

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL

**VARIABLE ROTOR SPEED STRATEGY FOR FUEL
EFFICIENCY IN HELICOPTERS**

CAN ESEN

**A THESIS OF MASTER OF SCIENCE
DEPARTMENT OF ENERGY TECHNOLOGIES
APPLIED PROPULSION SYSTEM DESIGN AND
ENGINEERING FOR AEROSPACE TECHNOLOGIES
PROGRAM**

ADVISOR: PROF. DR. ALİ MURAT SOYDAN

JULY 2025

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL

**VARIABLE ROTOR SPEED STRATEGY FOR FUEL
EFFICIENCY IN HELICOPTERS**

CAN ESEN

A THESIS OF MASTER OF SCIENCE
DEPARTMENT OF ENERGY TECHNOLOGIES
APPLIED PROPULSION SYSTEM DESIGN AND
ENGINEERING FOR AEROSPACE TECHNOLOGIES
PROGRAM

ADVISOR: PROF. DR. ALİ MURAT SOYDAN

JULY 2025

T.C.
GEBZE TEKNİK ÜNİVERSİTESİ
LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

HELİKOPTERLERDE YAKIT VERİMLİLİĞİ İÇİN
DEĞİŞKEN ROTOR HIZI STRATEJİSİ

CAN ESEN

YÜKSEK LİSANS TEZİ
ENERJİ TEKNOLOJİLERİ ANABİLİM DALI
HAVACILIK VE UZAY TEKNOLOJİLERİNDE
UYGULAMALI İTKİ SİSTEMİ TASARIMI
MÜHENDİSLİĞİ PROGRAMI

DANIŞMAN: PROF. DR. ALİ MURAT SOYDAN

TEMMUZ 2025

MASTER of SCIENCE JURY APPROVAL FORM

A thesis submitted by Can ESEn, defended on 08/07/2025 before the jury formed with the 30/06/2025 date and 2025/34 numbered decision of the GTU Graduate Administration Board, has been accepted as a MASTER of SCIENCE thesis in the Department of Energy Technologies, Applied Propulsion System Design and Engineering for Aerospace Technologies Program.

JURY

MEMBER

(THESIS ADVISOR) : Prof. Dr. Ali Murat SOYDAN

MEMBER

: Prof. Dr. Ali ATA

MEMBER

: Assoc. Prof. Dr. Ümran KURTAN

APPROVAL

Gebze Technical University Graduate Administration Board

...../...../..... date and/..... numbered decision.

SIGNATURE/SEAL



*To my
Family ...*

ABSTRACT

This thesis presents a comprehensive investigation into the application of variable main rotor speed (N_R) strategies to improve fuel efficiency in helicopters. Unlike conventional rotorcraft operations that employ a fixed N_R across the flight envelope, the proposed approach explores the dynamic adjustment of rotor speed in accordance with varying flight conditions, enabled by the integration of Full Authority Digital Engine Control (FADEC) systems. This concept allows for performance optimization without requiring significant hardware modifications. A generic light-class utility helicopter, equipped with 1500 hp class turboshaft engine, is considered as the baseline configuration. The configuration reflects typical mission requirements encountered in military and civilian rotorcraft operations worldwide. The study employs a validated multidisciplinary simulation framework, combining a high-fidelity rotorcraft dynamics model and an engine performance model. The goal is to identify optimal N_R schedules that provide measurable improvements in fuel economy while satisfying operational constraints. The methodology focuses on evaluating critical flight parameters affected by N_R variation, such as fuel consumption, rotor control margin, blade stall boundaries, tip Mach number, never-exceed-velocity (V_{NE}), power turbine speed (N_P). A parametric analysis across the full flight envelope is carried out using multi-objective optimization techniques to define rotor speed profiles that enhance overall mission efficiency. Findings reveal that variable N_R operation can yield significant reductions in fuel burn and engine workload, particularly in cruise and hover phases, while maintaining acceptable handling and safety margins. The study concludes with a set of recommendations for implementing variable N_R strategies in future rotorcraft systems, supported by simulation-based validation and performance trade-off analyses.

Keywords: Variable Rotor Speed, Fuel Efficiency, Helicopter Performance Optimization, FADEC, Helicopter Flight Envelope

ÖZET

Bu tez, helikopterlerde yakıt verimliliğini artırmak amacıyla değişken ana rotor hızı (N_R) stratejilerinin uygulanmasına yönelik kapsamlı bir araştırma sunmaktadır. Konvansiyonel helikopter operasyonlarında uçuş zarfı boyunca sabit N_R değerleri tercih edilmekteyken, bu çalışmada, uçuş koşullarına bağlı olarak rotor hızının dinamik şekilde ayarlanması yaklaşımı ele alınmakta ve bu stratejinin uygulanabilirliği, Tam Yetkili Dijital Motor Kontrolü (FADEC) sistemlerinin entegrasyonu sayesinde değerlendirilmektedir. Bu kavram, önemli bir donanımsal değişikliğe ihtiyaç duymaksızın sistem performansının optimize edilmesine imkân tanımaktadır. Temel yapılandırma olarak, 1500 beygir gücü sınıfı turboşaft motorla donatılmış, genel maksat görevlerini yerine getirebilecek hafif sınıf bir helikopter modeli esas alınmıştır. Helikopter modeli, dünya genelindeki askeri ve sivil helikopter operasyonlarında karşılaşılan tipik görev gereksinimlerini yansıtmaktadır. Çalışmada, yüksek doğrulukta rotor sistem dinamiği ve motor performans modeli içeren, doğrulanmış çok disiplinli bir simülasyon altyapısı kullanılmıştır. Hedef, operasyonel kısıtları sağlarken yakıt ekonomisinde somut iyileştirmeler sunabilecek optimal rotor hızı profillerinin belirlenmesidir. Yöntem, değişken N_R 'nin etkilediği kritik uçuş parametrelerinin değerlendirilmesine odaklanmaktadır. Bunlar arasında yakıt tüketimi, rotor kontrol marjı, pal stall sınırları, uç hızlarında uç nokta Mach sayısı, azami izin verilen hız (V_{NE}) ve güç türbini devri (N_P) gibi parametreler yer almaktadır. Uçuş zarfının tamamında çok amaçlı optimizasyon teknikleri kullanılarak parametrik analizler yürütülmüş ve görev verimliliğini artıracak rotor hızı profilleri tanımlanmıştır. Elde edilen bulgular, değişken N_R operasyonunun özellikle seyir ve askıda kalma gibi uçuş evrelerinde, yakıt tüketimi ve motor yükünde anlamlı azalmalar sağladığını ve bunun yanında kabul edilebilir kontrol ve emniyet sınırları içerisinde kalındığını ortaya koymaktadır. Çalışma, simülasyon tabanlı doğrulama ve performans denge analizleri ile desteklenen, gelecekteki rotor sistemlerinde değişken N_R stratejilerinin uygulanmasına yönelik bir dizi öneri ile sonuçlandırılmıştır.

Anahtar Kelimeler: Tam Yetkili Dijital Motor Kontrolü, Değişken Rotor Devri, Uçuş Zarfı, Helikopter Aerodinamik Verimliliği

ACKNOWLEDGEMENTS

First and foremost, I would like to express my sincere gratitude to my thesis advisor, Prof. Dr. Ali Murat Soydan for their invaluable guidance, constructive feedback, and continuous support throughout the preparation of this thesis. Their expertise, patience, and motivation were instrumental in shaping the direction and quality of this research. I would also like to thank the academic and technical staff of the Institute of Energy Technologies at Gebze Technical University, whose insightful comments and encouragement greatly enriched my academic experience.

I am also grateful to Dr. Murat Şenipek, whose thesis provided the foundation for the helicopter model used in this study.

Special thanks are extended to my colleagues and friends, especially Yağmur Bulut, for their support and the engaging discussions that helped clarify many of the complex concepts encountered during this work.

Lastly, I would like to thank my beloved family for their unwavering emotional support and belief in me during the entire course of my graduate studies. Without their encouragement, this journey would not have been possible.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	vi
ÖZET	vii
ACKNOWLEDGEMENTS	viii
TABLE OF CONTENTS	ix
LIST OF SYMBOLS AND ABBREVIATIONS	xi
LIST OF FIGURES	xii
LIST OF TABLES	xiii
1. INTRODUCTION	1
1.1. Purpose	1
1.2. Gas Turbine Engines and the Brayton Cycle	2
1.2.1. Ideal Brayton Cycle	3
1.2.2. Real Gas Turbine Cycle and Deviations	7
1.2.3. Application to Turboshift Engines	8
1.2.4. Engine Control and FADEC	9
1.3. Descriptions and Configurations of Turboshift Engine Components	10
1.3.1. Inlet	11
1.3.2. Compressor	12
1.3.3. Combustion Chamber	12
1.3.4. Turbine	13
1.3.5. Exhaust	14
1.4. Helicopter Rotor Dynamics Overview	14
1.4.1. Rotor Speed and Thrust Relationship	15
1.4.2. Rotor Speed and Power Requirements	16
1.4.3. Impact of Variable Rotor Speed on Flight Envelope	16
2. LITERATURE REVIEW	18
2.1. Overview of Rotor Speed Control in Helicopters	18
2.2. Benefits of Variable Rotor Speed (VRS)	18
2.2.1. Fuel Efficiency Improvement	18
2.2.2. Acoustic Signature Reduction	19
2.2.3. Mechanical and Thermal Benefits	19
2.3. Constraints and Challenges	19
2.3.1. Retreating Blade Stall Margin	20
2.3.2. Height-Velocity Diagram Effects	20
2.3.3. Control System Adaptation	20
2.4. Research and Industry Applications	21
3. METHODOLOGY	23
3.1. Helicopter Model Description	24
3.2. Flight Envelope Parametric Sweep	26
3.3. Trim Algorithm and Data Collection	27
3.4. Optimization Methodology	29
3.5. Engine Performance Assessment	32
3.5.1. Power Matching Between Rotor and Engine	34
3.5.2. Fuel Flow and Thermal Efficiency	34
3.5.3. Comparative Evaluation	35
3.5.4. Python Implementation	35

4. RESULTS AND DISCUSSION	36
4.1. Overview of Optimization Output	36
4.2. Optimized Rotor Speed Envelope	38
4.3. Power and Fuel Efficiency Gains from Variable Rotor Speed Strategy	39
4.3.1. Observed Efficiency Improvements	39
4.4. Discussion and Interpretation of Results	42
4.4.1. Energy Efficiency Trade-offs	42
4.4.2. Comparison with Literature	42
4.4.3. Integration Feasibility	43
5. CONCLUSION	44
5.1. Summary of Findings	44
5.2. Contributions of This Work	45
5.3. Limitations and Future Work	46
5.4. Final Remarks	46
REFERENCES	47
BIOGRAPHY	49
PUBLICATIONS AND PRESENTATIONS FROM THE THESIS	50
APPENDICES	51

LIST OF SYMBOLS AND ABBREVIATIONS

μ	: Advance Ratio
H	: Altitude
Ω	: Angular Velocity
A	: Area
C_T/σ	: Blade Loading
R_p	: Compressor Pressure Ratio
θ	: Degree
$^{\circ}\text{C}$	: Degree Celsius
ρ	: Density
H_D	: Density Altitude
η	: Efficiency
h	: Enthalpy
S	: Entropy
Ft	: Feet
F	: Force
Q	: Heat
H-V	: Height-Velocity
Hp	: Horse Power
I	: Inertia
M	: Mach Number
N_R	: Main Rotor Speed
m	: Mass
$\dot{m}$	: Mass Flow
V_{NE}	: Never-Exceed-Velocity
N_P	: Power Turbine
P	: Pressure
P-V	: Pressure-Volume
R	: Radius
γ	: Specific Heat Ratio
σ	: Solidity
T-s	: Temperature-Entropy
T	: Thrust
C_T	: Thrust Coefficient
t	: Time
C_Q	: Torque Coefficient
V	: Velocity
W	: Work
IR	: Infrared
KIAS	: Knot In Air Speed
LHV	: Lower Heating Value
M	: Mach Number
MR	: Main Rotor
MTOW	: Maximum Take Off Weight
NASA	: National Aeronautics and Space Administration
OEI	: One Engine Inoperative
RPM	: Revolutions Per Minute
TIT	: Turbine Inlet Temperature
TR	: Tail Rotor
V_{NE}	: Never-Exceed-Velocity
VRS	: Variable Rotor Speed

LIST OF FIGURES

	<u>Page</u>
Figure 1.1: Turboshaft Engine Schematic [9].	3
Figure 1.2: Simple Gas Turbine Cycle [10].	5
Figure 1.3: Schematic Configuration of a Typical Turboshaft Engine [12].	11
Figure 1.4: Helicopter Rotor Dynamics.	15
Figure 2.1: Variable NR approach (Eco and Plus modes) in AW 169.	21
Figure 3.1: Flowchart of the Determination of Optimal N_R .	23
Figure 3.2: McHugh Stall Boundary Curve.	31
Figure 3.3: Engine Performance Program Interface for Inputs.	33
Figure 4.1: $CT\sigma / \mu$ Plot.	37
Figure 4.2: Optimized Rotor Speed Envelope as a Function of KTAS and Hd.	38
Figure 5.1: Optimum N_R Scheduling.	45

LIST OF TABLES

	<u>Page</u>
Table 3.1: Specifications of H/C Model.	26
Table 4.1: Observed Efficiency and Results-1.	41
Table 4.2: Observed Efficiency and Results-2.	41



1. INTRODUCTION

1.1. Purpose

Conventional helicopters generally do not prefer to use variable N_R designs not to increase complexity and the testing activities for helicopter and systems level for the effects on aerodynamics, handling qualities, structure and fatigue. In addition, conventional engine control system are not capable to govern variable power turbine. Variable N_R brings additional effort for a helicopter project, which must be managed at the initial stage.

Although significant amount of testing and design activities are required, variable N_R option brings several advantages in terms of fuel efficiency, aerodynamics, loads, aeroacoustics and fatigue. UH-60 helicopter is analyzed for variable N_R in terms of efficiency, rotor controls and stall margin [1]. Optimal N_R determination is performed for fuel efficiency [2]. Some of the designed helicopters utilize variable N_R option such as H160 for compressibility effects on V_{NE} [3], T129 ATAK helicopter for low-speed flight regimes, Comanche RAH-66 project for maneuverability and silent mode [4]. Variable N_R approach (Eco and Plus modes) is presented in AW 169 helicopter [5].

Although the main objective is to reduce overall fuel consumption, critical parameters for helicopter operation should also be evaluated within the flight envelope. Changing N_R results in a change in blade loading (C_T/σ), advance ratio (μ), tip Mach number and rotational kinetic energy. These are the main parameters to be considered for whole flight envelope. In this thesis, variable N_R optimization by using a validated physical model for helicopter and engine system is aimed. Whole flight envelope including altitude, temperature, airspeed and weight will be assessed and optimal N_R settings will be determined.

Aim and objective of this thesis is to provide data regarding variable rotor speed strategy which, obtained via validated engine and helicopter model. Thereby, the thesis will contribute to literature within detailed information and optimization unlike other studies.

1.2. Gas Turbine Engines and the Brayton Cycle

Gas turbines represent a class of internal combustion engines in which the working fluid typically air is compressed, mixed with fuel, combusted, and expanded continuously. These engines operate by converting the chemical energy of fuel into mechanical energy using a steady-flow process. Unlike reciprocating engines where compression, combustion, and expansion occur sequentially in the same chamber, gas turbines perform these processes simultaneously in distinct components of the engine [6].

A typical gas turbine engine consists of three fundamental sections:

- Compressor
- Combustion chamber
- Turbine

Depending on their configuration and application, gas turbines are commonly categorized into four types [7]:

- Turbojet
- Turbofan
- Turboprop
- Turboshift

The basic operating cycle begins as ambient air enters the engine through the intake and is directed to the compressor, where it undergoes a pressure increase. The pressurized air is then channeled into the combustion chamber. Within this section, fuel is injected and ignited, substantially increasing the thermal energy of the flow. This high-energy stream is then expanded through the turbine section, beginning with the gas generator turbine (or high-pressure turbine), which extracts energy to drive the compressor.

However, to maintain a net energy output sufficient for propulsion, additional fuel is required beyond what is needed merely to sustain compressor work. The excess energy is produced in turboshaft engines via a power turbine (low-pressure turbine), which is mechanically separated from the gas generator. The power turbine converts the remaining energy into useful shaft power, which is transmitted through a reduction gearbox and driveline to power helicopter rotors [8].

This architectural distinction the addition of a free power turbine differentiates turboshaft engines from turbojet or turbofan configurations. In helicopter applications, this setup is particularly advantageous because it enables the decoupling of rotor speed from the gas generator speed, a key feature that makes variable rotor speed strategies technically feasible.

This thesis focuses on dual-spool turboshaft engines, where both the compressor and turbine sections are split into high-pressure and low-pressure spools, allowing greater flexibility in performance optimization. A simplified schematic of a typical turboshaft engine is shown in **Figure 1.1**.

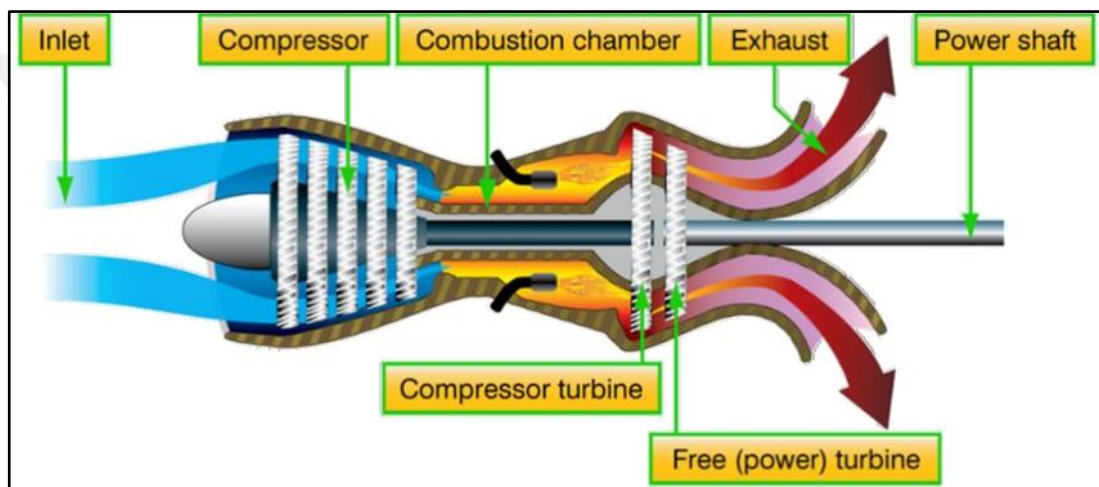


Figure 1.1: Turboshaft Engine Schematic [9].

To better understand the thermodynamic basis of gas turbine operation, it is essential to consider the Brayton cycle. Brayton cycle models the ideal behavior of the working fluid assuming steady, isentropic, and reversible processes and provides the analytical foundation for evaluating the efficiency and performance of real engines. Subsequent sections will build on this cycle to develop a comprehensive understanding of how rotor speed influences engine behavior and fuel consumption.

1.2.1. Ideal Brayton Cycle

The ideal gas turbine cycle, typically represented by the Brayton cycle, is used as baseline for understanding thermodynamics of gas turbine engines. It provides insights into the potential performance of a cycle under ideal conditions, where all processes are assumed to be reversible and adiabatic (isentropic), with no losses in the

components. These ideal assumptions allow for simplified analysis and offer a theoretical upper limit for the performance of gas turbines [6].

Assumptions of the Ideal Cycle

In ideal cycle, several assumptions are made to simplify the analysis:

- Compression and Expansion are Isentropic: Both processes occur without any loss of energy due to friction or heat dissipation.
- Negligible Kinetic Energy Changes: The change in kinetic energy of the working fluid as it moves through the components is considered insignificant.
- No Pressure Losses: There are no losses in pressure due to flow through the inlet, combustion chamber, exhaust, and ducts connecting the components.
- Perfect Gas: The working fluid behaves as an ideal gas with constant specific heats throughout the cycle.
- Constant Mass Flow: The mass flow of the gas is assumed to be constant at all stages of the cycle.
- Complete Heat Transfer in Heat Exchanger: In the case of a heat-exchange cycle, it is assumed that heat transfer occurs perfectly, with the temperature rise on the cold side of the heat exchanger exactly matching the temperature drop on the hot side.

These idealized assumptions are often used to establish performance baselines before introducing real world inefficiencies.

These processes are conventionally represented on a Temperature-Entropy (T-s) diagram and a Pressure-Volume (P-v) diagram, which help visualize the thermodynamic transformations throughout the cycle as shown in **Figure 1.2**.

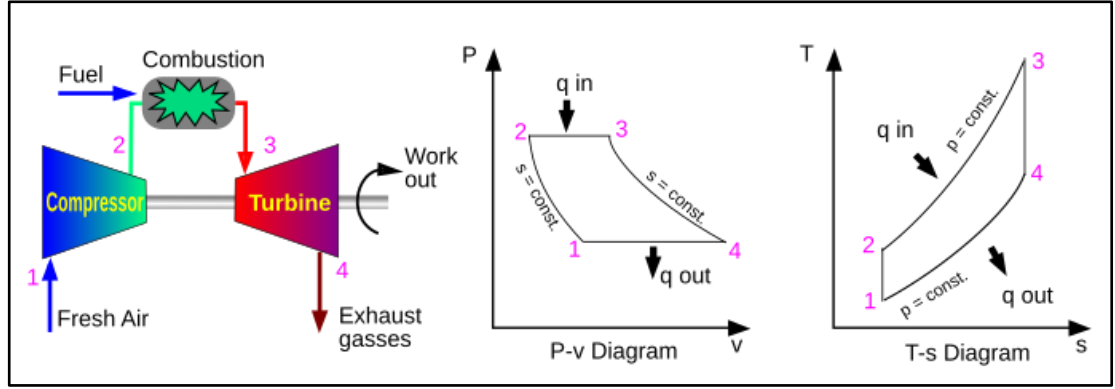


Figure 1.2: Simple Gas Turbine Cycle [10].

Efficiency of the Ideal Cycle

The efficiency of the ideal cycle is influenced by the pressure ratio across the compressor and turbine, as well as the maximum cycle temperature. The cycle efficiency (η) is defined as the ratio of net work output to the heat input, and can be expressed as:

$$\eta_{th,ideal} = 1 - \left(\frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \right) \quad (1.1)$$

Where:

$\eta_{th,ideal}$ = Ideal thermal efficiency

r_p = Compressor pressure ratio (P_2/P_1)

γ = Specific heat ratio (c_p/c_v)

As shown by the equation, increasing the pressure ratio r_p results in an increase in the cycle thermal efficiency. However, practical limitations such as material strength, cooling needs, and aerodynamic losses restrict how high the pressure ratio can realistically be pushed.

The Brayton cycle processes are conventionally illustrated on a Temperature–Entropy (T–s) diagram. In this diagram as shown in **Figure 1.2** [10]:

- Process 1–2 represents isentropic compression (vertical upward line),
- Process 2–3 shows constant-pressure heat addition (horizontal rightward line),

- Process 3–4 represents isentropic expansion (vertical downward line),
- Process 4–1 shows constant-pressure heat rejection (horizontal leftward line).

This visualization is crucial for understanding how energy is added and extracted from the working fluid in idealized conditions.

In the ideal Brayton cycle, the work done by the compressor and the turbine can be expressed using the following relations:

Compressor Work:

$$w_c = c_p(T_2 - T_1) \quad (1.2)$$

Turbine Work:

$$w_t = c_p(T_3 - T_4) \quad (1.3)$$

Net Work Output:

$$w_{net} = w_t - w_c \quad (1.4)$$

Thermal Efficiency:

$$\eta_{th,ideal} = \frac{w_{net}}{q_{in}} \quad (1.5)$$

Where:

w_c = Work input to compressor

w_t = Work output from turbine

w_{net} = Net work output

q_{in} = Heat added during combustion

1.2.2. Real Gas Turbine Cycle and Deviations

In practical applications, the operation of gas turbines significantly deviates from the assumptions made in the ideal Brayton cycle. These deviations primarily result from mechanical losses, non-ideal fluid behavior, pressure drops in ducts and components, and the non-isentropic nature of compression and expansion processes [6].

Real gas turbines experience the following major imperfections:

- The compression and expansion processes are not perfectly isentropic, meaning there is an increase in entropy and energy losses.
- Combustion does not occur at constant pressure due to pressure drops across the combustion chamber.
- Mechanical friction, leakage, and heat loss to the surroundings reduce the net energy conversion efficiency.

Thus, actual gas turbines must be evaluated using isentropic efficiencies for both the compressor and turbine [8].

The isentropic efficiency of the compressor is defined as:

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \quad (1.6)$$

Where:

h_{2s} = Enthalpy after ideal isentropic compression

h_2 = Enthalpy after real (actual) compression

h_1 = Enthalpy at compressor inlet

Similarly, the isentropic efficiency of the turbine is given by:

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \quad (1.7)$$

Where:

h_3 = Enthalpy at turbine inlet

h_4 = Enthalpy after real (actual) expansion

h_{4s} = Enthalpy after ideal isentropic expansion

1.2.3. Application to Turboshaft Engines

Turboshaft engines represent a specific subclass of gas turbines, designed to deliver mechanical shaft power rather than producing thrust. These engines are particularly suited for helicopter applications, where the primary objective is to efficiently drive the main and tail rotors.

The overall architecture of a turboshaft engine typically includes two main turbine stages:

- The gas generator turbine (also called the high-pressure turbine),
- The power turbine (or free turbine).

The gas generator comprises the compressor, the combustion chamber, and the high-pressure turbine. Its main function is to maintain the continuous flow of high-energy gases necessary for engine operation.

After expansion through the gas generator turbine, the hot gases still retain significant energy. This residual energy is then extracted by the power turbine, which is mechanically decoupled from the compressor and responsible for delivering torque to the helicopter's transmission system. This free turbine architecture allows the gas generator speed and the power turbine (and hence rotor) speed to be independently controlled [8].

The rotational speeds of the gas generator N_G and the power turbine N_P are thus separate, enabling greater operational flexibility, especially under varying flight conditions.

The mechanical power output of the turboshaft engine can be expressed as [7]:

Shaft Power Output:

$$P_{shaft} = \dot{m} \times (h_3 - h_4) \quad (1.8)$$

Where:

P_{shaft} = Mechanical power delivered to the transmission system

$\dot{m}$ = Mass flow rate of the working fluid

h_3 = Specific enthalpy at power turbine inlet

h_4 = Specific enthalpy at power turbine outlet

The separation between the gas generator and the power turbine offers several operational advantages:

- Independent control of rotor speed and engine core speed,
- Reduced mechanical complexity in adapting engine performance to different flight phases,
- Enhanced opportunities for implementing variable rotor speed (VRS) strategies to optimize fuel efficiency during cruise or descent.

This architectural flexibility is fundamental for enabling the variable rotor speed operation approach discussed throughout this thesis.

1.2.4. Engine Control and FADEC

In modern helicopter propulsion systems, engine operation and rotor performance are regulated by highly sophisticated electronic control units known as Full Authority Digital Engine Control (FADEC) systems. FADEC represents a technological leap over traditional hydromechanical control systems by enabling complete automation of engine management across all flight conditions.

A typical FADEC system performs the following critical functions:

- Monitors and adjusts fuel flow, compressor settings, turbine parameters, and rotor speed,
- Protects the engine from exceeding operational limits such as maximum turbine inlet temperature (TIT), maximum rotor speed (N_R), or power turbine overspeed (N_P),
- Enables real-time adaptation of engine settings based on environmental conditions (altitude, temperature) and mission demands.

In the context of variable rotor speed (VRS) operations, FADEC is essential. It provides the necessary control fidelity to adjust the main rotor speed (N_R) dynamically without compromising flight safety or engine reliability.

The FADEC system regulates fuel flow to maintain a desired power output, ensuring that turbine inlet temperature and rotor torque remain within allowable boundaries.

The fuel flow ($\dot{m}_f$) required to achieve a certain shaft power output (P_{shaft}) can be approximated as [11]:

$$\dot{m}_f = \frac{P_{shaft}}{\eta_{overall} \times LHV} \quad (1.9)$$

Where:

$\dot{m}_f$ = Fuel mass flow rate (kg/s)

P_{shaft} = Shaft power output (W)

$\eta_{overall}$ = Overall engine efficiency

LHV = Lower Heating Value of the fuel (J/kg)

1.3. Descriptions and Configurations of Turboshaft Engine Components

A gas turbine engine is composed of several key components, each playing a critical role in the energy conversion process. The configuration and performance of these components directly influence the overall efficiency, specific fuel consumption, and operational flexibility of the engine. In the context of helicopter turboshaft engines, the following major elements are typically identified and shown in **Figure 1.3**.

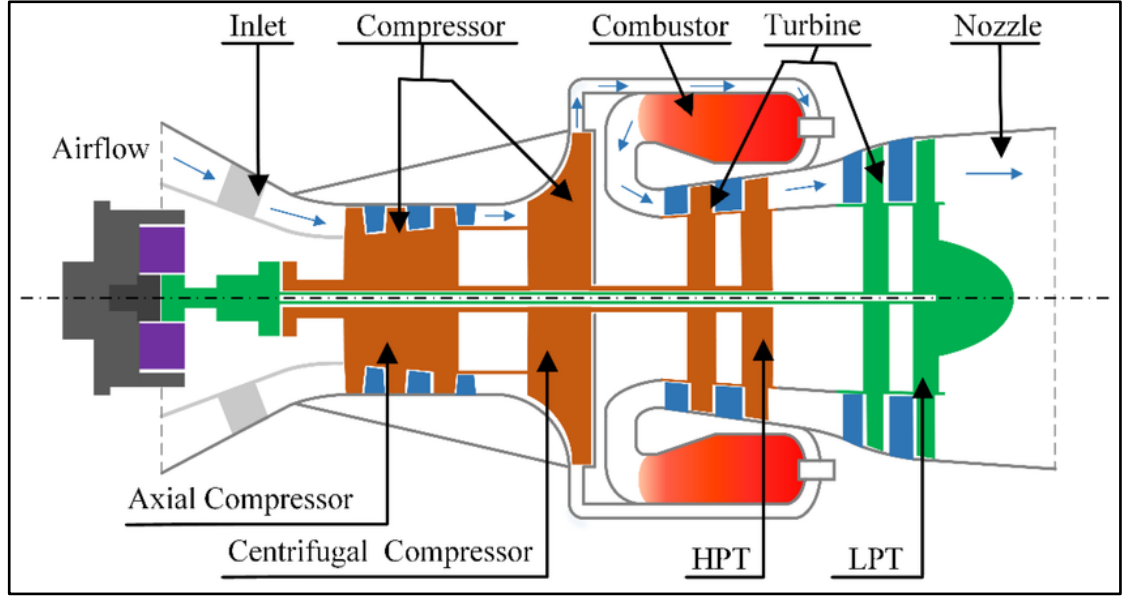


Figure 1.3: Schematic Configuration of a Typical Turboshaft Engine [12].

1.3.1. Inlet

The engine inlet is designed to deliver ambient air to the compressor with minimal pressure loss. For helicopter applications, where forward flight speeds are generally lower than in fixed-wing aircraft, the inlet geometry must ensure efficient air capture under a variety of flight conditions including hover, climb, and cruise.

Inlet performance is often characterized by the inlet pressure recovery factor π_d , defined as:

$$\pi_d = \frac{P_{02}}{P_{01}} \quad (1.10)$$

Where:

P_{01} = Ambient total pressure

P_{02} = Total pressure at compressor face

An ideal inlet has $\pi_d \approx 1$, but practical inlets exhibit small pressure losses due to boundary layer effects and installation disturbances [6].

1.3.2. Compressor

The compressor increases the pressure and temperature of the incoming air to prepare it for efficient combustion. In turboshaft engines, axial compressors are commonly used due to their compactness and high efficiency.

The compressor pressure ratio r_p is defined as:

$$r_p = \frac{P_2}{P_1} \quad (1.11)$$

Where:

P_2 = Pressure after compression

P_1 = Pressure before compression

The compressor work input w_c can be calculated by:

$$w_c = c_p(T_2 - T_1) \quad (1.12)$$

Where c_p is the specific heat at constant pressure, and (T_1) and (T_2) are the inlet and outlet total temperatures, respectively.

Typical axial compressors operate with efficiencies ranging from 85% to 92% depending on design and operating conditions [8].

1.3.3. Combustion Chamber

The combustion chamber is the section where chemical energy from the fuel is converted into thermal energy, significantly increasing the enthalpy of the working fluid. In helicopter turboshaft engines, combustion chambers are designed to ensure efficient fuel-air mixing, stable flame propagation, and minimal pressure loss [7].

The energy addition at constant pressure results in a substantial temperature rise. The heat input q_{in} to the working fluid can be calculated as:

$$q_{in} = c_p(T_3 - T_2) \quad (1.13)$$

Where:

q_{in} = Heat added during combustion

c_p = Specific heat at constant pressure

T_2 = Total temperature after compression

T_3 = Total temperature after combustion

Efficient combustor design aims to minimize the pressure loss coefficient ΔP_c , which is often expressed as a fraction of the inlet pressure:

$$\Delta P_c = \frac{P_2 - P_3}{P_2} \quad (1.14)$$

Where:

P_2 = Total pressure at combustor inlet

P_3 = Total pressure at combustor outlet

1.3.4. Turbine

The turbine section extracts mechanical work from the high-temperature, high-pressure gas flow. In turboshaft engines, the turbine is divided into two sections: the gas generator turbine and the free (power) turbine.

The turbine work output w_t can be expressed as:

$$w_t = c_p(T_3 - T_4) \quad (1.15)$$

Where:

w_t = Work\produced by turbine expansion

c_p = Specific heat at constant pressure

T_3 = Turbine inlet total temperature

T_4 = Turbine outlet total temperature

The gas generator turbine drives the compressor, maintaining the airflow through the engine, while the free turbine delivers torque to the main rotor system independently [8].

The isentropic efficiency of the turbine η_t is defined as:

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} \quad (1.16)$$

1.3.5. Exhaust

After expansion in the turbine stages, the exhaust gases are expelled into the atmosphere. Although the exhaust in a turboshaft engine does not contribute to thrust generation, it plays an important role in the thermal management and mass flow balancing of the propulsion system.

The residual kinetic energy in the exhaust is minimized in turboshaft designs to ensure that most of the energy has been extracted by the turbines. The exit velocity V_{exit} of the gases can be approximated using the energy balance:

$$\frac{V_{exit}^2}{2} = c_p(T_4 - T_5) \quad (1.17)$$

Where:

V_{exit} = Exit velocity

T_4 = Turbine exit total temperature

T_5 = Exhaust exit static temperature

Proper exhaust system design also assists in reducing infrared (IR) signatures and noise emissions, which are critical parameters for military helicopter applications [7].

1.4. Helicopter Rotor Dynamics Overview

The helicopter main rotor is the primary element responsible for producing both lift and propulsion. The aerodynamic behavior of the main rotor directly influences critical flight characteristics such as thrust, power consumption, control authority, and overall flight stability [13].

Unlike fixed-wing aircraft, helicopters achieve vertical lift and directional control by dynamically adjusting the pitch angle of the rotor blades. The combination of

collective and cyclic pitch inputs enables variations in lift force distribution and aircraft attitude.

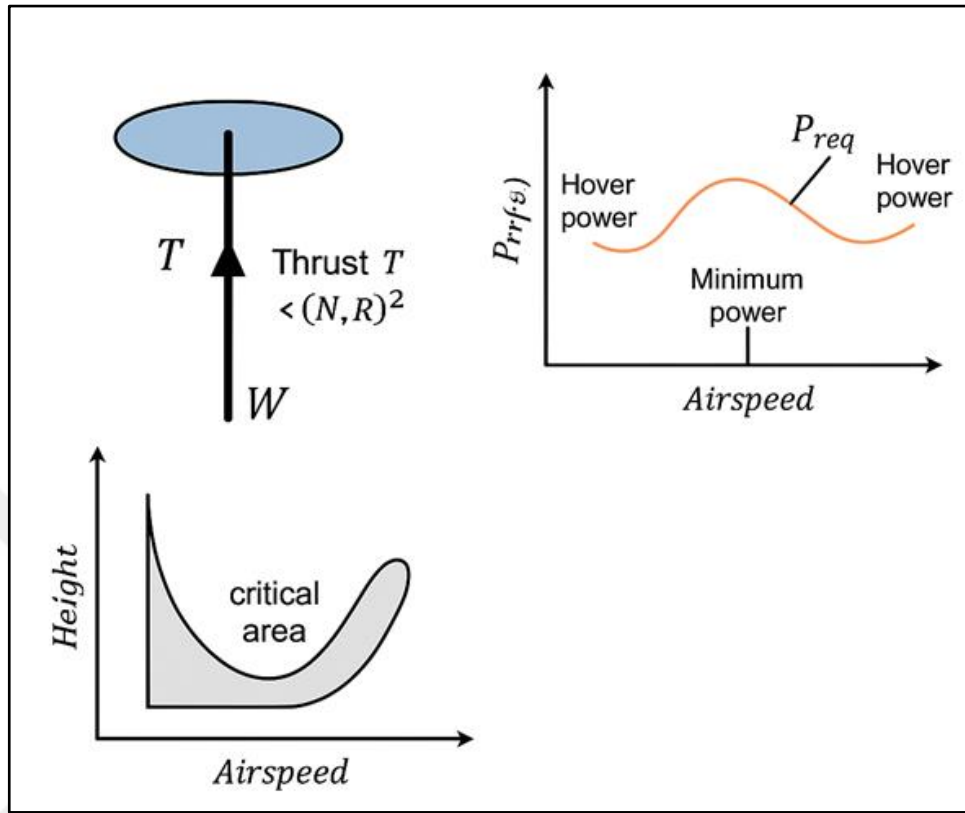


Figure 1.4: Helicopter Rotor Dynamics.

1.4.1. Rotor Speed and Thrust Relationship

The thrust T produced by the main rotor is proportional to the square of the rotor speed N_R . This relationship can be approximated by the following expression:

$$T \propto \rho A (N_R R)^2 \quad (1.18)$$

Where:

ρ = Air density

A = Rotor disk area (πR^2)

R = Rotor radius

Thus, small reductions in rotor speed result in a significant decrease in available thrust. Maintaining an adequate thrust margin is crucial, particularly during hover and autorotation maneuvers [13].

1.4.2. Rotor Speed and Power Requirements

The power required P by a helicopter rotor is heavily influenced by rotor speed. Total power can be decomposed into three primary components [14]:

1. Induced Power P_{ind}
2. Profile Power P_{pro}
3. Parasite Power P_{par}

At hover, induced power dominates, while at higher forward speeds, parasite power becomes more significant.

The induced power required at hover can be approximated by:

$$P_{ind} = \frac{T^{3/2}}{\sqrt{2\rho A}} \quad (1.19)$$

Since thrust T is proportional to N_R^2 , and induced power depends on $T^{3/2}$, a reduction in rotor speed has a nonlinear but significant effect on the total power consumption.

1.4.3. Impact of Variable Rotor Speed on Flight Envelope

Reducing N_R during forward flight phases such as cruise can lead to reduced profile and parasite drag, thereby improving fuel efficiency. However, lowering the rotor speed also affects key operational limits:

- Retreating Blade Stall Margin decreases, particularly at high-speed flight [13].
- Height-Velocity (H-V) Envelope boundaries shift, affecting safe autorotation zones [15].
- Control margin at low rotor speeds can become reduced, especially for lateral and longitudinal cyclic authority.

These trade-offs necessitate a carefully optimized rotor speed schedule across different phases of flight.

The height-velocity diagram, also known as the "dead man's curve," defines the combinations of altitude and airspeed where a safe autorotative landing is not possible. Variable rotor speed operation must ensure that the H-V envelope remains acceptable throughout the mission profile.

Understanding the relationship between rotor speed, thrust production, and power consumption is vital for implementing variable rotor speed strategies. Optimizing N_R dynamically during different flight regimes can yield significant fuel savings without compromising flight safety, provided that appropriate control margins and flight envelope boundaries are maintained [13][14].

2. LITERATURE REVIEW

2.1. Overview of Rotor Speed Control in Helicopters

Historically, conventional helicopters have been designed to operate with a constant main rotor speed N_R cross all phases of flight. This design simplifies control laws, reduces pilot workload, and ensures structural stability. However, maintaining a fixed rotor speed in varying flight conditions leads to suboptimal engine operation, especially during cruise, descent, and loitering phases [13].

The development of Full Authority Digital Engine Control (FADEC) systems in modern turboshaft engines has revolutionized rotor speed management by enabling precise and automated adjustments to N_R . FADEC systems continuously monitor engine parameters and environmental conditions to allow adaptive rotor speed scheduling [11].

This evolution in control technology opened the door for exploring Variable Rotor Speed (VRS) strategies aimed at improving fuel efficiency, noise reduction, and mission flexibility.

2.2. Benefits of Variable Rotor Speed (VRS)

The concept of VRS revolves around adjusting the rotor speed dynamically based on flight conditions, mission profile, and payload requirements. Numerous academic and experimental studies have demonstrated tangible benefits of reducing N_R during specific flight segments, particularly cruise, descent, and loiter.

2.2.1. Fuel Efficiency Improvement

Reducing rotor speed lowers the tip Mach number, which reduces profile drag and induced power. This in turn decreases the power required from the engine, thereby reducing fuel consumption [13]. According to NASA test campaigns [16], rotor speed reductions of 10–15% in cruise resulted in fuel savings of up to 10.6% without major control degradation.

Miste's thesis at the University of Padova [17] used a validated simulation. The findings indicated that VRS could yield fuel savings of up to about 7-8% within $\pm 15\%$ range of nominal rotor speed in optimized cruise conditions.

2.2.2. Acoustic Signature Reduction

Another advantage of reducing N_R is the lowering of the tip speed, which directly reduces rotor noise. The tip Mach number, M_{tip} , is defined as:

$$M_{tip} = \frac{N_R \cdot R \cdot 2\pi}{a} \quad (2.1)$$

Where;

N_R = Rotor speed (rev/s)

R = Rotor radius (m)

a = Speed of sound (m/s)

Since acoustic energy increases with, M_{tip}^6 even small reductions in rotor speed lead to substantial noise attenuation [13].

The Airbus Bluecopter demonstrator operated with reduced rotor speed in descent, achieving up to 5 dB reduction in community noise levels [18].

2.2.3. Mechanical and Thermal Benefits

Operating engines in part-load regimes (enabled by VRS) also reduces turbine inlet temperatures, improving engine life and reducing maintenance costs [7]. Lower rotor torque in cruise also leads to reduced loads on gearbox and rotor hub components, extending airframe durability.

2.3. Constraints and Challenges

Despite its advantages, VRS implementation presents significant engineering and certification challenges, particularly concerning flight safety, aeroelastic stability, and handling qualities.

2.3.1. Retreating Blade Stall Margin

At lower N_R , rotor tip speed decreases. While this reduces profile drag, it also reduces the available stall margin for the retreating blade during high-speed forward flight. The stall onset is characterized by adverse pressure gradients and flow separation, potentially leading to asymmetric lift and vibration [15].

Retreating blade stall is particularly critical above advance ratios $\mu > 0.4$, which limits high-speed performance in VRS configurations.

2.3.2. Height-Velocity Diagram Effects

The Height-Velocity (H-V) envelope defines the combinations of altitude and forward speed where safe autorotation is feasible following engine failure. Reduced N_R alters this envelope by affecting rotor inertia and autorotative RPM buildup [19].

Rotor inertia I_R and its influence on recovery time t_r can be modeled by:

$$t_r \propto \frac{I_R \cdot N_R}{T_{auto}} \quad (2.2)$$

A reduction in N_R , increases the required reaction time during emergency autorotation, possibly violating Category A certification criteria [19].

2.3.3. Control System Adaptation

To fully utilize VRS without increasing pilot workload, adaptive control laws must be integrated into FADEC and flight control systems. This requires robust rotorcraft system identification, gain scheduling, and real-time tuning to preserve stability margins [20].

Studies from Georgia Tech have shown that linear quadratic regulator (LQR)-based control augmentation is effective for preserving response quality under varying N_R regimes [21].

2.4. Research and Industry Applications

Numerous organizations have pursued VRS-related R&D projects to understand its feasibility:

- NASA's Variable Rotor Speed (VRS) project demonstrated up to 12% fuel savings in compound rotorcraft simulations [16].
- Airbus Bluecopter tested automatic rotor speed reduction during descent and approach for noise and fuel consumption reduction [18].
- ONERA, DLR, and AVX Aircraft have developed control strategies and variable-speed rotors for tiltrotor and compound platforms [22].
- Variable NR approach (Eco and Plus modes) is presented in AW 169 helicopter as illustrated in **Figure 2.1** [5].

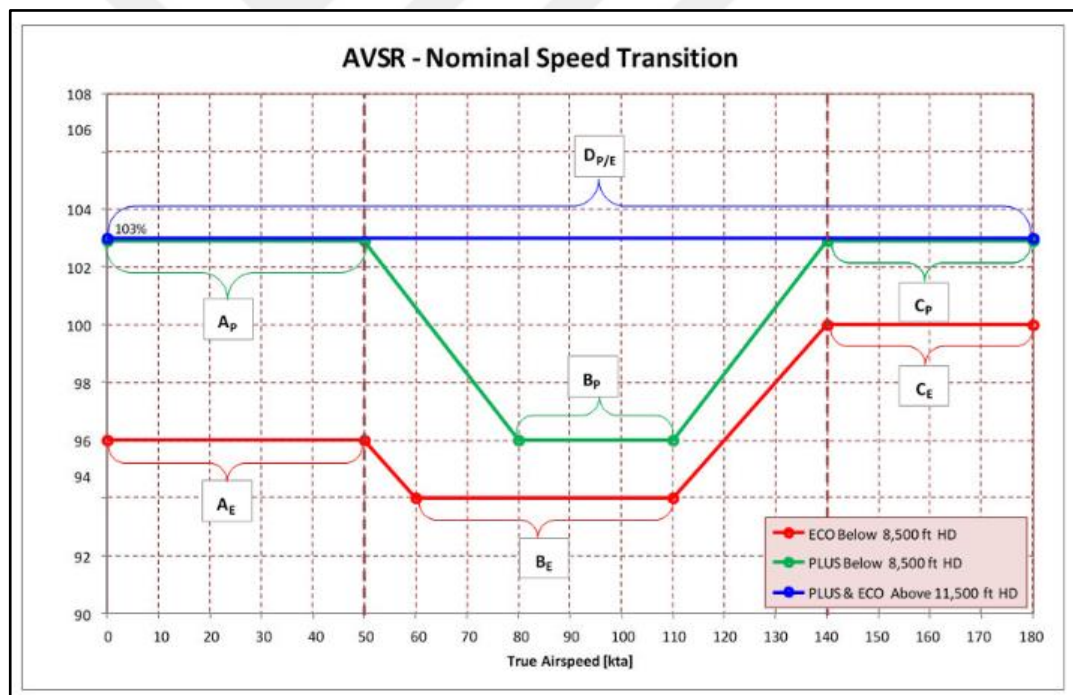


Figure 2.1: Variable NR approach (Eco and Plus modes) in AW 169.

Despite the growing literature on VRS, several research gaps persist:

- Most studies focus on large-class rotorcraft, less is known about light-class utility helicopters.
- Integrated engine-rotor modeling is rarely validated with real flight parameters.

- There's limited exploration of VRS effects across the entire flight envelope, particularly hover, climb, and autorotation transitions.

This thesis aims to address these gaps by:

- Simulating full-envelope mission profiles for a light utility helicopter
- Generating and examining up to 500.000 trim conditions
- Using a validated turboshaft engine model,
- Proposing an optimized rotor speed schedule that balances fuel efficiency with control margins and blade stall constraints.



3. METHODOLOGY

All flight envelope trim conditions are analyzed by GAVM and engine performance program. Helicopter and engine performance metrics are obtained. NR optimization on the whole flight envelope are performed by using multi-variable multi-objective algorithm in Python. The general methodology and flowchart of the study is provided in **Figure 3.1**.

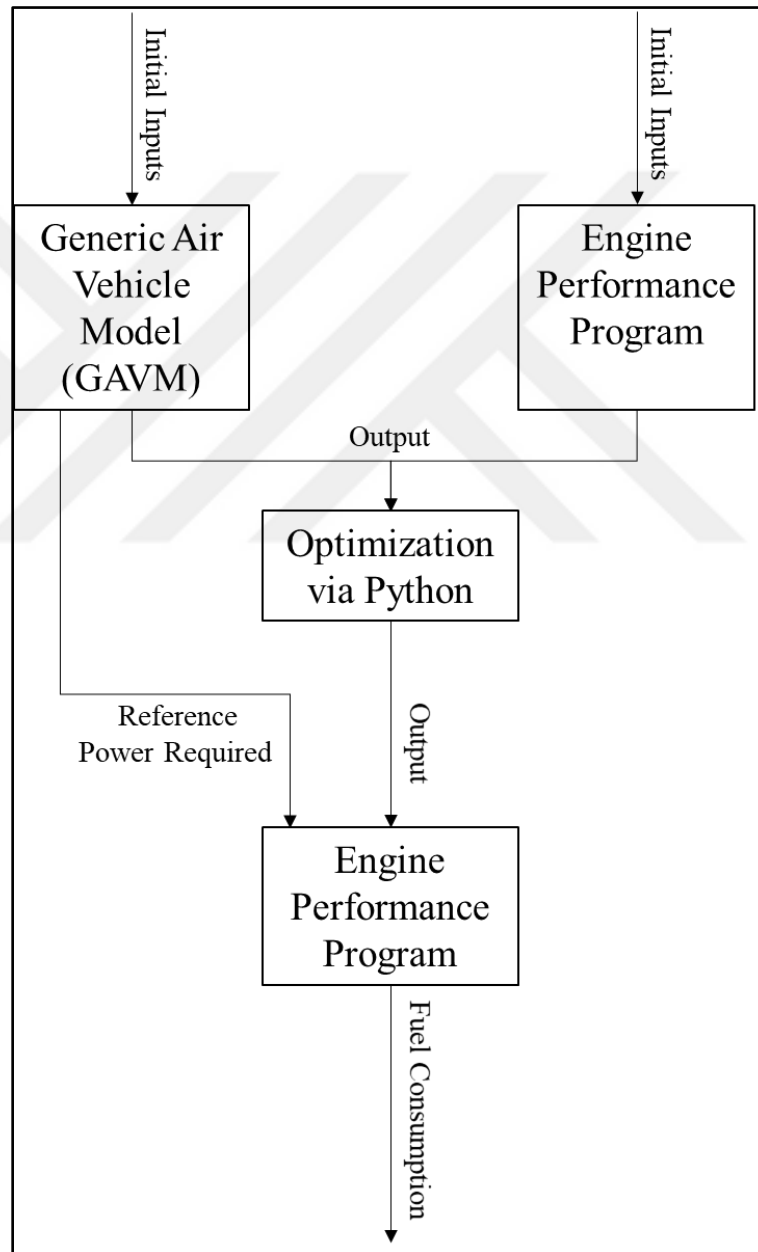


Figure 3.1: Flowchart of the Determination of Optimal N_R .

3.1. Helicopter Model Description

In this study, the helicopter dynamics are modeled based on the Generic Air Vehicle Model (GAVM) developed by M. Şenipek [23]. The GAVM is an object-oriented, modular, and multi-disciplinary simulation framework written in C++ that allows modeling and analysis of various air vehicle types, including rotorcraft. It is capable of simulating 6-DOF flight dynamics, rotor aerodynamics, propulsion system response, and flight control characteristics.

The model is composed of several interacting subsystems:

- Airframe dynamics
- Main and tail rotor models
- Actuator and control systems
- Environmental conditions
- Mass and inertia properties

The helicopter configuration used in this study includes a single main rotor and tail rotor layout, with a conventional control system (cyclic, collective, and pedal). The dynamics are governed by the Newton-Euler equations in the body-fixed frame:

$$F_{total} = m \cdot \dot{V} + \omega \times (m \cdot V) \quad (3.1)$$

$$M_{total} = I \cdot \dot{\omega} + \omega \times (I \cdot \omega) \quad (3.2)$$

Where;

F_{total} = Total external force vector

M_{total} = Total moment vector

V = Velocity vector in body frame

ω = Angular velocity vector in body frame

m = Vehicle mass

I = Inertia matrix

Main rotor forces and moments are calculated using a combination of blade element theory and momentum theory. The nondimensional thrust and torque coefficients are given by:

$$C_T = \frac{T}{\rho A (\Omega R)^2} \quad (3.3)$$

$$C_Q = \frac{Q}{\rho A R (\Omega R)^2} \quad (3.4)$$

Where;

C_T = Thrust coefficient (–)

C_Q = Torque coefficient (–)

T = Main rotor thrust (N)

Q = Main rotor torque (Nm)

ρ = Air density

$A = \pi R^2$ = Rotor disk area (m^2)

Ω = Main rotor angular velocity (rad/s)

R = Main rotor radius (m)

A trim solution is computed for each flight condition by solving the equilibrium equations:

$$F_{total} = 0 \quad (3.5)$$

$$M_{total} = 0 \quad (3.6)$$

Trimmed flight data include values such as thrust, torque, required power P_{req} , thrust coefficient C_T , rotor solidity σ , blade loading $\frac{C_T}{\sigma}$, and control inputs.

Full helicopter mathematical model of the conventional helicopter configuration given in **Table 3.1** is modeled via GAVM.

Table 3.1: Specifications of H/C Model.

Property	Value	Unit
MTOW	3000	kg
# of MR Blades	4	-
Main Rotor Radius	5.7	m
Main Rotor Chord	0.3280	m
Main Rotor Tip Speed	220	m/s
Fuselage Drag	58	ft ²
# of TR Blades	2	-
Tail Rotor Radius	1.0	m
Tail Rotor Chord	0.27	m
Tail Rotor Tip Speed	210	m/s

3.2. Flight Envelope Parametric Sweep

To investigate the effect of variable rotor speed on helicopter performance, a comprehensive flight envelope sweep was conducted. The simulation was implemented in MATLAB using the validated GAVM-based helicopter model. Five key input parameters were varied to cover a wide range of flight conditions.

The parameter ranges used in the sweep are:

- Forward flight speed: $\mathbf{V} \in [0,140]\text{kt}$ with increments of 10 kt
- Pressure Altitude: $\mathbf{h_p} \in [0,20000]\text{ft}$ with increments of 1000 ft
- Ambient temperature: $\mathbf{T_{amb}} \in [-40^\circ\text{C}, +50^\circ\text{C}]$ with increments of 5°C
- Gross weight: $\mathbf{W} \in [2500,3500]\text{kg}$ with increments of 250 kg
- Main rotor speed: $\mathbf{N_R} \in [90\%,105\%]$ with increments of 1%

The total number of evaluated combinations was:

$$N_{\text{total}} = N_V \cdot N_h \cdot N_T \cdot N_W \cdot N_{N_R} = 15 \cdot 21 \cdot 19 \cdot 5 \cdot 16 = 478800 \quad (3.7)$$

Each point represents a unique trimmed condition.

Trim Equilibrium Conditions

For each flight point, a numerical trim algorithm is executed to find steady-state control inputs where total forces and moments are balanced:

$$\mathbf{F_{nt}} = \mathbf{F_{ar}} + \mathbf{F_{gaiy}} + \mathbf{F_{po}} = 0 \quad (3.8)$$

$$M_{nt} = M_{ar} + M_{po} = 0 \quad (3.9)$$

Where;

F_{nt} = Total force vector in body axes (N)

M_{nt} = Total moment vector in body axes (Nm)

F_{ar} = Aerodynamic forces

$F_{gaiy} = m \cdot g$ = Gravitational force vector

F_{po} = Propulsive force (main and tail rotor)

M_{po} = Associated moments

MATLAB Implementation

The sweep is implemented using nested loops in MATLAB. A simplified version of the code structure is provided in APPENDICES.

Output Parameters

For each converged trim point, the following outputs are recorded:

- Rotor thrust: T
- Torque: Q
- Required power: P_{req}
- Main rotor speed: N_R
- Thrust coefficient: C_T
- Rotor solidity: σ
- Blade loading: $\frac{C_T}{\sigma}$
- Control inputs (*collective, cyclic, pedal*)

3.3. Trim Algorithm and Data Collection

In this study, a comprehensive trim analysis was performed to obtain equilibrium flight data for a wide range of operating conditions. The helicopter simulation model,

adapted from Şenipek's work [23], was implemented in MATLAB and iteratively solved for force and moment balance equations using numerical methods.

To define steady-state flight conditions (trimmed states), the model solves the nonlinear equations that represent the translational and rotational force and moment equilibrium.

Force Balance:

$$F_{total} = 0 \quad (3.10)$$

Moment Balance:

$$F_{total} = 0 \quad (3.11)$$

$$M_{total} = 0 \quad (3.12)$$

These equations are numerically solved for a set of control inputs, including collective pitch (θ_0), longitudinal cyclic (θ_{1c}), lateral cyclic (θ_{1s}), and tail rotor pitch (θ_t). The solver iteratively adjusts these control inputs until both translational and rotational equilibrium is achieved.

To perform a flight envelope sweep, the following flight conditions were considered:

- Forward flight speed: $\mathbf{V} \in [0,140]\text{kt}$ with increments of 10 kt
- Pressure Altitude: $\mathbf{h_p} \in [0,20000]\text{ft}$ with increments of 1000 ft
- Ambient temperature: $\mathbf{T_{amb}} \in [-40^\circ\text{C}, +50^\circ\text{C}]$ with increments of 5°C
- Gross weight: $\mathbf{W} \in [2500,3500]\text{kg}$ with increments of 250 kg
- Main rotor speed: $\mathbf{N_R} \in [90\%, 105\%]$ with increments of 1%

At each grid point in this five-dimensional flight envelope, the MATLAB-based solver performs trim analysis and logs the resulting parameters such as control inputs, power required, induced velocities, rotor thrust and torque, blade flapping angles, fuselage attitude, and net aerodynamic forces and moments.

To formalize the trim solution, the following objective function was used:

Objective Function:

- minimize {Required power: P_{req} }

This objective ensures that required power are minimized in the solution. The optimization algorithm is typically implemented using nonlinear solvers.

The trimmed parameters obtained from each simulation run were logged and stored in a structured data format for further post-processing. This dataset constitutes the foundation for performance optimization in the subsequent sections.

3.4. Optimization Methodology

In this study, a custom optimization framework was developed in Python to determine the optimal main rotor speed (N_R) across various flight conditions. The main goal is to minimize the power required (P_{req}) while ensuring adequate aerodynamic stall margin and control margins across the helicopter's flight envelope.

The trim data previously generated via MATLAB was used as a database for this optimization. Each data point corresponds to a unique combination of forward speed, density altitude, ambient temperature, gross weight, and rotor speed, from which performance parameters were extracted.

The optimization algorithm focuses on identifying the minimum power required (P_{req}) rotor speed subject to two main constraints:

Stall Margin Constraint via CT/σ vs McHugh Stall Boundary Curve Envelope

To prevent main rotor blade stall at high blade loading conditions, a stall margin constraint is implemented using a non-dimensional thrust loading parameter $\frac{C_T}{\sigma}$.

The McHugh curve defines a critical aerodynamic boundary in the domain of rotorcraft flight performance. It maps the relationship between the advance ratio μ and the blade loading coefficient $\frac{C_T}{\sigma}$, offering a criterion to avoid stall onset and ensure safe rotor operation during high-speed forward flight. The curve is widely used in helicopter performance studies, particularly for identifying the retreating blade stall or reverse flow boundaries.

The advance ratio μ and blade loading coefficient $\frac{C_T}{\sigma}$ are defined as:

$$C_T = \frac{T}{\rho A (\Omega R)^2} \quad (3.13)$$

$$\sigma = \frac{Ncc}{\pi R} \quad (3.14)$$

$$\mu = \frac{V}{\Omega R} \quad (3.15)$$

- C_T : Rotor thrust coefficient
- σ : Rotor solidity
- μ : Advance ratio
- McHugh: *Stall boundary curve defined from experimental data*
- V : forward airspeed [m/s]
- Ω : rotor angular velocity [rad/s]
- R rotor radius [m]
- T : total rotor thrust [N]
- ρ : air density [kg/m³]
- $A = \pi R^2$: rotor disk area [m²]
- $\sigma = \frac{Ncc}{\pi R}$: rotor solidity, with N as the number of blades and cc as the blade chord.

Originally developed through test data of helicopters such as the UH-60A Black Hawk, the McHugh curve was introduced by R. McHugh and later refined and validated in multiple NASA research efforts. It is most especially presented in the NASA technical publication “Rotorcraft Airloads Measurements—Extraordinary Costs, Extraordinary Benefits” [24], which established the curve’s value in identifying safe operational boundaries for rotor systems.

In this thesis, the McHugh curve serves as a strict filtering constraint within the rotor performance optimization routine. All flight conditions, obtained from the full mission envelope sweep using a MATLAB-based helicopter model, were evaluated in the $\mu - \frac{C_T}{\sigma}$ domain. Any trimmed flight state exceeding the McHugh boundary was directly excluded from further processing.

This hard exclusion approach ensures that the optimized rotor speed schedule N_R remains entirely within the aerodynamically stable region. By operating only below the McHugh curve, the optimization, executed via Python, reliably identifies the minimum power-required operating points without risking blade stall or performance instability.

$$\frac{C_T}{\sigma} \leq \left(\frac{C_T}{\sigma} \right)_{McHugh \text{ Stall Boundary}} \quad (3.16)$$

McHugh Stall Boundary Curve is provided **Figure 3.2**.

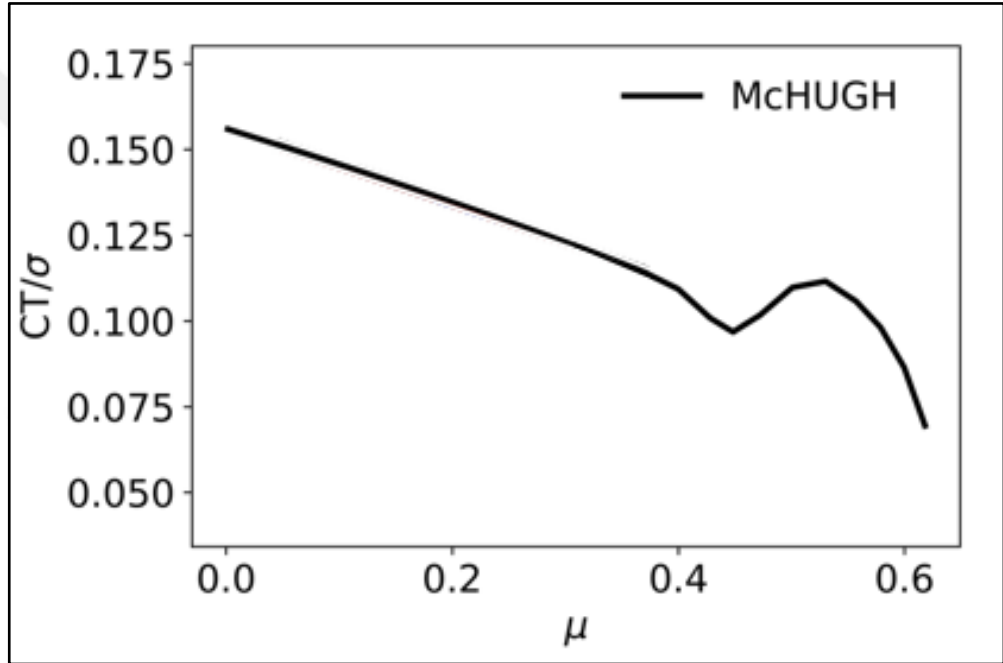


Figure 3.2: McHugh Stall Boundary Curve.

Control Margin Constraint

The second constraint addresses the available control margin, especially at high speeds or heavy gross weights where collective and pedal control authority becomes limiting. This is evaluated from the trim output data using rotor control input limits.

To ensure the reliability and safety of flight control surfaces during the variable rotor speed optimization process, practical limits were imposed on the primary pilot control inputs. Specifically, the maximum allowable collective pitch angle was constrained to 18° , and the pedal input (tail rotor collective) was limited to 26° . These values

represent approximately 20% below the full authority limits, consistent with industry practices and previous experimental studies.

To ensure controllability and safe flight handling, following constraint is applied as;

$$\text{Collective} \leq 18^\circ \quad (3.17)$$

$$\text{Pedal} \leq 26^\circ \quad (3.18)$$

A similar control margin methodology was employed in NASA's flight control research on the UH-60 Black Hawk, where margins of up to 20% were introduced to maintain safe and predictable response within the certified flight envelope [24].

Objective Function – Power Minimization

The primary objective is to minimize the power required by the helicopter for each condition:

$$\min_{N_R} P_{req}(V, H_{DA}, N_R): \quad (3.19)$$

- V : True airspeed
- H_D : *Density altitude*
- N_R : *Rotor speed(decision variable)*
- P_{req} : *Total power required at given condition*

The output of this optimization process is an optimal N_R schedule defined across a two-dimensional map of density altitude and forward speed. This forms a variable rotor speed envelope, which will later be used as input for engine performance analysis and fuel flow estimation.

3.5. Engine Performance Assessment

Following the rotorcraft-level optimization of main rotor speed (N_R) for minimum power requirement, the resulting flight conditions were evaluated using a validated engine performance model to assess the impact of variable rotor speed on engine-level metrics.

The turboshaft engine considered in this study is a dual-spool, free-turbine type representative of the 1500 hp class. The performance assessment focuses on steady-state cruise and hover conditions across the optimized rotor speed envelope.

Engine Performance Program, which as illustrated in **Figure 3.3**, is used evaluate fuel consumption reduction with the outcome from optimization tool and reference (100% N_R) power required information from GAVM.

		Minimum Value 1	0
Altitude (ft)		Maximum Value 1	0
		Value 1 Step Size	1000
		Minimum Value 2	15
Isa+ Amb. Temperature (C)		Maximum Value 2	15
		Value 2 Step Size	103
Constant Inputs	Altitude	0	(ft)
	ISA+ Amb Temperature	0	(°C)
	Amb Temperature	0.00	(°C)
	LP Bleed Air Ratio	0	%
	HP Bleed Air Ratio	0	%
	Inducer Vent Inflow Loss	0	%
	Inducer Vent Outflow Loss	0	%
	True Air Speed	0	(knots)
	RMS Swirl	0.00	
	Bulk Swirl	0.00	
	Pressure Distortion	0	
	Power	0	
Power Extraction (hp)	0		
PTIT ©	0		
Loss	Constant Pressure Recovery	1	
	Constant Exhaust Loss (%)	0	
Engine	Engine Selection	Uninstalled Eng	
	Rating Code	Max Cont. Powe	
	Engine Situation	New	
	Limitations	Mechanical Limi	
Anti-icing Selection	OFF		
NP Selection	☒ 100 %	-	
	☐ 102 %	-	
	☐ Other (%)	95	

Figure 3.3: Engine Performance Program Interface for Inputs.

The engine performance program, which is validated via available test data, calculates steady-state engine operating characteristics, which includes output power, gas generator speed, power turbine speed, power turbine inlet temperature, etc. for turboshaft engines, based on the input parameters such as pressure altitude, outside air

temperature, power rating, power turbine shaft speed. By using these inputs / outputs, power available and fuel consumption can be generated.

3.5.1. Power Matching Between Rotor and Engine

At each flight condition, the optimized N_R corresponds to a specific rotor power demand P_{req} . In a free turbine architecture, the gas generator operates independently of the power turbine, allowing the engine to respond flexibly to rotor speed variations. In this study, transmission losses are not taken into consideration. That is why, the power balance is defined as:

$$P_{turbine} = P_{req} \quad (3.20)$$

Where:

- $P_{turbine}$: Output power from the power turbine
- P_{req} : Required power to sustain trimmed rotor flight

3.5.2. Fuel Flow and Thermal Efficiency

The primary engine metric analyzed is the fuel flow rate $\dot{m}_f$, which is a function of power demand, altitude, ambient temperature, and rotor speed. The following approximation is used:

$$\dot{m}_f \approx \frac{P_{turbine}}{\eta_{th} \cdot Q_{LHV}} \quad (3.21)$$

Where:

- η_{th} : Thermal efficiency of the engine
- Q_{LHV} : Lower heating value of the fuel (e.g., Jet-A)

As rotor speed decreases, the induced drag and required power also decrease, resulting in lower turbine loading and fuel consumption. However, the reduction in gas generator speed may also reduce thermal efficiency.

3.5.3. Comparative Evaluation

Each optimized flight point is compared against a baseline case where $N_R = 100$. The relative benefit is expressed in terms of fuel savings:

$$\Delta \dot{m}_f = \frac{\dot{m}_{f_{100}} - \dot{m}_{f_{opt}}}{\dot{m}_{f_{100}}} \times 100\% \quad (3.22)$$

Where:

- $\dot{m}_{f_{100}}$: Fuel flow at fixed 100% N_R
- $\dot{m}_{f_{opt}}$: Fuel flow at optimized N_R

3.5.4. Python Implementation

The optimization is implemented in Python. McHugh Curve, $\frac{C_T}{\sigma}$ / μ data, control margin elimination, minimum P_{req} and optimum N_R is obtained. A simplified version of the code structure is provided in APPENDICES.

4. RESULTS AND DISCUSSION

4.1. Overview of Optimization Output

In this study, an extensive mission envelope scan was performed to evaluate the potential benefits of implementing a variable rotor speed (N_R) strategy for a turboshaft-powered helicopter. The scan covered a wide range of operating conditions, including:

- **True Airspeed (TAS):** 0 – 140 knots (in 10 kt increments)
- **Altitude:** –1000 ft to 20,000 ft (in 1000 ft increments)
- **Ambient Temperature:** –40 °C to +50 °C (in 5 °C increments)
- **Gross Weight:** 2500 kg to 3500 kg (in 250 kg increments)
- **Rotor Speed Range:** 90% to 105% N_R (in 1% increments)

For each condition, trimmed flight data were obtained using a validated helicopter simulation model adapted from [23] and stored in an Excel database. These trimmed conditions were then optimized using blade stall and control margin constraints. Specifically:

- **Blade stall constraint** was enforced using the McHUGH curve as a hard boundary. All data points falling above the original McHUGH $\frac{C_T}{\sigma} / \mu$ envelope were excluded.
- A **control margin constraint** was applied by eliminating any data where the collective pitch angles exceeded predefined thresholds (e.g., $Coll > 18^\circ$, $Pedal > 26^\circ$).

To ensure aerodynamic safety, above McHugh curve was subtracted from the boundary, setting the following constraint as shown in **Figure 4.1**. Black region in the figure shows that the points which are subtracted from data base for aerodynamic safety.

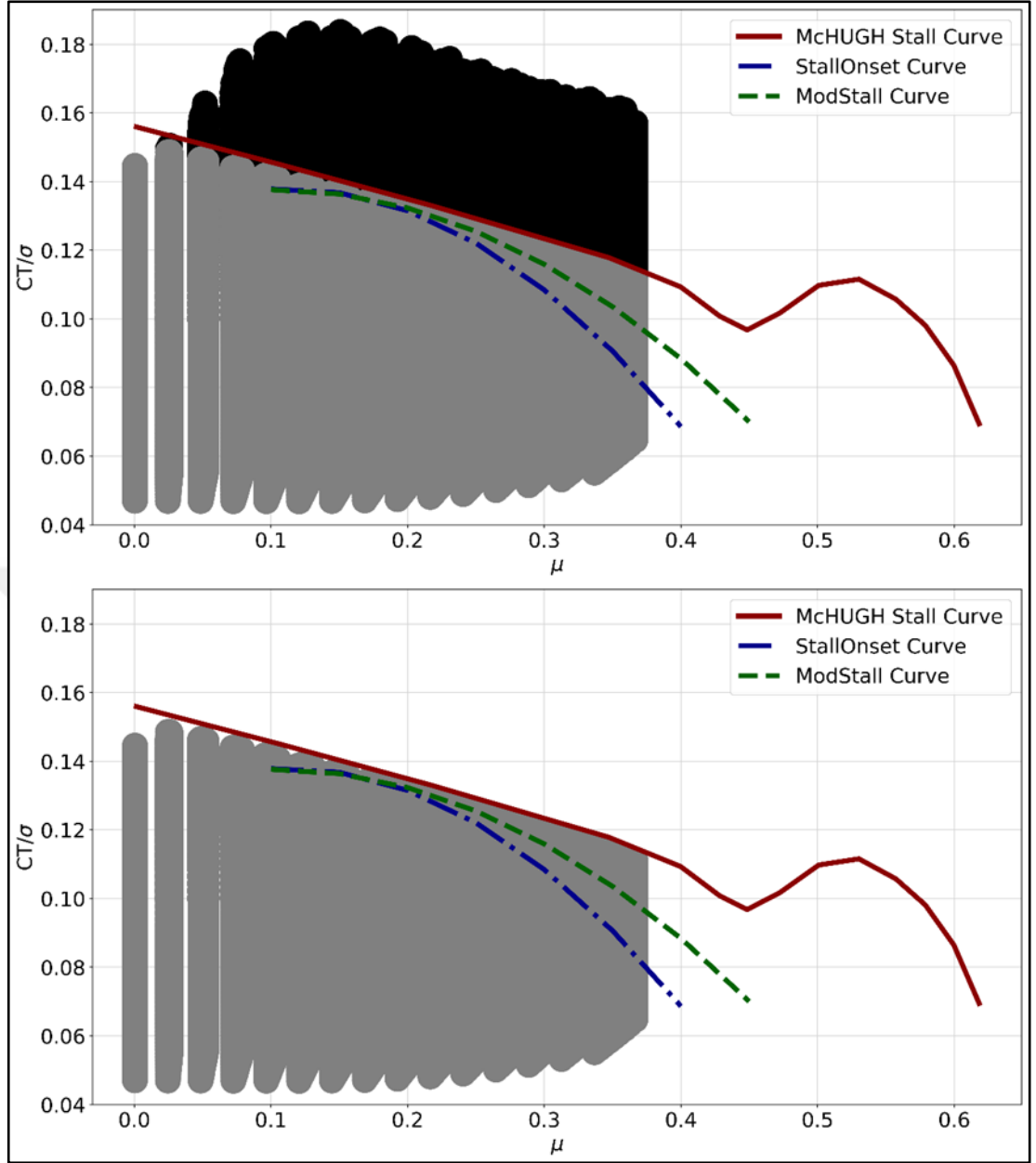


Figure 4.1: $\frac{C_T}{\sigma} / \mu$ Plot.

The resulting data were used to perform a power-based optimization at each flight point, selecting the minimum power-required rotor speed N_R . A visual envelope of this optimization is presented in the following section.

The selection of data for selected N_R and 100% N_R are implemented in MATLAB.

A simplified version of the code structure is provided in APPENDICES

4.2. Optimized Rotor Speed Envelope

The optimized rotor speed envelope is shown in **Figure 4.2**, which illustrates the distribution of optimal N_R values with respect to **true airspeed** and **density altitude**. This figure is derived from the filtered dataset using the Python script detailed in APPENDICES.

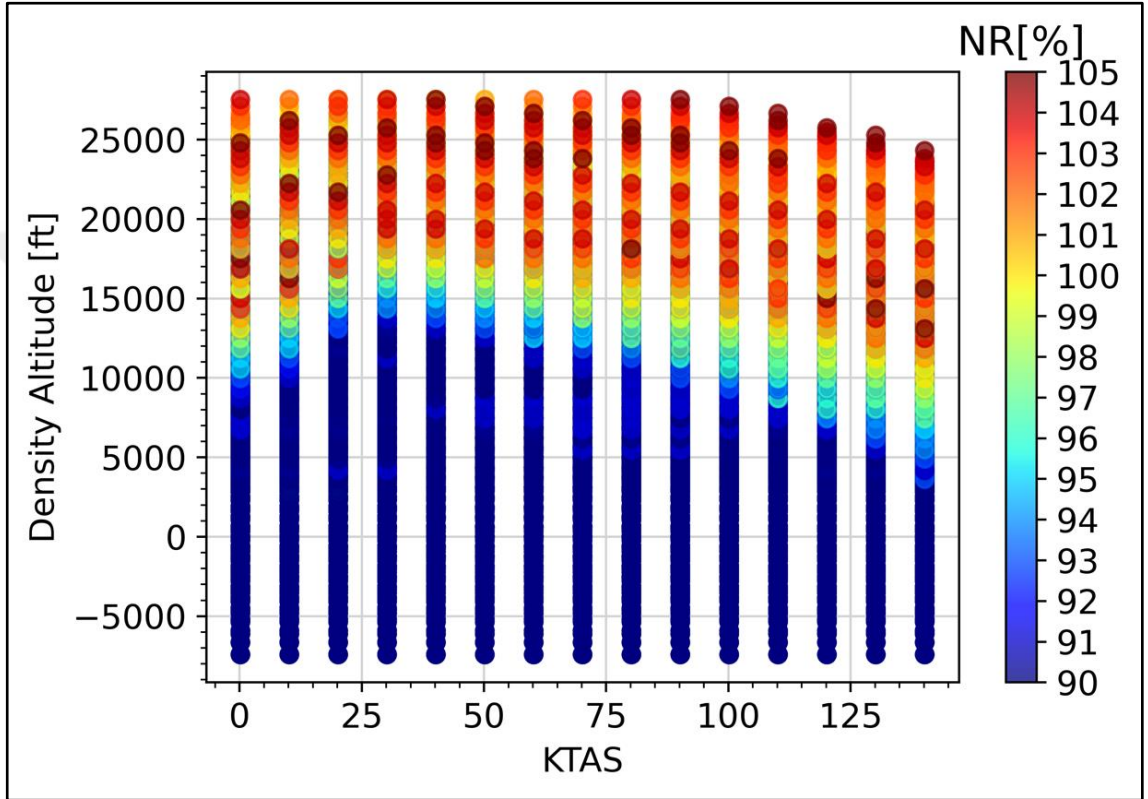


Figure 4.2: Optimized Rotor Speed Envelope as a Function of KTAS and Hd.

Each color corresponds to a discrete N_R value between 90% and 105%.

This graph clearly shows that rotor speeds can be reduced at lower density altitudes without violating stall or control margin limits. For instance:

- At **low density altitude**, the optimal N_R tends to remain close to 90%–95% to minimize power required.
- At **high-density altitudes (above 10,000 ft)**, the optimal rotor speed often increase above 100% N_R , indicating the aerodynamic safety and stall margin.

These findings support the hypothesis that significant power savings are achievable through adaptive rotor speed strategies tailored to mission profiles.

4.3. Power and Fuel Efficiency Gains from Variable Rotor Speed Strategy

After determining the optimal rotor speed (N_R) at each flight condition within the filtered mission envelope, the power required and fuel consumption were compared against a baseline case with fixed $N_R=100\%$.

For this comparison, each selected optimal configuration was evaluated using a performance prediction model, ensuring consistency in environmental and weight conditions. The fuel flow was approximated using the following relation derived from thermodynamic principles:

$$\dot{m}_f \approx \frac{P_{shaft}}{\eta_{overall} \cdot Q_{LHV}} \quad (4.1)$$

Where:

$\dot{m}_f$: Fuel mass flow rate [kg/s]

P_{shaft} : Shaft power required [W]

η_{th} : Thermal efficiency of the engine

Q_{LHV} : Lower heating value of the fuel [J/kg]

4.3.1. Observed Efficiency Improvements

Across the mission envelope, implementing the variable rotor speed strategy resulted in the following average improvements compared to the fixed-speed (100%) baseline:

- **Power saving:** $\approx 7\%$ reduction in power required
- **Fuel saving:** $\approx 6\%$ reduction in fuel flow

In some mission profiles such as (15°C - 4924 ft - 50 Knot & -20°C - 5625ft - 70.2 Knot) rotor speed reductions of 10% in cruise resulted in **power saving** up to **11%** and **fuel savings** of up to **10%**.

The performance improvements were particularly remarkable at low density altitudes (-5000 to 10000 ft), where reduced main rotor speeds significantly lowered rotor-induced power requirements.

In order to provide realistic estimation of power and fuel efficiency gains, a weighting factor (gain) was introduced. A gain value of 0.8 was applied for operations up to 15000 ft density altitude, whereas a gain of 0.2 was applied for the altitude range between 15000 ft and 25000 ft.

In general, the helicopter operates at or below 15000 ft density altitude throughout the majority of its mission envelope.

27 points are determined in order to sweep all conditions such as pressure and density altitude, outside ambient temperature, forward speed and all NR selection between 90-105%.

Thus, efficiency in power and fuel consumption are evaluated by considering each NR settings, all ambient conditions and mission envelope.

All results including; Usage (gain value), Pressure Altitude [ft], OAT [°C], Density Altitude [ft], Forward Speed [KIAS], Total Power Required for selected NR [shp], NR Selected [%], Total Power Required for 100% NR [shp], Fuel Flow for Selected NR [kg/s], Fuel Flow for 100% NR [kg/s], Power Required Difference [%], Fuel Flow Difference [%], Fuel Flow for Selected NR [%], Fuel Flow for 100% NR [%], Sigma, Density Altitude [ft], are provided in **Table 4.1** and

Table 4.2.

Table 4.1: Observed Efficiency and Results-1.

Usage (Gain Value)	Pressure Altitude [ft]	OAT [°C]	Density Altitude [ft]	Forward Speed [KIAS]	Total Power Required for selected NR [shp]	NR Selected [%]	Total Power Required for 100% NR [shp]
80%	0	-30	-5920	0.2	649	90	696
	0	15	0	0.2	679	90	716
	0	45	3346	0.2	700	90	732
	1000	30	2943	10.2	647	90	681
	2000	-5	29	30.2	442	90	482
	3000	0	1904	40.2	379	90	415
	5000	-10	3148	20.2	558	90	594
	4000	15	4924	50.2	350	90	383
	8000	-20	5625	70.2	321	90	355
	6000	15	7374	80.2	327	90	359
	7000	10	8032	60.2	335	90	365
	10000	-10	9378	100.2	370	90	401
	11000	-5	11212	140.2	614	96	633
	12000	0	13020	110.2	403	93	422
	12000	15	14673	90.2	346	91	366
20%	15000	-10	15554	110.2	410	97	419
	14000	10	16561	90.2	358	95	370
	14000	15	17090	100.2	379	98	384
	15000	10	17771	120.2	450	101	447
	15000	10	17771	130.2	503	102	497
	16000	0	17905	50.2	450	94	464
	17000	-10	18009	40.2	521	92	537
	16000	10	18978	80.2	377	99	379
	18000	-10	19233	130.2	538	105	512
	18000	5	20867	60.2	449	100	449
	19000	-5	21007	90.2	405	103	399
	20000	-20	20543	110.2	451	104	440

Table 4.2: Observed Efficiency and Results-2.

Usage (Gain Value)	Pressure Altitude [ft]	OAT [°C]	Density Altitude [ft]	Forward Speed [KIAS]	Fuel Flow for Selected NR [%]	Fuel Flow for 100 % NR [%]	Power Required Difference [%]	Fuel Flow Difference [%]
80%	0	-30	-5920	0.2	93.6	100.0	-7.2	-6.4
	0	15	0	0.2	95.3	100.0	-5.4	-4.7
	0	45	3346	0.2	95.6	100.0	-4.6	-4.4
	1000	30	2943	10.2	96.0	100.0	-5.3	-4.0
	2000	-5	29	30.2	92.2	100.0	-9.0	-7.8
	3000	0	1904	40.2	91.0	100.0	-9.5	-9.0
	5000	-10	3148	20.2	94.2	100.0	-6.5	-5.8
	4000	15	4924	50.2	91.0	100.0	-9.5	-9.0
	8000	-20	5625	70.2	90.1	100.0	-10.8	-9.9
	6000	15	7374	80.2	90.8	100.0	-9.6	-9.2
	7000	10	8032	60.2	91.5	100.0	-9.1	-8.5
	10000	-10	9378	100.2	92.0	100.0	-8.4	-8.0
	11000	-5	11212	140.2	97.1	100.0	-3.0	-2.9

Table 4.2(continued): Observed Efficiency and Results-2.

	12000	0	13020	110.2	96.1	100.0	-4.7	-3.9
	12000	15	14673	90.2	94.8	100.0	-5.8	-5.2
20%	15000	-10	15554	110.2	98.2	100.0	-2.0	-1.8
	14000	10	16561	90.2	96.8	100.0	-3.2	-3.2
	14000	15	17090	100.2	99.0	100.0	-1.2	-1.0
	15000	10	17771	120.2	100.3	100.0	0.5	0.3
	15000	10	17771	130.2	100.8	100.0	1.1	0.8
	16000	0	17905	50.2	97.2	100.0	-3.0	-2.8
	17000	-10	18009	40.2	97.4	100.0	-3.1	-2.6
	16000	10	18978	80.2	99.7	100.0	-0.6	-0.3
	18000	-10	19233	130.2	103.4	100.0	4.8	3.4
	18000	5	20867	60.2	100.0	100.0	0.0	0.0
	19000	-5	21007	90.2	101.0	100.0	1.6	1.0
	20000	-20	20543	110.2	101.5	100.0	2.4	1.5
Average Difference							-6.71	-6.14

4.4. Discussion and Interpretation of Results

The implementation of a Variable Rotor Speed (VRS) strategy, supported by blade loading constraints ($\frac{C_T}{\sigma}$) and control margin filtering, demonstrated clear advantages over the traditional fixed-speed rotor operation. The optimal N_R map, obtained from a dense parametric scan and McHUGH-based filtering, effectively aligned rotor dynamics with mission demands.

4.4.1. Energy Efficiency Trade-offs

Operating the rotor at reduced speeds yielded a significant reduction in power demand, primarily through decreased profile and induced power components. This effect was particularly evident at lower air densities.

Despite the benefit, lower N_R increases the required collective pitch for a given thrust, which may push the rotor closer to stall or control saturation. This necessitated the careful exclusion of configurations near or beyond the McHUGH boundary.

4.4.2. Comparison with Literature

Similar studies have been conducted regarding variable rotor speed for decreasing the power required, thereby reducing fuel consumption. According to NASA test campaigns, rotor speed reductions of 10–15% in cruise resulted in fuel savings of up to 10.6% without major control degradation [16].

Miste's thesis at the University of Padova [17] used a validated simulation model.. The findings indicated that VRS could yield fuel savings of about 7-8% within $\pm 15\%$ range of nominal rotor speed in optimized cruise conditions.

4.4.3. Integration Feasibility

Although performance gains are promising, practical integration of VRS strategies into real flight control systems requires:

- Real-time trim and performance prediction models
- Rotor speed governors capable of adaptive control
- Validation via engine response modeling to assess surge margins and mechanical limits
- Flight Test Campaign with each N_R setting to ensure safe operation
- Rotor and Transmission design with each N_R setting
- Structure, vibration and load analysis with each N_R setting

5. CONCLUSION

This study has investigated the implementation of a **Variable Rotor Speed (VRS)** strategy for fuel-efficient helicopter operation, with particular emphasis on the integration of rotor aerodynamics, engine performance, and flight envelope constraints. The methodology involved a high-fidelity parametric sweep based on a validated rotorcraft model, followed by constraint filtering using aerodynamic (e.g., $\frac{C_T}{\sigma}$) and control margin limits, ultimately determining an optimal rotor speed (N_R) map across a wide range of flight conditions.

5.1. Summary of Findings

- A wide envelope scan in altitude (−1000 ft to 20000 ft), temperature (−40 °C to +50 °C), speed (0 to 140 KTAS), weight (2500–3500 kg), and rotor speed (90–105% N_R) was conducted using a MATLAB-based model derived from Şenipek’s thesis [23].
- The **McHUGH curve** was used to impose blade loading constraints; all data points above this boundary were excluded to ensure stall-free operation.
- Additional control margin filters (e.g., limiting collective pitch) ensured practical control authority remained available across the envelope.
- Optimization revealed that rotor speed reductions of up to 10% in certain regions provided **up to 7% power savings** and **6% fuel savings** compared to the nominal 100% N_R baseline.

In this thesis, variable N_R optimization by using a validated physical model for helicopter and engine system is aimed. Whole flight envelope including altitude, temperature, airspeed and weight are assessed and optimal N_R settings are determined. N_R scheduling is proposed as given in **Figure 5.1**.

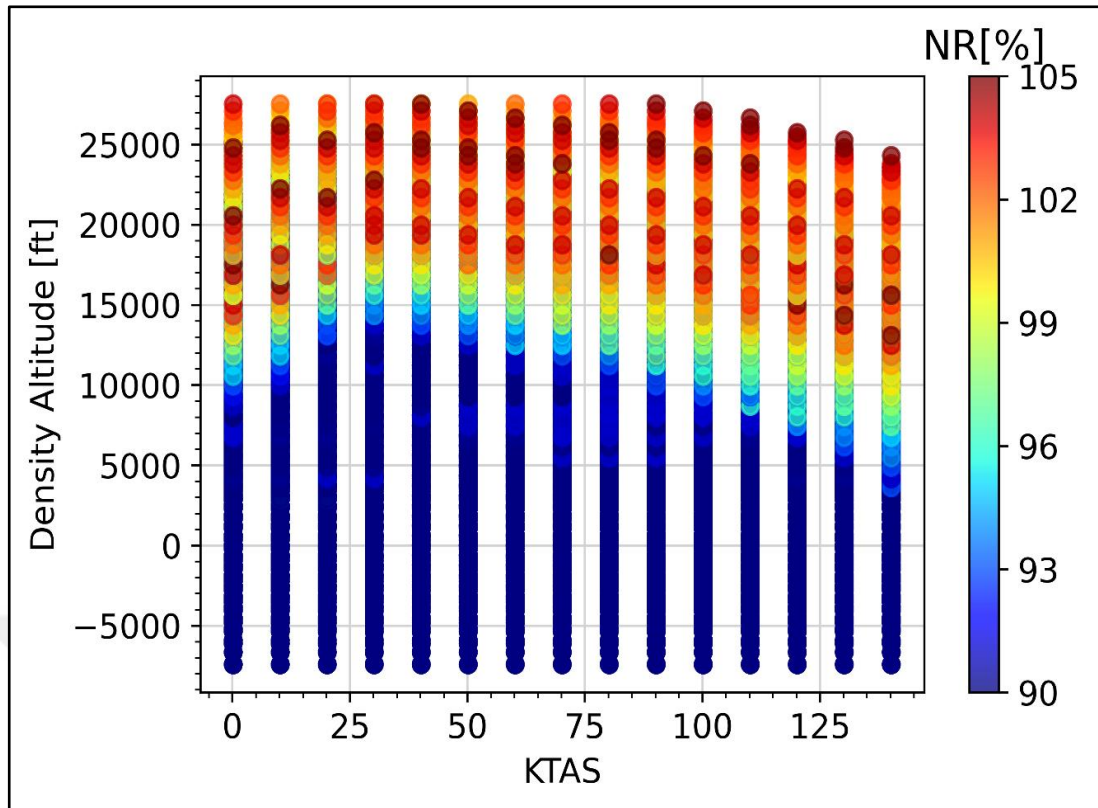


Figure 5.1: Optimum N_R Scheduling.

5.2. Contributions of This Work

- Developed a data-driven optimization framework combining rotor trim, aerodynamic constraints, and power minimization.
- Optimum N_R setting are obtained by analyzing whole envelope (approximately 500.000 trim condition)
- Demonstrated mission-aware rotor speed mapping using realistic flight envelope coverage.
- Highlighted the operational trade-offs and constraints (e.g., stall margins) associated with variable-speed rotorcraft design.

5.3. Limitations and Future Work

Despite the promising outcomes, the current study has several limitations:

- Engine transient response was not modeled; future work should incorporate dynamic surge margin modeling.
- The current study assumes idealized control behavior; real-time implementation may require fly-by-wire rotor speed control architectures.
- The optimization could be improved by applying multi-objective algorithms, simultaneously considering power, fuel, vibration, noise metrics, CAT-A operations and height-velocity (H-V) envelope.

Future work can aim to implement this strategy in a closed-loop flight simulation environment, integrate with turboshaft engine transient models, and evaluate safety-critical conditions such as OEI (One Engine Inoperative), autorotation entries under VRS, vibration, noise metrics, CAT-A operations and height-velocity (H-V) envelope. Also, future work can aim implementing the Optimum N_R Scheduling to engine control unit, which will provide automatic N_R changing according to density altitude and forward speed of helicopter. This will reduce pilot workload.

5.4. Final Remarks

The VRS approach presents a promising route toward reducing fuel burn and operational costs in rotorcraft missions. With advances in fly-by-wire controls and engine-governor integration, it may be feasible to deploy adaptive rotor speed strategies across a range of civil and military platforms. The results presented in this thesis provide a strong foundation for continued development and experimental validation of such systems.

REFERENCES

- [1] Steiner, J., Gandhi, F., Yoshizaki, Y., (2008), “An Investigation of Variable Rotor RPM on Performance and Trim”, AHS 64th Annual Forum, Montreal, Canada, March 28.
- [2] Misté G. A., Garavello A., Benini E., Gonzalez-Alcoy M., (2013), “A New Methodology for Determining the Optimal Rotational Speed of a Variable RPM Main Rotor/Turboshaft Engine System”, AHS 69th Annual Forum, May.
- [3] Sklandanek, Y., Hocquette, J., and Cranga, P., (2019), “H160 Dynamics Development: Setting New Standards”, VFS 75th Annual Forum & Technology Display, Philadelphia, US., May.
- [4] Linden A. W., Comanche Team, (2021), “The RAH-66 Comanche Helicopter: Technical Accomplishment, Program Frustration”, The American Institute of Aeronautics and Astronautics Inc., Reston, VA.
- [5] Air Accidents Investigation Branch (AAIB), (2023), “Report on the accident to Leonardo AW169, Leicester on 27 October 2018”, UK Department for Transport, September 6.
- [6] Saravanamuttoo H. I. H., Rogers G. F. C., Cohen H., Straznicky P. V., (2009), “Gas Turbine Theory”, 6th Edition, Pearson Education Limited, Harlow, UK.
- [7] Mattingly J. D., (2005). “Elements of Gas Turbine Propulsion”, AIAA Education Series, Reston, VA.
- [8] Hill P., Peterson C., (1992). “Mechanics and Thermodynamics of Propulsion”, 2nd Edition, Addison-Wesley Publishing Company, USA.
- [9] Sadıkoğlu U., (2020). “Investigation of Turbojet Engine Performance Using Real Gas Effects and Modeling of the Cycle with MATLAB/Simulink”, Master Thesis, Faculty of Mechanical Engineering, Middle East Technical University, Ankara, Turkey.
- [10] National Aeronautics and Space Administration (NASA), (2014), “Gas Turbine Propulsion”.
- [11] Rolls-Royce plc, (2005), “The Jet Engine”, 5th Edition, Rolls-Royce plc, Derby, UK.
- [12] Liu X., Tang H., Zhang X., Chen M., (2022). “Gaussian Process Model-Based Performance Uncertainty Quantification of a Typical Turboshaft Engine”. *Energies*, 15(1), 136.
- [13] Leishman J.G., (2006), “Principles of Helicopter Aerodynamics”, 2nd Edition, Cambridge University Press, UK.

- [14] Padfield G. D., (2007), "Helicopter Flight Dynamics: The Theory and Application of Flying Qualities and Simulation Modelling", 2nd Edition, Blackwell Publishing, Oxford, UK.
- [15] Prouty R. W., (2002), "Helicopter Performance, Stability, and Control", 2nd Edition, Krieger Publishing Company, Melbourne, FL.
- [16] Johnson A., Gordon J. E., Wells T. G., (2018), "Variable Rotor Speed Concepts for Future Rotorcraft", NASA TM-2018-219816.
- [17] Misté G. A., (2015), "Variable Speed Rotor Helicopters: Optimization of Main Rotor–Turboshaft Engine Integration", Master's Thesis, Department of Aerospace Engineering, University of Padova, Italy.
- [18] Airbus Helicopters, (2015), "Bluecopter Demonstrator Program Overview", Airbus Technical Whitepaper.
- [19] Federal Aviation Administration (FAA), (2016), "Rotorcraft Flying Handbook", FAA-H-8083-21A, U.S. Department of Transportation, Washington, DC.
- [20] Tischler M. B., (2006), "Flight Control Systems: Practical Issues in Design and Implementation", AIAA Education Series, Reston, VA.
- [21] Celi R., (2003), "Development of a Flight Control Design and Evaluation Methodology for Rotorcraft Operating with Variable Rotor Speed", AHS 59th Annual Forum, Phoenix, AZ.
- [22] Gennaretti G., Nibbi L., Marsili C., (2016), "Model-based Assessment of VRS Efficiency in Medium Rotorcraft", Journal of the American Helicopter Society, 61 (3), 44–52.
- [23] Senipek M., (2017), "Development of an Object-Oriented Design, Analysis and Simulation Software for a Generic Air Vehicle," M.S. thesis, Aerospace Eng. Dept., Middle East Technical University, Ankara, Türkiye.
- [24] Bousman B. G., (2014), "Rotorcraft Airloads Measurements – Extraordinary Costs, Extraordinary Benefits" NASA TP-2014-218374.

BIOGRAPHY

Can ESEN graduated from Ege University Mechanical Engineer department as an honor student. He has been working as propulsion systems lead engineer at Turkish Aerospace for more than 6 years and as powerplant and fuel system compliance verification engineer for about 2 year. He mainly focused on engine control system integration and engine operability. During his professional career, he has worked in many engine manufacturer worldwide companies, carried out discussions with the suppliers, actively involved in certification of Engine Installation on many Helicopter and Aeroplane platforms.



PUBLICATIONS AND PRESENTATIONS FROM THE THESIS

Esen, C., Soydan, A. M., (2025), “Variable Rotor Speed Strategy for Fuel Efficiency in Helicopters”, Proceedings of the 11th International Azerbaijan Congress on Life, Engineering, Mathematical, and Applied Sciences, Baku, Azerbaijan



APPENDICES

APPENDIX-A: MATLAB CODE FOR SWEEP

```
clc
```

```
clear all
```

```
close all
```

```
xDoc = xmlread('RUNXML/example_powerReq_sample.xml');
```

```
xRoPP = xDoc.getChildNodes.item(0);
```

```
xPerf = xRoPP.getChildNodes.item(3);
```

```
xTarget = xPerf.getChildNodes.item(3);
```

```
xTargetALT = xTarget.getChildNodes.item(11);
```

```
xTargetTEMP = xTarget.getChildNodes.item(13);
```

```
xTargetWEIGHT = xTarget.getChildNodes.item(19);
```

```
xDoc2 = xmlread('C:\Users\Desktop\Thesis\MODELS\HELICOPTERS\  
\Model.xml');
```

```
xRoPP2 = xDoc2.getChildNodes.item(0);
```

```
xHelicopter = xRoPP2.getChildNodes.item(1);
```

```
XMainRotor = xHelicopter.getChildNodes.item(7);
```

```
xMRInput = XMainRotor.getChildNodes.item(19);
```

```
xMRrpm = xMRInput.getChildNodes.item(13); 366.216
```

```
XTailRotor = xHelicopter.getChildNodes.item(9);
```

```
xTRInput = XTailRotor.getChildNodes.item(19);
```

```
xTRrpm = xTRInput.getChildNodes.item(9); 2025
```

```

ParametersTotal = table();

for l=90:105

    for i=0:20

        for j=1:19

            for k=0:4

                newValue1 = num2str(i*1000);

                newValue2 = num2str(-45+j*5);

                newValue3 = num2str(2500+k*250);

                newValue4 = num2str(l/100*366.216);

                newValue5 = num2str(l/100*2025);

                xTargetALT.setAttribute('value', newValue1);

                xTargetTEMP.setAttribute('value', newValue2);

                xTargetWEIGHT.setAttribute('value', newValue3);

                xMRrpm.setAttribute('value', newValue4);

                xTRrpm.setAttribute('value', newValue5);

                xmlwrite('RUNXML/example_powerReq.xml',xDoc); % XML gncellemesi

                system('run_TRIM.bat'); % TRIM altrma

                txtfile = fileread('C:\Users\Desktop\Thesis\MODELS\HELICOPTERS\
\out.txt');

                txtfile = replace(txtfile,"Trim point cannot be found.,"NaN | NaN | NaN |
NaN | NaN | NaN | NaN | NaN |NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |NaN | NaN | NaN | NaN |
NaN | NaN | NaN | NaN |NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
NaN ");

```

```

folder = 'C:\Users\ \Desktop\Thesis\OUTPUT_example_TRIM';

fullpath = fullfile(folder,'out2.txt');

xx = fopen(fullpath,'w');

fprintf(xx,txtfile)

fclose(xx)


Output = readtable('C:\Users\Desktop\Thesis\MODELS\HELICOPTERS\
\out2.txt');

currentParameters = Output(:, [1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43]); % Bu
dngde elde edilen satrlaral

ParametersTotal = [ParametersTotal; currentParameters]; % Sonularbirletir

end

end

end

ParametersTotalMatrix = table2array(ParametersTotal);

xlswrite('output_data.xlsx', ParametersTotalMatrix); % 'output_data.xlsx' dosyasna
yazdr

end

```

APPENDIX-B: PYTHON CODE FOR OPTIMIZATION

```
import pandas as pd
```

```
import numpy as np
```

```
import os
```

```
import matplotlib.pyplot as plt
```

```
from scipy.interpolate import interp1d
```

```
pathy = r"C:\Users\Desktop\Thesis\Python"
```

```
df_ALL = pd.read_excel(r"C:\Users\Desktop\Thesis\Python")
```

```
### Literature Curves
```

```
McHUGHX = np.array([0.001544, 0.104961, 0.216869, 0.347299, 0.399779,  
0.428335, 0.448401, 0.472326, 0.500882, 0.530209, 0.557222, 0.578831, 0.599669,  
0.618192])
```

```
McHUGHY = np.array([0.155914, 0.145161, 0.133065, 0.117832, 0.109319,  
0.100806, 0.096774, 0.101703, 0.109767, 0.111559, 0.105735, 0.098118, 0.08647,  
0.069444])
```

```
ModStallX = np.array([0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45])
```

```
ModStallY = np.array([0.137646, 0.136373, 0.132325, 0.125503, 0.115907,  
0.103537, 0.088393, 0.07])
```

```
StallOnsetX = np.array([0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4])
```

```
StallOnsetY = np.array([0.13784, 0.136769, 0.131511, 0.122067, 0.108436, 0.090619,  
0.068616])
```

```
McHugh_interp = interp1d(McHUGHX, McHUGHY, kind = 'linear', fill_value =  
'extrapolate')
```

```
margin = 0.03
```

```
### Constants
```

```
N = 4
```

```
c = 0.3280
```

```
R = 5.7
```

```
A = np.pi*R**2
```

```
### Inputs
```

```
IAS = df_ALL[' IAS [kts] ']
```

```
TAS = df_ALL[' TAS [kts] ']
```

```
RPM = df_ALL[' r0_RPM_[r/m] ']
```

```
Thrust = df_ALL[' r0_Thrust[N] ']
```

```
Temp = df_ALL[' Temp.[C] ']
```

```
df_ALL = df_ALL[df_ALL[' r0_Coll_[deg] ' ] < 18]
```

```
df_ALL = df_ALL[df_ALL[' r1_Coll_[deg] ' ] < 26]
```

```
df_ALL.index = range(len(df_ALL))
```

```
Omega = RPM*np.pi/30
```

```
V_tip = Omega*R
```

```
speed_sound = 331.3*np.sqrt(1+Temp/273.15)
```

```
Mach = V_tip/speed_sound
```

Mach Filtered

```
df_ALL['Mach'] = Mach  
df_ALL = df_ALL[df_ALL['Mach'] < 1]  
df_ALL.index = range(len(df_ALL))
```

Filtered Inputs

```
IAS = df_ALL[' IAS [kts] ']  
TAS = df_ALL[' TAS [kts] ']  
RPM = df_ALL[' r0_RPM_[r/m] ']  
Thrust = df_ALL[' r0_Thrust[N] ']  
Temp = df_ALL[' Temp.[C] ']  
  
rho = (IAS / TAS)**2*1.225  
mu = TAS*0.5144/V_tip  
sigma = N*c/np.pi/R  
CT = Thrust/(rho*A*V_tip**2)  
CT_sigma = CT/sigma
```

CT Curve Filtered Excel

```
filtered_mu = []  
filtered_CT_sigma = []  
inside_env_ind = []  
  
outside_mu = []  
outside_CT_sigma = []  
outside_env_ind = []
```

```

for i in range(len(mu)):
    if CT_sigma[i] < McHugh_interp(mu[i])*(1-margin):
        filtered_mu.append(mu[i])
        filtered_CT_sigma.append(CT_sigma[i])
        inside_env_ind.append(i)
    else:
        outside_mu.append(mu[i])
        outside_CT_sigma.append(CT_sigma[i])
        outside_env_ind.append(i)

df_ALL = df_ALL.loc[inside_env_ind]
df_ALL.index = range(len(df_ALL))

Vel = df_ALL[' TAS [kts] ']
Mass = df_ALL[' Weight[kg] ']
Altitude = df_ALL[' Altitude[ft] ']
temp = df_ALL[' Temp.[C] ']
P_required = df_ALL[' Ptot [hp] ']

unq_Altitude = np.unique(df_ALL[' Altitude[ft] '])
unq_P_required = np.unique(df_ALL[' Ptot [hp] '])

indices = []

for i_alt in unq_Altitude:
    df_current = df_ALL[df_ALL[' Altitude[ft] ']== i_alt]

```

```

unq_Vel      = np.unique(df_current[' TAS [kts] '])

for i_kts in unq_Vel:
    df_current_kts    = df_current[df_current[' TAS [kts] ']== i_kts]
    unq_Mass          = np.unique(df_current_kts[' Weight[kg] '])

    for i_kg in unq_Mass:
        df_current_kg    = df_current_kts[df_current_kts[' Weight[kg] ']== i_kg]
        unq_temp         = np.unique(df_current_kg[' Temp.[C] '])

        for i_Temp in unq_temp:
            df_current_temp = df_current_kg[df_current_kg[' Temp.[C] ']== i_Temp]

            if len(df_current_temp) > 0:
                min_index = df_current_temp[' Ptot [hp] '].idxmin()
                indices.append(min_index)

            else:
                indices.append("NA")

            print(f"ALT: {i_alt} \n TAS: {i_kts} \n Weight: {i_kg} \n Temp:
{i_Temp} ")

df_filtered = df_ALL.loc[indices]
df_filtered.to_excel("DF_Filtered.xlsx")

### Plotting

plt.figure()

plt.scatter(filtered_mu, filtered_CT_sigma, color ='maroon', s = 5)

```

```

plt.xlabel(r"$\mu$")
plt.ylabel(r"$CT/\sigma$")
plt.tight_layout()
plt.plot(McHUGHX,McHUGHY,color='black',label='McHUGH',linewidth= 2.5)
plt.plot(McHUGHX,McHUGHY*0.97,color='green',label='McHUGH*0.97',linewidth
h= 1.5)
plt.plot(ModStallX,ModStallY,'--',color='navy',label='ModStall',linewidth= 2.5)
plt.rcParams.update({'font.size':12})
plt.legend(fontsize = 10)
plt.savefig('.\Output3.png', dpi=600)
plt.close()

df_filtered['Sigma-HD'] = ((1-6.876/1000000*df_filtered[' Altitude[ft]
'])**5.2557813)/((df_filtered[' Temp.[C] ']+273.15)/288.15)
df_filtered['HD'] = 145442.156*(1-df_filtered['Sigma-HD']**0.234969)

x = df_filtered[' TAS [kts] ']
y = df_filtered[' Altitude[ft] ']
z = df_filtered[' r0_RPM_[r/m] ']
z_un = np.unique(z)

color_list = ['black','gray','rosybrown','maroon','red','lightsalmon','peru','bisque',
              'orange','gold','yellow','greenyellow','aqua','deepskyblue','purple','pink']

nr_perc = np.arange(90,106,1)
fig, ax = plt.subplots(figsize=(6,6))
for j in range(len(z_un)):

```

```

df_curr = df_filtered[df_filtered[' r0_RPM_[r/m] ']== z_un[j]]

x_curr = df_curr[' TAS [kts] ']

y_curr = df_curr['HD']


plt.scatter(x_curr,y_curr,label= fr'NR: {nr_perc[j]}%', color=color_list[j])

plt.xlabel('KTAS')

plt.ylabel('Density Altitude [ft]')

plt.legend(ncol = 4, loc="upper right", fontsize=10, bbox_to_anchor = (1.02,-0.1))

plt.savefig('.\Output4.png', dpi=600,bbox_inches="tight")

plt.close()

```

APPENDIX-C: MATLAB CODE FOR DATA SELECTION (FOR SELECTED NR AND 100% NR)

```
clc
```

```
clear all
```

```
MainData = readtable('C:\Users\Desktop\Thesis\Selection\output_data_ALL.xlsx');
```

```
Missions = readtable('C:\Users\Desktop\Thesis\Selection\Mission_Mix.xlsx');
```

```
LongTable = Missions(1,:);
```

```
LongTable(1,:)=[];
```

```
% LongTable.Power=[];
```

```
% LongTable.NR=[];
```

```
LongTable.Power=NaN(0,1);
```

```
LongTable.NR=NaN(0,1);
```

```
rowCounter=1;
```

```
missionCols = Missions.Properties.VariableNames;
```

```
for i = 1:height(Missions)
```

```
    OAT = Missions.OAT__C_(i);
```

```
    alt = Missions.PressureAltitude_ft_(i);
```

```
    speed = Missions.ForwardSpeed_KIAS_(i);
```

```
    idx = find(MainData.Temp__C_ == OAT & MainData.Altitude_ft_ == alt &  
MainData.TAS_kts_ == speed & MainData.Weight_kg_ == 3000 &  
MainData.r0_RPM__r_m_ < 367 & MainData.r0_RPM__r_m_ > 365 );
```

```

if ~isempty(idx)

    for j = 1:length(idx)

        LongTable(rowCounter, missionCols) = Missions(i,:);

        LongTable.Power(rowCounter,1)= MainData.Ptot_hp_(idx(j));

        % LongTable.NR(rowCounter,1)= MainData.r0_RPM__r_m_(idx(j));

        rowCounter = rowCounter+1;

    end

else

    LongTable(rowCounter,missionCols) = Missions(i,:);

    LongTable.Power(rowCounter,1)= NaN;

    % LongTable.NR(rowCounter,1)= NaN;

    rowCounter = rowCounter+1;

end

end

writetable(LongTable, 'output_with_power_nr.xlsx');

```