin et al., 2014] is used for time series with non-normal distributions such as cryptocurrencies are

mostly reliable. Therefore, it has been concluded that the studies on the effects of cryptocurrencies on the economy and finance using the NARDL model are mostly accurate and reliable. An opposite finding was reached only when the skew-t distribution had a very high kurtosis and a positive high skewness. In these cases, the tests showing the short-term asymmetric relationship have a very low power.

- Monte Carlo simulation experiments for the comparison of R and STATA have revealed that the *nardl* package developed by [Zaghdoudi, 2017], which is used in NARDL analysis, is unreliable since the results from R appear as if there is a symmetrical relationship although there is an asymmetrical relationship. However, the reliability of studies with STATA has been demonstrated.

- In the empirical part, the effect of non-normally distributed cryptocurrencies on gold prices was examined with NARDL. As a result of the examinations, a symmetrical relationship with cryptocurrencies other than BTC has been revealed in the short and long term. It has only been discovered that the BTC price has an asymmetrical relationship with the gold price in the short run. Although the correlations among BTC, BSV, and XLM are high, the NARDL model only shows that there is an asymmetrical relationship between gold prices and BTC in the short run, which is in line with the inference obtained as a result of simulation experiments. For this reason, if the variables that do not have a normal distribution are included in the analysis, it has emerged that the test results showing the short-term asymmetric relationship should be approached with suspicion.

Future research may assess the NARDL model's dependability for variables with various non-normal distributions (besides from student-t and skew-t). Future studies may focus on assessing the dependability of open-source software, like Python, and applications utilized for the NARDL model.

REFERENCES

- Allen D. E., McAleer M., (2021), “A Nonlinear Autoregressive Distributed Lag (NARDL) Analysis of the FTSE and S&P500 Indexes”, *Risks*, 9, 195.
- Allen D. E., McAleer M., (2020), “A Nonlinear Autoregressive Distributed Lag (NARDL) Analysis of West Texas Intermediate Oil Prices and the DOW JONES Index”, *Energies* 2020, 13, 4011.
- Azzalini A., Capitanio A., (2014), “The Skew-Normal and Related Families (First)”, Cambridge University Press.
- Azzalini A., (2022), “The R package 'sn': The Skew-Normal and Related Distributions such as the Skew-t and the SUN”, Version 2.0.2.
- Banerjee A., Dolado J., Mestre R., (1998), “Error-correction mechanism tests for cointegration in a single-equation framework”, *Journal of Time Series Analysis*, 19 (3), 267–283.
- Braglia L., (2016), “RStata: A Bit of Glue Between R and Stata”, Version 1.1.1.
- Baur D. G., Hong K. H., Lee A. D., (2018), “Bitcoin: Medium of exchange or speculative assets?”, *Journal of International Financial Markets, Institutions and Money*, 54, 177–189.
- Becker R. A., Chambers J. M., Wilks A. R., (1988), “The New S Language”, Wadsworth & Brooks/Cole.
- Borgards O, (2021), “Dynamic time series momentum of cryptocurrencies”, *North American Journal of Economics and Finance*, 57.
- Bouri E., Gupta R., Lahiani A., Shahbaz M., (2018), “Testing for asymmetric nonlinear short- and long-run relationships between bitcoin, aggregate commodity, and gold prices”, *Resources Policy*, 57, 224–235.
- Box G. E., Pierce D. A. (1970), “Distribution of residual autocorrelations in autoregressive-integrated moving average time series models”, *Journal of the American Statistical Association*, 65 (332), 1509-1526.
- Breusch T. S., Pagan A. R., (1979), "A Simple Test for Heteroskedasticity and Random Coefficient Variation", *Econometrica*, 47 (5), 1287–1294.
- Canoz I., Dirican C., (2017), “The Cointegration Relationship Between Bitcoin Prices and Major World Stock Indices: An Analysis with ARDL Model Approach”, *Press academia*, 4(4), 377–392.
- Çaşkurlu E., Arslan C. B., (2021), “Blockchain Technology, Cryptocurrency and Financial Depensement: An Analysis on Turkey”, *Iğdır Üniversitesi Sosyal Bilimler Dergisi*, 28, 97–124.

Ciaian P., Rajcaniova M., Kancs d'Artis., (2018), "Virtual relationships: Short- and long-run evidence from Bitcoin and altcoin markets", *Journal of International Financial Markets, Institutions and Money*, 52, 173–195.

Contuk F. Y., (2021), "Covid-19 Sürecinde Altın ve Petrol Fiyatlarının Bitcoin Üzerindeki Asimetrik Etkisi", *Ankara Hacı Bayram Veli Üniversitesi İktisadi ve İdari Bilimler Fakültesi Dergisi*, 23(3), 911–926.

Corbet S., Lucey B., Urquhart A., Yarovaya L., (2019), "Cryptocurrencies as a financial asset: A systematic analysis", *International Review of Financial Analysis*, 62, 182–199.

De la O González M., Jareño F., Skinner F. S., (2020), "Nonlinear autoregressive distributed lag approach: An application on the connectedness between bitcoin returns and the other ten most relevant cryptocurrency returns", *Mathematics*, 8(5).

Demir E., Simonyan S., García-Gómez C. D., Lau C. K. M., (2020), "The asymmetric effect of bitcoin on altcoins: evidence from the nonlinear autoregressive distributed lag (NARDL) model", *Finance Research Letters*.

Dickey D. A., Fuller W. A., (1979). "Distribution of the Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, 74 (366), 427–431.

Dyhrberg A. H., (2016), "Bitcoin, gold and the dollar - A GARCH volatility analysis", *Finance Research Letters*, 16, 85–92.

Gaies B., Nakhli M. S., Sahut J. M., Guesmi K., (2021), "Is Bitcoin rooted in confidence? – Unraveling the determinants of globalized digital currencies", *Technological Forecasting and Social Change*, 172.

Ghorbel A., Frikha W., Manzli Y. S., (2022), "Testing for asymmetric non-linear short- and long-run relationships between crypto-currencies and stock markets", *Eurasian Economic Review*.

Jareño F., De La O González M., Belmonte P., (2022)., "Asymmetric interdependencies between cryptocurrency and commodity markets: the COVID-19 pandemic impact", *Quantitative Finance and Economics*, 6(1), 83–112.

Jareño F., González M. de la O., López R., Ramos A. R., (2021), "Cryptocurrencies and oil price shocks: A NARDL analysis in the COVID-19 pandemic", *Resources Policy*, 74.

Jareño F., González M. de la O., Tolentino M., Sierra K., (2020), "Bitcoin and gold price returns: A quantile regression and NARDL analysis", *Resources Policy*, 67.

Jarque C. M., Bera A. K., (1980), "Efficient tests for normality, homoscedasticity and serial independence of regression residuals", *Economics Letters*, 6 (3), 255–259.

Jeribi A., Jena S. K., Lahiani A., (2021), "Are cryptocurrencies a backstop for the stock market in a covid-19-led financial crisis? Evidence from the nardl approach", *International Journal of Financial Studies*, 9(3).

Johnson N. L., Kotz S., Balakrishnan N., (1995), "Continuous Univariate Distributions, (2)", Wiley.

Kwiatkowski D., Phillips P. C. B., Schmidt P., Shin Y., (1992), "Testing the null hypothesis of stationarity against the alternative of a unit root", *Journal of Econometrics*, 54 (1–3), 159–178.

Leirvik T., (2022), "Cryptocurrency returns and the volatility of liquidity", *Finance Research Letters*, 44.

Li X., Wang C. A., (2017), "The technology and economic determinants of cryptocurrency exchange rates: The case of Bitcoin", *Decision Support Systems*, 95, 49–60.

Lin M. Y., An C. L., (2021), "The relationship between Bitcoin and resource commodity futures: Evidence from NARDL approach", *Resources Policy*, 74.

Long S., Pei H., Tian H., Lang K., (2021), "Can both Bitcoin and gold serve as safe-haven assets? A comparative analysis based on the NARDL model", *International Review of Financial Analysis*, 78.

Moussa W., Mgadmi N., Béjaoui A., Regaieg R., (2021), "Exploring the dynamic relationship between Bitcoin and commodities: New insights through STECM model", *Resources Policy*, 74.

Nakamoto S., (2008), "Bitcoin: A Peer-to-Peer Electronic Cash System".

Pesaran M. H., Shin Y., Smith R. J., (2001), "Bounds testing approaches to the analysis of level relationships", *Journal of Applied Econometrics*, 16(3), 289–326.

Phillips P. C. B., Perron P. (1988), "Testing for a Unit Root in Time Series Regression", *Biometrika*, 75 (2), 335–346.

R Core Team, (2022), "R: A language and environment for statistical computing. R Foundation for Statistical Computing", Vienna, Austria.

Ramsey J. B., (1969), "Tests for Specification Errors in Classical Linear Least Squares Regression Analysis", *Journal of the Royal Statistical Society, Series B*, 31 (2), 350–371.

Sayed A. A., Ahmed F., Kamal, M. A., Ullah A., Ramos-Requena J. P., (2022), "Is There an Asymmetric Relationship between Economic Policy Uncertainty, Cryptocurrencies, and Global Green Bonds? Evidence from the United States of America", *Mathematics*, 10(5).

Shanaev S., Ghimire B., (2021), "A fitting return to fitting returns: Cryptocurrency distributions revisited".

Shin Y., Yu B., Greenwood-Nimmo Ma., (2014), "Modelling Asymmetric Cointegration and Dynamic Multipliers in a Nonlinear ARDL Framework", In R. C. Sickles & W. C. Horrace (Eds.), *Festschrift in Honor of Peter Schmidt; Econometric Methods and Applications*, 281–314.

Web 1, (2022), <https://coinmarketcap.com/tr/currencies/bitcoin/>, (Access Date: 29/08/ 2022).

Web 2, (n.d.), <http://www.marco-sunder.de/stata/>, (Access Date: 24/06/2021).

Zaghdoudi T., (2022), “Nardl: Nonlinear Cointegrating Autoregressive Distributed Lag Model; R Package Version 0.1.6.”

Zhang Z., Sun Q., Ma Y., (2022), "The hedge and safe haven properties of non-fungible tokens (NFTs): Evidence from the nonlinear autoregressive distributed lag (NARDL) model", Finance Research Letters, 50.

Zhu X., Niu Z., Zhang H., Huang J., Zuo X., (2022),"Can gold and bitcoin hedge against the COVID-19 related news sentiment risk? New evidence from a NARDL approach", Resources Policy, 79.



BIOGRAPHY

Abdlsamet AA completed his bachelor’s degree in Industrial Engineering and Management (Double Major) at İstanbul Şehir University in 2019. In the same year, he was accepted to Graduate Programme of Industrial and System Engineering at Institute of Natural and Applied Sciences in İstanbul Şehir University. He worked as a research assistant at Department of Industrial Engineering in Gebze Technical University from October 2020 to January 2023 and continued his Master of Science Program there. Since February 2023, he has been working at Information and Communication Technologies Authority as an informatics specialist assistant.



APPENDICES

Appendix A: Publications Based on This Thesis

Aça A., Dengeç K. D., (2022), “Cryptocurrencies and Non-linear Autoregressive Distributed Lag (NARDL) Model Under Non-normal Distributions” 5th International Conference on Data Science and Applications (ICONDATA'22), Muğla, Türkiye, 7-11 September

Appendix B: Monte Carlo Simulation Outputs

Table B1.1: Monte Carlo Simulation Results for Student-t Distribution ($\omega = 0.5$)

		T-Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = 0.5$	df=2.1	α	-0.019	0.240	0.240	α	-0.010	0.191	0.191	α	-0.015	0.135	0.135
		ρ	-0.070	0.076	0.103	ρ	-0.032	0.045	0.056	ρ	-0.016	0.028	0.032
		$\theta+$	-0.035	0.100	0.366	$\theta+$	-0.038	0.044	0.383	$\theta+$	-0.039	0.026	0.392
		$\theta-$	-0.072	0.127	0.729	$\theta-$	-0.076	0.061	0.767	$\theta-$	-0.078	0.036	0.784
		$\pi+$	0.050	0.247	0.555	$\pi+$	0.050	0.149	0.522	$\pi+$	0.050	0.104	0.511
		$\pi-$	0.051	0.217	0.555	$\pi-$	0.050	0.138	0.522	$\pi-$	0.050	0.102	0.512
		$\beta+$	0.031	0.312	0.313	$\beta+$	0.008	0.140	0.141	$\beta+$	0.006	0.088	0.088
		$\beta-$	-0.469	0.312	0.563	$\beta-$	-0.492	0.140	0.512	$\beta-$	-0.494	0.088	0.502
	df = 3	α	-0.047	0.379	0.382	α	-0.041	0.280	0.283	α	-0.034	0.205	0.208
		ρ	-0.072	0.078	0.106	ρ	-0.033	0.047	0.057	ρ	-0.017	0.030	0.035
		$\theta+$	-0.035	0.084	0.360	$\theta+$	-0.038	0.040	0.382	$\theta+$	-0.039	0.020	0.391
		$\theta-$	-0.071	0.114	0.723	$\theta-$	-0.076	0.059	0.766	$\theta-$	-0.078	0.034	0.783
		$\pi+$	0.051	0.182	0.538	$\pi+$	0.050	0.122	0.519	$\pi+$	0.050	0.080	0.508
		$\pi-$	0.050	0.189	0.535	$\pi-$	0.050	0.114	0.509	$\pi-$	0.050	0.079	0.506
		$\beta+$	0.039	0.232	0.235	$\beta+$	0.010	0.117	0.117	$\beta+$	0.004	0.053	0.053
		$\beta-$	-0.461	0.232	0.516	$\beta-$	-0.490	0.117	0.504	$\beta-$	-0.496	0.053	0.499
	df= 10	α	-0.040	0.401	0.403	α	-0.042	0.285	0.288	α	-0.041	0.213	0.216
		ρ	-0.069	0.077	0.104	ρ	-0.033	0.048	0.058	ρ	-0.016	0.030	0.034
		$\theta+$	-0.035	0.078	0.362	$\theta+$	-0.038	0.039	0.383	$\theta+$	-0.039	0.020	0.392
		$\theta-$	-0.072	0.110	0.727	$\theta-$	-0.076	0.060	0.767	$\theta-$	-0.078	0.034	0.785
		$\pi+$	0.050	0.175	0.529	$\pi+$	0.050	0.121	0.516	$\pi+$	0.050	0.084	0.507
		$\pi-$	0.050	0.175	0.532	$\pi-$	0.050	0.119	0.512	$\pi-$	0.049	0.084	0.502
		$\beta+$	0.030	0.215	0.217	$\beta+$	0.007	0.105	0.105	$\beta+$	0.002	0.051	0.051
		$\beta-$	-0.470	0.215	0.517	$\beta-$	-0.493	0.105	0.504	$\beta-$	-0.498	0.051	0.501

Table B1.2: Monte Carlo Simulation Results for Student-t Distribution ($\omega = -0.5$)

T-Distribution													
T = 100				T = 200				T = 400					
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = -0.5$	df = 2.1	α	-0.023	0.269	0.270	α	-0.016	0.180	0.181	α	-0.016	0.141	0.142
		ρ	-0.069	0.075	0.102	ρ	-0.031	0.044	0.054	ρ	-0.015	0.029	0.032
		$\theta+$	-0.038	0.079	0.386	$\theta+$	-0.039	0.047	0.391	$\theta+$	-0.039	0.020	0.394
		$\theta-$	-0.074	0.097	0.750	$\theta-$	-0.077	0.058	0.775	$\theta-$	-0.079	0.031	0.786
		$\pi+$	-0.050	0.227	0.549	$\pi+$	-0.050	0.185	0.533	$\pi+$	-0.050	0.107	0.512
		$\pi-$	-0.050	0.230	0.551	$\pi-$	-0.051	0.144	0.525	$\pi-$	-0.050	0.099	0.509
		$\beta+$	-0.032	0.290	0.292	$\beta+$	-0.014	0.182	0.182	$\beta+$	-0.003	0.069	0.069
		$\beta-$	-0.532	0.290	0.606	$\beta-$	-0.514	0.182	0.546	$\beta-$	-0.503	0.069	0.508
	df = 3	α	-0.040	0.375	0.377	α	-0.041	0.267	0.270	α	-0.032	0.213	0.215
		ρ	-0.069	0.076	0.103	ρ	-0.032	0.048	0.058	ρ	-0.016	0.030	0.034
		$\theta+$	-0.038	0.064	0.383	$\theta+$	-0.039	0.031	0.389	$\theta+$	-0.039	0.018	0.393
		$\theta-$	-0.074	0.084	0.748	$\theta-$	-0.077	0.049	0.773	$\theta-$	-0.078	0.031	0.785
		$\pi+$	-0.050	0.176	0.533	$\pi+$	-0.050	0.119	0.512	$\pi+$	-0.050	0.079	0.506
		$\pi-$	-0.050	0.183	0.530	$\pi-$	-0.050	0.121	0.518	$\pi-$	-0.050	0.079	0.507
		$\beta+$	-0.031	0.239	0.241	$\beta+$	-0.008	0.138	0.138	$\beta+$	-0.003	0.052	0.052
		$\beta-$	-0.531	0.239	0.583	$\beta-$	-0.508	0.138	0.526	$\beta-$	-0.503	0.052	0.505
	df = 10	α	-0.070	0.399	0.405	α	-0.036	0.282	0.285	α	-0.038	0.220	0.223
		ρ	-0.074	0.079	0.108	ρ	-0.034	0.048	0.058	ρ	-0.016	0.031	0.035
		$\theta+$	-0.038	0.060	0.381	$\theta+$	-0.039	0.031	0.388	$\theta+$	-0.039	0.017	0.394
		$\theta-$	-0.074	0.083	0.744	$\theta-$	-0.077	0.049	0.771	$\theta-$	-0.079	0.031	0.786
		$\pi+$	-0.050	0.179	0.533	$\pi+$	-0.050	0.121	0.517	$\pi+$	-0.050	0.083	0.507
		$\pi-$	-0.050	0.177	0.534	$\pi-$	-0.050	0.123	0.515	$\pi-$	-0.050	0.085	0.507
		$\beta+$	-0.030	0.221	0.223	$\beta+$	-0.011	0.107	0.107	$\beta+$	-0.005	0.051	0.051
		$\beta-$	-0.530	0.221	0.575	$\beta-$	-0.511	0.107	0.522	$\beta-$	-0.505	0.051	0.508

Table B1.3: Monte Carlo Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -2$)

Skew-t Distribution									
T = 100			T = 200			T = 400			
	Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$ $a_{\text{skew}} = -2$	df= 2.1		0.943	0.130		0.995	0.084	1.000	0.067
	df= 3	W_{LR}	0.972	0.128	W_{LR}	0.999	0.099	1.000	0.082
	df=10		0.991	0.124		1.000	0.095	1.000	0.081
	df= 2.1		0.195	0.057		0.307	0.053	0.514	0.055
	df= 3	W_{SR}	0.198	0.047	W_{SR}	0.354	0.050	0.601	0.046
	df=10		0.189	0.059		0.357	0.052	0.658	0.043
	df= 2.1		0.616	0.077		0.989	0.073	1.000	0.073
	df= 3	F_{PSS}	0.615	0.081	F_{PSS}	0.995	0.070	1.000	0.069
	df=10		0.598	0.080		0.996	0.072	1.000	0.068
$\omega = 0$ $a_{\text{skew}} = -2$	df= 2.1		0.925	0.117		0.992	0.092	0.999	0.068
	df= 3	W_{LR}	0.963	0.122	W_{LR}	1.000	0.088	1.000	0.074
	df=10		0.985	0.126		1.000	0.092	1.000	0.075
	df= 2.1		0.153	0.057		0.248	0.051	0.423	0.044
	df= 3	W_{SR}	0.152	0.059	W_{SR}	0.281	0.050	0.509	0.053
	df=10		0.162	0.056		0.288	0.049	0.544	0.055
	df= 2.1		0.403	0.073		0.965	0.077	1.000	0.073
	df= 3	F_{PSS}	0.404	0.082	F_{PSS}	0.960	0.078	1.000	0.066
	df=10		0.421	0.077		0.965	0.079	1.000	0.076
$\omega = 0.5$ $a_{\text{skew}} = -2$	df= 2.1		0.929	0.114		0.995	0.081	0.999	0.069
	df= 3	W_{LR}	0.974	0.115	W_{LR}	1.000	0.088	1.000	0.066
	df=10		0.988	0.121		1.000	0.078	1.000	0.065
	df= 2.1		0.183	0.052		0.319	0.053	0.497	0.050
	df= 3	W_{SR}	0.200	0.056	W_{SR}	0.372	0.044	0.624	0.055
	df=10		0.195	0.048		0.376	0.043	0.652	0.049
	df= 2.1		0.631	0.077		0.992	0.078	1.000	0.064
	df= 3	F_{PSS}	0.618	0.084	F_{PSS}	0.991	0.074	1.000	0.069
	df=10		0.570	0.082		0.991	0.069	1.000	0.074

Table B1.4: Monte Carlo Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -1$)

Skew-t Distribution										
		T = 100			T = 200			T = 400		
		Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$ $a_{\text{skew}} = -1$	df= 2.1	W_{LR}	0.943	0.117	W_{LR}	0.996	0.092	W_{LR}	1.000	0.074
	df= 3		0.974	0.126		1.000	0.099		1.000	0.075
	df=10		0.989	0.124		1.000	0.105		1.000	0.070
	df= 2.1	W_{SR}	0.236	0.049	W_{SR}	0.416	0.056	W_{SR}	0.663	0.048
	df= 3		0.228	0.053		0.436	0.058		0.714	0.053
	df=10		0.217	0.058		0.390	0.052		0.701	0.047
	df= 2.1	F_{PSS}	0.624	0.078	F_{PSS}	0.990	0.076	F_{PSS}	1.000	0.067
	df= 3		0.617	0.082		0.993	0.071		1.000	0.069
	df=10		0.598	0.075		0.995	0.068		1.000	0.063
$\omega = 0$ $a_{\text{skew}} = -1$	df= 2.1	W_{LR}	0.930	0.120	W_{LR}	0.994	0.093	W_{LR}	1.000	0.068
	df= 3		0.973	0.127		0.999	0.091		1.000	0.075
	df=10		0.986	0.124		1.000	0.096		1.000	0.073
	df= 2.1	W_{SR}	0.190	0.053	W_{SR}	0.329	0.047	W_{SR}	0.551	0.052
	df= 3		0.192	0.063		0.352	0.052		0.608	0.048
	df=10		0.156	0.049		0.316	0.053		0.561	0.052
	df= 2.1	F_{PSS}	0.411	0.082	F_{PSS}	0.967	0.072	F_{PSS}	1.000	0.073
	df= 3		0.410	0.076		0.962	0.075		1.000	0.069
	df=10		0.413	0.076		0.961	0.071		1.000	0.070
$\omega = 0.5$ $a_{\text{skew}} = -1$	df= 2.1	W_{LR}	0.938	0.118	W_{LR}	0.994	0.078	W_{LR}	0.999	0.064
	df= 3		0.980	0.112		1.000	0.080		1.000	0.065
	df=10		0.991	0.136		1.000	0.091		1.000	0.062
	df= 2.1	W_{SR}	0.226	0.061	W_{SR}	0.402	0.055	W_{SR}	0.658	0.044
	df= 3		0.228	0.047		0.424	0.053		0.733	0.051
	df=10		0.209	0.042		0.388	0.049		0.693	0.049
	df= 2.1	F_{PSS}	0.624	0.088	F_{PSS}	0.990	0.066	F_{PSS}	1.000	0.076
	df= 3		0.626	0.080		0.992	0.072		1.000	0.068
	df=10		0.591	0.079		0.993	0.071		1.000	0.075

Table B1.5: Monte Carlo Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 1$)

Skew-t Distribution										
		T = 100			T = 200			T = 400		
		Test	Power	Size	Test	Power	Size	Test	Power	Size
$\omega = -0.5$ $a_{\text{skew}} = 1$	df= 2.1	W_{LR}	0.944	0.134	W_{LR}	0.997	0.092	W_{LR}	1.000	0.066
	df= 3		0.977	0.127		1.000	0.090		1.000	0.065
	df=10		0.993	0.125		1.000	0.098		1.000	0.072
	df= 2.1	W_{SR}	0.233	0.066	W_{SR}	0.430	0.056	W_{SR}	0.680	0.053
	df= 3		0.229	0.062		0.457	0.056		0.734	0.058
	df=10		0.207	0.056		0.401	0.057		0.697	0.053
	df= 2.1	F_{PSS}	0.603	0.081	F_{PSS}	0.989	0.076	F_{PSS}	1.000	0.076
	df= 3		0.612	0.075		0.993	0.071		1.000	0.067
	df=10		0.595	0.076		0.993	0.073		1.000	0.066
$\omega = 0$ $a_{\text{skew}} = 1$	df= 2.1	W_{LR}	0.925	0.114	W_{LR}	0.995	0.093	W_{LR}	0.999	0.087
	df= 3		0.971	0.124		0.999	0.095		1.000	0.078
	df=10		0.987	0.123		1.000	0.087		1.000	0.079
	df= 2.1	W_{SR}	0.194	0.055	W_{SR}	0.356	0.056	W_{SR}	0.584	0.047
	df= 3		0.200	0.057		0.352	0.052		0.615	0.043
	df=10		0.162	0.065		0.311	0.061		0.565	0.052
	df= 2.1	F_{PSS}	0.383	0.079	F_{PSS}	0.969	0.075	F_{PSS}	1.000	0.068
	df= 3		0.411	0.076		0.958	0.077		1.000	0.064
	df=10		0.427	0.082		0.965	0.078		1.000	0.078
$\omega = 0.5$ $a_{\text{skew}} = 1$	df= 2.1	W_{LR}	0.940	0.127	W_{LR}	0.996	0.089	W_{LR}	0.998	0.074
	df= 3		0.974	0.119		1.000	0.083		1.000	0.071
	df=10		0.988	0.133		1.000	0.087		1.000	0.062
	df= 2.1	W_{SR}	0.236	0.052	W_{SR}	0.439	0.054	W_{SR}	0.691	0.054
	df= 3		0.240	0.060		0.431	0.054		0.729	0.051
	df=10		0.209	0.058		0.394	0.054		0.709	0.047
	df= 2.1	F_{PSS}	0.615	0.075	F_{PSS}	0.990	0.076	F_{PSS}	1.000	0.075
	df= 3		0.614	0.076		0.993	0.079		1.000	0.072
	df=10		0.584	0.077		0.994	0.070		1.000	0.074

Table B1.6: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 2$ and $\omega = 0$)

Skew-t Distribution												
T = 100					T = 200				T = 400			
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
df = 2.1	α	-0.015	0.277	0.277	α	-0.019	0.182	0.183	α	-0.015	0.129	0.130
	ρ	-0.085	0.078	0.115	ρ	-0.039	0.047	0.061	ρ	-0.020	0.031	0.037
	$\theta+$	-0.357	0.123	0.378	$\theta+$	-0.382	0.048	0.385	$\theta+$	-0.390	0.024	0.391
	$\theta-$	-0.714	0.122	0.725	$\theta-$	-0.762	0.060	0.764	$\theta-$	-0.780	0.036	0.781
	$\pi+$	-0.001	0.206	0.206	$\pi+$	-0.002	0.138	0.138	$\pi+$	-0.003	0.078	0.078
	$\pi-$	0.005	0.486	0.486	$\pi-$	0.005	0.352	0.352	$\pi-$	0.008	0.226	0.226
	$\beta+$	-0.002	0.408	0.408	$\beta+$	-0.004	0.168	0.168	$\beta+$	0.001	0.080	0.080
df = 3	$\beta-$	-0.502	0.408	0.647	$\beta-$	-0.504	0.168	0.531	$\beta-$	-0.499	0.080	0.505
	α	-0.052	0.450	0.453	α	-0.032	0.292	0.294	α	-0.036	0.214	0.217
	ρ	-0.085	0.084	0.120	ρ	-0.042	0.051	0.066	ρ	-0.020	0.033	0.039
	$\theta+$	-0.358	0.091	0.370	$\theta+$	-0.379	0.040	0.381	$\theta+$	-0.390	0.022	0.390
	$\theta-$	-0.716	0.112	0.725	$\theta-$	-0.758	0.059	0.760	$\theta-$	-0.780	0.036	0.780
	$\pi+$	0.003	0.228	0.228	$\pi+$	0.002	0.114	0.114	$\pi+$	0.002	0.085	0.085
	$\pi-$	0.007	0.316	0.316	$\pi-$	-0.008	0.219	0.219	$\pi-$	-0.001	0.155	0.155
df = 10	$\beta+$	-0.001	0.297	0.297	$\beta+$	0.000	0.129	0.129	$\beta+$	0.001	0.062	0.062
	$\beta-$	-0.501	0.297	0.582	$\beta-$	-0.500	0.129	0.516	$\beta-$	-0.499	0.062	0.503
	α	-0.062	0.452	0.456	α	-0.051	0.308	0.312	α	-0.050	0.228	0.234
	ρ	-0.086	0.087	0.122	ρ	-0.041	0.053	0.067	ρ	-0.020	0.034	0.039
	$\theta+$	-0.357	0.079	0.365	$\theta+$	-0.379	0.039	0.381	$\theta+$	-0.390	0.021	0.391
	$\theta-$	-0.714	0.108	0.722	$\theta-$	-0.759	0.060	0.761	$\theta-$	-0.780	0.036	0.781
	$\pi+$	0.000	0.185	0.185	$\pi+$	-0.001	0.126	0.126	$\pi+$	-0.002	0.085	0.085
	$\pi-$	0.002	0.238	0.238	$\pi-$	0.003	0.163	0.163	$\pi-$	0.001	0.112	0.112
	$\beta+$	0.000	0.244	0.244	$\beta+$	0.001	0.118	0.118	$\beta+$	0.000	0.058	0.058
	$\beta-$	-0.500	0.244	0.556	$\beta-$	-0.499	0.118	0.512	$\beta-$	-0.500	0.058	0.504

Table B1.7: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 2$ and $\omega = 0.5$)

Skew-t Distribution													
T = 100					T = 200				T = 400				
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = 0.5$ $a_{\text{skew}} = 2$	df = 2.1	α	-0.017	0.258	0.258	α	-0.010	0.208	0.208	α	-0.016	0.114	0.115
		ρ	-0.065	0.072	0.097	ρ	-0.031	0.042	0.052	ρ	-0.014	0.026	0.030
		$\theta+$	-0.355	0.124	0.376	$\theta+$	-0.383	0.078	0.391	$\theta+$	-0.392	0.021	0.393
		$\theta-$	-0.723	0.128	0.734	$\theta-$	-0.767	0.069	0.770	$\theta-$	-0.785	0.031	0.786
		$\pi+$	0.496	0.265	0.562	$\pi+$	0.506	0.289	0.583	$\pi+$	0.501	0.087	0.508
		$\pi-$	0.509	0.554	0.753	$\pi-$	0.502	0.429	0.660	$\pi-$	0.504	0.196	0.541
		$\beta+$	0.032	0.553	0.553	$\beta+$	0.004	0.323	0.323	$\beta+$	0.002	0.069	0.069
	$\beta-$	-0.468	0.553	0.724	$\beta-$	-0.496	0.323	0.592	$\beta-$	-0.498	0.069	0.502	
	df = 3	α	-0.040	0.405	0.407	α	-0.038	0.247	0.250	α	-0.034	0.195	0.198
		ρ	-0.070	0.073	0.101	ρ	-0.033	0.046	0.056	ρ	-0.015	0.029	0.033
		$\theta+$	-0.354	0.084	0.364	$\theta+$	-0.380	0.040	0.382	$\theta+$	-0.392	0.020	0.392
		$\theta-$	-0.719	0.109	0.728	$\theta-$	-0.764	0.058	0.766	$\theta-$	-0.784	0.033	0.785
		$\pi+$	0.503	0.159	0.527	$\pi+$	0.500	0.103	0.510	$\pi+$	0.501	0.066	0.505
		$\pi-$	0.506	0.271	0.574	$\pi-$	0.503	0.183	0.535	$\pi-$	0.495	0.130	0.512
$\beta+$		0.026	0.243	0.245	$\beta+$	0.010	0.117	0.117	$\beta+$	0.003	0.055	0.055	
$\beta-$	-0.474	0.243	0.532	$\beta-$	-0.490	0.117	0.503	$\beta-$	-0.497	0.055	0.500		
df=10	α	-0.030	0.415	0.417	α	-0.028	0.257	0.260	α	-0.024	0.205	0.208	
	ρ	-0.060	0.083	0.111	ρ	-0.023	0.056	0.066	ρ	-0.005	0.039	0.043	
	$\theta+$	-0.344	0.094	0.374	$\theta+$	-0.370	0.050	0.392	$\theta+$	-0.382	0.030	0.402	
	$\theta-$	-0.709	0.119	0.738	$\theta-$	-0.754	0.068	0.776	$\theta-$	-0.774	0.043	0.795	
	$\pi+$	0.513	0.169	0.537	$\pi+$	0.510	0.113	0.520	$\pi+$	0.511	0.076	0.515	
	$\pi-$	0.516	0.281	0.584	$\pi-$	0.513	0.193	0.545	$\pi-$	0.505	0.140	0.522	
	$\beta+$	0.036	0.253	0.255	$\beta+$	0.020	0.127	0.127	$\beta+$	0.013	0.065	0.065	
$\beta-$	-0.464	0.253	0.542	$\beta-$	-0.480	0.127	0.513	$\beta-$	-0.487	0.065	0.510		

Table B1.8: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 1$ and $\omega = -0.5$)

Skew-t Distribution													
T = 100				T = 200				T = 400					
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = -0.5$ $a_{\text{skew}} = 1$	df = 2.1	α	-0.019	0.269	0.270	α	-0.014	0.164	0.164	α	-0.012	0.134	0.135
		ρ	-0.067	0.072	0.098	ρ	-0.032	0.045	0.055	ρ	-0.014	0.028	0.032
		$\theta+$	-0.379	0.108	0.394	$\theta+$	-0.388	0.040	0.390	$\theta+$	-0.394	0.037	0.396
		$\theta-$	-0.745	0.107	0.752	$\theta-$	-0.772	0.051	0.773	$\theta-$	-0.787	0.035	0.788
		$\pi+$	-0.498	0.218	0.544	$\pi+$	-0.500	0.111	0.512	$\pi+$	-0.502	0.087	0.510
		$\pi-$	-0.512	0.404	0.652	$\pi-$	-0.497	0.210	0.540	$\pi-$	-0.499	0.163	0.525
		$\beta+$	-0.032	0.362	0.364	$\beta+$	-0.014	0.152	0.152	$\beta+$	-0.006	0.155	0.155
	$\beta-$	-0.532	0.362	0.644	$\beta-$	-0.514	0.152	0.536	$\beta-$	-0.506	0.155	0.529	
	df = 3	α	-0.043	0.382	0.385	α	-0.039	0.267	0.270	α	-0.039	0.192	0.196
		ρ	-0.073	0.079	0.108	ρ	-0.032	0.045	0.056	ρ	-0.016	0.030	0.034
		$\theta+$	-0.379	0.067	0.385	$\theta+$	-0.387	0.033	0.388	$\theta+$	-0.393	0.018	0.393
		$\theta-$	-0.742	0.084	0.747	$\theta-$	-0.771	0.048	0.772	$\theta-$	-0.785	0.031	0.786
		$\pi+$	-0.504	0.159	0.528	$\pi+$	-0.499	0.106	0.511	$\pi+$	-0.502	0.068	0.506
		$\pi-$	-0.513	0.234	0.564	$\pi-$	-0.501	0.160	0.526	$\pi-$	-0.502	0.103	0.512
		$\beta+$	-0.040	0.249	0.252	$\beta+$	-0.007	0.122	0.122	$\beta+$	-0.002	0.055	0.055
	$\beta-$	-0.540	0.249	0.595	$\beta-$	-0.507	0.122	0.522	$\beta-$	-0.502	0.055	0.505	
	df= 10	α	-0.069	0.401	0.407	α	-0.043	0.274	0.277	α	-0.043	0.215	0.219
		ρ	-0.075	0.080	0.109	ρ	-0.033	0.047	0.058	ρ	-0.016	0.031	0.035
		$\theta+$	-0.377	0.061	0.381	$\theta+$	-0.387	0.030	0.388	$\theta+$	-0.393	0.018	0.393
		$\theta-$	-0.740	0.082	0.744	$\theta-$	-0.770	0.048	0.771	$\theta-$	-0.785	0.031	0.786
		$\pi+$	-0.499	0.175	0.529	$\pi+$	-0.502	0.112	0.514	$\pi+$	-0.504	0.079	0.510
		$\pi-$	-0.502	0.195	0.538	$\pi-$	-0.498	0.128	0.514	$\pi-$	-0.498	0.089	0.506
		$\beta+$	-0.034	0.223	0.226	$\beta+$	-0.010	0.105	0.105	$\beta+$	-0.004	0.051	0.051
	$\beta-$	-0.534	0.223	0.579	$\beta-$	-0.510	0.105	0.520	$\beta-$	-0.504	0.051	0.506	

Table B1.9: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 1$ and $\omega = 0$)

Skew-t Distribution													
T = 100					T = 200				T = 400				
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = 0$ $a_{\text{skew}} = 1$	df = 2.1	α	-0.021	0.279	0.280	α	-0.018	0.172	0.173	α	-0.017	0.154	0.155
		ρ	-0.081	0.081	0.114	ρ	-0.041	0.049	0.064	ρ	-0.019	0.032	0.037
		$\theta+$	-0.360	0.124	0.380	$\theta+$	-0.379	0.045	0.382	$\theta+$	-0.391	0.023	0.391
		$\theta-$	-0.719	0.132	0.731	$\theta-$	-0.759	0.059	0.761	$\theta-$	-0.781	0.036	0.782
		$\pi+$	-0.002	0.265	0.265	$\pi+$	-0.003	0.136	0.136	$\pi+$	-0.001	0.090	0.090
		$\pi-$	-0.009	0.398	0.398	$\pi-$	0.001	0.239	0.239	$\pi-$	-0.005	0.404	0.404
		$\beta+$	0.002	0.400	0.400	$\beta+$	0.001	0.160	0.160	$\beta+$	-0.001	0.077	0.077
	$\beta-$	-0.498	0.400	0.639	$\beta-$	-0.499	0.160	0.523	$\beta-$	-0.501	0.077	0.507	
	df = 3	α	-0.055	0.449	0.452	α	-0.036	0.291	0.294	α	-0.036	0.217	0.219
		ρ	-0.084	0.084	0.118	ρ	-0.040	0.051	0.064	ρ	-0.019	0.034	0.039
		$\theta+$	-0.357	0.084	0.366	$\theta+$	-0.380	0.042	0.383	$\theta+$	-0.391	0.022	0.391
		$\theta-$	-0.715	0.108	0.723	$\theta-$	-0.760	0.061	0.763	$\theta-$	-0.781	0.036	0.782
		$\pi+$	-0.002	0.193	0.193	$\pi+$	-0.001	0.126	0.126	$\pi+$	0.001	0.081	0.081
		$\pi-$	-0.003	0.272	0.272	$\pi-$	0.001	0.181	0.181	$\pi-$	0.004	0.117	0.117
		$\beta+$	0.008	0.265	0.265	$\beta+$	-0.001	0.132	0.132	$\beta+$	-0.001	0.062	0.062
	$\beta-$	-0.492	0.265	0.559	$\beta-$	-0.501	0.132	0.518	$\beta-$	-0.501	0.062	0.505	
	df=10	α	-0.055	0.463	0.467	α	-0.049	0.304	0.308	α	-0.039	0.231	0.234
		ρ	-0.088	0.087	0.124	ρ	-0.042	0.053	0.067	ρ	-0.020	0.035	0.040
		$\theta+$	-0.357	0.079	0.366	$\theta+$	-0.379	0.039	0.381	$\theta+$	-0.390	0.022	0.391
		$\theta-$	-0.712	0.109	0.721	$\theta-$	-0.758	0.060	0.760	$\theta-$	-0.780	0.038	0.781
		$\pi+$	0.002	0.198	0.198	$\pi+$	0.002	0.131	0.131	$\pi+$	0.002	0.091	0.091
		$\pi-$	-0.001	0.223	0.223	$\pi-$	-0.001	0.149	0.149	$\pi-$	8.532	0.104	0.104
		$\beta+$	-0.003	0.242	0.242	$\beta+$	0.000	0.120	0.120	$\beta+$	0.000	0.059	0.059
	$\beta-$	-0.503	0.242	0.558	$\beta-$	-0.500	0.120	0.514	$\beta-$	-0.500	0.059	0.504	

Table B1.10: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = 1$ and $\omega = 0.5$)

Skew-t Distribution												
T = 100					T = 200				T = 400			
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
df = 2.1	α	-0.017	0.229	0.230	α	-0.009	0.550	0.550	α	-0.014	0.125	0.126
	ρ	-0.068	0.075	0.101	ρ	-0.032	0.044	0.055	ρ	-0.015	0.028	0.032
	$\theta+$	-0.355	0.135	0.380	$\theta+$	-0.382	0.051	0.385	$\theta+$	-0.391	0.037	0.393
	$\theta-$	-0.720	0.132	0.732	$\theta-$	-0.765	0.061	0.768	$\theta-$	-0.784	0.043	0.785
	$\pi+$	0.500	0.260	0.563	$\pi+$	0.501	0.130	0.518	$\pi+$	0.500	0.113	0.513
	$\pi-$	0.497	0.412	0.646	$\pi-$	0.484	0.296	0.567	$\pi-$	0.505	0.170	0.533
	$\beta+$	-0.028	2.876	2.876	$\beta+$	0.007	0.189	0.189	$\beta+$	0.004	0.150	0.150
	$\beta-$	-0.528	2.876	2.924	$\beta-$	-0.493	0.189	0.528	$\beta-$	-0.496	0.150	0.518
$\omega=0.5$ $a_{skew}=1$ df = 3	α	-0.031	0.385	0.386	α	-0.035	0.258	0.260	α	-0.032	0.205	0.208
	ρ	-0.074	0.079	0.108	ρ	-0.033	0.046	0.057	ρ	-0.015	0.029	0.033
	$\theta+$	-0.351	0.090	0.363	$\theta+$	-0.382	0.039	0.384	$\theta+$	-0.391	0.021	0.392
	$\theta-$	-0.715	0.117	0.725	$\theta-$	-0.765	0.059	0.768	$\theta-$	-0.783	0.033	0.784
	$\pi+$	0.501	0.163	0.527	$\pi+$	0.502	0.106	0.513	$\pi+$	0.499	0.068	0.504
	$\pi-$	0.500	0.227	0.549	$\pi-$	0.497	0.151	0.520	$\pi-$	0.499	0.100	0.509
	$\beta+$	0.025	0.255	0.256	$\beta+$	0.003	0.114	0.114	$\beta+$	0.005	0.056	0.056
	$\beta-$	-0.475	0.255	0.539	$\beta-$	-0.497	0.114	0.510	$\beta-$	-0.495	0.056	0.498
df=10	α	-0.055	0.407	0.410	α	-0.038	0.280	0.283	α	-0.041	0.220	0.224
	ρ	-0.072	0.078	0.107	ρ	-0.035	0.047	0.058	ρ	-0.016	0.031	0.035
	$\theta+$	-0.351	0.080	0.360	$\theta+$	-0.379	0.038	0.381	$\theta+$	-0.391	0.021	0.391
	$\theta-$	-0.715	0.112	0.724	$\theta-$	-0.762	0.058	0.764	$\theta-$	-0.783	0.035	0.784
	$\pi+$	0.503	0.173	0.532	$\pi+$	0.501	0.115	0.514	$\pi+$	0.499	0.078	0.505
	$\pi-$	0.500	0.196	0.537	$\pi-$	0.496	0.131	0.513	$\pi-$	0.505	0.090	0.513
	$\beta+$	0.031	0.227	0.229	$\beta+$	0.009	0.105	0.105	$\beta+$	0.004	0.052	0.052
	$\beta-$	-0.469	0.227	0.521	$\beta-$	-0.491	0.105	0.502	$\beta-$	-0.496	0.052	0.499

Table B1.11: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -2$ and $\omega = -0.5$)

Skew-t Distribution													
T = 100					T = 200				T = 400				
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = -0.5$ $a_{\text{skew}} = -2$	df = 2.1	α	-0.012	0.267	0.267	α	-0.011	0.154	0.154	α	-0.012	0.146	0.147
		ρ	-0.066	0.073	0.098	ρ	-0.030	0.044	0.053	ρ	-0.014	0.028	0.032
		$\theta+$	-0.381	0.080	0.389	$\theta+$	-0.388	0.036	0.390	$\theta+$	-0.393	0.020	0.394
		$\theta-$	-0.748	0.105	0.755	$\theta-$	-0.773	0.053	0.775	$\theta-$	-0.786	0.031	0.787
		$\pi+$	-0.501	0.466	0.684	$\pi+$	-0.504	0.275	0.574	$\pi+$	-0.501	0.198	0.539
		$\pi-$	-0.504	0.184	0.537	$\pi-$	-0.499	0.114	0.512	$\pi-$	-0.499	0.076	0.505
		$\beta+$	-0.038	0.286	0.288	$\beta+$	-0.010	0.135	0.135	$\beta+$	-0.002	0.068	0.068
		$\beta-$	-0.538	0.286	0.609	$\beta-$	-0.510	0.135	0.528	$\beta-$	-0.502	0.068	0.507
	df = 3	α	-0.040	0.420	0.421	α	-0.036	0.307	0.309	α	-0.036	0.233	0.236
		ρ	-0.070	0.077	0.104	ρ	-0.033	0.045	0.055	ρ	-0.016	0.030	0.034
		$\theta+$	-0.378	0.064	0.384	$\theta+$	-0.387	0.031	0.388	$\theta+$	-0.393	0.017	0.393
		$\theta-$	-0.743	0.088	0.748	$\theta-$	-0.771	0.047	0.772	$\theta-$	-0.785	0.030	0.785
		$\pi+$	-0.499	0.295	0.580	$\pi+$	-0.505	0.183	0.537	$\pi+$	-0.500	0.125	0.516
		$\pi-$	-0.500	0.192	0.535	$\pi-$	-0.500	0.102	0.511	$\pi-$	-0.502	0.064	0.506
		$\beta+$	-0.032	0.240	0.242	$\beta+$	-0.011	0.110	0.111	$\beta+$	-0.002	0.053	0.053
		$\beta-$	-0.532	0.240	0.584	$\beta-$	-0.511	0.110	0.523	$\beta-$	-0.502	0.053	0.505
	df= 10	α	-0.055	0.410	0.414	α	-0.045	0.289	0.293	α	-0.041	0.232	0.236
		ρ	-0.074	0.080	0.109	ρ	-0.032	0.047	0.057	ρ	-0.015	0.030	0.034
		$\theta+$	-0.376	0.060	0.381	$\theta+$	-0.387	0.030	0.388	$\theta+$	-0.393	0.017	0.393
		$\theta-$	-0.739	0.083	0.744	$\theta-$	-0.770	0.048	0.772	$\theta-$	-0.786	0.030	0.786
		$\pi+$	-0.505	0.211	0.547	$\pi+$	-0.494	0.140	0.514	$\pi+$	-0.498	0.096	0.507
		$\pi-$	-0.504	0.158	0.528	$\pi-$	-0.501	0.107	0.513	$\pi-$	-0.500	0.073	0.506
		$\beta+$	-0.033	0.218	0.221	$\beta+$	-0.009	0.105	0.106	$\beta+$	-0.002	0.051	0.051
		$\beta-$	-0.533	0.218	0.576	$\beta-$	-0.509	0.105	0.519	$\beta-$	-0.502	0.051	0.505

Table B1.12: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -2$ and $\omega = 0$)

Skew-t Distribution													
T = 100					T = 200				T = 400				
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega=0$ $a_{skew}=-2$	df = 2.1	α	-0.029	0.469	0.470	α	-0.019	0.246	0.247	α	-0.010	0.126	0.126
		ρ	-0.082	0.082	0.116	ρ	-0.040	0.047	0.062	ρ	-0.019	0.031	0.037
		$\theta+$	-0.360	0.102	0.374	$\theta+$	-0.379	0.052	0.383	$\theta+$	-0.391	0.022	0.392
		$\theta-$	-0.719	0.146	0.733	$\theta-$	-0.760	0.071	0.763	$\theta-$	-0.782	0.036	0.782
		$\pi+$	0.008	0.462	0.462	$\pi+$	-0.006	0.405	0.405	$\pi+$	0.001	0.233	0.233
		$\pi-$	0.005	0.245	0.245	$\pi-$	-0.003	0.132	0.132	$\pi-$	-0.002	0.086	0.086
		$\beta+$	0.006	0.410	0.410	$\beta+$	0.003	0.192	0.192	$\beta+$	-0.003	0.071	0.071
	$\beta-$	-0.494	0.410	0.642	$\beta-$	-0.497	0.192	0.533	$\beta-$	-0.503	0.071	0.508	
	df = 3	α	-0.035	0.460	0.462	α	-0.042	0.323	0.326	α	-0.031	0.237	0.238
		ρ	-0.084	0.084	0.119	ρ	-0.041	0.050	0.065	ρ	-0.020	0.033	0.039
		$\theta+$	-0.359	0.085	0.369	$\theta+$	-0.380	0.040	0.382	$\theta+$	-0.390	0.021	0.391
		$\theta-$	-0.716	0.116	0.725	$\theta-$	-0.760	0.059	0.762	$\theta-$	-0.780	0.036	0.781
		$\pi+$	-0.004	0.324	0.324	$\pi+$	-0.006	0.217	0.217	$\pi+$	0.004	0.143	0.143
		$\pi-$	-0.002	0.193	0.193	$\pi-$	0.002	0.111	0.111	$\pi-$	-0.001	0.072	0.072
$\beta+$		-0.004	0.283	0.283	$\beta+$	-0.003	0.126	0.126	$\beta+$	3.152	0.061	0.061	
$\beta-$	-0.504	0.283	0.577	$\beta-$	-0.503	0.126	0.518	$\beta-$	-0.500	0.061	0.504		
df=10	α	-0.051	0.473	0.475	α	-0.063	0.328	0.334	α	-0.043	0.244	0.248	
	ρ	-0.087	0.086	0.122	ρ	-0.042	0.052	0.067	ρ	-0.020	0.035	0.040	
	$\theta+$	-0.357	0.078	0.365	$\theta+$	-0.379	0.039	0.381	$\theta+$	-0.390	0.022	0.391	
	$\theta-$	-0.713	0.108	0.721	$\theta-$	-0.759	0.059	0.761	$\theta-$	-0.780	0.037	0.781	
	$\pi+$	-0.003	0.240	0.240	$\pi+$	0.005	0.158	0.159	$\pi+$	-0.001	0.114	0.114	
	$\pi-$	-0.006	0.184	0.184	$\pi-$	-0.002	0.122	0.122	$\pi-$	0.002	0.086	0.086	
	$\beta+$	0.001	0.242	0.242	$\beta+$	-5.140	0.119	0.119	$\beta+$	8.535	0.059	0.059	
$\beta-$	-0.499	0.242	0.555	$\beta-$	-0.500	0.119	0.514	$\beta-$	-0.500	0.059	0.503		

Table B1.13: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -2$ and $\omega = 0.5$)

Skew-t Distribution													
T = 100					T = 200				T = 400				
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	
$\omega = 0.5$ $a_{\text{skew}} = -2$	df = 2.1	α	-0.016	0.258	0.258	α	-0.020	0.185	0.186	α	-0.014	0.133	0.134
		ρ	-0.069	0.073	0.101	ρ	-0.032	0.043	0.054	ρ	-0.014	0.027	0.031
		$\theta+$	-0.356	0.105	0.371	$\theta+$	-0.382	0.042	0.384	$\theta+$	-0.392	0.021	0.393
		$\theta-$	-0.720	0.150	0.736	$\theta-$	-0.766	0.061	0.769	$\theta-$	-0.785	0.033	0.786
		$\pi+$	0.503	0.517	0.722	$\pi+$	0.500	0.270	0.569	$\pi+$	0.504	0.200	0.542
		$\pi-$	0.495	0.370	0.618	$\pi-$	0.496	0.109	0.508	$\pi-$	0.498	0.077	0.504
		$\beta+$	0.017	0.353	0.354	$\beta+$	0.006	0.140	0.141	$\beta+$	0.003	0.066	0.066
	$\beta-$	-0.483	0.353	0.599	$\beta-$	-0.494	0.140	0.514	$\beta-$	-0.497	0.066	0.502	
	df = 3	α	-0.039	0.421	0.423	α	-0.036	0.290	0.292	α	-0.031	0.275	0.277
		ρ	-0.071	0.076	0.104	ρ	-0.032	0.045	0.055	ρ	-0.015	0.029	0.033
		$\theta+$	-0.354	0.086	0.364	$\theta+$	-0.382	0.038	0.383	$\theta+$	-0.392	0.020	0.392
		$\theta-$	-0.718	0.118	0.728	$\theta-$	-0.766	0.056	0.768	$\theta-$	-0.784	0.033	0.785
		$\pi+$	0.495	0.272	0.565	$\pi+$	0.496	0.193	0.532	$\pi+$	0.500	0.129	0.516
		$\pi-$	0.498	0.164	0.524	$\pi-$	0.500	0.101	0.510	$\pi-$	0.501	0.067	0.505
		$\beta+$	-0.072	5.163	5.162	$\beta+$	0.008	0.111	0.111	$\beta+$	0.003	0.053	0.053
	$\beta-$	-0.572	5.163	5.194	$\beta-$	-0.492	0.111	0.505	$\beta-$	-0.497	0.053	0.500	
	df=10	α	-0.057	0.415	0.419	α	-0.048	0.299	0.302	α	-0.045	0.235	0.239
		ρ	-0.070	0.079	0.106	ρ	-0.035	0.048	0.059	ρ	-0.016	0.031	0.035
		$\theta+$	-0.353	0.080	0.362	$\theta+$	-0.379	0.038	0.381	$\theta+$	-0.392	0.020	0.392
		$\theta-$	-0.719	0.112	0.727	$\theta-$	-0.762	0.059	0.764	$\theta-$	-0.784	0.034	0.785
		$\pi+$	0.501	0.206	0.542	$\pi+$	0.501	0.142	0.521	$\pi+$	0.501	0.100	0.511
		$\pi-$	0.501	0.163	0.527	$\pi-$	0.501	0.108	0.513	$\pi-$	0.500	0.074	0.505
		$\beta+$	0.027	0.227	0.229	$\beta+$	0.011	0.106	0.106	$\beta+$	0.001	0.051	0.051
	$\beta-$	-0.473	0.227	0.524	$\beta-$	-0.489	0.106	0.500	$\beta-$	-0.499	0.051	0.501	

Table B1.14: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -1$ and $\omega = -0.5$)

Skew-t Distribution													
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = -0.5$ $a_{\text{skew}} = -1$	df = 2.1	α	-0.019	0.300	0.301	α	-0.024	0.235	0.236	α	-0.019	0.186	0.187
		ρ	-0.068	0.073	0.100	ρ	-0.032	0.044	0.054	ρ	-0.016	0.029	0.033
		$\theta+$	-0.378	0.088	0.388	$\theta+$	-0.386	0.038	0.388	$\theta+$	-0.393	0.022	0.393
		$\theta-$	-0.745	0.118	0.754	$\theta-$	-0.770	0.053	0.772	$\theta-$	-0.785	0.032	0.785
		$\pi+$	-0.505	0.344	0.611	$\pi+$	-0.499	0.207	0.541	$\pi+$	-0.492	0.139	0.511
		$\pi-$	-0.499	0.224	0.546	$\pi-$	-0.496	0.130	0.513	$\pi-$	-0.501	0.095	0.510
		$\beta+$	-0.035	0.316	0.318	$\beta+$	-0.004	0.142	0.142	$\beta+$	-0.002	0.082	0.082
	df = 3	$\beta-$	-0.535	0.316	0.622	$\beta-$	-0.504	0.142	0.524	$\beta-$	-0.502	0.082	0.509
		α	-0.056	0.405	0.408	α	-0.033	0.285	0.286	α	-0.038	0.226	0.229
		ρ	-0.072	0.077	0.105	ρ	-0.033	0.046	0.057	ρ	-0.016	0.030	0.034
		$\theta+$	-0.376	0.077	0.384	$\theta+$	-0.387	0.031	0.388	$\theta+$	-0.393	0.018	0.393
		$\theta-$	-0.740	0.093	0.745	$\theta-$	-0.770	0.048	0.772	$\theta-$	-0.785	0.030	0.786
		$\pi+$	-0.500	0.265	0.566	$\pi+$	-0.503	0.159	0.527	$\pi+$	-0.498	0.106	0.509
		$\pi-$	-0.501	0.213	0.545	$\pi-$	-0.496	0.104	0.507	$\pi-$	-0.502	0.068	0.507
	df = 10	$\beta+$	-0.028	0.289	0.291	$\beta+$	-0.010	0.111	0.111	$\beta+$	-0.003	0.056	0.056
		$\beta-$	-0.528	0.289	0.602	$\beta-$	-0.510	0.111	0.522	$\beta-$	-0.503	0.056	0.506
		α	-0.040	0.404	0.406	α	-0.050	0.278	0.283	α	-0.038	0.221	0.224
		ρ	-0.074	0.081	0.109	ρ	-0.034	0.048	0.059	ρ	-0.016	0.031	0.035
		$\theta+$	-0.375	0.060	0.380	$\theta+$	-0.387	0.030	0.388	$\theta+$	-0.393	0.018	0.393
		$\theta-$	-0.738	0.084	0.743	$\theta-$	-0.770	0.048	0.771	$\theta-$	-0.784	0.031	0.785
		$\pi+$	-0.503	0.190	0.538	$\pi+$	-0.500	0.130	0.517	$\pi+$	-0.502	0.087	0.510
		$\pi-$	-0.498	0.168	0.525	$\pi-$	-0.506	0.115	0.519	$\pi-$	-0.498	0.079	0.504
		$\beta+$	-0.026	0.224	0.225	$\beta+$	-0.012	0.101	0.102	$\beta+$	-0.002	0.051	0.051
		$\beta-$	-0.526	0.224	0.571	$\beta-$	-0.512	0.101	0.522	$\beta-$	-0.502	0.051	0.504

Table B1.15: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -1$ and $\omega = 0$)

Skew-t Distribution													
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = 0$ $a_{\text{skew}} = -1$	df = 2.1	α	-0.014	0.267	0.267	α	-0.011	0.172	0.172	α	-0.020	0.205	0.206
		ρ	-0.083	0.082	0.116	ρ	-0.040	0.048	0.063	ρ	-0.019	0.032	0.037
		$\theta+$	-0.362	0.093	0.374	$\theta+$	-0.380	0.045	0.383	$\theta+$	-0.391	0.023	0.391
		$\theta-$	-0.721	0.127	0.732	$\theta-$	-0.760	0.064	0.763	$\theta-$	-0.781	0.036	0.782
		$\pi+$	0.001	0.375	0.375	$\pi+$	-0.012	0.296	0.296	$\pi+$	0.007	0.163	0.163
		$\pi-$	-0.002	0.273	0.273	$\pi-$	0.001	0.142	0.142	$\pi-$	-0.002	0.085	0.085
		$\beta+$	-0.015	0.303	0.304	$\beta+$	-0.001	0.157	0.157	$\beta+$	-0.001	0.078	0.078
	df = 3	$\beta-$	-0.515	0.303	0.598	$\beta-$	-0.501	0.157	0.525	$\beta-$	-0.501	0.078	0.507
		α	-0.045	0.482	0.484	α	-0.039	0.313	0.315	α	-0.039	0.245	0.248
		ρ	-0.087	0.085	0.122	ρ	-0.041	0.051	0.065	ρ	-0.020	0.034	0.039
		$\theta+$	-0.355	0.081	0.365	$\theta+$	-0.380	0.040	0.382	$\theta+$	-0.390	0.022	0.390
		$\theta-$	-0.712	0.112	0.721	$\theta-$	-0.760	0.060	0.762	$\theta-$	-0.780	0.036	0.781
		$\pi+$	-0.012	0.267	0.267	$\pi+$	-0.005	0.172	0.172	$\pi+$	0.000	0.118	0.118
		$\pi-$	0.001	0.193	0.193	$\pi-$	0.001	0.125	0.125	$\pi-$	-0.001	0.076	0.076
	df = 10	$\beta+$	0.006	0.259	0.259	$\beta+$	-0.002	0.133	0.133	$\beta+$	0.002	0.059	0.059
		$\beta-$	-0.494	0.259	0.557	$\beta-$	-0.502	0.133	0.519	$\beta-$	-0.498	0.059	0.501
		α	-0.054	0.460	0.463	α	-0.053	0.322	0.326	α	-0.042	0.245	0.248
		ρ	-0.085	0.085	0.120	ρ	-0.041	0.055	0.069	ρ	-0.020	0.034	0.039
		$\theta+$	-0.357	0.076	0.365	$\theta+$	-0.379	0.040	0.381	$\theta+$	-0.390	0.021	0.391
		$\theta-$	-0.714	0.107	0.722	$\theta-$	-0.759	0.062	0.761	$\theta-$	-0.780	0.036	0.781
		$\pi+$	-0.005	0.218	0.218	$\pi+$	0.001	0.147	0.147	$\pi+$	0.002	0.103	0.103
		$\pi-$	0.002	0.198	0.198	$\pi-$	-0.001	0.130	0.130	$\pi-$	-0.002	0.091	0.091
		$\beta+$	0.005	0.233	0.233	$\beta+$	0.001	0.119	0.119	$\beta+$	-0.002	0.059	0.059
		$\beta-$	-0.495	0.233	0.548	$\beta-$	-0.499	0.119	0.513	$\beta-$	-0.502	0.059	0.506

Table B1.16: Simulation Results for Skew-t Distribution ($a_{\text{skew}} = -1$ and $\omega = 0.5$)

Skew-t Distribution												
T = 100					T = 200				T = 400			
	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
df =2.1	α	-0.019	0.249	0.250	α	-0.017	0.211	0.212	α	-0.015	0.156	0.156
	ρ	-0.070	0.074	0.102	ρ	-0.031	0.044	0.054	ρ	-0.015	0.028	0.032
	$\theta+$	-0.350	0.112	0.368	$\theta+$	-0.382	0.046	0.385	$\theta+$	-0.392	0.023	0.392
	$\theta-$	-0.714	0.146	0.729	$\theta-$	-0.767	0.065	0.769	$\theta-$	-0.784	0.035	0.785
	$\pi+$	0.509	0.387	0.639	$\pi+$	0.505	0.224	0.552	$\pi+$	0.501	0.160	0.526
	$\pi-$	0.497	0.199	0.535	$\pi-$	0.500	0.132	0.517	$\pi-$	0.498	0.080	0.505
	$\beta+$	0.040	0.344	0.347	$\beta+$	0.006	0.155	0.155	$\beta+$	0.002	0.075	0.075
	$\beta-$	-0.460	0.344	0.574	$\beta-$	-0.494	0.155	0.518	$\beta-$	-0.498	0.075	0.503
$\omega= 0.5$ $a_{skew}= -1$	α	-0.042	0.405	0.407	α	-0.033	0.276	0.278	α	-0.035	0.235	0.237
	ρ	-0.073	0.076	0.105	ρ	-0.032	0.046	0.056	ρ	-0.015	0.030	0.033
	$\theta+$	-0.351	0.084	0.360	$\theta+$	-0.380	0.038	0.382	$\theta+$	-0.392	0.020	0.393
	$\theta-$	-0.714	0.116	0.723	$\theta-$	-0.764	0.058	0.766	$\theta-$	-0.785	0.034	0.785
	$\pi+$	0.507	0.235	0.559	$\pi+$	0.502	0.152	0.525	$\pi+$	0.501	0.104	0.512
	$\pi-$	0.503	0.162	0.528	$\pi-$	0.502	0.103	0.512	$\pi-$	0.500	0.067	0.504
	$\beta+$	0.033	0.234	0.237	$\beta+$	0.012	0.111	0.112	$\beta+$	0.002	0.052	0.052
	$\beta-$	-0.467	0.234	0.522	$\beta-$	-0.488	0.111	0.501	$\beta-$	-0.498	0.052	0.501
df= 0	α	-0.062	0.415	0.420	α	-0.044	0.286	0.289	α	-0.040	0.226	0.229
	ρ	-0.074	0.079	0.108	ρ	-0.034	0.047	0.058	ρ	-0.016	0.031	0.035
	$\theta+$	-0.349	0.083	0.359	$\theta+$	-0.380	0.037	0.382	$\theta+$	-0.391	0.021	0.392
	$\theta-$	-0.712	0.116	0.722	$\theta-$	-0.764	0.058	0.766	$\theta-$	-0.783	0.035	0.784
	$\pi+$	0.496	0.193	0.532	$\pi+$	0.499	0.128	0.515	$\pi+$	0.501	0.088	0.509
	$\pi-$	0.505	0.168	0.532	$\pi-$	0.501	0.109	0.512	$\pi-$	0.500	0.078	0.506
	$\beta+$	0.035	0.222	0.225	$\beta+$	0.007	0.105	0.105	$\beta+$	0.003	0.051	0.051
	$\beta-$	-0.465	0.222	0.516	$\beta-$	-0.493	0.105	0.504	$\beta-$	-0.497	0.051	0.500

Table B1.17: Simulation Results for Normal and Student-t Distribution: ($\omega = 0.5$)

		Normal and T-Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
df = 2.1	α	-0.052	0.293	0.298	α	-0.048	0.246	0.251	α	-0.041	0.192	0.196	
	ρ	-0.050	0.065	0.082	ρ	-0.023	0.037	0.044	ρ	-0.011	0.024	0.026	
	$\theta+$	-0.367	0.060	0.372	$\theta+$	-0.386	0.029	0.387	$\theta+$	-0.394	0.015	0.394	
	$\theta-$	-0.742	0.090	0.747	$\theta-$	-0.775	0.047	0.776	$\theta-$	-0.788	0.027	0.789	
	$\pi+$	0.499	0.119	0.513	$\pi+$	0.501	0.091	0.510	$\pi+$	0.501	0.071	0.506	
	$\pi-$	0.504	0.116	0.517	$\pi-$	0.499	0.077	0.505	$\pi-$	0.502	0.057	0.505	
	$\beta+$	0.022	0.143	0.145	$\beta+$	0.008	0.075	0.076	$\beta+$	0.003	0.036	0.036	
	$\beta-$	-0.478	0.143	0.499	$\beta-$	-0.492	0.075	0.498	$\beta-$	-0.497	0.036	0.498	
$\omega = 0.5$ df = 3	α	-0.057	0.406	0.410	α	-0.055	0.283	0.288	α	-0.045	0.224	0.229	
	ρ	-0.069	0.076	0.103	ρ	-0.032	0.046	0.056	ρ	-0.015	0.029	0.033	
	$\theta+$	-0.354	0.077	0.363	$\theta+$	-0.381	0.038	0.383	$\theta+$	-0.392	0.019	0.392	
	$\theta-$	-0.720	0.108	0.728	$\theta-$	-0.765	0.058	0.767	$\theta-$	-0.784	0.033	0.785	
	$\pi+$	0.501	0.179	0.532	$\pi+$	0.503	0.119	0.517	$\pi+$	0.501	0.086	0.509	
	$\pi-$	0.498	0.182	0.531	$\pi-$	0.499	0.124	0.514	$\pi-$	0.499	0.085	0.506	
	$\beta+$	0.029	0.231	0.232	$\beta+$	0.010	0.106	0.106	$\beta+$	0.002	0.049	0.049	
	$\beta-$	-0.471	0.231	0.524	$\beta-$	-0.490	0.106	0.502	$\beta-$	-0.498	0.049	0.500	
df= 10	α	-0.049	0.423	0.426	α	-0.045	0.287	0.290	α	-0.041	0.220	0.224	
	ρ	-0.077	0.080	0.111	ρ	-0.034	0.047	0.058	ρ	-0.016	0.031	0.035	
	$\theta+$	-0.348	0.083	0.358	$\theta+$	-0.379	0.038	0.381	$\theta+$	-0.391	0.020	0.392	
	$\theta-$	-0.709	0.115	0.719	$\theta-$	-0.762	0.058	0.764	$\theta-$	-0.783	0.034	0.784	
	$\pi+$	0.502	0.177	0.532	$\pi+$	0.500	0.120	0.515	$\pi+$	0.502	0.084	0.509	
	$\pi-$	0.505	0.183	0.538	$\pi-$	0.497	0.124	0.512	$\pi-$	0.498	0.085	0.505	
	$\beta+$	0.031	0.224	0.226	$\beta+$	0.011	0.106	0.107	$\beta+$	0.003	0.050	0.050	
	$\beta-$	-0.469	0.224	0.520	$\beta-$	-0.489	0.106	0.500	$\beta-$	-0.497	0.050	0.500	

Table B1.17: Simulation Results for Normal and Student-t Distribution: ($\omega = -0.5$)

		Normal and T-Distribution											
		T = 100				T = 200				T = 400			
		Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE	Coef	Bias	STDE	RMSE
$\omega = -0.5$	df = 2.1	α	-0.045	0.331	0.334	α	-0.039	0.240	0.243	α	-0.037	0.199	0.203
		ρ	-0.050	0.064	0.081	ρ	-0.023	0.037	0.044	ρ	-0.011	0.024	0.027
		$\theta+$	-0.384	0.039	0.386	$\theta+$	-0.391	0.021	0.391	$\theta+$	-0.395	0.013	0.395
		$\theta-$	-0.758	0.061	0.761	$\theta-$	-0.779	0.036	0.780	$\theta-$	-0.790	0.024	0.790
		$\pi+$	-0.500	0.154	0.523	$\pi+$	-0.501	0.090	0.509	$\pi+$	-0.501	0.061	0.504
		$\pi-$	-0.502	0.165	0.528	$\pi-$	-0.502	0.087	0.509	$\pi-$	-0.499	0.063	0.503
		$\beta+$	-0.023	0.146	0.148	$\beta+$	-0.007	0.073	0.073	$\beta+$	-0.002	0.033	0.033
		$\beta-$	-0.523	0.146	0.543	$\beta-$	-0.507	0.073	0.512	$\beta-$	-0.502	0.033	0.503
	df = 3	α	-0.055	0.401	0.405	α	-0.050	0.274	0.279	α	-0.044	0.212	0.217
		ρ	-0.069	0.076	0.102	ρ	-0.031	0.045	0.055	ρ	-0.016	0.030	0.034
		$\theta+$	-0.379	0.060	0.383	$\theta+$	-0.387	0.029	0.388	$\theta+$	-0.393	0.017	0.393
		$\theta-$	-0.745	0.082	0.749	$\theta-$	-0.771	0.046	0.773	$\theta-$	-0.785	0.030	0.785
		$\pi+$	-0.501	0.185	0.534	$\pi+$	-0.501	0.122	0.516	$\pi+$	-0.499	0.084	0.506
		$\pi-$	-0.508	0.183	0.540	$\pi-$	-0.498	0.128	0.514	$\pi-$	-0.502	0.084	0.509
		$\beta+$	-0.032	0.221	0.223	$\beta+$	-0.007	0.101	0.101	$\beta+$	-0.003	0.051	0.051
		$\beta-$	-0.532	0.221	0.576	$\beta-$	-0.507	0.101	0.517	$\beta-$	-0.503	0.051	0.505
	df = 10	α	-0.056	0.404	0.408	α	-0.034	0.273	0.275	α	-0.043	0.220	0.225
		ρ	-0.072	0.079	0.107	ρ	-0.035	0.048	0.059	ρ	-0.016	0.030	0.034
		$\theta+$	-0.375	0.060	0.380	$\theta+$	-0.386	0.030	0.387	$\theta+$	-0.393	0.017	0.393
		$\theta-$	-0.738	0.084	0.743	$\theta-$	-0.768	0.049	0.770	$\theta-$	-0.785	0.030	0.786
		$\pi+$	-0.502	0.180	0.533	$\pi+$	-0.503	0.126	0.519	$\pi+$	-0.502	0.084	0.509
		$\pi-$	-0.495	0.181	0.527	$\pi-$	-0.501	0.125	0.517	$\pi-$	-0.500	0.088	0.507
		$\beta+$	-0.025	0.218	0.220	$\beta+$	-0.009	0.104	0.104	$\beta+$	-0.002	0.051	0.051
		$\beta-$	-0.525	0.218	0.569	$\beta-$	-0.509	0.104	0.519	$\beta-$	-0.502	0.051	0.505