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HAKAN AK
A THESIS SUBMITTED FOR THE DEGREE OF
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GEBZE TEKNİK ÜNİVERSİTESİ

FEN BİLİMLERİ ENSTİTÜSÜ

AĞIR KUYRUKLU DAĞILIMA SAHİP OTOKORELATİF SÜREÇLER İÇİN İSTATİSTİKSEL KONTROL ŞEMALARINDA UYGUN KONTROL LİMİTLERİNİN BELİRLENMESİ

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YÜKSEK LİSANS TEZİ

ENDÜSTRİ MÜHENDİSLİĞİ ANABİLİM DALI

DANIŞMANI

DR. ÖĞRETİM ÜYESİ KEMAL DİNÇER DİNGEÇ

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SUMMARY

In the thesis study, an autocorrelated process when the characteristic of interest is Pareto distributed, is handled. In order to test whether the process mean is under control, the $\bar{X}$ charts are used. By means of Monte Carlo simulations carried out in the R programming language, firstly samples are created, secondly the $\bar{X}$ charts of these samples are plotted, and finally the performance metrics of these $\bar{X}$ charts are estimated. Here, the ARTOP(1) model is used to handle the dependency in the process. And ARL_0 and ARL_1 are used as performance metrics. Thanks to the code written in R; In Pareto distributed autocorrelated processes, it is aimed to observe the performances of $\bar{X}$ control charts plotted with the default settings of the "qcc" function, which is used to plot control charts. And the performance metrics are simulated for five different cases of the two parameters of the Pareto distribution, and the change of the performance according to these parameters is desired to be observed. Another purpose of the thesis is to observe and interpret the effects of sample size (m), subgroup size (n), and autocorrelation coefficient (ϕ) changes on performance metrics. After the simulations are performed, the performance metrics are tabulated, interpreted and inferences are made. It has concluded that the "qcc" function in default settings did not perform well enough in detecting the mean of the process as under control for the stationary ARTOP(1) process under the conditions of this thesis.

Key Words: Monte Carlo Simulation, Statistical Process Control, Statistical Quality Control, Pareto Distribution, ARL, R.

ÖZET

Bu tezde, ilgi konusu karakteristiğinin Pareto dağıldığı otokorelatif bir süreç ele alınmaktadır. Süreç ortalamasının kontrol altında olup olmadığını test etmek için ise, $\bar{X}$ şemaları kullanılmaktadır. R programlama dilinde gerçekleştirilen Monte Carlo simülasyonları vasıtasıyla, ilk olarak örneklemler yaratılmakta, ikinci olarak bu örneklemlerin $\bar{X}$ şemaları çizdirilmekte, ve son olarak bu $\bar{X}$ şemalarının performans metrikleri tahminlenmektedir. Burada, süreçteki bağımlılığı ele almak adına, ARTOP(1) modelinden faydalanılmaktadır. Ve ARL_0 ve ARL_1, performans metrikleri olarak kullanılmaktadır. R'da yazılan kod sayesinde; Pareto dağılımlı otokorelatif süreçlerde, kontrol şemalarını çizmek için kullanılan "qcc" fonksiyonunun varsayılan ayarları ile çizilen $\bar{X}$ kontrol şemalarının performanslarının gözlemlenmesi amaçlanmaktadır. Pareto dağılımının iki parametresinin beş farklı durumu için performans metrikleri simüle edilerek, performansın bu parametrelere göre değişimi gözlenmek istenmektedir. Tezin bir diğer amacı da örneklem büyüklüğü (m), alt grup büyüklüğü (n) ve otokorelasyon katsayısı (ϕ) değişimlerinin performans metrikleri üzerindeki etkilerini gözlemlemek ve yorumlamaktır. Simülasyonlar gerçekleştirildikten sonra performans metrikleri tablolaştırılıp, yorumlanarak çıkarımlar yapılmaktadır. Tez koşulları altında durağan ARTOP(1) süreci için varsayılan ayarlardaki "qcc" fonksiyonunun, kontrol altındaki sürecin ortalamasını tespit etmede yeterince başarılı olmadığı sonucuna varılmıştır.

Anahtar Kelimeler: Monte Carlo Simülasyonu, İstatistiksel Süreç Kontrolü, İstatistiksel Kalite Kontrol, Pareto Dağılımı, ARL, R.

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LIST OF ABBREVIATIONS AND ACRONYMS

Abbreviations and Acronyms		**Explanations**
α	:	Type-1 Error
β	:	Type-2 Error
AR	:	Autoregressive
ARL	:	Average Run Length
ARTA	:	Autoregressive-to-Anything
ARTOP	:	Autoregressive-to-Pareto
CDF	:	Cumulative Distribution Function
CL	:	Center Line
EWMA	:	Exponentially Weighted Moving Average
FSI	:	Fixed Sampling Intervals
LAD	:	Least Absolute Deviations
LCL	:	Lower Control Limit
OLS	:	Ordinary Least Squares
PDF	:	Probability Density Function
UCL	:	Upper Control Limit
UWAVE-R	:	Unweighted Average of Subgroup Estimates Based On Subgroup Ranges
VSI	:	Variable Sampling Intervals

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1. INTRODUCTION

It can be said that the applicability of statistical process control has increased relatively with the ever-expanding consumption areas of humanity and the transition to Industry 4.0. These on-line activities, which can be carried out mostly in a computer environment in order to save time and resources, enable the necessary actions to be taken in a timely manner by making inferences about whether the characteristic of the process is under control or not. In this respect, statistical control charts are quite helpful tools that describe the situations of the processes in the statistical process control. One of them, $\bar{X}$ charts, is one of the tools that provide information about whether the mean for the characteristic of interest of the processes are under control, and are mainly used in normally distributed processes. In this context, the fundamental hypotheses and essential working logic of the $\bar{X}$ charts are given in section 1.1.

1.1. Essential Working Logic and the Fundamental Hypotheses of the $\bar{X}$ Charts

A typical $\bar{X}$ chart has composed of three key components. These components are the lower control limit (LCL), the upper control limit (UCL), and the center line (CL), as can be seen in Figure 1.1.

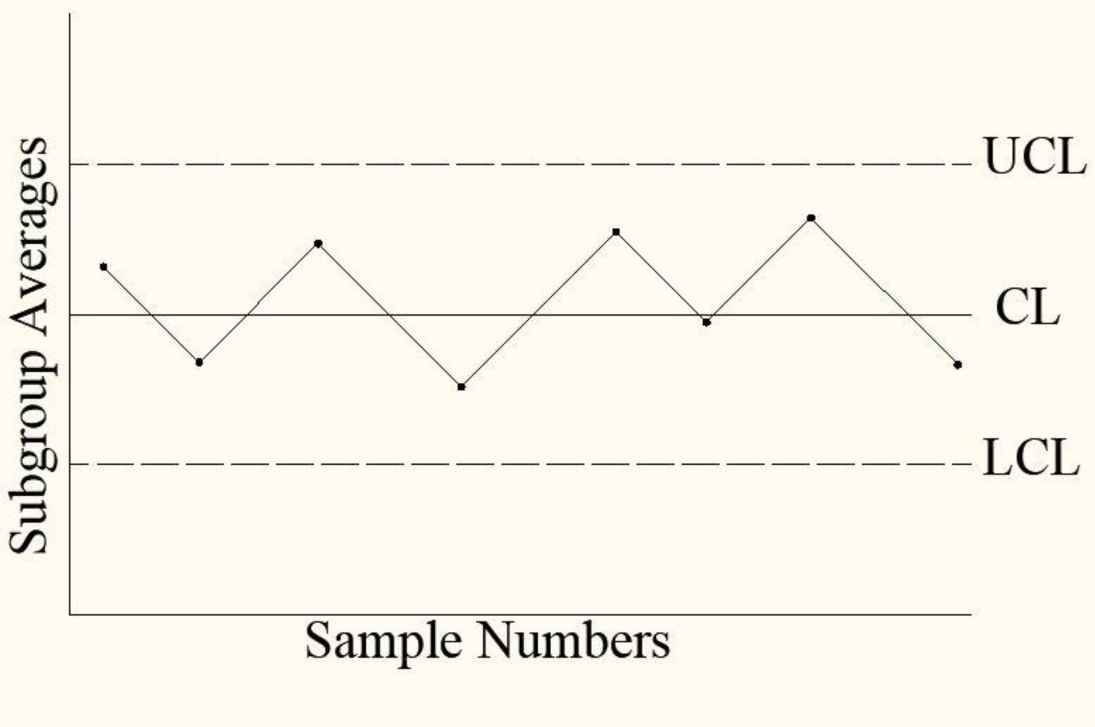

Figure 1.1: A typical $\bar{X}$ chart.

Suppose that m samples are obtained from the process, each consisting of n subgroups. Here, let the sample size be represented by m, and the size of the subgroups be represented by n. Each point in Figure 1.1 is the arithmetic mean of n samples in the corresponding m. Then, the means of these points gives the center line. While calculating UCL and LCL values, the method is usually used; First, the ranges of n values in each m are calculated and the average range is reached by taking the average of these values. Then, the UCL and LCL values are determined by means of this value. (Since this calculation is calculated on the computer by means of a function in the R programming language in this thesis, it is no need to elaborate. Similarly, there are also different methods used in the calculation of UCL and LCL in the literature. For instance; equations (1.1) and (1.2) are used when the population is known to distribute by the Normal distribution with population mean (μ) and standard error (σ). For more detailed information, may see Chapter 6 at Montgomery [2009])

$$UCL = \mu + 3\left(\frac{\sigma}{\sqrt{n}}\right) \tag{1.1}$$

$$LCL = \mu - 3\left(\frac{\sigma}{\sqrt{n}}\right) \tag{1.2}$$

Let H_o and H_1 hypotheses be defined as follows:

$$H_o: \textit{The event that the variable of interest is at an acceptable value} \quad (1.3)$$
$$(\textit{meaning the process is under control})$$

$$H_1: \textit{The event that the variable of interest is at an unacceptable value} \quad (1.4)$$
$$(\textit{meaning the process is out} - \textit{of} - \textit{control})$$

The Type-1 (α) error is an error type that occurs when the H_o hypothesis is rejected, assuming that the sample of interest is at an unacceptable value because it does not fall between LCL and UCL while is at an acceptable value actually. On the other hand, the Type-2 (β) error is an error type that occurs when the sample of interest is at an unacceptable value actually (meaning the H_1 hypothesis is true), despite falling between LCL and UCL. In other words, Type-2 error occurs when the H_0 hypothesis can not be rejected while is actually false. Additionally, $(1 - \beta)$ is called "Power of the Test".

1.2. Purpose, Scope, and Target Contribution of the Thesis

In some cases, the characteristics of interest of the processes that are needed to determine whether the mean is under control or not, are sometimes non-normally distributed. Moreover, these processes may have autocorrelation. Actually, it is quite likely to encounter autocorrelated processes in real life [Yang and Yang, 2005]. For instance, Yang and Yang [2005] emphasized that observations in the chemical and pharmaceutical industry processes are always autocorrelated. Then, they note that autocorrelation existence has a significant effect on the control chart performance. In this context, one of the goals of this thesis study is to handle an autocorrelated process and to observe the significant effect of autocorrelation on control chart performance as stated by Yang and Yang [2005]. Additionally, the characteristics of interest of the processes may also have a heavy-tailed structure. And, these autocorrelated processes are sometimes distributed as a heavy-tailed process. In this context, Thaga [2008] handled autocorrelated processes with a heavy-tailed t-distribution. Similarly, autocorrelated processes with a heavy-tailed distribution are handled in this thesis study. Intrinsically, Shewhart's control charts work very well in the analysis of normally distributed processes. Nonetheless, they are even usable for non-normally distributed processes even though it includes some issues. Hence, a great number of studies have grown up to tackle these issues, as mentioned in the next section too. In this context, since an autocorrelated process with the Pareto distribution has not been handled in control charts before, the Pareto distribution has been selected to be handled in the thesis. And another goal of the thesis study is, to observe the performances of the $\bar{X}$ control charts of Pareto distributed autocorrelated processes which are plotted by the default settings of the "qcc" function of the R programming language [Scrucca, 2004]. Note that, with the default settings of the "qcc" function, the "UWAVE-R" (Unweighted Average of subgroup estimates based on subgroup ranges) method is inherently used in the calculation of UCL and LCL. In order to investigate the methods that can be used to calculate the performances of these $\bar{X}$ control charts, the literature review is done in the next section. Then, it has been seen that ARL (Average Run Length) is usually used to evaluate the performance of control charts. For the same purpose, ARL has been also used in this thesis work. In addition, it has been observed that Monte Carlo Simulation is used to estimate performance indicators of charts, particularly in works regarding autocorrelated data. Likewise, Monte Carlo Simulation has been used also in this thesis work.

2. LITERATURE REVIEW

Firstly, Shewhart [1926] came up with his own quality control chart theory. The following discussions and recommendations about Shewhart's control chart theory and applications have taken place for decades in the statistics literature, particularly concerning quality control. In line with the thesis subject, the important works concerning autocorrelation, heavy-tailed distributions, and ARL concepts, in particular $\bar{X}$ charts, has given in this review to provide adequate insights into previous works.

Roberts [1958] proposed a method that estimates the two-sided $\bar{X}$ chart's ARL value by means of each of the upper and lower regions' estimated ARL values (under own their decision rules) to attain a reasonable estimation. Then, Champ and Woodall [1987] used the Markov Chain approach to acquire an accurate estimation of ARL value.

When it comes to the autocorrelation issue, obviously there are two types of fundamental approaches used in the literature for dealing with the autocorrelation problem;

In the first one, time series models are fitted to the data, and then the standard control charts are implemented to the residuals. On the other hand, in the second one, adjusted control limits regarding autocorrelation structure are implemented on the standard control charts [Prajapati and Singh, 2012]. For example; Montgomery and Mastrangelo [1991], Cheng and Thaga [2005], Lu and Reynolds [1999] used the latter method [Thaga, 2008]. Likewise, Alwan and Roberts [1988] noted that autocorrelations and other systematic time series effects are mostly non-negligible. And, they propounded that modifying the control chart model is needed to handle univariate autocorrelated processes. Also, they put forward and showed the approach of using time series to fit the acquired sample data for modifying Shewhart's control chart by the residuals of these fits. (By the way, do not forget that in this thesis work, the data has not been acquired by any kind of real life process. On the contrary, similarly to the latter approach, the sample data has been generated under some kind of autocorrelation circumstances. In essence, the control limits of the standard control charts have been determined through the default settings of the "qcc" package in the R programming language. In other words, the first method for the autocorrelation problem has not been handled in this thesis work. The methodology and details of this thesis work are described in the next sections thoroughly.)

Moreover, do not forget that using control charts for autocorrelated data should only be preferred as the last resort, particularly in the case of intrinsically autocorrelated processes (such as chemical processes) existence issue [Jensen et al., 2006]. Because processes in real life may have an unnatural autocorrelation that can not be associated with their intrinsic characteristics. And this indicates an out-of-control process generally. In this case, it is key to investigate the root cause triggering the out-of-control situation first in an attempt to comprehend the nature of the process, so that can make use of appropriate statistical process control methods and implement them properly. On the other hand; in this thesis, it is evident that generated sample data has autocorrelation because it is formed by the nature of generation structure in the R programming language.

Subsequently, Alwan [1992] studied the capability levels of control charts for individual observations with fixed control limits when autocorrelation exists, so that to describe extreme points.

Afterwards, Charnes [1995] pointed out that to-date papers proposed time-series-based statistical models for constructing control charts that are appropriate for autocorrelated processes. And also, used the Monte Carlo Simulation method with a multivariate autocorrelated process, to scrutinize the effects of misleading presume of serial independence. Similarly, in this thesis work, Monte Carlo Simulation has been used to estimate performance indicators of the charts. It is explained in the next sections of the thesis.

Then, Atienza et al. [1997] handled sample autocorrelation charts (SACC) with Monte Carlo experiments to analyze the ARL properties of SACC.

Also in 1997, Prybutok et al. [1997] compared the FSI (Fixed Sampling Intervals) and VSI (Variable Sampling Intervals) techniques in the sample-acquiring step from the population which has an autocorrelated process. They studied on $\bar{X}$ charts for sample size one (1) and they performed a simulation study to compare FSI and VSI techniques' performance. In the VSI technique, contrary to the FSI technique, the sampling time between samples varies [Prybutok et al., 1997]. Also, they noted that VSI is better than FSI when it comes to autocorrelated processes.

English et al. [2000] used control charts with fixed control limits for residuals to monitor the performance of a process yielding time-dependent data subject to shifts in the mean and the autocorrelation structure. They used ARL value to measure the performance of $\bar{X}$ and EWMA charts.

Kramer and Schmid [2000] scrutinized the behavior of the residual chart and the modified Shewhart chart for acquired process data with unknown parameters which they estimated. They pointed out that; even though both charts did very sensitive to parameter estimation of AR processes, a modified Shewhart chart with estimated parameters should be preferred in the presence of AR(1) processes with positive correlation. They also noted that the production processes they faced, apparently have positive correlations that are appropriate to use with modified Shewhart charts. Parallelly, the AR(1) model which has a positive correlation has been used as the base process of the using process model of the ARTOP (Autoregressive-to-Pareto) in this thesis work.

Loredo et al. [2002] offered a method for monitoring autocorrelated processes based on regression adjustment. They used the Monte Carlo Simulation method to compare the performance of the residual chart to the observation-based control chart with respect to ARL value.

Lada et al. [2007] used the Autoregressive-to-Pareto process, which is used to attain the sample data in this thesis, for experimental performance evaluation of the "WASSP" method in their work presented to providing the opportunity to create an approximate confidence interval in the steady-state mean of a simulation output process.

Thaga [2008] used the standard error of the LAD (Least Absolute Deviations) estimator instead of the OLS (Ordinary Least Squares) estimator in order to estimate process variability more effectively and informatively. And, he offered a chart based on computing the control limits using the process mean and the standard error of the LAD for the case when the process quality characteristic has a heavy-tailed t-distribution. Moreover, he pointed out that even though using residuals to form control charts at high autocorrelation levels works well, residual charts are not very effective at detecting process changes at low autocorrelation levels. On the other hand, the other method using original observations to form control charts with determined control limits is particularly applicable at low autocorrelation levels, is deduced [Thaga, 2008].

Chen and Cheng [2009] tackled the sample data generally with an unknown marginal distribution yet a known covariance structure for the $\bar{X}$ chart. They offered two approaches to determine sample size and control limit factor (number of standard deviations from the center). The main goal was to minimize the out-of-control ARL (ARL_1) in the course of retaining the specified value at the in-control ARL (ARL_0). While the first idea was to neglect the autocorrelation by accepting sample means as independent normally distributed random variables, the second idea was to accept sample means as AR(1) process. The latter idea outweighed when mean shift and correlation were high. Then, they suggested that modify the second idea. And modified second idea outperformed the others, specifically at high correlation.

Prajapati and Singh [2012] offered a comprehensive literature review of control charts for autocorrelated processes.

Alexopoulos et al. [2019] presented their procedure called “Sequest”, which offers advanced point and confidence interval estimators for many processes, including the Autoregressive-to-Pareto process.

Kapase and Ghute [2022] offered a maximum likelihood estimator of the process change point (meaning to be a shift in the process parameters) when $\bar{X}$ chart for autocorrelated observations signals a change in the process mean. And they experimented with their own change point estimator’s performance in the $\bar{X}$ chart for AR(1) process via Monte Carlo Simulations. They found that the results were good with respect to the expected length and coverage probability of the estimator.

The methodology is explained in the following section of the thesis.

3. METHODOLOGY

In this fundamental chapter of the thesis, the design of the Pareto distributed base autoregressive model is described in the first section. Subsequently, the methodology for evaluating the control chart performance is presented in the second section.

3.1. Designing the Pareto-Distributed Base Autoregressive Model

The Pareto-distributed autocorrelated process handled in this section has identically distributed random variables. Because it has been planned to carry out Monte Carlo Simulations by assuming that all process data are acquired in the same manner. However, since there is autocorrelation among the acquired data, it cannot be said that the aforementioned random variables are exactly independent. In order to handle dependency, ARTOP (Autoregressive-to-Pareto) model from ARTA (Autoregressive-to-Anything) has been utilized.

Firstly, a stationary AR(1) model has been used as the base process of the ARTOP(1) model;

Understandably, let $lag = 1$ in the autoregression model. (It has been accepted as 1 throughout the thesis work. And also, note that sample intervals are not been a concern for the thesis work.)

Let the sample size be represented by m, and the size of the subgroups be represented by n. First of all, the initial datum denoted by Z_1 is randomly generated by the Normal distribution with a mean of 0 and variance of 1, as seen from equation (3.1).

$$Z_1 \sim N(0,1) \tag{3.1}$$

Then, in order to simulate the sample acquisition process, $m * n$ data corresponding between 2 to $(m * n) + 1$ in the AR(1) process is generated one by one taking as a basis of the initial datum Z_1. Here, note that Z_1 is the initial random data and, this datum is not going to be used as input for the sample acquisition in the next step. Let the autocorrelation coefficient be ϕ. With regard to equations (3.2) and (3.3), let Z_k in equation (3.4) be the iterative autoregression function.

$$\phi \in (0,1) \tag{3.2}$$

$$\{Z_k: 2 \leq k \leq ((m * n) + 1)\} \tag{3.3}$$

$$Z_k = \phi Z_{k-1} + \varepsilon_k \tag{3.4}$$

In equation (3.4), let ε_k represents a white noise process for the base stationary AR(1) process with regard to equations (3.5) and (3.6). Note that ε_k's are independent and identically distributed (iid) normal random variables with a mean of 0 and a variance of σ_ε^2, which is determined according to the autocorrelation coefficient (ϕ), as can be seen from equation (3.6) [Alexopoulos et al., 2019].

$$\{\varepsilon_k: 2 \leq k \leq ((m * n) + 1)\} \overset{iid}{\sim} N(0, \sigma_\varepsilon^2) \tag{3.5}$$

$$\sigma_\varepsilon^2 = 1 - \phi^2 \tag{3.6}$$

Thereby, the base process data corresponding to equation (3.3) is obtained with the AR(1) model. Consequently, the ARTOP(1) model constructed as follows;

The Pareto distribution has two parameters: Let the shape and location parameters denoted by θ and γ, respectively. The probability density function (PDF) of the Pareto distribution is given in equation (3.7).

$$f(x) = \begin{cases} \dfrac{\theta\gamma^\theta}{x^{\theta+1}} & for\ x \geq \gamma \\ 0 & for\ x < \gamma \end{cases} \tag{3.7}$$

Then, the cumulative distribution function (CDF) of the Pareto distribution is yielded by equation (3.7), as seen from equation (3.8).

$$F(x) = \begin{cases} 1 - \left(\dfrac{\gamma}{x}\right)^\theta & for\ x \geq \gamma \\ 0 & for\ x < \gamma \end{cases} \tag{3.8}$$

Let S_k be a sequence yielded by the CDF values of the Standard Normal distribution at Z_k values, which are obtained from the base AR(1) process, with regard to equation (3.9). (Do not confuse (capital) Φ with ϕ. While the second one represents the autoregression coefficient, the first one represents the CDF of the Standard Normal distribution.)

$$\{S_k = \Phi[Z_k]: 2 \leq k \leq ((m * n) + 1)\} \tag{3.9}$$

Here, it is obvious that the elements of the S_k are distributed in the continuous interval of $(0,1)$. Then, let X_k be a sequence of the acquired data yielded by the inverse of the CDF values of the Pareto distribution at S_k values with regard to equations (3.10) and (3.11) [Lada et al., 2007].

$$\{X_k: 2 \leq k \leq ((m * n) + 1)\} \tag{3.10}$$

$$\{X_k = F^{-1}(S_k) = F^{-1}(\Phi[Z_k])\} \tag{3.11}$$

Accordingly; the elements of the sequence X_k, described in equation (3.10), are going to be used as the sample data, which are assumed to be acquired in the same manner from a Pareto-Distributed autocorrelated process.

In the next section, the methodology for evaluating the performance of control charts, which are going to be formed with the help of the Pareto-Distributed Base Autoregressive Model, is described.

3.2. The Methodology of Evaluating the Control Chart Performance

In this section, it is explained how the performances of the control charts are evaluated, first. And secondly, all the necessary information about the simulations to be performed in the next section of the thesis for evaluating the control chart performances is shared.

3.2.1. How to Evaluate the Performances of the Control Charts?

As it has been mentioned earlier; in most of the studies in the literature, the performances of the control charts have been evaluated by their ARL values. For that reason, ARL has been used in this thesis to evaluate the control chart performances. There are two types of ARL, which are ARL_0 and ARL_1. ARL_1 used when the process is out-of-control, while ARL_0 used when the process is under control [Montgomery, 2009]. These are calculated by equations (3.14) and (3.15).

$$ARL_0 = \frac{1}{\alpha} \tag{3.14}$$

$$ARL_1 = \frac{1}{1-\beta} \tag{3.15}$$

ARL_0 and ARL_1 values are estimated for five cases which are determined as $(\gamma = 3, \theta = 3)$, $(\gamma = 3, \theta = 4)$, $(\gamma = 3, \theta = 5)$, $(\gamma = 4, \theta = 3)$ and $(\gamma = 5, \theta = 3)$. For each case, ARL_0 and ARL_1 values are estimated with regard to $n = 2, 4, 6, 8$ and $m = 10, 15, 20, 25, 30, 35$ when $\phi = 0.2, 0.4, 0.6$ and 0.8. According to equations (3.14) and (3.15), firstly Type-1 or Type-2 error is estimated in order to calculate ARL_0 or ARL_1 value, respectively. Understandably, both estimating Type-1 error when the process is under control and, estimating Type-2 error when the process is out-of-control are coherent endeavors. Here, there are two different situations required to estimate the aforementioned two types of errors on the Pareto-Distributed Base Autoregressive Process: Stationary state and non-stationary state. Therefore, these two different situations are addressed individually. One of them is a non-stationary process, which is an out-of-control process in which the H_1 hypothesis is true. On the other hand, the other is a stationary process, which is an under control process in which the H_o hypothesis is true. Here, the “qcc” and “EnvStats” packages are used in the “RStudio” program in order to execute the Monte Carlo Simulations to estimate the Type-1 and Type-2 errors [Scrucca, 2004; Millard, 2013]. In this context; $\bar{X}$ charts are plotted while Monte Carlo Simulations of these processes are carried out, enabling to estimate Type-1 and Type-2 errors for evaluation of the charts’ performance. Through the instrument of the Type-1 and Type-2 errors estimated from here, the performance metrics that are going to be used in the performance evaluation of the $\bar{X}$ charts are calculated. All the necessary information about the simulation and calculations are given in section 3.2.2.

3.2.2. The Structure of the Simulation

"qcc" is a package that helps for plotting and analysis of the control charts while "EnvStats" is helpful when it comes to statistics. Both of them are used here. First, the required parameters and vectors for Monte Carlo Simulations are determined as shown in Figure 3.1. While n and m are shown as the same named vectors; ϕ is shown as "fi1", γ is shown as "gama", and θ is shown as "teta" vectors. Here; with the help of a vector called "errtype", which is initially created in the R code and is planned to retain the value of "1" or "2", the performance of the control charts created concerning the situation of the process, is estimated utilizing the Monte Carlo Simulations. (Understandably, Type-1 error is represented by "1" and Type-2 error is represented by "2" here.)

On the simulations, by generating a certain number of Pareto-Distributed Base Autoregressive Model one by one, the number of samples that are not between LCL and UCL on the $\bar{X}$ charts are stored two different vectors as can be seen in Figure 3.2. (Note that the aforementioned certain number is 100,000, which is enough to identify the loop as a Monte Carlo Simulation, represented by a vector called "numrep1".) These two types of vectors called "alpha" and "beta" are used with respect to error type in the simulation of the stationary process and the non-stationary process, respectively. These vectors retain the numbers of samples which are outside of LCL and UCL for each run. Thus, these vectors make it possible to estimate Type-1 and Type-2 errors in the thesis work.

```
errtype<-1          #Which error type is going to be estimated? (Is changed manually)
numrep<-6           #The variable that allows m to be assigned as 10,15,20,25 and 30 thanks to the outmost "for" loop, respectively.
numrep1<-(10^5)     #How many charts are going to plot for each m value?
alpha<-rep(0,numrep1)
beta<-rep(0,numrep1)
allm<-rep(0,numrep)
err1<-rep(0,numrep)
ARL0<-rep(0,numrep)
err2<-rep(0,numrep)
poweroftest<-rep(0,numrep)
ARL1<-rep(0,numrep)
m<- 10              #Sample size. (First value of m)
n<- 2               #Subgroups. (n is going to run for 2,4,6 and 8) (Is changed manually)
fi1=0.2             #Autocorrelation coefficient(ϕ) for lag=1. (Is going to run for 0.2, 0.4, 0.6 and 0.8) (Is changed manually)
gama<-3             #Gama(γ) parameter for ARTOP(1) process.(Is changed manually)
teta<-3             #Teta(θ) parameter for ARTOP(1) process.(Is changed manually)
```

Figure 3.1: Initial Parameters.

Cardinally, every 100,000 simulations are run for every combination of n, m, ϕ, and “errtype” by their given values in section 3.2.1. And these simulations are executed for all γ and θ combinations given in mentioned section.

The goals of the first for loop are to increase m by five by fives in order to execute the simulation for all m’s and, calculate the ARL_0 or ARL_1 values after the simulation. Consequently; n, “fi1”, “gama”, “teta”, and “errtype” vectors are set manually to execute the simulations for all combinations. The goal of the second for loop is to execute the Monte Carlo Simulations and to retain “alpha” or “beta” values with respect to “errtype”. Accordingly, the base process of the ARTOP(1) model is attained first. In this respect, Z_1 is randomly generated regarding equation (3.1). Then, the third for loop enabling to attain base process is taken its place. In this context, the iterative autoregression function is constructed with regard to section 3.1. Here, the standard deviation of ε_k is calculated from the square root of the equation (3.6).

```
for (i in 1:numrep)        #Increasing m by five by five.
{
  for (j in 1:numrep1)      #Determining the "alpha" or "beta" via "numrep1" numbers of Monte Carlo Simulations.
  {
    x<- rep(0,((m*n)+1))    #Initially, all x's are zero.
    x[1]<- rnorm(1)         #Generating first value (Z1).

    for (k in 2:((m*n)+1)) #To attain base process with the help of ARTOP(1) model.
    {
      x[k]<- fi1*x[k-1]  + (rnorm(1, sd=sqrt(1-fi1^2))) #creating value k.

      if(errtype==2){       #Creating an impulse interval for m=5, 6, and 7 to attain a non-stationary process for Type-2 err. estimation.
        if(k==((4*n)+2)){
          x[k]<-x[k]+2.6
        }
        if(k==((7*n)+2)){
          x[k]<-x[k]-2.6
        }
      }
    }
    x<-x[-1]                            #Removing the x[1].
    X<- qpareto(pnorm(x), gama, teta) #Converting Base Process into Pareto Process.
    X[!is.finite(X)] <- median(X[is.finite(X)]) #The infinite values are accepted as the median of the other values, to get minimal effect on the mean of the Pareto Process.
    matx<-matrix(X, ncol=n, byrow=T)            #m*n number of values are grouped by "n".
    q<- qcc(matx,type="xbar", plot=F)           #Xbar charts are plotted.
    if(errtype==2){
      beta[j]<-length(beyond.limits(q))
    }else{
      alpha[j]<-length(beyond.limits(q))
    }
  }
  if(errtype==2)
  {
    err2[i]<- 1-((sum(beta)/(numrep1))/m) #Estimating the Type-2 error for each m.
    poweroftest[i]<- 1-err2[i]            #Estimating the power of test for each m.
    ARL1[i]<- 1/poweroftest[i]            #Estimating the ARL1 for each m. (ARL1 is used for out-of-control charts and is desirable to minimize.)
  }
  else
  {
    err1[i]<-((sum(alpha)/(numrep1))/m)   #Estimating the Type-1 error for each m.
    ARL0[i]<- 1/err1[i]                   #Estimating the ARL0 for each m. (ARL0 is used for under control charts and is desirable to maximize.)
  }
  allm[i]<-m
  m<- m+5
}
```

Figure 3.2: Execution of the Monte Carlo Simulations.

Since Z_1 and ε_k follow Normal distribution, "rnorm" function which generates normal random variables is used in both. And, the if loop in this for loop is used to create an impulse interval encompassing the fifth, sixth, and seventh values of m. It is accepted that the created impulse transforms the stationary model into a non-stationary model when "errtype" is equal "2". (Obviously, when the "errtype" is equal "1", the process stays stationary.) And, simulations and performance estimations are carried out in light of this assumption. Moreover, the value "2.6"; has been determined by trial and error to both enable the process goes out-of-control, and largely prevent the formation of infinite values directly in the "X" vector. (The "X" vector in R corresponds to X_k in section 3.1.) Because, infinite values has given rise to error in R. Actually, it has been observed that the formation of infinite values in "X" vector occurs much more frequently at larger values.

After the end of the loop, the initial datum generated by Standard Normal distribution is removed. Thus, with regard to equation (3.3), Z_k is represented by "x" hereupon. (Do not confuse "x" with "X".) Subsequently, with drawing on "EnvStats" package in R, "qpareto" function is used to convert the base process into Pareto process. Here; "pnorm" function is convert Z_k into S_k, which represents CDF values of the Standard Normal distribution at Z_k values, firstly. Then, "X" be the inverse of the CDF values of the Pareto distribution at S_k values, secondly. And then, the infinite values in "X" vector are accepted as the median of the other finite values, to get minimal effect on the mean of the Pareto Process. And finally; the values in the vector "X" are grouped in such a way that there are n of them for each m, respectively. Formed matrix is represented by "matx". Here; with the help of "byrow=T", the subgroup values for each m are determined from sequential values of "X". Thus; While $\bar{X}$ charts are plotted in R, they are handled in accordance with the nature of the subgroup data taken from real life autocorrelated processes. Then, with the help of the "qcc" function, $\bar{X}$ charts are plotted. Thanks to "plot=F", it is hindered to plot them visually in order to save time.

Thanks to the subsequent if loop, the aforementioned values are retained by "beta" or "alpha" vectors. After the second for loop is finished (i.e. after the Monte Carlo Simulation ended), performance metrics with respect to the value of the "errtype" vector are estimated, thanks to the if loop. If the "errtype" is equal to "2"; Type-2 error ("err2"), power of the test $(1-\beta)$ and, ARL_1 is calculated respectively.

Similarly if the “errtype” is not equal to “2” (meaning is equal to “1”); Type-1 error (“err1”) and ARL_0 is calculated respectively. In the Type-1 error calculation, the probability of a sample being an out-of-control value is estimated for the one m that represents the n subgroups. Similarly; in the Type-2 error calculation, the probability of a sample being an under control value is estimated for the one m that represents the n subgroups. Note that these calculations are estimations yielded by the Monte Carlo Simulations. And do not forget that before calculating the error rates, the Pareto-Distributed Base Autoregressive Model is designed as a stationary or non-stationary process by the error type which is planned to be estimated. If the goal is to design a non-stationary model allowing for an out-of-control process, the impulse mentioned above that provides to violate the stationarity is created. Obviously, estimations of the ARL_1 and ARL_0 are calculated through “err1” or “err2” with regard to equations (3.14) and (3.15). If the “errtype” is equal to “2”; power of the test (“poweroftest”) is calculated before the calculate of the ARL_1.

Then, the command which enables to increase of m by five by fives for executing the simulations is taken its place. And finally, it is enabled to show the ARL_1 or ARL_0 values yielded from simulations with the help of “cat” and “write” functions.

As explained, simulations are run through these codes and the tables in the next section are reached. Also in the next section, the obtained results are interpreted.

4. SIMULATION OUTPUTS

All the simulations described in section 3.2.2 were executed. Below are the tables (Tables 4.1, 4.2, 4.3, 4.4, and 4.5.) created to allow the comparison of the relevant data. In the next section of the thesis, the results are discussed and the effects of the changing parameters m, n, ϕ, θ, and γ on the performance of the "qcc" function under the conditions in the thesis are examined.

Table 4.1: ARL_0 and ARL_1 values for $\gamma = 3\ and\ \theta = 3$.

For $\gamma = 3\ and\ \theta = 3$									
$n = 2$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	20.44	14.88	10.32	6.16	**10**	12.21	9.69	6.8	3.45
15	19.98	14.99	10.69	5.96	**15**	15.25	12.38	8.79	4.06
20	20	15.2	10.92	5.98	**20**	17.28	14.13	10.16	4.66
25	20.04	15.27	11.16	6.1	**25**	18.63	15.19	11.2	5.13
30	20.07	15.36	11.31	6.2	**30**	19.49	15.9	11.8	5.51
35	20.02	15.42	11.43	6.29	**35**	20.14	16.38	12.28	5.81
$n = 4$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	21.81	14.96	10.04	4.84	**10**	13.71	11.15	8.54	3.91
15	20.4	14.58	10.08	4.73	**15**	16.01	13.03	10.01	4.46
20	20.05	14.43	10.27	4.72	**20**	17.42	13.94	10.77	4.81
25	19.86	14.47	10.34	4.75	**25**	18.39	14.5	11.15	5.01
30	19.78	14.4	10.41	4.76	**30**	18.92	14.85	11.38	5.15
35	19.78	14.44	10.47	4.81	**35**	19.39	15.87	11.52	5.24

Table 4.1 continued: ARL_0 and ARL_1 values for $\gamma = 3\ and\ \theta = 3$.

$\boldsymbol{n = 6}$									
$\boldsymbol{\phi}$									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	23.63	15.46	9.64	4.12	**10**	15.12	11.97	8.75	3.84
15	21.93	14.93	9.78	4.02	**15**	17.25	13.46	9.87	4.18
20	21.36	14.82	9.91	3.99	**20**	18.52	14.22	10.47	4.36
25	21.17	14.7	10.02	3.99	**25**	19.36	14.69	10.74	4.41
30	21.05	14.69	10.11	4	**30**	19.93	14.97	10.92	4.44
35	21.03	14.74	10.16	3.99	**35**	20.36	15.15	11.01	4.48
$\boldsymbol{n = 8}$									
$\boldsymbol{\phi}$									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	25.75	16.22	9.56	3.76	**10**	16.69	12.8	8.94	3.79
15	23.83	15.61	9.64	3.71	**15**	18.65	14.12	9.9	4.03
20	23.21	15.5	9.8	3.67	**20**	19.81	14.8	10.38	4.09
25	22.92	15.47	9.91	3.67	**25**	20.72	15.25	10.6	4.11
30	22.7	15.36	9.98	3.66	**30**	21.25	15.5	10.76	4.12
35	22.59	15.33	10.07	3.66	**35**	21.65	15.62	10.82	4.13

Table 4.2: ARL_0 and ARL_1 values for $\gamma = 4\ and\ \theta = 3$.

For $\gamma = 4\ and\ \theta = 3$

$n = 2$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	20.42	14.85	10.38	6.16	**10**	12.22	9.71	6.79	3.43
15	20.03	15.01	10.69	5.95	**15**	15.24	12.45	8.78	4.06
20	19.96	15.19	10.93	6.01	**20**	17.23	14.12	10.2	4.64
25	19.95	15.3	11.16	6.06	**25**	18.56	15.17	11.2	5.13
30	19.99	15.34	11.31	6.2	**30**	19.49	15.86	11.83	5.52
35	20.05	15.42	11.44	6.3	**35**	20.12	16.31	12.28	5.83
$n = 4$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	21.62	15.01	9.99	4.85	**10**	13.69	11.21	8.52	3.92
15	20.55	14.58	10.18	4.75	**15**	15.98	12.99	10.04	4.47
20	20.05	14.45	10.22	4.72	**20**	17.44	13.93	10.77	4.79
25	19.92	14.47	10.32	4.75	**25**	18.31	14.52	11.17	5.01
30	19.78	14.52	10.38	4.77	**30**	18.99	14.86	11.38	5.16
35	19.76	14.45	10.44	4.81	**35**	19.42	15.07	11.52	5.24

Table 4.2 continued: ARL_0 and ARL_1 values for $\gamma = 4\ and\ \theta = 3$.

$n = 6$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	23.5	15.44	9.67	4.09	**10**	15.1	11.99	8.77	3.84
15	21.99	15.01	9.79	4.01	**15**	17.24	13.48	9.91	4.19
20	21.36	14.85	9.93	4.01	**20**	18.54	14.2	10.47	4.35
25	21.15	14.79	10.02	3.99	**25**	19.39	14.67	10.77	4.41
30	21.05	14.75	10.11	3.99	**30**	19.9	14.97	10.91	4.44
35	21.03	14.7	10.16	4	**35**	20.31	15.13	11.02	4.47
$n = 8$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	25.65	16.24	9.57	3.76	**10**	16.67	12.79	8.94	3.79
15	23.74	15.61	9.69	3.69	**15**	18.65	14.12	9.9	4.03
20	23.13	15.47	9.81	3.68	**20**	19.97	14.85	10.39	4.1
25	22.86	15.42	9.91	3.67	**25**	20.63	15.29	10.63	4.13
30	22.71	15.38	10.01	3.67	**30**	21.26	15.5	10.73	4.12
35	22.59	15.37	10.06	3.67	**35**	21.65	15.63	10.84	4.12

Table 4.3: ARL_0 and ARL_1 values for $\gamma = 5\ and\ \theta = 3$.

For $\gamma = 5\ and\ \theta = 3$									
$n = 2$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	20.53	14.88	10.33	6.15	**10**	12.2	9.74	6.81	3.43
15	20.09	15.01	10.69	5.96	**15**	15.23	12.36	8.8	4.08
20	20	15.18	10.95	5.98	**20**	17.32	14.11	10.2	4.65
25	19.93	15.28	11.15	6.08	**25**	18.53	15.16	11.17	5.12
30	19.99	15.33	11.33	6.2	**30**	19.53	15.87	11.87	5.49
35	20.04	15.44	11.46	6.28	**35**	20.17	16.34	12.28	5.84
$n = 4$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	21.68	15.02	10.03	4.84	**10**	13.69	11.2	8.54	3.91
15	20.44	14.55	10.12	4.73	**15**	15.97	12.95	10.05	4.5
20	20.03	14.44	10.26	4.74	**20**	17.36	13.96	10.78	4.8
25	19.79	14.47	10.32	4.75	**25**	18.35	14.49	11.19	5
30	19.78	14.42	10.38	4.77	**30**	18.96	14.88	11.39	5.14
35	19.78	14.47	10.45	4.82	**35**	19.34	15.05	11.51	5.25

Table 4.3 continued: ARL_0 and ARL_1 values for $\gamma = 5\ and\ \theta = 3$.

$\boldsymbol{n = 6}$									
$\boldsymbol{\phi}$									
$\boldsymbol{ARL_0}$ **by** $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**	$\boldsymbol{ARL_1}$ **by** $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**
10	23.39	15.43	9.68	4.1	**10**	15.1	11.93	8.76	3.85
15	21.87	15.02	9.81	4.02	**15**	17.16	13.46	9.9	4.18
20	21.38	14.79	9.94	4	**20**	18.52	14.24	10.49	4.35
25	21.08	14.76	10.03	3.99	**25**	19.34	14.72	10.78	4.4
30	21.06	14.75	10.12	4	**30**	19.9	14.94	10.91	4.46
35	20.97	14.75	10.15	3.99	**35**	20.25	15.21	11.01	4.47
$\boldsymbol{n = 8}$									
$\boldsymbol{\phi}$									
$\boldsymbol{ARL_0}$ **by** $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**	$\boldsymbol{ARL_1}$ **by** $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**
10	25.55	16.16	9.53	3.76	**10**	16.65	12.8	8.91	3.8
15	23.75	15.65	9.7	3.7	**15**	18.68	14.13	9.89	4.02
20	23.11	15.45	9.83	3.66	**20**	19.82	14.8	10.35	4.1
25	22.77	15.41	9.92	3.67	**25**	20.76	15.26	10.61	4.11
30	22.67	15.35	10	3.67	**30**	21.26	15.47	10.74	4.13
35	22.57	15.34	10.07	3.66	**35**	21.67	15.69	10.85	4.12

Table 4.4: ARL_0 and ARL_1 values for $\gamma = 3$ and $\theta = 4$.

For $\gamma = 3$ and $\theta = 4$									
$n = 2$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	22.26	15.31	10.31	5.95	**10**	12.72	9.76	6.6	3.09
15	21.57	15.48	10.54	5.62	**15**	15.73	12.53	8.48	3.64
20	21.31	15.58	10.8	5.6	**20**	17.72	14.12	9.81	4.14
25	21.18	15.59	10.95	5.59	**25**	19.01	15.09	10.68	4.54
30	21.16	15.54	11.1	5.65	**30**	19.84	15.69	11.29	4.83
35	21.22	15.58	11.2	5.69	**35**	20.47	16.02	11.67	5.09
$n = 4$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	24.33	15.84	9.93	4.49	**10**	14.83	11.56	8.27	3.41
15	22.7	15.21	9.97	4.29	**15**	17.04	13.22	9.62	3.79
20	22.09	15.07	10.06	4.23	**20**	18.41	14.08	10.29	4
25	21.76	14.98	10.13	4.19	**25**	19.4	14.54	10.63	4.12
30	21.68	14.86	10.17	4.18	**30**	19.92	14.86	10.81	4.21
35	21.6	14.89	10.24	4.16	**35**	20.33	15.05	10.88	4.23

Table 4.4 continued: ARL_0 and ARL_1 values for $\gamma = 3$ and $\theta = 4$.

$n = 6$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	27.12	16.3	9.47	3.8	**10**	16.87	12.46	8.45	3.35
15	24.76	15.71	9.53	3.66	**15**	18.78	13.81	9.46	3.57
20	24.06	15.43	9.63	3.59	**20**	20.06	14.6	9.93	3.66
25	23.7	15.37	9.72	3.56	**25**	20.97	14.93	10.16	3.69
30	23.4	15.31	9.76	3.54	**30**	21.46	15.12	10.28	3.69
35	23.31	15.23	9.82	3.53	**35**	21.79	15.25	10.37	3.7

$n = 8$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	29.47	17.23	9.25	3.49	**10**	18.84	13.46	8.62	3.32
15	27.28	16.44	9.26	3.38	**15**	20.83	14.62	9.38	3.46
20	26.47	16.22	9.34	3.33	**20**	21.94	15.3	9.76	3.49
25	25.85	16.07	9.43	3.3	**25**	22.71	15.6	9.95	3.48
30	25.56	15.98	9.48	3.29	**30**	23.2	15.75	10.03	3.47
35	25.29	15.96	9.51	3.28	**35**	23.61	15.86	10.09	3.46

Table 4.5: ARL_0 and ARL_1 values for $\gamma = 3 \; and \; \theta = 5$.

For $\gamma = 3 \; and \; \theta = 5$									
$n = 2$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	23.11	15.74	10.24	5.77	**10**	13.12	9.85	6.42	2.9
15	22.57	15.86	10.45	5.41	**15**	16.11	12.43	8.27	3.38
20	22.4	15.76	10.64	5.34	**20**	17.99	14.06	9.57	3.84
25	22.08	15.79	10.83	5.32	**25**	19.35	14.98	10.38	4.19
30	22.05	15.75	10.94	5.34	**30**	20.19	15.48	10.91	4.43
35	22.09	15.76	11.07	5.35	**35**	20.73	15.82	11.22	4.65
$n = 4$									
ϕ									
ARL_0 by m	**0.2**	**0.4**	**0.6**	**0.8**	**ARL_1 by m**	**0.2**	**0.4**	**0.6**	**0.8**
10	26.49	16.39	9.8	4.3	**10**	15.64	11.75	8.08	3.11
15	24.57	15.68	9.82	4.07	**15**	17.88	13.37	9.32	3.44
20	23.7	15.46	9.91	3.97	**20**	19.33	14.17	9.94	3.61
25	23.24	15.31	9.99	3.92	**25**	20.14	14.62	10.23	3.69
30	23.07	15.25	10.03	3.89	**30**	20.78	14.85	10.41	3.75
35	22.93	15.17	10.08	3.86	**35**	21.14	15.05	10.49	3.78

Table 4.5 continued: ARL_0 and ARL_1 values for $\gamma = 3\ and\ \theta = 5$.

$\boldsymbol{n = 6}$									
$\boldsymbol{\phi}$									
$\boldsymbol{ARL_0}$ by $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**	**$\boldsymbol{ARL_1}$ by $\boldsymbol{m}$**	**0.2**	**0.4**	**0.6**	**0.8**
10	29.5	16.94	9.31	3.65	**10**	18.27	12.84	8.23	3.1
15	27.03	16.26	9.3	3.49	**15**	20.02	14.17	9.07	3.27
20	26.04	15.92	9.37	3.41	**20**	21.34	14.73	9.54	3.33
25	25.53	15.77	9.45	3.37	**25**	22.21	15.07	9.75	3.35
30	25.23	15.7	9.5	3.34	**30**	22.68	15.31	9.85	3.36
35	25	15.64	9.54	3.32	**35**	22.94	15.4	9.93	3.36
$\boldsymbol{n = 8}$									
$\boldsymbol{\phi}$									
$\boldsymbol{ARL_0}$ by $\boldsymbol{m}$	**0.2**	**0.4**	**0.6**	**0.8**	**$\boldsymbol{ARL_1}$ by $\boldsymbol{m}$**	**0.2**	**0.4**	**0.6**	**0.8**
10	32.59	17.69	9.03	3.35	**10**	20.63	14.07	8.37	3.08
15	29.69	16.99	8.97	3.23	**15**	22.45	15.04	9.03	3.18
20	28.69	16.74	9.02	3.18	**20**	23.56	15.59	9.3	3.21
25	28.08	16.54	9.07	3.15	**25**	24.29	15.85	9.44	3.2
30	27.78	16.39	9.11	3.13	**30**	24.78	16.01	9.5	3.19
35	27.41	16.41	9.15	3.11	**35**	25.12	16.12	9.55	3.18

5. RESULTS

Firstly; Considering that the value of ARL_0 will be around 370 on a 3-sigma process in the $\bar{X}$ charts, it is clear that the “qcc” function in default settings did not perform well enough in detecting the mean of the process as under control for the stationary ARTOP(1) process under the conditions of this thesis (See Tables 4.1, 4.2, 4.3, 4.4, and 4.5). In essence, relatively higher Type-1 error rates and therefore relatively lower ARL_0 values were encountered. In a similar vein, high Type-2 error rates and therefore high ARL_1 values were encountered. In other words, high Type-2 error rates mean that it is more difficult to detect that the process mean is out-of-control.

Secondly, as can be explicitly seen at Tables 4.1, 4.2, 4.3, 4.4, and 4.5; the ARL_1 and ARL_0 values are decreased when ϕ is increasing. As known, intrinsically it was expected that there is a tradeoff between ARL_1 and ARL_0 performance metrics. In this context; as ϕ increased, the “qcc” function with the default settings has showed worse performance on Type-1 error rates of the $\bar{X}$ charts for the stationary ARTOP(1) process. In other words, relatively better ARL_0 values were obtained at low autocorrelation rates. On the other hand; as ϕ increased, the “qcc” function with the default settings has shown better performance on Type-2 error rates of the $\bar{X}$ charts for the non-stationary process. (In fact, decrease in the Type-2 error rates has facilitated the detection of the out-of-control process.) In a nutshell; When ϕ is increased, ARL_1 showed better performance while ARL_0 showed worse performance.

Then; In order to describe the occurring changes at ARL_1 and ARL_0 as m increases, correlation analyses between m vs. ARL_1, and m vs. ARL_0 are performed in the “Minitab” program and the results are presented below in Tables 5.1, 5.2, 5.3, 5.4, and 5.5. Bold correlation coefficients in these tables mark pairs with a p-value less than 0.05 in the correlation analysis result, that is, having a significant correlation. (Additionally, the “Minitab” results can be seen in Appendix B including Figures B5.1, B5.2, B5.3,..., and Figure B5.20.)

Table 5.1: Correlation Coefficients between m vs. ARL_0 and m vs. ARL_1 for $\gamma = 3$ and $\theta = 3$.

For $\boldsymbol{\gamma = 3}$ $\boldsymbol{and}$ $\boldsymbol{\theta = 3}$

$\boldsymbol{\phi}$									
Corr. Coeff. between $\boldsymbol{m}$ vs. $\boldsymbol{ARL_0}$	**0.2**	**0.4**	**0.6**	**0.8**	**Corr. Coeff. between $\boldsymbol{m}$ vs. $\boldsymbol{ARL_1}$**	**0.2**	**0.4**	**0.6**	**0.8**
$\boldsymbol{n = 2}$	-0.542	**0.978**	**0.982**	0.608	$\boldsymbol{n = 2}$	**0.961**	**0.954**	**0.967**	**0.991**
$\boldsymbol{n = 4}$	**-0.829**	-0.785	**0.981**	-0.001	$\boldsymbol{n = 4}$	**0.955**	**0.965**	**0.918**	**0.95**
$\boldsymbol{n = 6}$	**-0.842**	-0.81	**0.989**	-0.76	$\boldsymbol{n = 6}$	**0.957**	**0.933**	**0.913**	**0.888**
$\boldsymbol{n = 8}$	**-0.875**	**-0.849**	**0.993**	**-0.859**	$\boldsymbol{n = 8}$	**0.961**	**0.932**	**0.912**	**0.817**

Table 5.2: Correlation Coefficients between m vs. ARL_0 and m vs. ARL_1 for $\gamma = 4$ and $\theta = 3$.

For $\boldsymbol{\gamma = 4}$ $\boldsymbol{and}$ $\boldsymbol{\theta = 3}$

$\boldsymbol{\phi}$									
Corr. Coeff. between $\boldsymbol{m}$ vs. $\boldsymbol{ARL_0}$	**0.2**	**0.4**	**0.6**	**0.8**	**Corr. Coeff. between $\boldsymbol{m}$ vs. $\boldsymbol{ARL_1}$**	**0.2**	**0.4**	**0.6**	**0.8**
$\boldsymbol{n = 2}$	-0.602	**0.975**	**0.989**	0.605	$\boldsymbol{n = 2}$	**0.963**	**0.951**	**0.966**	**0.992**
$\boldsymbol{n = 4}$	**-0.875**	-0.732	**0.976**	-0.046	$\boldsymbol{n = 4}$	**0.957**	**0.935**	**0.915**	**0.954**
$\boldsymbol{n = 6}$	**-0.856**	**-0.882**	**0.989**	-0.762	$\boldsymbol{n = 6}$	**0.953**	**0.933**	**0.914**	**0.885**
$\boldsymbol{n = 8}$	**-0.864**	**-0.815**	**0.994**	-0.796	$\boldsymbol{n = 8}$	**0.957**	**0.928**	**0.911**	0.796

Table 5.3: Correlation Coefficients between m vs. ARL_0 and m vs. ARL_1 for $\gamma = 5\ and\ \theta = 3$.

For $\gamma = 5\ and\ \theta = 3$

ϕ									
Corr. Coeff. between m vs. ARL_0	**0.2**	**0.4**	**0.6**	**0.8**	**Corr. Coeff. between m vs. ARL_1**	**0.2**	**0.4**	**0.6**	**0.8**
$n = 2$	-0.689	**0.986**	**0.985**	0.626	$n = 2$	**0.963**	**0.955**	**0.967**	**0.992**
$n = 4$	**-0.839**	-0.72	**0.989**	0.05	$n = 4$	**0.956**	**0.934**	**0.913**	**0.949**
$n = 6$	**-0.857**	**-0.828**	**0.981**	-0.763	$n = 6$	**0.954**	**0.933**	**0.909**	**0.894**
$n = 8$	**-0.87**	**-0.856**	**0.987**	-0.791	$n = 8$	**0.959**	**0.937**	**0.916**	**0.828**

Table 5.4: Correlation Coefficients between m vs. ARL_0 and m vs. ARL_1 for $\gamma = 3\ and\ \theta = 4$.

For $\gamma = 3\ and\ \theta = 4$

ϕ									
Corr. Coeff. between m vs. ARL_0	**0.2**	**0.4**	**0.6**	**0.8**	**Corr. Coeff. between m vs. ARL_1**	**0.2**	**0.4**	**0.6**	**0.8**
$n = 2$	**-0.828**	0.763	**0.986**	-0.478	$n = 2$	**0.96**	**0.94**	**0.964**	**0.989**
$n = 4$	**-0.87**	**-0.864**	**0.993**	**-0.871**	$n = 4$	**0.955**	**0.929**	**0.906**	**0.929**
$n = 6$	**-0.874**	**-0.886**	**0.99**	**-0.906**	$n = 6$	**0.957**	**0.915**	**0.905**	**0.845**
$n = 8$	**-0.919**	**-0.876**	**0.983**	**-0.918**	$n = 8$	**0.955**	**0.915**	**0.903**	0.598

Table 5.5: Correlation Coefficients between m vs. ARL_0 and m vs. ARL_1 for $\gamma = 3\ and\ \theta = 5$.

For $\gamma = 3\ and\ \theta = 5$									
ϕ									
Corr. Coeff. between m vs. ARL_0	**0.2**	**0.4**	**0.6**	**0.8**	**Corr. Coeff. between m vs. ARL_1**	**0.2**	**0.4**	**0.6**	**0.8**
$n = 2$	**-0.901**	-0.256	**0.993**	-0.723	$n = 2$	**0.959**	**0.938**	**0.957**	**0.987**
$n = 4$	**-0.896**	**-0.89**	**0.989**	**-0.911**	$n = 4$	**0.953**	**0.922**	**0.908**	**0.918**
$n = 6$	**-0.9**	**-0.9**	**0.978**	**-0.926**	$n = 6$	**0.955**	**0.917**	**0.909**	**0.833**
$n = 8$	**-0.9**	**-0.911**	**0.893**	**-0.925**	$n = 8$	**0.956**	**0.924**	**0.892**	0.61

As can be seen clearly in Tables 5.1, 5.2, 5.3, 5.4, and 5.5; all correlation coefficients between m and ARL_1 indicate a significant correlation with a few exceptions. In addition, these correlation coefficients are greater than 0.8. Thus, it can be said that there is a strong positive correlation between m and ARL_1. In other words, as m increases, the ARL_1 value also increases significantly, and therefore, ARL_1 starts to perform relatively worse with increasing m values. In addition, some of the significant correlation coefficients between m and ARL_0 indicate a strong positive correlation, while others indicate a strong negative correlation. From this point of view, it can be concluded that there is no significant correlation between m and ARL_0 since ARL_0 did not change in a certain direction with the increase of only m in general. In brief; As a result of the correlation analyses performed in the "Minitab" program, it has been observed that the "qcc" function in the default settings remains relatively weaker in terms of its power to detect that the mean of the process is out-of-control as m increases.

Then, to be able to compare the performance changes of ARL_1 and ARL_0 metrics by n, by θ, and by γ, Tables 5.6, 5.7, 5.8, 5.9, and 5.10 were created by calculating the mean values of these values for all m's.

Table 5.6: Mean ARL_0 and ARL_1 values by n for $\gamma = 3\ and\ \theta = 3$.

For $\gamma = 3\ and\ \theta = 3$

ϕ									
Mean ARL_0 by n	**0.2**	**0.4**	**0.6**	**0.8**	**Mean ARL_1 by n**	**0.2**	**0.4**	**0.6**	**0.8**
2	20.09	15.19	10.97	6.12	**2**	17.17	13.94	10.17	4.77
4	20.28	14.55	10.27	4.77	**4**	17.31	13.89	10.56	4.76
6	21.7	14.89	9.94	4.02	**6**	18.42	14.08	10.29	4.28
8	23.5	15.58	9.83	3.69	**8**	19.79	14.68	10.23	4.05

Table 5.7: Mean ARL_0 and ARL_1 values by n for $\gamma = 4\ and\ \theta = 3$.

For $\gamma = 4\ and\ \theta = 3$

ϕ									
Mean ARL_0 by n	**0.2**	**0.4**	**0.6**	**0.8**	**Mean ARL_1 by n**	**0.2**	**0.4**	**0.6**	**0.8**
2	20.07	15.18	10.99	6.11	**2**	17.14	13.94	10.18	4.77
4	20.28	14.58	10.26	4.77	**4**	17.31	13.76	10.57	4.76
6	21.68	14.92	9.95	4.01	**6**	18.41	14.08	10.31	4.28
8	23.45	15.58	9.84	3.69	**8**	19.81	14.7	10.24	4.05

Table 5.8: Mean ARL_0 and ARL_1 values by n for $\gamma = 5\ and\ \theta = 3$.

For $\gamma = 5\ and\ \theta = 3$

ϕ									
Mean ARL_0 by n	**0.2**	**0.4**	**0.6**	**0.8**	**Mean ARL_1 by n**	**0.2**	**0.4**	**0.6**	**0.8**
2	20.1	15.19	10.99	6.11	**2**	17.16	13.93	10.19	4.77
4	20.25	14.56	10.26	4.78	**4**	17.28	13.75	10.58	4.77
6	21.62	14.92	9.96	4.02	**6**	18.38	14.08	10.31	4.29
8	23.4	15.56	9.84	3.69	**8**	19.81	14.69	10.22	4.05

Table 5.9: Mean ARL_0 and ARL_1 values by n for $\gamma = 3\ and\ \theta = 4$.

For $\gamma = 3\ and\ \theta = 4$

ϕ									
Mean ARL_0 by n	**0.2**	**0.4**	**0.6**	**0.8**	**Mean ARL_1 by n**	**0.2**	**0.4**	**0.6**	**0.8**
2	21.45	15.52	10.82	5.68	**2**	17.58	13.87	9.75	4.22
4	22.36	15.14	10.08	4.26	**4**	18.32	13.89	10.08	3.96
6	24.39	15.56	9.65	3.61	**6**	19.99	14.36	9.77	3.61
8	26.65	16.32	9.38	3.35	**8**	21.86	15.1	9.64	3.45

Table 5.10: Mean ARL_0 and ARL_1 values by n for $\gamma = 3\ and\ \theta = 5$.

For $\gamma = 3\ and\ \theta = 5$

ϕ									
Mean ARL_0 by n	**0.2**	**0.4**	**0.6**	**0.8**	**Mean ARL_1 by n**	**0.2**	**0.4**	**0.6**	**0.8**
2	22.38	15.78	10.69	5.42	**2**	17.92	13.77	9.46	3.9
4	24	15.54	9.94	4	**4**	19.15	13.97	9.74	3.56
6	26.39	16.04	9.41	3.43	**6**	21.24	14.59	9.39	3.29
8	29.04	16.79	9.06	3.19	**8**	23.47	15.44	9.2	3.17

When looking at the tables, it is understood that there is not a very big change in the values of ARL_0 and ARL_1 with the increase of n, but a few situations draw attention. One of them is that ARL_0 and ARL_1 have increased while ϕ is 0,2 in all tables, that is, the performance of ARL_0 has increased relatively, while the performance of ARL_1 has decreased relatively. Another is that ARL_1 and ARL_0 decrease when the ϕ is 0,8, that is, the performance of ARL_0 decreases relatively, while the performance of ARL_1 increases relatively. In other words, it can be said that with increasing n and a high autocorrelation coefficient (ϕ), the ARL_1 value decreases relatively, making it easier to detect that the mean of the process is out-of-control. In a similar vein, it can also be said that the ARL_0 value increases relatively with increasing n and a low autocorrelation coefficient, making it easier to determine that the mean of the stationary ARTOP(1) process is under control.

Then; As it becomes more difficult to see the extreme values in the Pareto distribution with the increase of the θ parameter, it is expected that the Type-1 error will decrease and the ARL_0 value will increase. As a matter of fact; As can be seen in Tables 5.6, 5.9, and 5.10, when the θ increased while the ϕ was 0.2 or 0.4, this happened. However, when the autocorrelation coefficient increased to 0.6 or 0.8, on the contrary, a decrease in ARL_0 value was observed. From here; it is understood that the default settings of the "qcc" function with Pareto distribution perform much worse at $\phi = 0.6$ and above, compared to ϕ values of less than 0.4 or $\phi = 0.4$ concerning the power of the detection that the mean of the process is under control. Then unsurprisingly, the ARL_1 value increased at low autocorrelation rates such as 0.2 and 0.4 in general, whereas it decreased at high autocorrelation rates such as 0.6 and 0.8. Thus, it is understood that the default settings of the “qcc” function with Pareto distribution perform better at $\phi = 0.6$ and above, compared to ϕ values of less than 0.4 or $\phi = 0.4$ concerning the power of the detection that the mean of the process is out-of-control.

As for the effects of changing γ on the performance of the “qcc” function, the values of the ARL_1 and ARL_0 metrics did not change significantly, as can be seen in Tables 5.6, 5.7, and 5.8. From this point of view, it can be said that no significant change in performance was observed with the change of γ under the conditions of the thesis study.

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BIOGRAPHY

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APPENDICES

Appendix A: Publications Within the Scope of Thesis Study

Ak H., Dingeç K.D., (2022), “Pareto dağılımlı otokorelatif prosesler için istatistiksel kontrol şemalarında uygun kontrol limitlerinin belirlenmesi”, 4th International Eurasian Conference on Science, Engineering and Technology, 71, Ankara, Türkiye, 14-16 December.

Appendix B: Comprehensive Results of the Correlation Analyses Conducted in “Minitab” to Comprehend the Effects of Changing m Over ARL_0 and ARL_1 Performance Metrics

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,542 0,267					
ARL0(0.4)	0,978 0,001	-0,594 0,214				
ARL0(0.6)	0,982 0,000	-0,658 0,155	0,986 0,000			
ARL0(0.8)	0,608 0,200	0,313 0,546	0,497 0,316	0,463 0,355		
ARL1(0.2)	0,961 0,002	-0,711 0,113	0,984 0,000	0,995 0,000	0,379 0,459	
ARL1(0.4)	0,954 0,003	-0,726 0,102	0,980 0,001	0,992 0,000	0,354 0,491	1,000 0,000
ARL1(0.6)	0,967 0,002	-0,693 0,127	0,986 0,000	0,997 0,000	0,403 0,429	1,000 0,000
ARL1(0.8)	0,991 0,000	-0,609 0,199	0,993 0,000	0,997 0,000	0,516 0,295	0,988 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999 0,000	
ARL1(0.8)	0,984 0,000	0,992 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.1: Correlation analysis for $\gamma = 3,\ \theta = 3$ and $n = 2$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,829 0,041					
ARL0(0.4)	-0,785 0,065	0,989 0,000				
ARL0(0.6)	0,981 0,001	-0,835 0,039	-0,804 0,054			
ARL0(0.8)	-0,001 0,998	0,554 0,254	0,616 0,193	-0,052 0,922		
ARL1(0.2)	0,955 0,003	-0,953 0,003	-0,924 0,008	0,960 0,002	-0,289 0,578	
ARL1(0.4)	0,965 0,002	-0,936 0,006	-0,904 0,013	0,953 0,003	-0,233 0,656	0,989 0,000
ARL1(0.6)	0,918 0,010	-0,980 0,001	-0,960 0,002	0,928 0,008	-0,392 0,442	0,994 0,000
ARL1(0.8)	0,950 0,004	-0,957 0,003	-0,931 0,007	0,957 0,003	-0,304 0,558	1,000 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,979 0,001	
ARL1(0.8)	0,988 0,000	0,995 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.2: Correlation analysis for $\gamma = 3,\ \theta = 3$ and $n = 4$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,842 0,035					
ARL0(0.4)	-0,810 0,051	0,994 0,000				
ARL0(0.6)	0,989 0,000	-0,903 0,014	-0,880 0,021			
ARL0(0.8)	-0,760 0,079	0,983 0,000	0,977 0,001	-0,831 0,041		
ARL1(0.2)	0,957 0,003	-0,960 0,002	-0,942 0,005	0,987 0,000	-0,908 0,012	
ARL1(0.4)	0,933 0,007	-0,978 0,001	-0,965 0,002	0,972 0,001	-0,936 0,006	0,997 0,000
ARL1(0.6)	0,913 0,011	-0,987 0,000	-0,974 0,001	0,959 0,002	-0,950 0,004	0,992 0,000
ARL1(0.8)	0,888 0,018	-0,993 0,000	-0,979 0,001	0,939 0,006	-0,969 0,001	0,982 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999 0,000	
ARL1(0.8)	0,993 0,000	0,997 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.3: Correlation analysis for $\gamma = 3$, $\theta = 3$ and $n = 6$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,875 0,022					
ARL0(0.4)	-0,849 0,032	0,994 0,000				
ARL0(0.6)	0,993 0,000	-0,890 0,018	-0,854 0,030			
ARL0(0.8)	-0,859 0,028	0,991 0,000	0,980 0,001	-0,884 0,019		
ARL1(0.2)	0,961 0,002	-0,971 0,001	-0,949 0,004	0,972 0,001	-0,964 0,002	
ARL1(0.4)	0,932 0,007	-0,988 0,000	-0,970 0,001	0,947 0,004	-0,983 0,000	0,996 0,000
ARL1(0.6)	0,912 0,011	-0,995 0,000	-0,980 0,001	0,928 0,007	-0,991 0,000	0,989 0,000
ARL1(0.8)	0,817 0,047	-0,993 0,000	-0,992 0,000	0,834 0,039	-0,983 0,000	0,938 0,006

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	0,965 0,002	0,977 0,001

Cell Contents: Pearson correlation
P-Value

Figure B5.4: Correlation analysis for $\gamma = 3,\ \theta = 3$ and $n = 8$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,602 0,206					
ARL0(0.4)	0,975 0,001	-0,736 0,096				
ARL0(0.6)	0,989 0,000	-0,706 0,117	0,995 0,000			
ARL0(0.8)	0,605 0,203	0,267 0,609	0,446 0,375	0,488 0,326		
ARL1(0.2)	0,963 0,002	-0,790 0,061	0,994 0,000	0,992 0,000	0,377 0,461	
ARL1(0.4)	0,951 0,004	-0,815 0,048	0,990 0,000	0,985 0,000	0,338 0,512	0,999 0,000
ARL1(0.6)	0,966 0,002	-0,779 0,068	0,996 0,000	0,994 0,000	0,392 0,442	1,000 0,000
ARL1(0.8)	0,992 0,000	-0,689 0,130	0,994 0,000	1,000 0,000	0,511 0,300	0,988 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	0,981 0,001	0,991 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.5: Correlation analysis for $\gamma = 4,\ \theta = 3$ and $n = 2$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,875 0,022					
ARL0(0.4)	-0,732 0,098	0,958 0,003				
ARL0(0.6)	0,976 0,001	-0,945 0,005	-0,842 0,035			
ARL0(0.8)	-0,046 0,931	0,524 0,286	0,690 0,129	-0,230 0,661		
ARL1(0.2)	0,957 0,003	-0,976 0,001	-0,884 0,019	0,987 0,000	-0,330 0,523	
ARL1(0.4)	0,935 0,006	-0,989 0,000	-0,911 0,011	0,979 0,001	-0,394 0,440	0,997 0,000
ARL1(0.6)	0,915 0,010	-0,995 0,000	-0,932 0,007	0,970 0,001	-0,442 0,380	0,992 0,000
ARL1(0.8)	0,954 0,003	-0,979 0,001	-0,888 0,018	0,987 0,000	-0,341 0,508	1,000 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	0,998 0,000	0,994 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.6: Correlation analysis for $\gamma = 4,\ \theta = 3$ and $n = 4$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,856					
	0,030					
ARL0(0.4)	-0,882	0,998				
	0,020	0,000				
ARL0(0.6)	0,989	-0,909	-0,926			
	0,000	0,012	0,008			
ARL0(0.8)	-0,762	0,966	0,961	-0,812		
	0,078	0,002	0,002	0,050		
ARL1(0.2)	0,953	-0,971	-0,980	0,981	-0,905	
	0,003	0,001	0,001	0,001	0,013	
ARL1(0.4)	0,933	-0,984	-0,990	0,967	-0,930	0,998
	0,007	0,000	0,000	0,002	0,007	0,000
ARL1(0.6)	0,914	-0,992	-0,995	0,954	-0,941	0,994
	0,011	0,000	0,000	0,003	0,005	0,000
ARL1(0.8)	0,885	-0,998	-0,999	0,931	-0,955	0,983
	0,019	0,000	0,000	0,007	0,003	0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998	
	0,000	
ARL1(0.8)	0,992	0,997
	0,000	0,000

Figure B5.7: Correlation analysis for $\gamma = 4,\ \theta = 3$ and $n = 6$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,864					
	0,026					
ARL0(0.4)	-0,815	0,995				
	0,048	0,000				
ARL0(0.6)	0,994	-0,901	-0,855			
	0,000	0,014	0,030			
ARL0(0.8)	-0,796	0,990	0,998	-0,837		
	0,058	0,000	0,000	0,038		
ARL1(0.2)	0,957	-0,970	-0,941	0,979	-0,927	
	0,003	0,001	0,005	0,001	0,008	
ARL1(0.4)	0,928	-0,986	-0,964	0,957	-0,954	0,995
	0,008	0,000	0,002	0,003	0,003	0,000
ARL1(0.6)	0,911	-0,992	-0,975	0,943	-0,967	0,991
	0,012	0,000	0,001	0,005	0,002	0,000
ARL1(0.8)	0,796	-0,992	-0,997	0,842	-0,996	0,934
	0,058	0,000	0,000	0,036	0,000	0,006

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999	
	0,000	
ARL1(0.8)	0,961	0,973
	0,002	0,001

Cell Contents: Pearson correlation
P-Value

Figure B5.8: Correlation analysis for $\gamma = 4,\ \theta = 3$ and $n = 8$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,689					
	0,130					
ARL0(0.4)	0,986	-0,776				
	0,000	0,070				
ARL0(0.6)	0,985	-0,801	0,995			
	0,000	0,055	0,000			
ARL0(0.8)	0,626	0,124	0,513	0,488		
	0,184	0,815	0,298	0,326		
ARL1(0.2)	0,963	-0,854	0,989	0,995	0,401	
	0,002	0,030	0,000	0,000	0,431	
ARL1(0.4)	0,955	-0,867	0,985	0,992	0,378	1,000
	0,003	0,025	0,000	0,000	0,460	0,000
ARL1(0.6)	0,967	-0,845	0,990	0,997	0,421	1,000
	0,002	0,034	0,000	0,000	0,406	0,000
ARL1(0.8)	0,992	-0,770	0,997	0,999	0,531	0,989
	0,000	0,073	0,000	0,000	0,278	0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999	
	0,000	
ARL1(0.8)	0,985	0,991
	0,000	0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.9: Correlation analysis for $\gamma = 5,\ \theta = 3$ and $n = 2$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,839					
	0,037					
ARL0(0.4)	-0,720	0,973				
	0,107	0,001				
ARL0(0.6)	0,989	-0,889	-0,782			
	0,000	0,018	0,066			
ARL0(0.8)	0,050	0,496	0,650	-0,047		
	0,925	0,317	0,162	0,930		
ARL1(0.2)	0,956	-0,960	-0,882	0,980	-0,237	
	0,003	0,002	0,020	0,001	0,651	
ARL1(0.4)	0,934	-0,976	-0,911	0,966	-0,300	0,998
	0,006	0,001	0,011	0,002	0,564	0,000
ARL1(0.6)	0,913	-0,988	-0,933	0,950	-0,355	0,992
	0,011	0,000	0,007	0,004	0,490	0,000
ARL1(0.8)	0,949	-0,968	-0,897	0,974	-0,264	0,999
	0,004	0,002	0,015	0,001	0,614	0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998	
	0,000	
ARL1(0.8)	0,999	0,995
	0,000	0,000

Figure B5.10: Correlation analysis for $\gamma = 5,\ \theta = 3$ and $n = 4$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,857 0,029					
ARL0(0.4)	-0,828 0,042	0,994 0,000				
ARL0(0.6)	0,981 0,001	-0,924 0,009	-0,908 0,012			
ARL0(0.8)	-0,763 0,077	0,983 0,000	0,981 0,001	-0,844 0,035		
ARL1(0.2)	0,954 0,003	-0,968 0,002	-0,957 0,003	0,990 0,000	-0,910 0,012	
ARL1(0.4)	0,933 0,007	-0,984 0,000	-0,973 0,001	0,976 0,001	-0,939 0,005	0,997 0,000
ARL1(0.6)	0,909 0,012	-0,992 0,000	-0,986 0,000	0,963 0,002	-0,955 0,003	0,991 0,000
ARL1(0.8)	0,894 0,016	-0,994 0,000	-0,990 0,000	0,955 0,003	-0,963 0,002	0,986 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	0,994 0,000	0,999 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.11: Correlation analysis for $\gamma = 5,\ \theta = 3$ and $n = 6$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,870 0,024					
ARL0(0.4)	-0,856 0,030	0,998 0,000				
ARL0(0.6)	0,987 0,000	-0,936 0,006	-0,926 0,008			
ARL0(0.8)	-0,791 0,061	0,978 0,001	0,985 0,000	-0,873 0,023		
ARL1(0.2)	0,959 0,003	-0,972 0,001	-0,964 0,002	0,992 0,000	-0,920 0,009	
ARL1(0.4)	0,937 0,006	-0,987 0,000	-0,980 0,001	0,980 0,001	-0,945 0,004	0,997 0,000
ARL1(0.6)	0,916 0,010	-0,994 0,000	-0,990 0,000	0,968 0,002	-0,961 0,002	0,992 0,000
ARL1(0.8)	0,828 0,042	-0,996 0,000	-0,999 0,000	0,905 0,013	-0,988 0,000	0,950 0,004

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	0,970 0,001	0,982 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.12: Correlation analysis for $\gamma = 5,\ \theta = 3$ and $n = 8$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,828 0,042					
ARL0(0.4)	0,763 0,078	-0,970 0,001				
ARL0(0.6)	0,986 0,000	-0,900 0,014	0,845 0,034			
ARL0(0.8)	-0,478 0,338	0,881 0,020	-0,895 0,016	-0,591 0,217		
ARL1(0.2)	0,960 0,002	-0,951 0,004	0,903 0,014	0,990 0,000	-0,696 0,124	
ARL1(0.4)	0,940 0,005	-0,969 0,001	0,924 0,009	0,980 0,001	-0,740 0,092	0,998 0,000
ARL1(0.6)	0,964 0,002	-0,945 0,004	0,894 0,016	0,993 0,000	-0,682 0,136	1,000 0,000
ARL1(0.8)	0,989 0,000	-0,898 0,015	0,840 0,036	0,999 0,000	-0,589 0,219	0,990 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,997 0,000	
ARL1(0.8)	0,979 0,001	0,992 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.13: Correlation analysis for $\gamma = 3,\ \theta = 4$ and $n = 2$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,870 0,024					
ARL0(0.4)	-0,864 0,027	0,994 0,000				
ARL0(0.6)	0,993 0,000	-0,859 0,029	-0,839 0,037			
ARL0(0.8)	-0,871 0,024	0,999 0,000	0,995 0,000	-0,857 0,029		
ARL1(0.2)	0,955 0,003	-0,975 0,001	-0,966 0,002	0,949 0,004	-0,973 0,001	
ARL1(0.4)	0,929 0,007	-0,990 0,000	-0,983 0,000	0,920 0,009	-0,989 0,000	0,996 0,000
ARL1(0.6)	0,906 0,013	-0,996 0,000	-0,989 0,000	0,897 0,015	-0,995 0,000	0,990 0,000
ARL1(0.8)	0,929 0,007	-0,988 0,000	-0,983 0,000	0,920 0,009	-0,987 0,000	0,996 0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998 0,000	
ARL1(0.8)	1,000 0,000	0,998 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.14: Correlation analysis for $\gamma = 3,\ \theta = 4$ and $n = 4$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,874 0,023					
ARL0(0.4)	-0,886 0,019	0,996 0,000				
ARL0(0.6)	0,990 0,000	-0,882 0,020	-0,897 0,015			
ARL0(0.8)	-0,906 0,013	0,996 0,000	0,997 0,000	-0,918 0,010		
ARL1(0.2)	0,957 0,003	-0,970 0,001	-0,976 0,001	0,969 0,001	-0,987 0,000	
ARL1(0.4)	0,915 0,010	-0,991 0,000	-0,995 0,000	0,931 0,007	-0,999 0,000	0,992 0,000
ARL1(0.6)	0,905 0,013	-0,996 0,000	-0,997 0,000	0,919 0,010	-1,000 0,000	0,987 0,000
ARL1(0.8)	0,845 0,034	-0,995 0,000	-0,995 0,000	0,866 0,026	-0,990 0,000	0,960 0,002

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999 0,000	
ARL1(0.8)	0,988 0,000	0,992 0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.15: Correlation analysis for $\gamma = 3,\ \theta = 4$ and $n = 6$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,919					
	0,010					
ARL0(0.4)	-0,876	0,995				
	0,022	0,000				
ARL0(0.6)	0,983	-0,879	-0,829			
	0,000	0,021	0,042			
ARL0(0.8)	-0,918	0,999	0,994	-0,881		
	0,010	0,000	0,000	0,020		
ARL1(0.2)	0,955	-0,994	-0,978	0,926	-0,994	
	0,003	0,000	0,001	0,008	0,000	
ARL1(0.4)	0,915	-0,997	-0,991	0,883	-0,999	0,993
	0,011	0,000	0,000	0,020	0,000	0,000
ARL1(0.6)	0,903	-0,998	-0,995	0,868	-0,999	0,990
	0,014	0,000	0,000	0,025	0,000	0,000
ARL1(0.8)	0,598	-0,863	-0,907	0,522	-0,863	0,803
	0,209	0,027	0,013	0,288	0,027	0,054

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999	
	0,000	
ARL1(0.8)	0,860	0,877
	0,028	0,022

Cell Contents: Pearson correlation
P-Value

Figure B5.16: Correlation analysis for $\gamma = 3,\ \theta = 4$ and $n = 8$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,901					
	0,014					
ARL0(0.4)	-0,256	0,008				
	0,624	0,988				
ARL0(0.6)	0,993	-0,944	-0,215			
	0,000	0,005	0,683			
ARL0(0.8)	-0,723	0,919	-0,226	-0,788		
	0,105	0,009	0,667	0,063		
ARL1(0.2)	0,959	-0,982	-0,125	0,984	-0,880	
	0,002	0,000	0,813	0,000	0,021	
ARL1(0.4)	0,938	-0,986	-0,099	0,971	-0,908	0,998
	0,006	0,000	0,852	0,001	0,012	0,000
ARL1(0.6)	0,957	-0,982	-0,136	0,983	-0,881	1,000
	0,003	0,001	0,797	0,000	0,020	0,000
ARL1(0.8)	0,987	-0,955	-0,212	0,999	-0,811	0,991
	0,000	0,003	0,687	0,000	0,050	0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,998	
	0,000	
ARL1(0.8)	0,981	0,991
	0,001	0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.17: Correlation analysis for $\gamma = 3,\ \theta = 5$ and $n = 2$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,896					
	0,016					
ARL0(0.4)	-0,890	0,998				
	0,018	0,000				
ARL0(0.6)	0,989	-0,876	-0,861			
	0,000	0,022	0,028			
ARL0(0.8)	-0,911	0,999	0,997	-0,890		
	0,011	0,000	0,000	0,018		
ARL1(0.2)	0,953	-0,988	-0,982	0,940	-0,992	
	0,003	0,000	0,001	0,005	0,000	
ARL1(0.4)	0,922	-0,998	-0,995	0,904	-0,999	0,995
	0,009	0,000	0,000	0,013	0,000	0,000
ARL1(0.6)	0,908	-0,999	-0,996	0,890	-1,000	0,992
	0,012	0,000	0,000	0,017	0,000	0,000
ARL1(0.8)	0,918	-0,998	-0,995	0,900	-1,000	0,995
	0,010	0,000	0,000	0,014	0,000	0,000

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999	
	0,000	
ARL1(0.8)	1,000	1,000
	0,000	0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.18: Correlation analysis for $\gamma = 3,\ \theta = 5$ and $n = 4$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,900					
	0,014					
ARL0(0.4)	-0,900	0,999				
	0,014	0,000				
ARL0(0.6)	0,978	-0,814	-0,819			
	0,001	0,049	0,046			
ARL0(0.8)	-0,926	0,997	0,998	-0,851		
	0,008	0,000	0,000	0,032		
ARL1(0.2)	0,955	-0,982	-0,985	0,905	-0,992	
	0,003	0,000	0,000	0,013	0,000	
ARL1(0.4)	0,917	-0,999	-0,998	0,839	-0,999	0,989
	0,010	0,000	0,000	0,037	0,000	0,000
ARL1(0.6)	0,909	-0,998	-1,000	0,832	-0,999	0,989
	0,012	0,000	0,000	0,040	0,000	0,000
ARL1(0.8)	0,833	-0,990	-0,989	0,727	-0,979	0,949
	0,040	0,000	0,000	0,102	0,001	0,004

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,999	
	0,000	
ARL1(0.8)	0,983	0,985
	0,000	0,000

Cell Contents: Pearson correlation
P-Value

Figure B5.19: Correlation analysis for $\gamma = 3,\ \theta = 5$ and $n = 6$.

Correlations: m; ARL0(0.2); ARL0(0.4); ARL0(0.6); ARL0(0.8); ARL1(0.2); ...

	m	ARL0(0.2)	ARL0(0.4)	ARL0(0.6)	ARL0(0.8)	ARL1(0.2)
ARL0(0.2)	-0,900 0,014					
ARL0(0.4)	-0,911 0,012	0,996 0,000				
ARL0(0.6)	0,893 0,016	-0,622 0,187	-0,650 0,162			
ARL0(0.8)	-0,925 0,008	0,998 0,000	0,997 0,000	-0,668 0,147		
ARL1(0.2)	0,956 0,003	-0,985 0,000	-0,990 0,000	0,743 0,091	-0,994 0,000	
ARL1(0.4)	0,924 0,008	-0,995 0,000	-0,996 0,000	0,679 0,138	-0,998 0,000	0,995 0,000
ARL1(0.6)	0,892 0,017	-0,999 0,000	-0,996 0,000	0,614 0,195	-0,997 0,000	0,984 0,000
ARL1(0.8)	0,610 0,198	-0,893 0,016	-0,873 0,023	0,215 0,682	-0,865 0,026	0,811 0,050

	ARL1(0.4)	ARL1(0.6)
ARL1(0.6)	0,996 0,000	
ARL1(0.8)	0,862 0,027	0,901 0,014

Cell Contents: Pearson correlation
P-Value

Figure B5.20: Correlation analysis for $\gamma = 3$, $\theta = 5$ and $n = 8$.