

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**EXACT EVALUATION OF THE PO AND EEC INTEGRALS
IN TIME DOMAIN USING RADON TRANSFORM
INTERPRETATION**

ASLIHAN AKTEPE
A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
DEPARTMENT OF ELECTRONICS ENGINEERING

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ASSOC. PROF. DR. HÜSEYİN ARDA ÜLKÜ

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T.C.
GEBZE TEKNİK ÜNİVERSİTESİ
FEN BİLİMLERİ ENSTİTÜSÜ

FO VE EKA İNTEGRALLERİNİN RADON
DÖNÜŞÜMÜ KULLANILARAK ZAMAN
UZAYINDA HESAPLANMASI

ASLIHAN AKTEPE
YÜKSEK LİSANS TEZİ
ELEKTRONİK MÜHENDİSLİĞİ ANABİLİM DALI

DANIŞMANI
DOÇ. DR. HÜSEYİN ARDA ÜLKÜ

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2021



YÜKSEK LİSANS JÜRİ ONAY FORMU

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SUMMARY

In this thesis, schemes for exact evaluation of the time domain physical optics (PO) and equivalent edge current (EEC) (radiation) integrals on linear and quadratic triangular patches and edges, respectively, are developed, and application of the developed schemes to the analysis of transient scattering from arbitrary perfect electrically conducting (PEC) and impedance surface scatterers is presented. The exact evaluation schemes are developed based on the Radon transform interpretation of the time domain radiation integrals. It is shown that the PO integral on a surface of a linear or quadratic triangular patch is reduced to a line integral on the intersection of the patch and the plane formed by the propagation direction of the incident plane wave and the observation direction. Specifically, for linear triangular patches the PO integral is evaluated in closed-form, and for quadratic triangles the PO integral is evaluated exactly by applying an appropriate coordinate transformation (e.g. elliptic coordinates if the intersecting curve is an ellipse in barycentric coordinates of the triangle) and using suitable order Gauss-Legendre quadrature rule (GLQR). Similarly, the EEC integral on an edge of a quadratic triangular patch is evaluated in closed-form at the intersecting point of the edge and the formed plane. In addition, a formula for a special case that occurs if there is no intersection of the formed plane and all scatterer, called under-sampled reflection (USR) case, is suggested. The accuracy and validity of the developed schemes and expressions are demonstrated via numerical examples.

Keywords: Equivalent Edge Currents (EEC), Impedance Surfaces, Linear and Quadratic Triangular Patches, Physical Optics (PO) Approximation, Radon Transform Interpretation, Time Domain Analysis.

ÖZET

Bu tez çalışmasında, lineer ve ikinci dereceden üçgenler ile ayrıklaştırılmış mükemmel elektriksel iletken saçıcılar için fiziksel optik (FO) ve eşdeğer kenar akımları (EKA) (ışırma) integrallerinin zaman uzayında tam (exact) olarak hesaplanması ve lineer üçgenler kullanılarak modellenen empedans yüzeylerin zaman uzayında FO yaklaşımı ile analizi için yöntemler geliştirilmiştir. Işıma integrallerinin Radon dönüşümü olarak yorumlanmasıyla; lineer veya ikinci dereceden üçgen yüzeyi üzerinden olan FO integralinin, gelen alan yönü ve gözlem yönünün oluşturduğu düzlem ile saçıcıyı modelleyen üçgenin kesişimi sonucu oluşan eğri üzerinden bir integrale indirgenebileceği gösterilmiştir. Özel olarak, lineer üçgenler için kesişim düz bir çizgi olduğundan, indirgenen FO integralinin kapalı-form ifadesi belirlenmiştir. İkinci dereceden üçgenler için ise bu kesişim üçgenin alan koordinatlarında ikinci dereceden bir eğri oluşturur ve eğrinin türüne (elips, hiperbol vs.) uygun bir koordinat dönüşümü (örneğin elips için eliptik koordinatlar gibi) ve uygun dereceden Gauss-Legendre integrasyon kuralı kullanılarak FO integrali tam olarak hesaplanmıştır. Benzer olarak, kırınım katkılarının belirlenmesi amacıyla ikinci dereceden üçgenler ile modellenen saçıcıların kenarları üzerinden olan EKA integralinin, ikinci dereceden üçgenin kenarı ve düzlemin kesişim noktasında, kapalı-formda hesaplanabileceği gösterilmiştir. Ek olarak, düzlem ile saçıcı arasında hiç kesişim olmadığı ve saçıcının tamamının atlandığı özel durumda saçılan alanların FO yaklaşımı ile zaman uzayında belirlenmesi için bir formül önerilmiştir. Geliştirilen yöntemlerin ve ifadelerin doğruluğu ve geçerliliği nümerik örnekler aracılığıyla gösterilmiştir.

Anahtar Kelimeler: Empedans Yüzeyler, Eşdeğer Kenar Akımları (EKA), Fiziksel Optik (FO) Yaklaşımı, Lineer ve İkinci Dereceden Üçgenler, Radon Dönüşümü, Zaman Uzay Analizi.

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I want to dedicate this thesis to my father Mesut AKTEPE, who recently deceased – you are missed.

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LIST OF SYMBOLS AND ABBREVIATIONS

<u>Symbols and Abbreviations</u>	<u>Explanations</u>
S	: Surface of the PEC scatterer
S^{imp}	: Surface of the impedance scatterer
$S_{\text{lit}}, S_{\text{sha}}$	: Illuminated and shadowed surfaces of the scatterer S
$S_{\text{lit}}^{\text{imp}}, S_{\text{sha}}^{\text{imp}}$	: Illuminated and shadowed surfaces of the scatterer S^{imp}
S_n	: n^{th} quadratic patch of the discretized surface of the scatterer
$\tilde{S}_n$	: n^{th} linear patch of the discretized surface of the scatterer
l	: Edges of the illuminated triangles
l_n	: Discretized triangle edges
$\mathbf{r}$	: Position vector
$\hat{\mathbf{k}}_{\text{i}}$	: Propagation direction of the incident field
$\hat{\mathbf{k}}_{\text{s}}$	: Observation direction of the scattered far-field
$\mathbf{k}_{\text{r}}$	: Radon transform direction
$\hat{\mathbf{p}}$	: Polarization direction
c	: Speed of light
k	: Wave number
ε_0	: Permittivity of the vacuum
ε	: Permittivity of the medium
μ_0	: Permeability of the vacuum
μ	: Permeability of the medium
η_0	: Intrinsic impedance of the vacuum
η	: Intrinsic impedance of the medium
t	: Time
f	: Frequency
f_0	: Center frequency
f_{bw}	: Effective bandwidth
$\delta(\cdot)$	: Dirac delta function
$\hat{\mathbf{n}}(\mathbf{r})$	: The outward pointing unit normal vector of S or S^{imp}

<u>Symbols and Abbreviations</u>	<u>Explanations</u>
$\hat{\mathbf{t}}_n(\mathbf{r})$	: Unit tangent vector along the edge
$\mathbf{r}'_p$	: Intersection point of the edge and $\mathbf{k}_r$ -plane
$\mathbf{J}(\mathbf{r}, t)$	: Time domain induced current density
$\mathbf{E}^{\text{inc}}(\mathbf{r}, t)$	: Time domain electric field component of the incident wave
$\mathbf{H}^{\text{inc}}(\mathbf{r}, t)$	: Time domain magnetic field component of the incident wave
$\mathbf{H}^{\text{inc}}(\mathbf{r}, \omega)$	: Frequency domain magnetic field component of the incident wave
$\mathbf{E}^{\text{sca}}(\mathbf{r}, t)$	: Time domain scattered electric field
$\mathbf{E}_{\text{tot}}^{\text{sca}}(\mathbf{r}, t)$	: Time domain total scattered electric field
$\mathbf{E}^{\text{sca}}(\mathbf{r}, \omega)$	: Frequency domain scattered electric field
$\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$	: Range-corrected scattered electric field for time domain PO
$\bar{\mathbf{E}}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$	: Range-corrected scattered electric field for time domain EEC
$\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, \omega)$	: Frequency domain range-corrected scattered electric field
$\mathbf{H}^{\text{tot}}(\mathbf{r}, t)$	: Time domain total magnetic field
$h_n(t)$	: Time domain PO integral for linear triangular patches
$\mathbf{h}_n(t)$	: Time domain PO integral for quadratic triangular patches
$\mathbf{s}_n(t)$	: Time domain EEC integral for quadratic edges
$Z(t)$	: Time domain surface impedance
$Z(\omega)$	: Frequency domain surface impedance
$G(t)$	: Modulated Gaussian pulse
$J(\alpha, \beta)$	: Jacobean
$\tilde{J}_{\text{elp}}(a_{\text{elp}}, \phi)$	: Jacobean for the elliptic coordinate transformation
$\tilde{J}_{\text{hyp}}(c_{\text{hyp}}, \phi)$	: Jacobean for the hyperbolic coordinate transformation
(x, y, z)	: Local Cartesian coordinate system
(x_p, y_p, z_p)	: Rotated Cartesian coordinate system
(α, β, γ)	: Local barycentric coordinate system
(u, v)	: Transformed barycentric coordinate system
$(a_{\text{elp}}, b_{\text{elp}})$	: Major and minor axes of the ellipse
$(a_{\text{hyp}}, b_{\text{hyp}})$	: Major and minor axes of the hyperbola
(a_{elp}, ϕ)	: Elliptic coordinates
$c_{\text{elp}}, c_{\text{hyp}}$	: Focal length of the ellipse and hyperbola

<u>Symbols and Abbreviations</u>	<u>Explanations</u>
$(a_{\min}, a_{\max})$	: Minimum and maximum major axis sizes at intersection
$(c_{\text{hyp}}, \theta, \phi)$	: Elliptic cylindrical coordinates
$(c_{\min}, c_{\max})$	: Minimum and maximum focal length sizes at intersection
$(u_{\min}, u_{\max})$	: Minimum and maximum lines at intersection
A_n	: Area of n^{th} patch of the linear triangle
j	: Imaginary unit
t_i	: Intersecting time
Δt	: Time step size
Δf	: Frequency step size
Δx	: Spatial step size
∂_t	: Derivative w.r.t. time
$*$	: Temporal convolution
λ	: Wave length
σ	: RCS values
σ_v	: Duration of the modulated Gaussian pulse
ps	: Picoseconds
ns	: Nanoseconds
N_R	: Complex refractive index
N_{os}	: Oversampling factor
N	: Number of triangular patches
N_t	: Maximum number of time steps
N_T	: Total number of time steps
N_q	: GLQR points
p_q	: Quadrature nodes
w_q	: Quadrature weights
N_p^g	: Number of poles for rational function fit
a_i^g, b_i^g	: Pole and residue pairs for rational function fit
$\Phi(m)$	: Samples of the interested functions for the error calculation
$\Phi^{\text{wrt}}(m)$	: Comparision values for the error calculation
$D_{\{\text{e,h}\},n}^I, D_{\text{h},n}^M$	: Diffraction coefficients

<u>Symbols and Abbreviations</u>	<u>Explanations</u>
$R_{\parallel}(t)$	: Parallel reflection coefficients in time domain
$R_{\perp}(t)$	: Perpendicular reflection coefficients in time domain
$R_{\parallel}(\omega)$	: Parallel reflection coefficients in frequency domain
$R_{\perp}(\omega)$	: Perpendicular reflection coefficient in frequency domain
$\mathbf{u}(\mathbf{r}, t), \tilde{\mathbf{u}}(\mathbf{r}, t)$	: Functions of reflection coefficients
$\hat{\mathbf{e}}_{\parallel}(\mathbf{r}), \hat{\mathbf{e}}_{\perp}(\mathbf{r})$	: Unit vectors in parallel and perpendicular directions
EEC	: Equivalent Edge Current
FEM	: Finite Element Method
FD	: Frequency Domain
FDTD	: Finite-Difference Time-Domain
FRVF	: Fast Relaxed Vector Fitting
GLQR	: Gauss-Legendre Quadrature Rule
GO	: Geometric Optics
GTD	: Geometric Theory of Diffraction
LE	: Linear Edge
LT	: Linear Triangle
MoM	: Method of Moments
MOT	: Marching On-in-Time
NURBS	: Non-Uniform Rational B-Spline
PEC	: Perfect Electrically Conducting
PO	: Physical Optics
PTD	: Physical Theory of Diffraction
RCS	: Radar Cross Section
RT	: Radon Transform
TD	: Time Domain
TDIE	: Time Domain Integral Equations
QE	: Quadratic Edge
QT	: Quadratic Triangle
USR	: Under-Sampled Reflection

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1. INTRODUCTION

In the field of electromagnetics, finding location and/or determining properties of scatterers such as planes, ships, spacecraft, vehicles, people, and natural environments with the help of the scattered electromagnetic waves is one of the important and common problems. Radar (Radio Detection And Ranging), which is a detection system based on electromagnetic waves [1], is used for these purposes. In the military applications, radar is used for surveillance, navigation, and weapon guidance for land, sea and air vehicles. In civil life, it has been used as police radar, weather radar (meteorological radar), air traffic control radar in airports, spaceborne or airborne radar, which is used to determine the earth topology and environmental characteristics [2]. The basic working principle of radar is very similar to the working principle of the sound wave reflection: an echo occurs if someone shouts towards an object that can reflect the sound. If the speed of sound, i.e. the propagation speed of the sound in the medium, is known, the distance and direction of the object can be calculated. Similar to this, in a radar system, this is done by sending a specific electromagnetic wave signal, which is generated by the transmitter antenna of the radar. The generated electromagnetic wave, called incident electromagnetic wave, interacts with objects, called scatterers, then spreads in different directions (in other words, it scatters), which is called scattered electromagnetic wave. Furthermore, the scattered electromagnetic waves can propagate, interact, and scatter from other objects as scatterers. Thus, the scattered electromagnetic waves play an important role in detection of the objects. In the literature, the criterion of radar detectability of the scatterers is defined by the Radar Cross Section (RCS) [3]. Therefore, RCS value of an object is important in the design and development phase. RCS of a prototype or object can be determined by measurement [3], once it is fabricated. However, fabricating and measuring every prototype is extremely inefficient and expensive. Therefore, it is important to predict the RCS of an object via solving Maxwell's equations and using computers. In general, RCS is a complex value that depends on the geometric shape, orientation, and material of the scatterer, as well as the frequency and polarization of the incident wave [3–5]. Due to this complexity and lack of the technology, RCS was estimated analytically, but with the development of the technology, numerical methods have started to gain importance in order to provide

rapid and accurate solutions to electromagnetic radiation and scattering problems, involving the RCS estimation. As a result, various procedures have been developed for the simulation of the electromagnetic problems and the numerical side of the field has begun to emerge. This emerged field is called computational electromagnetics.

The methods to solve the electromagnetic problems can be categorized in two groups: full-wave solvers and asymptotic techniques. The full-wave solvers such as finite element method (FEM) [6], finite difference time domain (FDTD) method [7], and method of moments (MoM) [8], are generally used in low frequency problems and the solution obtained by applying them consist the effects of all scattering mechanisms, e.g. reflection, diffraction, surface waves, etc. However, as the physical size of the object becomes much larger compared to wavelength, called electrically large scatterers, the size of the computational domain increases and these methods require very high computational resources. The asymptotic techniques, such as geometric optics (GO) and physical optics (PO) approximations [9], are obtained from asymptotic evaluation of the Maxwell's equations under the high frequency assumption and they are used efficiently in various high frequency problems involving the analysis of the large scatterers. Compared to full-wave solvers, asymptotic techniques provide reasonable accuracy for the solution of electrically large scatterers in reasonable time using less computational resources. Nevertheless, the aforementioned asymptotic techniques can only model the reflection of the electromagnetic waves, the other scattering mechanisms such as multiple reflection, diffraction, i.e. scattering from edges, sharp wedges, etc., must be determined separately and added to the solution. Specifically to determine the diffraction effects, asymptotic techniques such as physical theory of diffraction (PTD), geometric theory of diffraction (GTD), and equivalent edge current (EEC), are developed as complimentary techniques to PO and GO [9–12].

In electromagnetic scattering, the PO approximation is one of the most preferred high frequency technique to analyze scattering and radiation from electrically large scatterers. The PO approximation usually involves the analysis of perfect electrically conducting (PEC) scatterers. The PO approximation for the PEC scatterers assumes that an equivalent electric current density, which is roughly twice of the tangential component (w.r.t. scatterer's surface) of the incident magnetic field, is induced only on the illuminated part of the scatterer, whereas the current density on the shadowed

region is assumed to be zero. Then the PO approximated current density creates the scattered electromagnetic fields. On the other hand, the PO approximation can be used for the analysis of scatterers with the impedance surfaces [3, 13–16]. For the analysis of the scatterers with the impedance surfaces, PO approximation assumes that the electric and magnetic equivalent currents are induced on the illuminated part of the surface of the scatterer. In the analysis using the PO approximation, once the current densities are approximated, the calculation of the scattered fields requires evaluation of a highly oscillatory radiation integral, which is called PO integral, over the illuminated surface. Although the PO approximation is a high frequency technique, the numerical evaluation of the PO integral is usually not efficient, since a huge number of samples should be taken into account to obtain accurate results for electrically large scatterers. Therefore, finding an analytical or an approximate expression to the PO integral is crucial to preserve the benefits of the PO approximation. In order to determine the scattered fields, the scatterer must be discretized using geometric patches. The surface of the scatterer can be discretized with different surface models, such as linear triangles [17], linear polygons [18–21], quadratic (second order) triangles [22] or NURBS surfaces [23–25], etc. In this case the scattered fields can be determined by evaluating the PO integral over the one of these surface models. In the literature, closed-form or approximate expressions for the PO integral over these surface models are developed:

- i) In frequency domain, the PO integral for linear triangular and polygonal patches are evaluated analytically resulting in the well-known Gordon formula [18, 19] or approximate expressions are available using the Levin's [26] or Ludwig's [27] integral methods. In time domain, analytical expression is also developed for linear triangular patches using the Radon transform interpretation in [17].
- ii) For a quadratic triangular patch, approximate expressions are developed in frequency domain [22, 28, 29], however in time domain there is no closed-form/approximate expressions or exact evaluation scheme available for quadratic triangles. Note that similar to the Radon transform interpretation, the closed-form expressions in time domain are developed for paraboloidal, ellipsoidal, and hyperboloidal surfaces [30–32]. These quadratic surfaces are defined in 3-dimensional space, whereas the quadratic triangles have a second order behavior in barycen-

tric coordinates. This limits the application of the analytical expressions in [30–32], since general 3-dimensional quadratic surfaces are not a common way to discretize arbitrary surfaces, however quadratic triangular meshes can be generated using open source and commercial software. In addition, the quasi-analytic expressions in [30] are obtained for a point source instead of a plane wave excitation.

- iii) For the NURBS surfaces, it is possible to reduce the PO integral over the NURBS surface to a line integral, both in time and frequency domains [23, 24, 33–35].

Note that the same PO integral on one of the surface models can be used in the analysis of the impedance surfaces using the PO approximation, whereas the physical theory of diffraction (PTD)–equivalent edge currents (EEC) [11, 12] requires evaluation of a highly oscillatory radiation integral, which is called EEC integral in this thesis, on the edge of the aforementioned surface models.

In general, all radiation integrals can be interpreted as a kind of a Radon transform, which allows to reduce the dimension of the integration domain. Especially in time domain this reduction is straightforward and can be performed analytically in most cases [17, 23, 36–38]. Since the argument of the Dirac delta function in the radiation integral defines a surface related to the observation direction/point and incident wave, the domain of the radiation integral can be reduced to the intersection of the surface defined by the Dirac delta function and the scatterer’s surface. As a result, the PO integrals for near/far-field scattering due to different source configurations for linear triangular patches [17, 36, 37], radiation integrals in time domain surface and volume integral equations, for basis functions defined on linear triangular patches [38–40] and linear tetrahedral elements [41], respectively, can be determined in closed-form.

The main purpose of this thesis is to develop schemes for exact evaluation of the PO and EEC integrals on second order triangular patches and edges, respectively, in time domain using Radon transform interpretation. For this purpose, the closed-form evaluation of the PO integral for linear triangular patches is revisited and a formula for the special case that can only occurs in time domain simulations is developed. Then a scheme for the exact evaluation of PO integrals on quadratic triangles and closed-form evaluation of the EEC integral on quadratic edges are developed. At last, transient

analysis of the impedance surfaces using time domain PO approximation and linear triangular patches is presented.

This thesis is organized as follows: In Chapter 2, the formulation of the closed-form evaluation of the PO integral for a linear triangular patch in time domain step by step without any interpretation is presented and the special case is investigated. In Chapter 3, a scheme for exact evaluation of the PO integral in time domain over quadratic triangular patches using Radon transform interpretation is presented. In Chapter 4, the closed-form evaluation of the EEC integral for the quadratic edges of the quadratic triangular patches in time domain is presented. In Chapter 5, the analysis of transient scattering from impedance surfaces using the PO approximation and linear triangular patches is presented. In Chapter 6, the conclusions are drawn. The publications of the thesis are presented in Appendix A. Frequently used properties of the Dirac delta function in this thesis is presented in Appendix B, definitions for quadratic triangles are given in Appendix C, quadratic curve types are given in Appendix D, and lastly diffraction coefficients for EEC are given in Appendix E.

2. LINEAR TRIANGLES

In this chapter, the closed-form evaluation of the PO integral for linear triangular patches in time domain, which is presented in [17], will be re-obtained mathematically step by step without any interpretation. In [17], the closed-form expression of the PO integral for linear triangular patches in time domain (and also Fourier transformed to frequency domain) is evaluated using the Radon transform interpretation. Specifically, in [17], the PO integral over a linear triangular surface is reduced to a line integral over the intersection of the linear triangle and the plane formed by the propagation direction of the incident plane wave and the observation direction, called $\mathbf{k}_r$ -plane, then it is concluded that the closed-form expression to the line integral should be a triangle function since the domain of the PO integral is a linear triangle. However, the closed-form expression presented in [17] can be obtained mathematically, which will lead to extension of the procedure for exact evaluation of the PO integral on high order surfaces. For this reason, in Section 2.1.1, formulation of the closed-form evaluation of the PO integral for a linear triangular patch is presented step by step without any interpretation in time domain. In addition, for a time domain simulation, it is possible to encounter with a case such that all scatterer can be completely fit into two consecutive time samples, resulting in no intersection. This case is not the specular reflection case (i.e. the special case that the $\mathbf{k}_r$ -plane is parallel to the triangular patch) and strongly depends on the choice of the time step size, the incidence and observation directions. This case is called under-sampled reflection (USR) case in this thesis. Note that, USR case has not been encountered in [17] since all examples in [17] are produced in frequency domain even the formulation is partially in time domain. Accordingly, in Section 2.1.2, a formula, which has a rectangular shape in time, is proposed for the USR case, and in Section 2.1.3 computational complexity is presented. Numerical examples are shown in Section 2.2. It is shown in numerical examples that the accuracy of the scattered field obtained using the time domain PO approximation depends on the norm of $\mathbf{k}_r$, besides the time step size. Note that this observation might be crucial to maintain the accuracy at the same level for all incident/observation directions. Conclusions are drawn in Section 2.3.

2.1. Formulation

Let S denote the surface of a PEC scatterer that resides in an infinite homogeneous background medium with permittivity ε_0 and permeability μ_0 . The electric and magnetic field components of the impulsively excited incident plane wave

$$\mathbf{E}^{\text{inc}}(\mathbf{r}, t) = \hat{\mathbf{p}}\delta(t - \hat{\mathbf{k}}_i \cdot \mathbf{r}/c), \quad (2.1)$$

$$\mathbf{H}^{\text{inc}}(\mathbf{r}, t) = \eta_0^{-1}(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}})\delta(t - \hat{\mathbf{k}}_i \cdot \mathbf{r}/c), \quad (2.2)$$

respectively, where $\hat{\mathbf{k}}_i$ and $\hat{\mathbf{p}}$ are the propagation direction and polarization of the incident wave, respectively, $\delta(\cdot)$ denotes the Dirac delta function, and $c = 1/\sqrt{\varepsilon_0\mu_0}$ and $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ are the speed of wave and the intrinsic impedance of the medium, respectively. In the PO approximation, the current density is assumed as $\mathbf{J}(\mathbf{r}, t) = 2\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{\text{inc}}(\mathbf{r}, t)$; $\mathbf{r} \in S_{\text{lit}}$, and $\mathbf{J}(\mathbf{r}, t) = 0$; $\mathbf{r} \in S_{\text{sha}}$, where $\hat{\mathbf{n}}(\mathbf{r})$ denotes the outward pointing unit normal vector of S , S_{lit} and S_{sha} denote the illuminated and shadowed parts of S . Assuming that S_{lit} is discretized with linear triangular patches, $S_{\text{lit}} \simeq \bigcup_{n=1}^N \tilde{S}_n$, the scattered electric field at far-zone can be determined as

$$\mathbf{E}^{\text{sca}}(\mathbf{r}, t) = -\frac{1}{2\pi c} \frac{\partial_t \delta(t - r/c)}{r} * \left[\hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \times \sum_{n=1}^N \hat{\mathbf{n}}_n h_n(t) \right], \quad (2.3)$$

where $\mathbf{r} = r\hat{\mathbf{k}}_s$ and $\hat{\mathbf{k}}_s$ is the observation direction, ∂_t and “*” denote the time derivative and convolution, respectively, $\hat{\mathbf{n}}_n = \hat{\mathbf{n}}(\mathbf{r})$; $\mathbf{r} \in \tilde{S}_n$, denotes the unit normal vector of the n^{th} triangular patch, and $h_n(t)$ denotes the PO integral:

$$h_n(t) = \int_{\tilde{S}_n} \delta\left(t - \frac{2}{c} \mathbf{k}_r \cdot \mathbf{r}'\right) d\mathbf{r}'. \quad (2.4)$$

Here $\mathbf{k}_r = (\hat{\mathbf{k}}_i - \hat{\mathbf{k}}_s)/2$. Note that in (2.4) the argument of the Dirac delta function defines a plane with a time dependent location $t = 2c^{-1}\mathbf{k}_r \cdot \mathbf{r}'$, called $\mathbf{k}_r$ -plane. Considering the infinite time integral of $h_n(t)$ yields the area of the triangular surface $\tilde{S}_n$,

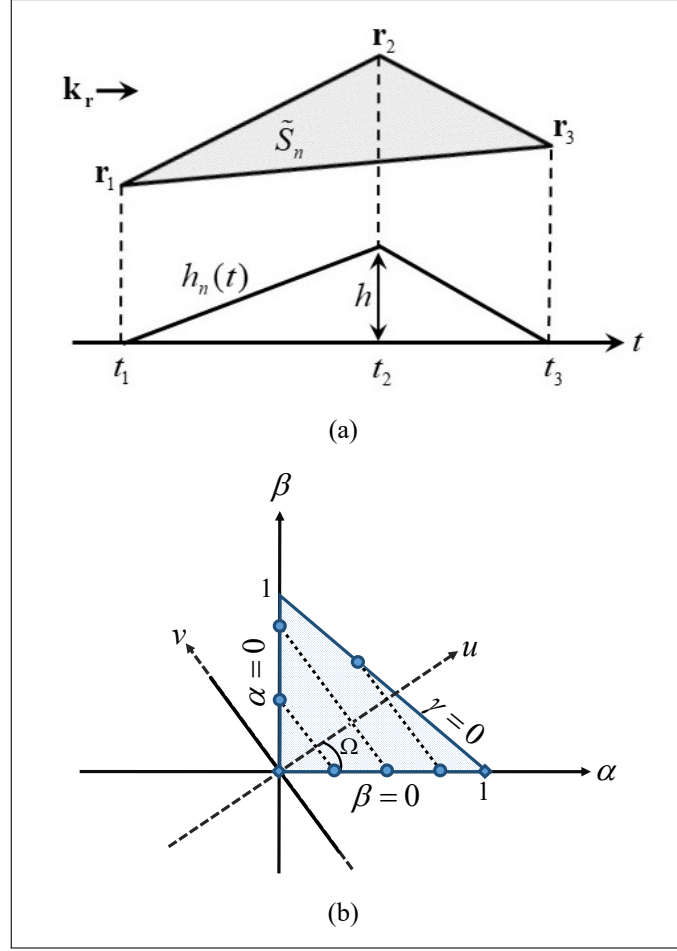


Figure 2.1: (a) Radon transform of the triangle and (b) intersections on the triangle.

time samples of $h_n(t)$ are the intersection of the $\mathbf{k}_r$ -plane and the triangular surface $\tilde{S}_n$. Since $\tilde{S}_n$ is a linear triangular surface, the result of the integral should be a triangle function [17] (see Fig. 1 in [17] or Fig. 2.1(a)). Next the closed-form evaluation of $h_n(t)$ is shown.

2.1.1. Evaluation of $h_n(t)$

A linear triangular patch is defined by three nodes $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and assume that these nodes are ordered in intersection time of the node and the $\mathbf{k}_r$ -plane as $t_3 > t_2 > t_1$, where $t_i = 2c^{-1}\mathbf{k}_r \cdot \mathbf{r}_i$, $i = 1, 2, 3$, as shown in Fig. 2.1(a). Any point on $\tilde{S}_n$ can be pointed using barycentric coordinates of $\tilde{S}_n$, i.e. $\mathbf{r}' = (\mathbf{r}_2 - \mathbf{r}_1)\alpha + (\mathbf{r}_3 - \mathbf{r}_1)\beta + \mathbf{r}_1$, where (α, β) denote the barycentric coordinates as shown in Fig. 2.1(b). The coordinate system can be rotated as $(x, y, z) \rightarrow (x_p, y_p, z_p)$ such that $\mathbf{k}_r \parallel \hat{\mathbf{x}}_p$. As a result intersection time can be given as $t_i = 2c^{-1}|\mathbf{k}_r| x_{p,i}$, $i = 1, 2, 3$, and using the

Dirac delta function's property given in (B.1), $h_n(t)$ in (2.4) can be written as

$$h_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{\tilde{S}_n} \delta\left(\frac{ct}{2|\mathbf{k}_r|} - x'_{p,i}\right) d\mathbf{r}'_{\mathbf{p}}. \quad (2.5)$$

The argument of the Dirac delta function in (2.5) determines the location of the $\mathbf{k}_r$ -plane:

$$0 = \frac{ct}{2|\mathbf{k}_r|} - x'_{p,i} = \frac{ct}{2|\mathbf{k}_r|} - [(x_{p,2} - x_{p,1})\alpha + (x_{p,3} - x_{p,1})\beta + x_{p,1}]. \quad (2.6)$$

A coordinate transformation as shown in Fig. 2.1(b) is applied to (2.5) and (2.6),

$$\alpha = u \cos \Omega - v \sin \Omega \quad (2.7)$$

$$\beta = u \sin \Omega + v \cos \Omega,$$

where $\Omega = \tan^{-1}[(x_{p,3} - x_{p,1})/(x_{p,2} - x_{p,1})]$. Note that since $t_3 > t_2 > t_1$ (also $x_{p,3} > x_{p,2} > x_{p,1}$), $\Omega \in [0, \pi/2)$. Substituting (2.7) into (2.6) yields

$$u = \frac{1}{\tau} \left[\frac{ct}{2|\mathbf{k}_r|} - x_{p,1} \right], \quad (2.8)$$

where $\tau = \sqrt{(x_{p,2} - x_{p,1})^2 + (x_{p,3} - x_{p,1})^2}$. Applying the coordinate transformation in (2.7) to the PO integral (2.5) yields

$$h_n(t) = \frac{ctA_n}{\tau|\mathbf{k}_r|} \int_u \delta\left(u - \frac{ct}{2|\mathbf{k}_r|} \tau + \frac{x_{p,1}}{\tau}\right) \int_{v(u)} dv du. \quad (2.9)$$

Here, $A_n = 0.5 |(\mathbf{r}_3 - \mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)|$ denotes the area of the triangle, $2A_n$ is the Jacobean of the transformation. The integral w.r.t. u in (2.9) can be evaluated analytically using the Dirac delta function's property given in (B.3) as

$$h_n(t) = \frac{ctA_n}{\tau|\mathbf{k}_r|} \int_{v(u)} dv \bigg|_{u=\frac{ct}{2|\mathbf{k}_r|} \tau - \frac{x_{p,1}}{\tau}}. \quad (2.10)$$

Note that in (u, v) , $\mathbf{k}_r \parallel v$ and the integral in (2.10) represents the line that is formed as the result of the intersection of the $\mathbf{k}_r$ -plane and $\tilde{S}_n$. In addition, since $t_3 > t_2 > t_1$; the $\mathbf{k}_r$ -plane first intersects with $\mathbf{r}_1$ (where $\alpha = 0$ and $\beta = 0$), then intersects with $\mathbf{r}_2$ (where $\alpha = 1$ and $\beta = 0$), and lastly intersects with $\mathbf{r}_3$ (where $\alpha = 0$ and $\beta = 1$). This order determines the limits of the integral in (2.10). Hence the integral in (2.10) should be evaluated for two regions: for $t_2 > t > t_1$ the lower and upper limits are on $\beta = 0$ and $\alpha = 0$ lines, respectively. For $t_3 > t > t_2$ they are on $\alpha = 0$ and $\gamma = 1 - \alpha - \beta = 0$ lines, respectively. For a given u , intersection of the v axis with $\beta = 0$, $\alpha = 0$, and $\gamma = 1 - \alpha - \beta = 0$ lines can be found using the inverse transformation of (2.7) as

$$v_\beta(u) = \frac{-u \sin \Omega}{\cos \Omega} = \frac{-u(x_{p,3} - x_{p,1})}{(x_{p,2} - x_{p,1})}, \quad (2.11)$$

$$v_\alpha(u) = \frac{u \cos \Omega}{\sin \Omega} = \frac{u(x_{p,2} - x_{p,1})}{(x_{p,3} - x_{p,1})}, \quad (2.12)$$

$$\begin{aligned} v_\gamma(u) &= \frac{1 - u[\cos \Omega + \sin \Omega]}{\cos \Omega - \sin \Omega} \\ &= \frac{\tau - u[x_{p,2} + x_{p,3} - 2x_{p,1}]}{x_{p,2} - x_{p,3}}, \end{aligned} \quad (2.13)$$

respectively. As a result, the closed-form expression of $h_n(t)$ in (2.10) can be determined for $t_2 > t > t_1$ as

$$\begin{aligned} h_n(t) &= \frac{ctA_n}{|\mathbf{k}_r| \tau} [v_\alpha(u) - v_\beta(u)]_{u=\frac{ct}{2|\mathbf{k}_r|\tau} - \frac{x_{p,1}}{\tau}} \\ &= \frac{ctA_n}{|\mathbf{k}_r| \tau^2} \left[\frac{ct}{2|\mathbf{k}_r|} - x_{p,1} \right] \left[\frac{\tau^2}{(x_{p,3} - x_{p,1})(x_{p,2} - x_{p,1})} \right] \\ &= \frac{ctA_n}{|\mathbf{k}_r|} \frac{\frac{ct}{2|\mathbf{k}_r|} - x_{p,1}}{(x_{p,3} - x_{p,1})(x_{p,2} - x_{p,1})} \\ &= 2A_n \frac{t - t_1}{(t_3 - t_1)(t_2 - t_1)}, \end{aligned} \quad (2.14)$$

and for $t_3 > t > t_2$ as

$$\begin{aligned}
h_n(t) &= \frac{ctA_n}{|\mathbf{k}_r| \tau} [v_\gamma(u) - v_\alpha(u)]_{u=\frac{ct}{2|\mathbf{k}_r|\tau} - \frac{x_{p,1}}{\tau}} \\
&= \frac{ctA_n}{|\mathbf{k}_r|} \frac{\frac{ct}{2|\mathbf{k}_r|} - x_{p,3}}{(x_{p,3} - x_{p,1})(x_{p,2} - x_{p,3})} \\
&= 2A_n \frac{t - t_3}{(t_3 - t_1)(t_2 - t_3)}.
\end{aligned} \tag{2.15}$$

If $t > t_3$ or $t_1 > t$, then there is no intersection and $h_n(t) = 0$. Consequently, $h_n(t)$ can be given as follows

$$h_n(t) = \begin{cases} \frac{2A_n(t-t_1)}{(t_2-t_1)(t_3-t_1)}, & (t_1 < t < t_2) \\ \frac{2A_n(t-t_3)}{(t_2-t_3)(t_3-t_1)}, & (t_2 < t < t_3) \\ 0, & \text{elsewhere.} \end{cases} \tag{2.16}$$

As a special case, if $t_1 = t_2 = t_3$, i.e. the specular reflection case, where $\hat{\mathbf{n}}_n \parallel \mathbf{k}_r$ and $\mathbf{k}_r \cdot \mathbf{r}'$ is constant all over $\tilde{S}_n$, (2.4) can be written as

$$\begin{aligned}
h_n(t) &= \int_{\tilde{S}_n} \delta(t - 2\mathbf{k}_r \cdot \mathbf{r}'/c) d\mathbf{r}' \\
&= \delta(t - t_1) \int_{\tilde{S}_n} d\mathbf{r}' \\
&= A_n \delta(t - t_1).
\end{aligned} \tag{2.17}$$

With (2.16) and (2.17), the closed-form expressions in [17] are obtained.

2.1.2. Under-sampled Reflection (USR) Case

For a time domain simulation a time step size Δt should be selected considering the desired higher limit of the frequency band of the simulation, i.e. $f_{\max}$. The intersection of the scatterer and the $\mathbf{k}_r$ -plane occurs in a time interval $t \in [t_{\min}, t_{\max}]$, where $t_{\min} = \min\{2c^{-1}\mathbf{k}_r \cdot \mathbf{r}\}$ and $t_{\max} = \max\{2c^{-1}\mathbf{k}_r \cdot \mathbf{r}\}$, $\forall \mathbf{r} \in S$. Note that $t_{\min}$ and $t_{\max}$ depend on the $\mathbf{k}_r$ -direction, which is not a unitary vector and also depends on the incidence and observation directions. If the time step size Δt samples the interval

$[t_{\min}, t_{\max}]$, i.e. the $\mathbf{k}_r$ -plane intersects with the scatterer at any time sample, then the closed-form expressions given in the previous subsection can be used. However, if $(m+1)\Delta t > t_{\max} > t_{\min} > m\Delta t$ for an integer m , then samples of the $\mathbf{k}_r$ -plane has no intersection with the scatterer. As a result, the scatterer is totally neglected as it does not exist. This case is named as USR case in this thesis.

In order to remedy this problem, a simple formula is suggested considering the PO integral $h_n(t)$ for a triangle patch is in triangle form in time as in (2.16) and its temporal integral yields the area of the patch, then $h_n(t)$ can be represented by

$$h_n(t) = \begin{cases} \frac{A_n}{t_3 - t_1}, & t_3 > t > t_1 \\ 0, & \text{elsewhere,} \end{cases} \quad (2.18)$$

where the temporal integral of (2.18) yields A_n and the Fourier transform of (2.18) is similar to the Fourier transform of (2.16) for the frequency $f < f_{\max}$, since Δt is large enough [42].

2.1.3. Computational Complexity

The computational complexity and memory requirement of the PO approximation are different than the marching on-in-time (MOT) based time domain integral equation solvers [43,44], since in PO approximation the solution is obtained without solving a matrix equation. The solution with time domain PO approximation requires $O(N_t N)$ operations and $O(N_T)$ memory, where N is the number of triangles, N_t is the maximum number of time steps that the $\mathbf{k}_r$ -plane and a triangle intersects, N_T is the total number of time steps that samples the scatterer in $[t_{\min}, t_{\max}]$, in other words total number of time steps that the $\mathbf{k}_r$ -plane and the scatterer intersects. For the USR case, both the computational complexity and the memory requirement are $O(N)$. N , N_t , and N_T depend on Δt , size of the scatterer, norm of $\mathbf{k}_r$, and discretization size. Also it should be emphasized that USR case might be used if the scatterer and the $\mathbf{k}_r$ -plane does not intersect, in other words $N_T = N_t = 0$, therefore it might not be conclusive to compare the computational complexity and memory requirement of the USR case given in Section 2.1.2 and the time domain PO formulation in Section 2.1.1.

2.2. Numerical Examples

In all examples presented in this section, a $\hat{\mathbf{p}} = \hat{\mathbf{x}}$ polarized plane wave traveling to $\hat{\mathbf{k}}_i = -\hat{\mathbf{z}}$ direction illuminates the scatterers. The upper frequency limit for simulations $f_{\max} = 2$ GHz (the minimum wave length $\lambda_{\min} = 0.1499$ m) and the time step size is determined by $\Delta t = 1/(2f_{\max}N_{\text{os}})$, where N_{os} denotes the oversampling factor. When $N_{\text{os}} = 1$, $\Delta t = 250$ ps is the Nyquist sampling limit for 2 GHz. In all figures, TD denotes the time domain results obtained using (2.16) and (2.17), FD denotes the frequency domain results obtained using the formulas given in [17], and USR denotes the case in Section 2.1.2. Analytical PO solutions for the unit sphere and rectangular plate are given in [45] and [9], respectively. While calculating errors, discrete Fourier transformed TD, FD, and analytical results are calculated at $f = [10 \text{ MHz}, 2 \text{ GHz}]$ with $\Delta f = 10$ MHz interval, then norm of the relative error in RCS, where all the frequency samples are taken into account, is determined.

2.2.1. Unit Sphere

First example analyzes the change of the error in the backscattered RCS ($\hat{\mathbf{k}}_s = -\hat{\mathbf{k}}_i$ and $\mathbf{k}_r = \hat{\mathbf{k}}_i$) w.r.t. analytical PO solution as the number of triangles increases. The illuminated part of the unit sphere is discretized with 1188 to 172700 triangles, where the average edge lengths change from $0.77\lambda_{\min}$ to $0.06\lambda_{\min}$. Fig. 2.2 shows the relative errors of FD and TD results for $\Delta t = \{0.55, 50, 100, 250\}$ ps. Note that the relative errors in TD for $\Delta t = \{50, 100, 250\}$ ps saturate and do not decrease as the number of triangles increases, even though time step sizes satisfy the Nyquist sampling criterion. This error is not caused by the discretization since the error of FD results, where time step size has no effect, is lower. The behavior of the error for $\Delta t = \{50, 100, 250\}$ ps is caused because most of the triangles are not taken into account since these triangles do not intersect with the $\mathbf{k}_r$ -plane. If the time step size is chosen as small as there is an intersection with all triangles, i.e. TD result for $\Delta t = 0.55$ ps, then the error levels coincide with FD results. Fig. 2.3 shows the relative error in backscattered RCS obtained by TD w.r.t. FD results to investigate the dependence of the accuracy to $\mathbf{k}_r$ (or $\hat{\mathbf{k}}_s$). $\hat{\mathbf{k}}_s$ is swept between $\theta = [0^\circ, 180^\circ]$ ($\phi = 0^\circ$) and the illuminated part of the unit

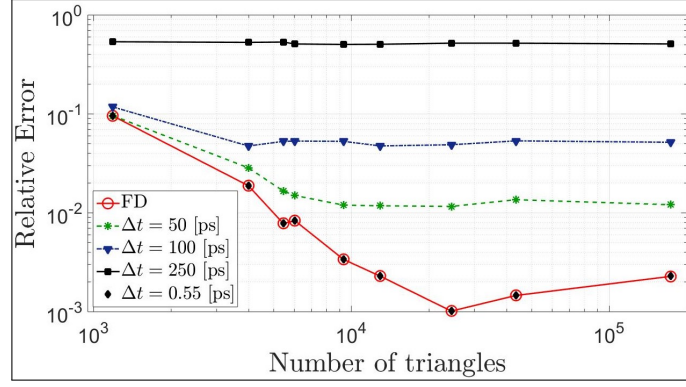


Figure 2.2: Relative error in the backscattered RCS of the unit sphere.

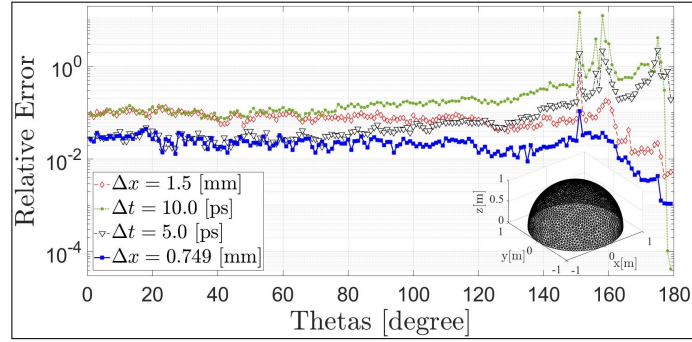


Figure 2.3: Relative error for $\theta = [0, 180]^\circ$ interval in the backscattered RCS.

sphere is discretized with 9307 triangles as shown in Fig. 2.3. Note that $|\mathbf{k}_r|$ changes from 1 to 0 with θ and the spatial step that the $\mathbf{k}_r$ -plane takes in a time step changes as $\Delta x = 0.5c\Delta t|\mathbf{k}_r|^{-1}$. Fig. 2.3 plots the relative error for $\Delta t = \{5.0, 10.0\}$ ps and $\Delta x = \{0.749, 1.5\}$ mm. The former results are obtained by keeping Δt fixed and for the latter ones, Δt is updated to keep Δx fixed, also in these cases no triangles are omitted. It can be seen in Fig. 2.3 that for $\theta < 80^\circ$ the error levels for associated Δt and Δx are coinciding but for $\theta > 80^\circ$ the error for the fixed Δt increases as the error for the fixed Δx remains in the same level.

2.2.2. Inclined Plate

As a second example, backscattered fields ($\hat{\mathbf{k}}_s = -\hat{\mathbf{k}}_i$) of an inclined plate with corners $\mathbf{r}_1 = (-0.5, -0.5, 0)$, $\mathbf{r}_2 = (0.5, -0.5, 0)$, $\mathbf{r}_3 = (0.5, 0.5, 0.1)$, and $\mathbf{r}_4 = (-0.5, 0.5, 0.1)$, modeled with 4 linear triangles (see inset of Fig. 2.4) is analyzed. Fig. 2.4 plots the time domain behavior of $\sum \hat{\mathbf{n}}_n h_n(t)$ for different Δt . It is known that the total PO integral should be in the form of a rectangle function for the plate [9,23] and

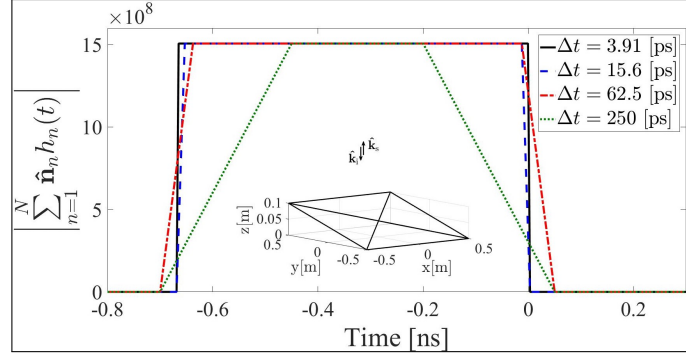


Figure 2.4: Norm of the total PO integral of the plate.

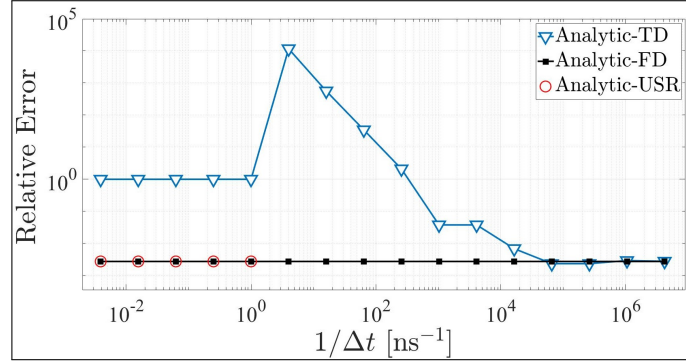


Figure 2.5: Comparison of the relative error in the backscattered RCS.

as it can be seen from Fig. 2.4 that the TD results are not satisfactory until $\Delta t = 3.91$ ps ($N_{\text{os}} = 64$). Fig. 2.5 shows the relative error in backscattered RCS obtained by TD results w.r.t. the analytical PO solution as the time step size changes. The relative errors for USR and FD results are also plotted. Note that USR and FD results do not change with Δt . It can be seen from Fig. 2.5 that as the time step size increases, the TD results begin to deteriorate and for $\Delta t > 0.667$ ns the scatterer fits in two sequential time steps, TD results become inconclusive, and the error is unitary, since there is no intersection of the $\mathbf{k}_r$ -plane and the plate, i.e. $h_n(t) = 0$. For $\Delta t > 0.667$ ns, USR result, which coincides with the FD result very well, can be used in time domain. The error level of TD results coincide with FD result for $\Delta t < 0.0153$ ps ($N_{\text{os}} > 16384$). In addition, this limit is higher as the inclination of the plate decreases to obtain enough intersections. Eventually, if the plate is normal to $\mathbf{k}_r$, the formula for the specular reflection case in (2.17) should be used.

2.3. Conclusions

In this chapter, the complete formulation of the closed-form evaluation of the PO integral on linear triangular patches in time domain is presented. Numerical examples show that even the closed-form expressions are used, time step size should be chosen considering the triangle sizes, incident and observation directions (or spatial steps), since for a fixed time step size, the accuracy of the solution depends on the norm of $\mathbf{k}_r$. If the scatterer fits into two consecutive time steps and the $\mathbf{k}_r$ -plane does not intersect with the scatterer at all, the formula for the USR case can be used.

3. QUADRATIC TRIANGLES

In this chapter, a scheme for exact evaluation of the PO integral in time domain on quadratic triangular patches using Radon transform interpretation is presented. Similar to the previous chapter, the scatterer is assumed to be illuminated by a plane wave and the scattered fields are observed at far-zone. Specifically, the argument of the Dirac delta function defines a plane and the dimension of the surface integral on a quadratic triangle can be reduced to the intersection of the plane and the quadratic triangle. As a result, the surface integral is reduced to a line integral. To determine the analytical expression of the intersecting curve (not the integral), the equation that defines the quadratic triangle and the equation of the plane defined by the Dirac delta function are solved together in barycentric coordinates of the associated quadratic triangle. The resulting intersection, is in the form of a quadratic curve, which can be an ellipse, a hyperbola, a parabola or a line/lines. Once the type of the curve is determined, then a suitable coordinate transformation is applied to parametrize the intersecting curve, e.g. elliptic coordinate system is used for an ellipse. At this point, instead of searching for a closed-form expression to the line integral, the Gauss-Legendre quadrature rule (GLQR) is applied to determine the PO integral over the parametrized curve. Due to the quadratic nature of the intersecting curve, the PO integral can be determined exactly using an appropriate order GLQR. The resulting scheme has two main advantages: (i) although the incident field is assumed to be band limited in time domain simulations, mathematically in time domain PO approximation it is possible to use an incident field with a Dirac delta time signature, which has infinite bandwidth. In this case the upper frequency to obtain accurate results is limited by the time step size and discretization efficiency of the scatterer's surface. If results for a specific time signature is required, they can be obtained by convolving the desired time signature with the scattered fields found for the Dirac delta excitation (see Chapter 5). (ii) Discretizing with quadratic triangles models the scatterer's surface much more accurate with less number of triangles than using linear ones, especially for curved surfaces. For a curved scatterer discretized with the same number of linear and quadratic triangular elements, the upper frequency limit for the quadratic triangles is much higher than the limit for the linear triangles.

The rest of the chapter is organized as follows: firstly, the formulation of the

PO approximation is presented in Section 3.1, and the evaluation of the PO integral on quadratic triangles is shown. Specifically, in Sections 3.1.1 and 3.1.2 the formulation of the PO approximation and evaluation of the PO integral is presented. The formulations for determining the analytical expression of the parametrized intersecting curve types is shown in Sections 3.1.3, 3.1.4, and 3.1.5, for an ellipse, a hyperbola, and a line/lines cases, respectively. The application of the GLQR over the parametrized curve, is shown in Section 3.1.6. In Section 3.2, numerical examples that demonstrate the accuracy and validity of the proposed scheme are presented, and in Section 3.3 conclusions are drawn. Additionally, the properties of the Dirac delta function, the parameters of the quadratic triangle definition and the parameters for the triangle plane intersection in barycentric coordinates, and variables for the quadratic curve types, which are frequently used throughout this chapter, are given in Appendices B, C, and D, respectively.

3.1. Formulation

3.1.1. PO Approximation

Let S denote the surface of a PEC scatterer, residing in an unbounded medium with permittivity ε_0 and permeability μ_0 . S is illuminated by an incident plane wave with a Dirac delta time dependence. The electric and magnetic field components of the incident wave are given in (2.1) and (2.2), respectively. As mentioned in the Section 2.1, the PO approximated surface current density is assumed as $\mathbf{J}(\mathbf{r}, t) = 2\hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{\text{inc}}(\mathbf{r}, t)$; $\mathbf{r} \in S_{\text{lit}}$, and $\mathbf{J}(\mathbf{r}, t) = 0$; $\mathbf{r} \in S_{\text{sha}}$, where $\hat{\mathbf{n}}(\mathbf{r})$ is the outward pointing unit normal vector on S , S_{lit} and S_{sha} denote the illuminated and shadowed part of S , respectively. The scattered electric field at the far-zone can be found as [46, 47]:

$$\mathbf{E}^{\text{sca}}(\mathbf{r}, t) = -\frac{1}{2\pi c} \frac{\partial_t \delta(t - r/c)}{r} * \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t). \quad (3.1)$$

Here $\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$ is the range-corrected scattered electric field [46, 47]:

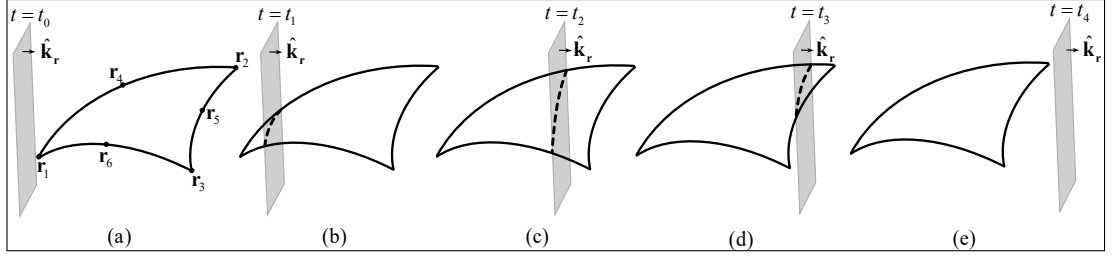


Figure 3.1: The intersection of the $\mathbf{k}_r$ -plane and the quadratic triangle: $t_0 < t_1 < t_2 < t_3 < t_4$.

$$\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = -\frac{\eta}{2} \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \int_S \mathbf{J}(\mathbf{r}', t) * \delta(t + \hat{\mathbf{k}}_s \cdot \mathbf{r}'/c) d\mathbf{r}'. \quad (3.2)$$

Substituting the PO approximated current density into (3.2) and assuming that the illuminated surface S_{lit} is discretized with quadratic triangles, i.e. $S_{\text{lit}} \simeq \bigcup_{n=1}^N S_n$, $\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$ can be written as:

$$\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \times \sum_{n=1}^N \mathbf{h}_n(t), \quad (3.3)$$

where $\mathbf{h}_n(t)$ is called the PO integral in this section and defined as:

$$\mathbf{h}_n(t) = \int_{S_n} \hat{\mathbf{n}}(\mathbf{r}') \delta(t - 2\mathbf{k}_r \cdot \mathbf{r}'/c) d\mathbf{r}'. \quad (3.4)$$

The main difference of (3.4) from the formulation given in Chapter 2 or in [17] for linear triangles is that $\hat{\mathbf{n}}(\mathbf{r})$ is not constant throughout the integration domain S_n . Therefore $\hat{\mathbf{n}}(\mathbf{r})$ should be taken into account while evaluating the integral, hence $\mathbf{h}_n(t)$ is a vector function.

3.1.2. Evaluation of $\mathbf{h}_n(t)$

The PO integral $\mathbf{h}_n(t)$ given in (3.4), can be interpreted as a planar Radon transform due to the set of points that makes the argument of the Dirac delta function zero is a plane with the surface normal $\hat{\mathbf{k}}_r = \mathbf{k}_r/|\mathbf{k}_r|$ (called $\mathbf{k}_r$ -plane), and the position of the plane depends on time, as shown in Fig. 3.1. Integral domain in (3.4) is a quadratic triangle S_n , which is defined by 6 nodes, $\mathbf{r}_i$, $i = 1, \dots, 6$, as shown in Fig. 3.1(a).

Depending on the position of the $\mathbf{k}_r$ -plane, quadratic triangle, and time, the integral in $\mathbf{h}_n(t)$ can be reduced to a line integral that is formed by the intersection of the $\mathbf{k}_r$ -plane and S_n , as shown in Figs. 3.1(b)-(d). If there is no intersection then $\mathbf{h}_n(t) = 0$, as shown in Fig. 3.1(e). To find the expression of the intersecting line, the expression that defines the surface of the quadratic triangle should be substituted into the equation of the $\mathbf{k}_r$ -plane and the resulting equation should be solved.

As the first step, it is assumed that the coordinate system is rotated $(x, y, z) \rightarrow (x_p, y_p, z_p)$ such that $\hat{\mathbf{k}}_r = \hat{\mathbf{x}}_p$. Since this is a simple rotation operation, it brings unitary Jacobean, moreover $\mathbf{k}_r \cdot \mathbf{r}'_p = |\mathbf{k}_r| x'_p$ and using the Dirac delta function's property given in (B.1), (3.4) can be written as:

$$\begin{aligned} \mathbf{h}_n(t) &= \int_{S_n} \hat{\mathbf{n}}(\mathbf{r}'_p) \delta(t - 2\mathbf{k}_r \cdot \mathbf{r}'_p/c) d\mathbf{r}'_p \\ &= \frac{c}{2|\mathbf{k}_r|} \int_{S_n} \hat{\mathbf{n}}(\mathbf{r}'_p) \delta\left(\frac{ct}{2|\mathbf{k}_r|} - x'_p\right) d\mathbf{r}'_p. \end{aligned} \quad (3.5)$$

If the definition in (C.1) that defines the location of any point on S_n , is substituted in the equation of the $\mathbf{k}_r$ -plane in (3.5), expression of the intersecting curve in barycentric coordinates can be found as:

$$\begin{aligned} 0 &= \frac{ct}{2|\mathbf{k}_r|} - x_p(\alpha, \beta) = \frac{ct}{2|\mathbf{k}_r|} - \sum_{i=1}^6 N_i(\alpha, \beta) x_{p,i} \\ &= \tilde{a}\alpha^2 + \tilde{b}\beta^2 + \tilde{c}\alpha + \tilde{d}\beta + \tilde{e}\alpha\beta - \tilde{f} = F(\alpha, \beta). \end{aligned} \quad (3.6)$$

Here the equation in the second line is a general quadratic curve equation [48], and depending on the coefficients it can be an ellipse, a hyperbola, a parabola or a line/lines. The determination of the curve type can be found in [48] and is briefly given in Appendix D for the appropriate notation in (3.6). The explicit expressions of $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}, \tilde{f}$ are given in (C.5). Note that even it is not explicitly shown, the coefficients $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}, \tilde{f}$ are functions of the nodes of S_n ($x_{p,i}, i = 1, \dots, 6$), in addition $\tilde{f}$ is a function of time. The variables of the PO integral in (3.5) can be changed to the barycentric coordinates

(α, β) as

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{S_n} \mathbf{n}(\mathbf{r}'_p(\alpha, \beta)) \delta\left(\frac{ct}{2|\mathbf{k}_r|} - x'_p(\alpha, \beta)\right) d\alpha d\beta, \quad (3.7)$$

where the Jacobean $J(\alpha, \beta) = |\mathbf{n}(\mathbf{r}'_p(\alpha, \beta))|$ is the norm of the normal vector, which is defined in (C.3) for S_n .

Hitherto, the variable of the PO integral is changed to barycentric coordinates and the type of the intersecting curve is determined. The next step is parametrizing the intersecting curve for the ellipse, hyperbola, and line cases. Note that parabola case is not common and has never been encountered in simulations, even if (3.6) can represent a parabola. Therefore the formulation for the parabola case is not given here, nevertheless the expressions for the parabola case can be derived applying the similar steps in the following sections.

3.1.3. Ellipse Case

If $\Delta \neq 0$, $\Delta/I < 0$, and $\zeta > 0$, as defined in Appendix D and Table D.1, the quadratic curve is an ellipse and at a certain time the quadratic curve equation in (3.6) represents, in general, a rotated ellipse with the center $(\hat{g}, \hat{h})$. (3.6) can be reorganized as

$$1 = \hat{a}(\alpha - \hat{g})^2 + \hat{b}(\beta - \hat{h})^2 + \hat{e}(\alpha - \hat{g})(\beta - \hat{h}), \quad (3.8)$$

where coefficients $\hat{a}, \hat{b}, \hat{e}, \hat{g}, \hat{h}$ are defined in Table 3.1. A coordinate transformation, where the major and minor axes of the ellipse coincide with u, v axes, is applied to (3.8) as shown in Fig. 3.2:

$$\begin{aligned} \alpha - \hat{g} &= u \cos \Omega - v \sin \Omega \\ \beta - \hat{h} &= u \sin \Omega + v \cos \Omega. \end{aligned} \quad (3.9)$$

Table 3.1: Coefficients of the reorganized quadratic curve equation for different type of curves.

Coefficient	Ellipse/Hyperbola Cases	Coinciding/Parallel Lines Cases
$\hat{a}$	$\tilde{a}/\hat{f}$	$\tilde{a} + \tilde{b}$
$\hat{b}$	$\tilde{b}/\hat{f}$	$\sqrt{\tilde{c} + \tilde{d}}$
$\hat{e}$	$\tilde{e}/\hat{f}$	—
$\hat{f}$	$\tilde{a}\hat{g}^2 + \tilde{b}\hat{h}^2 + \tilde{e}\hat{g}\hat{h} - \tilde{f}$	$-\tilde{f}$
$\hat{g}$	$[-2\tilde{b}\tilde{c} + \tilde{d}\tilde{e}]/[4\tilde{a}\tilde{b} - \tilde{e}^2]$	—
$\hat{h}$	$[\tilde{c}\tilde{e} - 2\tilde{a}\tilde{d}]/[4\tilde{a}\tilde{b} - \tilde{e}^2]$	—
Ω	$0.5\arctan(\hat{e}/(\hat{a} - \hat{b}))$	$0.5\arctan(\tilde{d}/\tilde{c})$

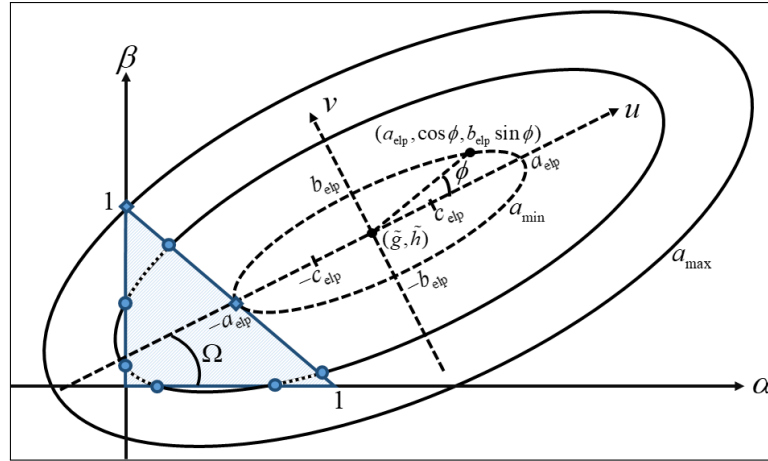


Figure 3.2: Intersection of the ellipse and the quadratic triangle in barycentric coordinates, and coordinate definitions for the ellipse case.

Here the rotation angle $\Omega = 0.5 \arctan(\hat{e}/(\hat{a} - \hat{b}))$. As a result (3.8) is converted to a standard ellipse equation

$$\frac{u^2}{a_{\text{elp}}^2} + \frac{v^2}{b_{\text{elp}}^2} = 1, \quad (3.10)$$

where a_{elp} and b_{elp} are major and minor axes of the ellipse, respectively. Note that the coordinate transformation in (3.9) results in a unitary Jacobean in (3.7), also a_{elp} and b_{elp} are function of time due to $\hat{f}$. To parametrize the ellipse, Cartesian coordinates u, v

are transformed to elliptic coordinates (a_{elp}, ϕ) as

$$\begin{aligned} u &= a_{\text{elp}} \cos(\phi) \\ v &= \sqrt{a_{\text{elp}}^2 - c_{\text{elp}}^2} \sin(\phi), \end{aligned} \quad (3.11)$$

where $a_{\text{elp}} \in [0, \infty)$, $\phi \in [0, 2\pi)$, $c_{\text{elp}} = \sqrt{a_{\text{elp}}^2 - b_{\text{elp}}^2}$ is the focal length of the ellipse. If (3.9) and (3.11) are applied to (3.7), $\mathbf{h}_n(t)$ can be given as

$$\begin{aligned} \mathbf{h}_n(t) &= \frac{c}{2|\mathbf{k}_r|} \int_{a_{\min}}^{a_{\max}} \int_{\phi(a_{\text{elp}})} \mathbf{n}(\mathbf{r}'_p(a_{\text{elp}}, \phi)) \\ &\quad \times \delta\left(\frac{ct}{2|\mathbf{k}_r|} - x'_p(a_{\text{elp}}, \phi)\right) \tilde{J}_{\text{elp}}(a_{\text{elp}}, \phi) d\phi da_{\text{elp}}, \end{aligned} \quad (3.12)$$

where $a_{\min}$ and $a_{\max}$ are the minimum and maximum major axis sizes that intersect with the quadratic triangle in barycentric coordinate system, as shown in Fig. 3.2. If $a_{\text{elp}} < a_{\min}$ or $a_{\max} < a_{\text{elp}}$, then $\mathbf{h}_n(t) = 0$, i.e. there is no intersection. The Jacobean for the coordinate transformation is

$$\tilde{J}_{\text{elp}}(a_{\text{elp}}, \phi) = \frac{a_{\text{elp}}^2 + b_{\text{elp}}^2 - c_{\text{elp}}^2 \cos(2\phi)}{2b_{\text{elp}}}. \quad (3.13)$$

As the last step, to use (B.3), the argument of the Dirac delta function should be given in terms of a_{elp} . This is achieved by using the property (B.2) as

$$\begin{aligned} \mathbf{h}_n(t) &= \frac{c}{2|\mathbf{k}_r|} \int_{a_{\min}}^{a_{\max}} \int_{\phi(a_{\text{elp}})} \frac{\mathbf{n}(\mathbf{r}'_p(a_{\text{elp}}, \phi))}{|\Psi_{\text{elp}}(a_0, \phi)|} \\ &\quad \times \delta(a_{\text{elp}} - a_0) \tilde{J}_{\text{elp}}(a_{\text{elp}}, \phi) d\phi da_{\text{elp}}, \end{aligned} \quad (3.14)$$

where $\Psi_{\text{elp}}(a_0, \phi) = \partial F(a_{\text{elp}}, \phi) / \partial a_{\text{elp}}|_{a_{\text{elp}}=a_0}$ and a_0 ($a_{\min} < a_0 < a_{\max}$) is the root of the argument of the Dirac delta function in (3.12). Note that $\Psi_{\text{elp}}(a_0, \phi)$ can be determined using a symbolic mathematical calculator, explicit expression of $\Psi_{\text{elp}}(a_0, \phi)$ is not given here since it occupies too much space. Due to the second order behavior,

there are two roots of the argument of the Dirac delta function in (3.12):

$$a_{0,1} = \pm \sqrt{(\hat{a}\cos^2\Omega + \hat{b}\sin^2\Omega + \hat{e}\cos\Omega\sin\Omega)^{-1}} \quad (3.15)$$

and there should be two integrals w.r.t. a_{elp} , however only a_0 satisfies the condition $a_{\min} < a_0 < a_{\max}$, since $a_{\min} > 0$ and $a_{\max} > 0$. As a result the integral for root a_1 is zero due to the Dirac delta function's property in (B.3). Using (B.3), $\mathbf{h}_n(t)$ can be found as:

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{\phi(a_0)} \frac{\mathbf{n}(\mathbf{r}'_p(a_0, \phi))}{|\Psi_{\text{elp}}(a_0, \phi)|} \tilde{J}_{\text{elp}}(a_0, \phi) d\phi, \quad (3.16)$$

where the major and minor axes are defined as:

$$a_0 = \sqrt{(\hat{a}\cos^2\Omega + \hat{b}\sin^2\Omega + \hat{e}\cos\Omega\sin\Omega)^{-1}}, \quad (3.17)$$

$$b_0 = \sqrt{(\hat{a}\cos^2\Omega + \hat{b}\sin^2\Omega - \hat{e}\cos\Omega\sin\Omega)^{-1}}, \quad (3.18)$$

respectively. It should be emphasized that a_0 and b_0 are time dependent variables. Hitherto, line integral representation of $\mathbf{h}_n(t)$ for the ellipse case is determined.

3.1.4. Hyperbola Case

The steps for the hyperbola case is very similar to the ellipse case. If ($\Delta \neq 0$ and $\zeta < 0$) as defined in Appendix D and Table D.1, the quadratic curve is a hyperbola and at a certain time the quadratic curve equation in (3.6) represents, in general, a rotated hyperbola with four branches centered at $(\hat{g}, \hat{h})$. The same reorganization in (3.8) and the coordinate transformation given in (3.9) can be applied to (3.6) as in the previous subsection. As a result (3.6) is converted to a standard hyperbola equation

$$\frac{u^2}{a_{\text{hyp}}^2} - \frac{v^2}{b_{\text{hyp}}^2} = 1, \quad (3.19)$$

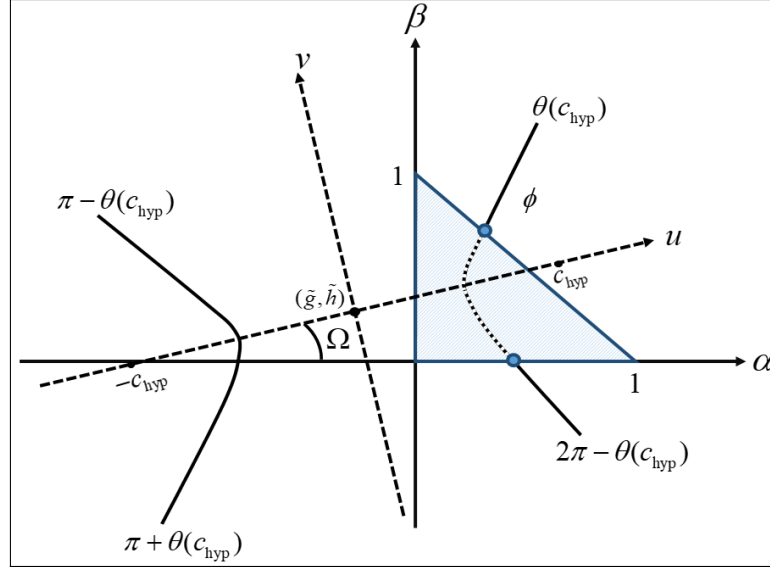


Figure 3.3: Intersection of the hyperbola and the quadratic triangle in barycentric coordinates, and coordinate definitions for the hyperbola case.

where a_{hyp} and b_{hyp} are transverse and conjugate axes of the hyperbola, respectively. To parametrize the hyperbola, Cartesian coordinate u, v is transformed to elliptic cylindrical coordinates [48] $(c_{\text{hyp}}, \theta, \phi)$:

$$\begin{aligned} u &= c_{\text{hyp}} \cos(\theta) \cosh(\phi) \\ v &= c_{\text{hyp}} \sin(\theta) \sinh(\phi), \end{aligned} \quad (3.20)$$

where $c_{\text{hyp}} = \sqrt{a_{\text{hyp}}^2 + b_{\text{hyp}}^2} \in [0, \infty)$ is the focal length of the hyperbola, $\theta \in [0, 2\pi)$ denotes the branches of the hyperbola with the focal length c_{hyp} on four quadrants, and $\phi \in (0, \infty)$, as shown in Fig. 3.3. If (3.9) and (3.20) are applied to (3.7) and the similar steps in previous subsection for ellipse case is repeated, $\mathbf{h}_n(t)$ can be found as

$$\begin{aligned} \mathbf{h}_n(t) &= \frac{c}{2|\mathbf{k}_r|} \int_{c_{\min}}^{c_{\max}} \int_{\phi(c_{\text{hyp}})} \frac{\mathbf{n}(\mathbf{r}'_p(c_{\text{hyp}}, \theta, \phi))}{|\Psi_{\text{hyp}}(c_0, \theta, \phi)|} \\ &\quad \times \delta(c_{\text{hyp}} - c_0) \tilde{J}_{\text{hyp}}(c_{\text{hyp}}, \phi) d\phi dc_{\text{hyp}}, \end{aligned} \quad (3.21)$$

where $c_{\min}$ and $c_{\max}$ are the minimum and maximum focal length sizes that intersect with the quadratic triangle in barycentric coordinate system. If $c_{\text{hyp}} < c_{\min}$ or $c_{\max} < c_{\text{hyp}}$, then $\mathbf{h}_n(t) = 0$, i.e. there is no intersection. The Jacobean for the coordinate transformation is $\tilde{J}_{\text{hyp}}(c_{\text{hyp}}, \phi) = 0.5c_{\text{hyp}}^2(\cos(2\phi) - \cosh(2\phi))$, $\Psi_{\text{hyp}}(c_0, \theta, \phi) =$

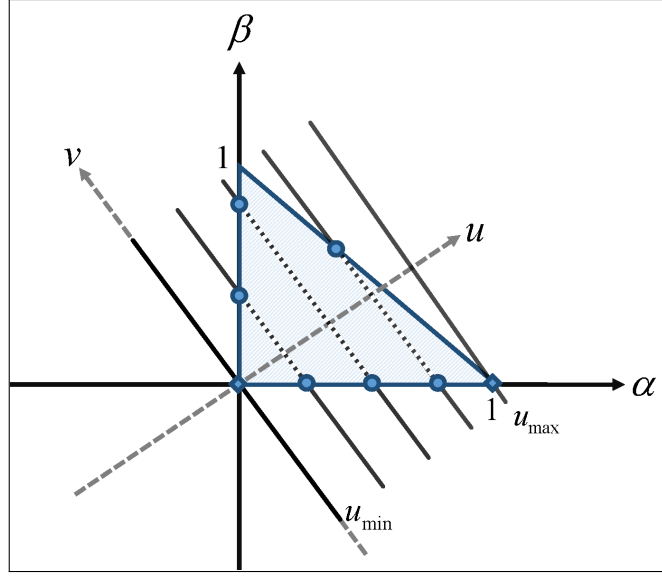


Figure 3.4: Intersection of the line and the quadratic triangle in barycentric coordinates, and coordinate definitions for the line case.

$\partial F(c_{\text{hyp}}, \theta, \phi) / \partial c_{\text{hyp}}|_{c_{\text{hyp}}=c_0}$ and c_0 ($c_{\min} < c_0 < c_{\max}$) is the root of the argument of the Dirac delta function:

$$c_{0,1} = \pm \sqrt{\frac{2(\hat{a}\cos^2\Omega + \hat{b}\sin^2\Omega)}{(\hat{a}\cos^2\Omega + \hat{b}\sin^2\Omega)^2 + (\hat{e}\cos\Omega\sin\Omega)^2}}. \quad (3.22)$$

Only the positive root c_0 satisfies the condition $c_{\min} < c_0 < c_{\max}$, and the integral for root c_1 is zero due to the Dirac delta function's property in (B.3). Note that for a given hyperbola with the focal length c_0 , θ can have four different values that represent the branches of the hyperbola, e.g. $\theta \in \{\theta_0, \pi - \theta_0, \pi + \theta_0, 2\pi - \theta_0\}$ where $\theta_0 \in [0, \pi/2]$. Using the property in (B.3), $\mathbf{h}_n(t)$ can be found as

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{\phi(c_0)} \frac{\mathbf{n}(\mathbf{r}'_p(c_0, \theta, \phi))}{|\Psi_{\text{hyp}}(c_0, \theta, \phi)|} \tilde{j}_{\text{hyp}}(c_0, \phi) d\phi, \quad (3.23)$$

where the focal length is given in (3.22). It should be emphasized that c_0 is a time dependent variable. Hitherto, line integral representation of $\mathbf{h}_n(t)$ for the hyperbola case is determined.

3.1.5. Line Case

If $\Delta = 0$, as defined in Appendix D and Table D.1, the quadratic curve equation in (3.6) denotes, in general, rotated and shifted two lines. These lines can be intersecting, coinciding or parallel lines. As a special case, if $\tilde{a} = 0, \tilde{b} = 0, \tilde{c} = 0$, then (3.6) denotes a single rotated and shifted line. Here, coinciding and parallel lines cases in Line Case 1 and single line case in Line Case 2 are considered. Intersecting lines case has not been encountered in simulations, nevertheless it can be considered as two separate single lines and the formulae given in this subsection can be extended easily.

• Line Case 1

If $\Delta = 0, \zeta = 0$, ($\tilde{a}, \tilde{b}, \tilde{c}$ are not zero all together), then (3.6) denotes coinciding or parallel lines. In this case, the coordinate transformation in (3.9) with $(\hat{g}, \hat{h}) = (0, 0)$ is applied to (3.6), where the rotation angle $\Omega = \arctan(\tilde{d}/\tilde{c})$, and $u, v \in (-\infty, \infty)$. As a result (3.6) is converted to a coinciding or parallel lines equation parallel to the v axis:

$$\hat{a}u^2 + \hat{b}u + \hat{f} = 0, \quad (3.24)$$

where coefficients $\hat{a}, \hat{b}, \hat{f}$ are given in Table 3.1. Applying the same transformation to (3.7) and following the similar steps as in ellipse case, $\mathbf{h}_n(t)$ can be found as

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{u_{\min}}^{u_{\max}} \int_{v(u)} \frac{\mathbf{n}(\mathbf{r}'_p(u, v))}{|\Psi_{\text{line}}(u_0, v)|} \delta(u - u_0) dv du, \quad (3.25)$$

where $u_{\min}$ and $u_{\max}$ are the minimum and maximum lines intersecting with the quadratic triangle as shown in Fig. 3.4 and $\Psi_{\text{line}}(u_0, v) = \partial F(u, v)/\partial u|_{u=u_0}$. Note that in line case there is no need for an additional coordinate transformation. In (3.25), u_0 ($u_{\min} < u_0 < u_{\max}$) is the root of the argument of the Dirac delta function. Due to the second order behavior, there are two roots:

$$u_{0,1} = \frac{-\hat{b} \pm \sqrt{\hat{b}^2 - 4\hat{a}\hat{f}}}{2\hat{a}}. \quad (3.26)$$

If the lines are coinciding then $u_0 = u_1$. As in previous cases, contribution of the two roots should be considered w.r.t. u . If $u_{0,1} < u_{\min}$ or $u_{\max} < u_{0,1}$, then $\mathbf{h}_n(t) = 0$, i.e. there is no intersection. At this point, it is assumed that only u_0 satisfies the condition $u_{\min} < u_0 < u_{\max}$, hence single integral due to u_0 contributes to $\mathbf{h}_n(t)$. Using (B.3), $\mathbf{h}_n(t)$ can be determined as:

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{v(u_0)} \frac{\mathbf{n}(\mathbf{r}'_p(u_0, v))}{|\Psi_{\text{line}}(u_0, v)|} dv. \quad (3.27)$$

If $u_{\min} < u_1 < u_{\max}$ then contribution of u_1 should be added to (3.27). Hitherto, line integral representation of $\mathbf{h}_n(t)$ for the coinciding or parallel lines cases is determined.

• Line Case 2

If $\tilde{a} = 0$, $\tilde{b} = 0$, and $\tilde{e} = 0$ in (3.6), then resulting expression denotes a single line. In this case, (3.6) reduces to

$$0 = \tilde{c}\alpha + \tilde{d}\beta - \tilde{f}. \quad (3.28)$$

Applying the same coordinate transformation to (3.6) results in a line equation parallel to v axis:

$$u = \tilde{f}/\sqrt{\tilde{c}^2 + \tilde{d}^2}. \quad (3.29)$$

Following the similar steps, $\mathbf{h}_n(t)$ can be found as in (3.27). Note that in single line case there is only one root, i.e. $u_0 = \tilde{f}/\sqrt{\tilde{c}^2 + \tilde{d}^2}$ ($u_{\min} < u_0 < u_{\max}$). If $u_0 < u_{\min}$ or $u_{\max} < u_0$, then $\mathbf{h}_n(t) = 0$, i.e. there is no intersection. Using (B.3), $\mathbf{h}_n(t)$ can be determined as in (3.27). Hitherto, line integral representation of $\mathbf{h}_n(t)$ for the single line case is determined.

3.1.6. Evaluation of the Line Integrals

Here, instead of searching for a closed-form expression to the integral in (3.16), (3.23), and (3.27), N_q point GLQR is applied to calculate the line integral over the

intersecting curve:

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \sum_{q=1}^{N_q} \frac{\mathbf{n}(\mathbf{r}'_p(a_0, \phi_q))}{|\Psi(a_0, \phi_q)|} \tilde{J}(a_0, \phi_q) w_q \Delta\phi, \quad (3.30)$$

where $\Delta\phi = \phi_{\max}(a_0) - \phi_{\min}(a_0)$, $\phi_q = \phi_{\min}(a_0) + p_q \Delta\phi$, p_q and w_q are the quadrature nodes and weights, respectively. In (3.30), ϕ and a_0 represent the integral variable and the root of variable of the Dirac delta function's argument, respectively. For example, for the ellipse case (3.30) can be written as

$$\mathbf{h}_n(t) = \frac{c}{2|\mathbf{k}_r|} \sum_{q=1}^{N_q} \frac{\mathbf{n}(\mathbf{r}'_p(a_0, \phi_q))}{|\Psi_{\text{elp}}(a_0, \phi_q)|} \tilde{J}_{\text{elp}}(a_0, \phi_q) w_q \Delta\phi, \quad (3.31)$$

where $\mathbf{r}'_p(a_0, \phi_q)$ can be determined using (C.1) and the barycentric coordinates

$$\begin{aligned} \alpha_q &= a_0 \cos(\phi_q) \cos \Omega - \sqrt{a_0^2 - c_{\text{elp}}^2} \sin(\phi_q) \sin \Omega + \hat{g} \\ \beta_q &= a_0 \cos(\phi_q) \sin \Omega + \sqrt{a_0^2 - c_{\text{elp}}^2} \sin(\phi_q) \cos \Omega + \hat{h}. \end{aligned} \quad (3.32)$$

Note that because of the second order behavior of the integrand in (3.7) and considering the effect of Jacobean and coordinate transformations, 4 point GLQR is enough to obtain exact results in the sense of machine precision as shown in numerical examples (see Figs. 3.5(b) and 3.6(b)). Hence there is no need to search for closed-form expressions of $\mathbf{h}_n(t)$.

3.2. Numerical Examples

In this section, numerical examples are presented to validate the proposed scheme in this section. In all examples, it is assumed that the scatterers are residing in an unbounded space with permittivity ε_0 , permeability μ_0 , and the speed of wave is $c = 1/\sqrt{\varepsilon_0\mu_0}$. A $\hat{\mathbf{p}}$ polarized plane wave travelling to $\hat{\mathbf{k}}_i$ direction, given in (2.1) and (2.2), is incident on the scatterers. The illuminated part of the scatterer is discretized with linear and quadratic triangles, and the PO integrals on linear triangles are calculated with the analytical expressions given in [17] directly in time domain, whereas the PO integrals

on quadratic triangles are calculated with the proposed scheme in this section. In all examples 4 point GLQR is chosen unless otherwise specified and the time step sizes are determined using $\Delta t = 1/(2f_{\max}N_{\text{os}})$, where $f_{\max}$ denotes the maximum frequency of interest and N_{os} denotes the oversampling factor ($5 \leq N_{\text{os}} \leq 7.5$). The relative error of a function $\Phi(m)$, $m = 1, \dots, M$, w.r.t. $\Phi^{\text{wrt}}(m)$ is calculated using

$$\text{Relative Error} = \sqrt{\frac{1}{M} \sum_{m=1}^M \frac{|\Phi(m) - \Phi^{\text{wrt}}(m)|^2}{|\Phi^{\text{wrt}}(m)|^2}}, \quad (3.33)$$

where $\Phi(m)$ can be time samples of $\mathbf{E}_{\text{rc}}^{\text{sca}}(\mathbf{r}, t)$ or $\mathbf{h}_n(t)$, frequency samples of RCS values at a given angle, while $\Phi^{\text{wrt}}(m)$ denotes the values used for comparison. All results in this section are produced in Matlab using a computer with Intel i5-4590 CPU and 4 GB RAM.

3.2.1. A Single Triangle

In this example, a single quadratic triangle with the nodes $\mathbf{r}_1 = (0.218, 0.705, 0.675)$, $\mathbf{r}_2 = (0.221, 0.531, 0.818)$, $\mathbf{r}_3 = (0.181, 0.3397, 0.923)$, $\mathbf{r}_4 = (0.409, 0.406, 0.817)$, $\mathbf{r}_5 = (0.6292, 0.356, 0.691)$, $\mathbf{r}_6 = (0.458, 0.5688, 0.6831)$ is analyzed to demonstrate the accuracy and validity of the proposed scheme. For this example, the time step size $\Delta t = 38.4$ ps, $\hat{\mathbf{p}} = \hat{\mathbf{x}}$, and $\hat{\mathbf{k}}_{\text{i}} = -\hat{\mathbf{z}}$. The observation direction $\hat{\mathbf{k}}_{\text{s}} = \hat{\mathbf{z}}$, resulting in $\mathbf{k}_{\text{r}} = \hat{\mathbf{z}}$. In this case, the intersection of the $\mathbf{k}_{\text{r}}$ -plane and the quadratic triangle yields hyperbolas.

Fig. 3.5(a) plots the $\hat{\mathbf{z}}$ -component of $\mathbf{h}_n(t)$ obtained for the quadratic triangle (QT-1), 6, 36, and 216 linear triangles (LT-6, LT-36, and LT-216). The linear triangles are obtained by repeatedly dividing the quadratic triangle to 6 linear and quadratic sub-triangles. Consequently, all linear triangles are obtained from quadratic sub-triangles. It can be seen in Fig. 3.5(a), the intersection starts and ends at $t = -6.158$ ns and $t = -4.505$ ns, respectively, and $\mathbf{h}_n(t)$ obtained for linear triangles converge to the result of the quadratic triangle as the number of linear triangles increases.

Next, the effect of the triangle size on the accuracy of the GLQR order is analyzed. The nodes of the quadratic triangle, denoted by $\text{QT} \times 1$, are multiplied by 10 and 100, denoted by $\text{QT} \times 10$ and $\text{QT} \times 100$, respectively, to expand the size of the triangle.

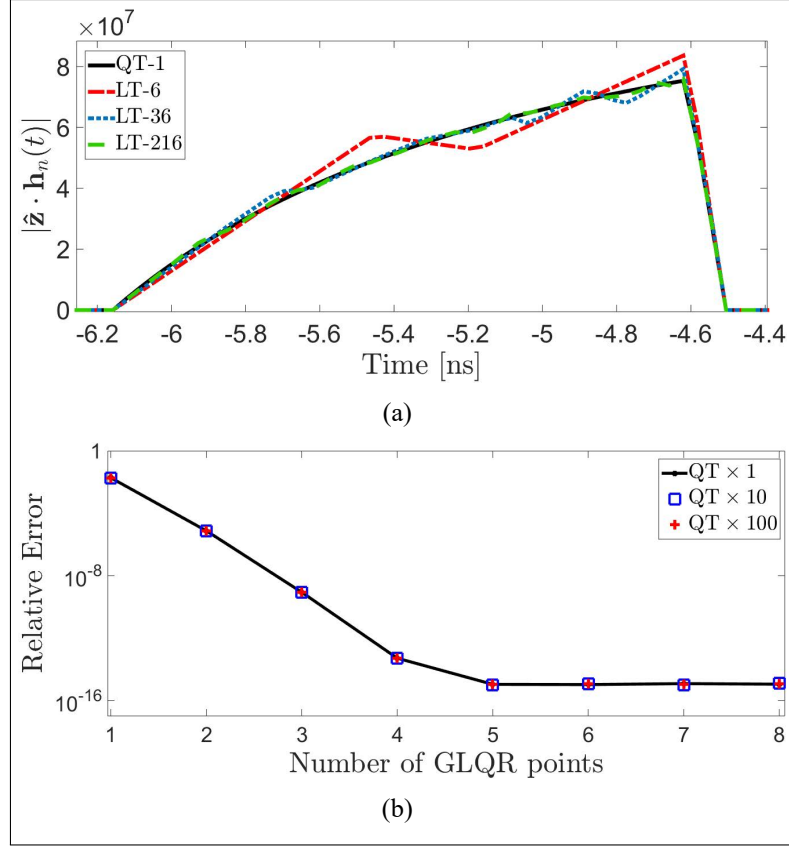


Figure 3.5: (a) $\hat{\mathbf{z}}$ -component of the PO integral and (b) change of the relative error in $\mathbf{h}_n(t)$ with the number of GLQR points for the single triangle.

Fig. 3.5(b) plots the relative error in $\mathbf{h}_n(t)$ for different number of GLQR points w.r.t. $\mathbf{h}_n(t)$ obtained for 9 point GLQR. In Fig. 3.5(b), the errors are calculated using 42, 429, and 4296 samples between $t \in [-6.158, -4.505]$ ns, $t \in [-61.58, -45.05]$ ns, and $t \in [-615.8, -450.5]$ ns for QT $\times$ 1, QT $\times$ 10, and QT $\times$ 100, respectively. It can be seen in Fig. 3.5(b) that after 4 point GLQR the relative error in $\mathbf{h}_n(t)$ is in order of machine precision and the size of the quadratic triangle does not have any effect on the accuracy of $\mathbf{h}_n(t)$. This example clearly shows that 4 point GLQR integrates $\mathbf{h}_n(t)$ exactly. It can be also shown that 2 point and 3 point GLQRs are enough to evaluate the PO integral exactly for lines and ellipses, respectively.

3.2.2. A Unit Sphere

In this example, scattering from a unit sphere centered to the origin is analyzed to examine the multiple curve types together. For this example, $\hat{\mathbf{k}}_i = -\hat{\mathbf{z}}$ and $\hat{\mathbf{p}} = \hat{\mathbf{x}}$, unless it is specified the observation direction $\hat{\mathbf{k}}_s = \hat{\mathbf{z}}$, hence $\mathbf{k}_r = \hat{\mathbf{z}}$. In this case, half

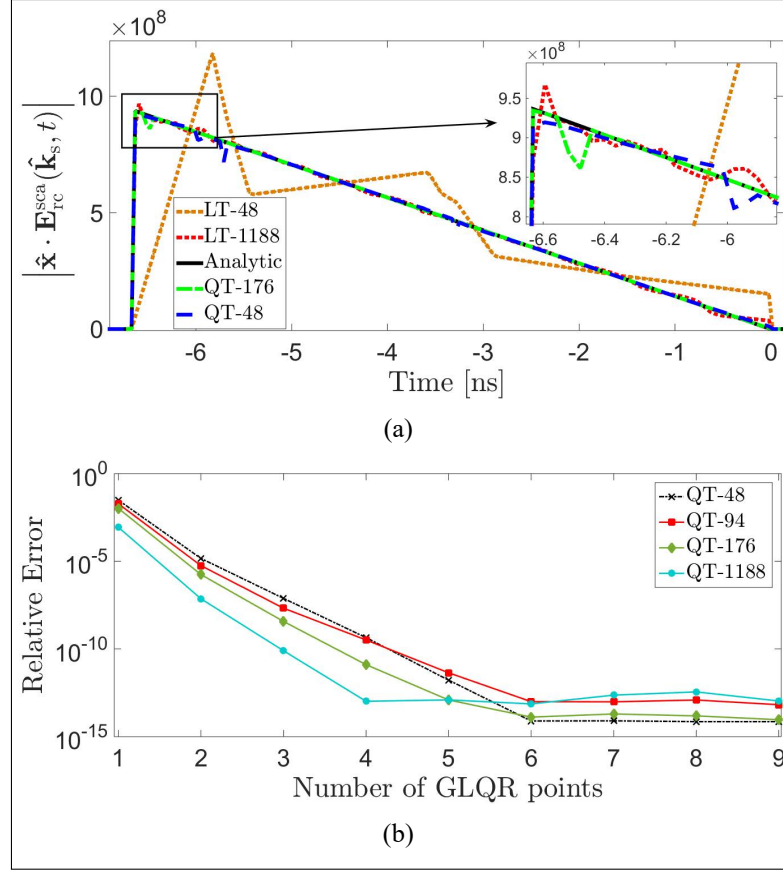


Figure 3.6: (a) $\hat{\mathbf{x}}$ -component of $\mathbf{E}_{\text{rc}}^{\text{sca}}(\mathbf{r}, t)$ and (b) change of the relative error in $\mathbf{E}_{\text{rc}}^{\text{sca}}(\mathbf{r}, t)$ with the number of GLQR points for the unit sphere.

of the sphere is illuminated, and analytical expression of the PO integral is available in frequency domain [45] and can be given in time domain as:

$$\mathbf{h}_{\text{analytic}}(t) = \begin{cases} \frac{-\pi c^2 t}{2} \hat{\mathbf{z}}, & 0 > \frac{ct}{2} > -1 \\ 0, & \text{otherwise.} \end{cases} \quad (3.34)$$

Fig. 3.6(a) shows the $\hat{\mathbf{x}}$ -component of the range-corrected scattered electric field $\mathbf{E}_{\text{rc}}^{\text{sca}}(\mathbf{r}, t)$ of the unit sphere, where the illuminated region of the sphere is modeled by 48 and 1188 linear triangles (LT-48 and LT-1188), 48 and 176 quadratic triangles (QT-48 and QT-176), and compares them with the analytical result obtained using (3.34) for the time step size $\Delta t = 38.4$ ps. The $\mathbf{k}_r$ -plane starts to intersect with the sphere at $t = -6.671$ ns. In Fig. 3.6(b), the effect of GLQR order is shown via comparing the relative error in $\mathbf{E}_{\text{rc}}^{\text{sca}}(\mathbf{r}, t)$ for quadratic triangles and different number of GLQR points. Relative error is calculated as in (3.33) w.r.t. the result obtained for 10 point GLQR

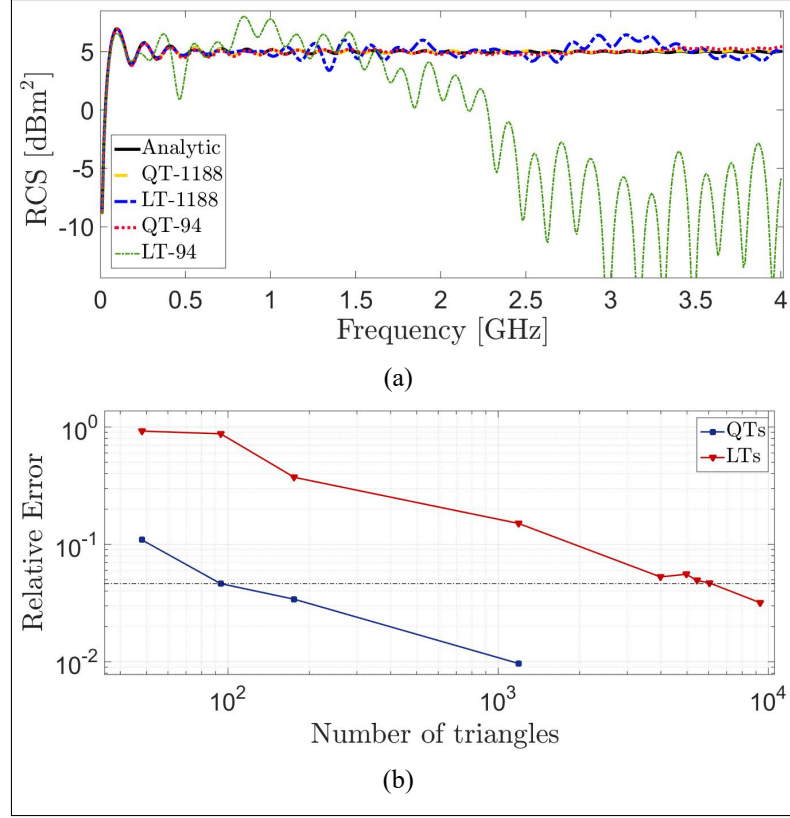


Figure 3.7: (a) Backscattered RCS of the unit sphere and (b) relative error in the backscattered RCS w.r.t. the analytical solution.

using the time samples between $t = -6.671$ ns and $t = 0$ s. Considering the results shown in Figs. 3.6(a) and (b), all errors are practically in machine precision above 4 point GLQR. It can be concluded that using 4 point GLQR is enough to integrate $\mathbf{h}_n(t)$ exactly.

Next the RCS of the unit sphere is analyzed. The RCS value (σ) at a given direction $\hat{\mathbf{k}}_s$ is obtained by discrete Fourier transforming the time samples of $\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$ for the time step size $\Delta t = 19.2$ ps, i.e. $\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, f)$, and using $\sigma = (k^2/\pi) \left| \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, f) \right|^2$, where $k = 2\pi f/c$ is the wave number, since the Fourier transform of the incident wave is unitary. In Fig. 3.7(a), the backscattered RCS values obtained using 94 and 1188 linear and quadratic triangles (LT-94, LT-1188, QT-94, and QT-1188) in 0-4 GHz frequency band with steps $\Delta f = 10$ MHz are compared with the analytical PO solution of the unit sphere given in [45], i.e. $\sigma_a = \pi r_a^2 |1 - \exp[-jkr_a] \text{sinc}(kr_a)|^2$ where r_a denotes the radius of the sphere. Note that the RCS results obtained using QT-94 and QT-1188 are coinciding with the analytical result roughly till 750 MHz and 3 GHz, respectively, whereas the results for LT-94 and LT-1188 are inaccurate above 250 MHz

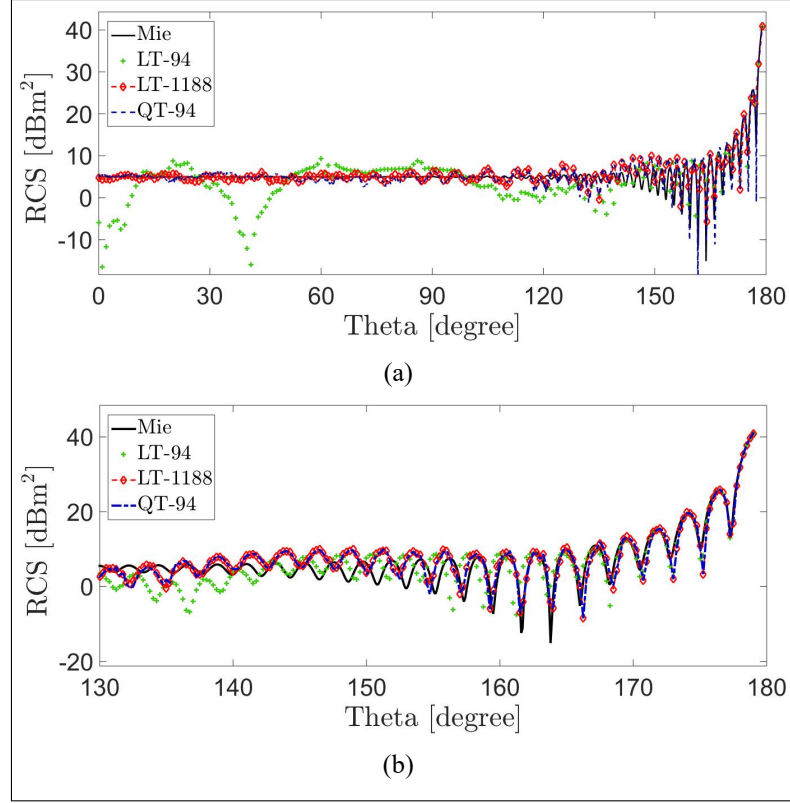


Figure 3.8: Bistatic RCS comparison of the unit sphere at 4 GHz:
(a) $\theta \in [0, 180]^\circ$ and (b) $\theta \in [130, 180]^\circ$.

and 1 GHz. Fig. 3.7(b) shows the relative error in RCS values calculated using the frequency samples in 2-4 GHz band w.r.t. the analytical result for different number of linear and quadratic triangles (LTs and QTs). It can be seen from Fig. 3.7(b) that the error obtained using 94 quadratic and 5444 linear triangles are in the same level, and the solution time for 94 quadratic triangles is 0.301 s, whereas the solution time for 5444 linear triangles is 0.631 s. Fig. 3.8 plots the bistatic RCS results at 4 GHz obtained using 94 and 1188 linear, and 94 quadratic triangles (LT-94, LT-1188, and QT-94) for the same incident field and compares them with the Mie series solution. The observation direction $\hat{\mathbf{k}}_s$ is chosen as $\theta \in [0, 180]^\circ$ and $\phi = 0^\circ$. This results in different $\mathbf{k}_r$ directions. It is seen from Figs. 3.7 and 3.8 that small number of quadratic triangles is enough to obtain accurate results.

3.2.3. A Generic Missile

In this subsection, a missile-like scatterer with the maximum length, width, and height of 3.325 m, 1.74 m, and 0.51 m, respectively, as shown in inset of Fig. 3.9(b), is

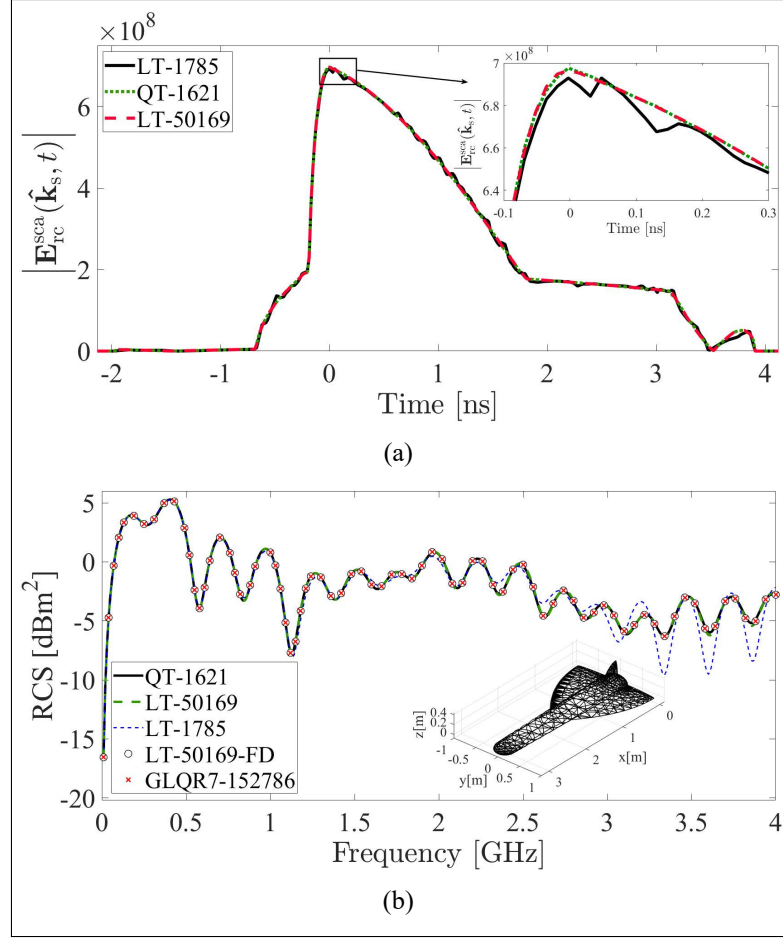


Figure 3.9: (a) Norm of $\mathbf{E}_{rc}^{sca}(\mathbf{r}, t)$ and (b) RCS comparison for the generic missile scatterer.

analyzed. Note that the scatterer does not have any flat parts. A $\hat{\mathbf{p}} = (0.866, 0, -0.5)$ polarized incident field illuminates the scatterer from $\hat{\mathbf{k}}_i = (-0.5, 0, -0.866)$ direction. The scattered fields are observed on $\hat{\mathbf{k}}_s = (-0.866, 0, 0.5)$ direction. In this case illuminated part of the scatterer's surface is modeled by 1621 quadratic triangles (QT-1621), 1785 and 50169 linear triangles (LT-1785 and LT-50169). The time step size $\Delta t = 16.667$ ps. Fig. 3.9(a) shows the norm of the range-corrected scattered electric field, where the $\mathbf{k}_r$ -plane starts to intersect with the scatterer at $t = -1.967$ ns and intersection ends at $t = 3.9$ ns. It can be seen that the results for QT-1621 and LT-50169 are coinciding very-well. The solution times for QT-1621, LT-1785, and LT-50169 are 3.978 s, 0.326 s, and 4.975 s, respectively. Fig. 3.9(b) shows the RCS values that are obtained from the time domain samples of $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$, as explained in previous subsection, in 0-4 GHz frequency band. In addition, in Fig. 3.9(b), the frequency domain PO solutions obtained using the closed-form expressions given in [17]

and 50169 linear triangles (LT-50169-FD), and using 7-point GLQR for triangles and 152876 linear triangles (GLQR7-152876) are included to compare the accuracy of the time domain results. It can be seen that all results except the results for LT-1785 are coinciding very-well. Note that the relative errors, which are calculated using (3.33), in the backscattered RCS values w.r.t. frequency domain GLQR7-152786 solution using frequency samples in 0-4 GHz frequency band with steps $\Delta f = 10$ MHz, are 0.0184, 0.164, and 0.0179 for QT-1621, LT-1785, and LT-50169, respectively.

3.3. Conclusions

In this chapter, the exact evaluation of the PO integral in time domain on quadratic triangular patches using Radon transform interpretation is presented. The scatterers are assumed to be illuminated by a plane wave with Dirac delta time signature and the scattered far-fields are observed. Using the Radon transform interpretation, the PO integral on quadratic triangles is reduced to a line integral over a quadratic curve, e.g. an ellipse, a hyperbola or a line/lines in barycentric coordinates. The quadratic curve is obtained by the intersection of the quadratic triangle's surface and the plane which is formed by the incident field direction and the observation direction. For the evaluation of the line integral, first an appropriate coordinate transformation (e.g. elliptic coordinates for ellipse) is applied to parametrize the quadratic curve, then, instead of searching for a closed-form expression, a suitable order GLQR is applied to determine the PO integral exactly. It is shown that using 4 point GLQR provides the PO integral in machine precision independent from the size of the quadratic triangle and the numerical errors caused by the evaluation of the PO integral are removed. In addition, when compared with linear triangles, modeling errors in representing the scatterer's surface are reduced because of the quadratic triangles.

4. QUADRATIC EDGES

In this chapter, the closed-form evaluation of the time domain equivalent edge current (EEC) contour integral on quadratic edges of quadratic triangular patches, is presented. In [11], the EEC representation is used to determine both the PO and diffraction contributions to the scattered field for linear surfaces, e.g. linear triangular patches, in addition, for linear edges of linear triangular patches the radiation integral in the EEC representation can be evaluated in closed-form in time domain using Radon transform interpretation [49]. However, in this chapter the quadratic surfaces and edges are used to model the surface of the scatterers. For the quadratic edges, the EEC formulation in [11] cannot be directly used to determine the PO contribution, however diffraction contributions can still be determined, since diffraction is a local phenomenon. Therefore, in this thesis, unlike the EEC formulation in [11], where the PO and EEC integrals are both expressed as contour integrals, the PO and EEC integrals are evaluated separately: the PO integral is evaluated on the surface of the quadratic triangular patch to determine the PO contribution as shown in Chapter 3, and the EEC integral is evaluated on the edge of the quadratic triangle to determine the diffraction contribution. Using a similar procedure presented in Chapter 3, the EEC integral on the quadratic edges are determined in closed-form using the Radon transform interpretation of the radiation integrals. Specifically, it is shown that the EEC integral in time domain can be determined in closed-form as a function evaluation at the intersection point of the edge of the quadratic triangle and the plane formed by the incident plane wave and observation direction.

The rest of the chapter is organized as follows: time domain EEC formulation is presented in Section 4.1.1, and in Section 4.1.2 closed-form evaluation of the EEC integral is shown. In Section 4.2, the numerical examples that demonstrate the accuracy and validity of the developed closed-form expressions in this chapter are presented. In Section 4.3, conclusions are drawn. Lastly, the diffraction coefficients and associated parameters used in this chapter are given in Appendix E.

4.1. Formulation

4.1.1. Time Domain EEC

Let S_{lit} and l denote the illuminated surface and edge of a PEC scatterer, residing in an unbounded medium with permittivity ε_0 and permeability μ_0 , by an incident plane wave with a Dirac delta time dependence, and S_{lit} is discretized with quadratic triangular patches as $S_{\text{lit}} \simeq \bigcup_{n=1}^N S_n$, as explained in Chapter 3. Hence, l can be represented by the quadratic edge segments l_n attached to S_n , as $l \simeq \bigcup_{n=1}^{\tilde{N}} S_n$. For the time domain EEC, the scattered electric far-field due to an impulsively excited plane wave is expressed as

$$\mathbf{E}^{\text{sca}}(\mathbf{r}, t) = \frac{\delta(t - r/c)}{4\pi r} * \bar{\mathbf{E}}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t), \quad (4.1)$$

where $\bar{\mathbf{E}}_{\text{rc}}^{\text{sca}}(t)$ is the range-corrected scattered electric field due to EEC, which is only related to diffraction. Note that the PO contribution should be added to the scattered electric field separately using (3.1). $\bar{\mathbf{E}}_{\text{rc}}^{\text{sca}}(t)$ can be expressed as

$$\bar{\mathbf{E}}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = \sum_{n=1}^{\tilde{N}} \mathbf{s}_n(t). \quad (4.2)$$

In (4.2), $\mathbf{s}_n(t)$ is called EEC integral in this thesis and defined as [11]

$$\mathbf{s}_n(t) = \int_{l_n} \delta\left(t - \frac{2}{c} \mathbf{k}_r \cdot \mathbf{r}'\right) \mathbf{A}(\mathbf{r}') dl', \quad (4.3)$$

where $\mathbf{A}(\mathbf{r}')$ is given as

$$\begin{aligned} \mathbf{A}(\mathbf{r}') = & \left\{ \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \hat{\mathbf{t}}_n(\mathbf{r}') \left[D_{\text{e},n}^I \hat{\mathbf{p}} \cdot \hat{\mathbf{t}}_n(\mathbf{r}') + D_{\text{h},n}^I (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{t}}_n(\mathbf{r}') \right] \right. \\ & \left. + \hat{\mathbf{k}}_s \times \hat{\mathbf{t}}_n(\mathbf{r}') \left[D_{\text{h},n}^M (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{t}}_n(\mathbf{r}') \right] \right\}. \end{aligned} \quad (4.4)$$

Here, $\hat{\mathbf{t}}_n(\mathbf{r})$ is the unit tangent vector along the edge l_n (see Fig. 4.1), the diffraction coefficients $D_{\text{e},n}^I$, $D_{\text{h},n}^I$, $D_{\text{h},n}^M$ are given in Appendix E, and parameters of the diffraction

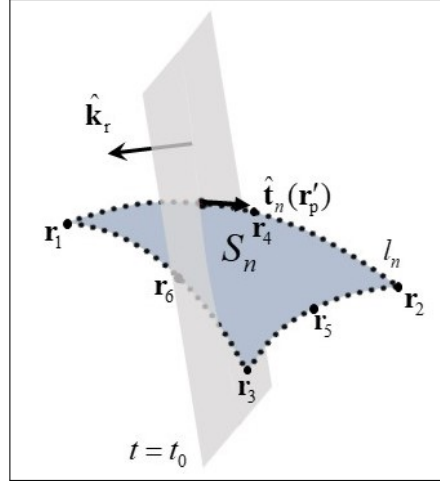


Figure 4.1: Intersection of the quadratic edge and the $\mathbf{k}_r$ -plane.

coefficients are shown in Fig. 4.2. Note that even it is not shown explicitly, $s_n(t)$ depends on $\hat{\mathbf{k}}_s$, $\hat{\mathbf{k}}_i$, $\hat{\mathbf{t}}_n(\mathbf{r})$, $D_{e,n}^I$, $D_{h,n}^I$, and $D_{h,n}^M$. For the edge of a linear triangular patch, $\hat{\mathbf{t}}_n(\mathbf{r})$ and the diffraction coefficients are constant along the edge, however for the quadratic edges, they are not constant throughout the segment l_n , therefore they should be calculated for every time instance while evaluating $s_n(t)$.

4.1.2. Evaluation of $s_n(t)$

The quadratic triangular patch is defined in barycentric coordinates using the shape functions given in Appendix C. Assuming that the nodes of the quadratic triangle S_n , i.e. $\mathbf{r}_i$, $i = 1, \dots, 6$, are ordered as shown in Fig. 4.1, at a given time $t = t_0$, $s_n(t)$ can be evaluated as follows:

Step 1: Rotate the order of the nodes such that the interested edge l_n is always defined by $\mathbf{r}_i$, $i = 1, 3, 6$, nodes of the quadratic triangular patch, i.e. the edge with the barycentric coordinate $\alpha = 0$. As a result, the EEC contour integral $s_n(t)$ can be evaluated for $\alpha = 0$.

Step 2: The Dirac delta function in (4.3) defines the $\mathbf{k}_r$ -plane at the given time $t = t_0$, i.e. $ct_0 = 2\mathbf{k}_r \cdot \mathbf{r}'$. Inserting the barycentric coordinates defined on the quadratic edge/triangle into the plane equation results in the quadratic equation in (3.6). The solution of (3.6) for $\alpha = 0$ yields the intersection point ($\mathbf{r}'_p$ and β_0 in barycentric coordinates) on the quadratic edge (see Fig 4.1). Note that,

if there is no intersection, then $s_n(t_0) = 0$.

Step 3: Using the Dirac delta function's property given in (B.1) and transforming the coordinates to barycentric coordinates of the quadratic triangular patch, the EEC integral $s_n(t)$ in (4.3) can be expressed as

$$\begin{aligned} s_n(t) &= \frac{c}{2|\mathbf{k}_r|} \int_{l_n} \delta \left(\frac{ct}{2|\mathbf{k}_r|} - x'_p(0, \beta) \right) \mathbf{A}(\mathbf{r}'_p(0, \beta)) J(\alpha, \beta) d\beta \\ &= \frac{c}{2|\mathbf{k}_r|} \int_{l_n} \delta \left(\frac{ct}{2|\mathbf{k}_r|} - x'_p(0, \beta) \right) \mathbf{A}(\mathbf{r}'_p(0, \beta)) |\mathbf{t}_n(\mathbf{r}'_p(0, \beta))| d\beta, \end{aligned} \quad (4.5)$$

where $J(\alpha, \beta)$ is Jacobian, defined as $J(\alpha, \beta) = |\mathbf{t}_n(\mathbf{r}'_p(0, \beta))|$. Note that in (4.5), $x'_p(0, \beta)$ is a quadratic expression in terms of β as given in (3.6).

Step 4: Using the Dirac delta function's property given in (B.2), (4.5) is reduced to

$$s_n(t) = \frac{c}{2|\mathbf{k}_r|} \int_{\beta} \frac{|\mathbf{t}_n(\mathbf{r}'_p(0, \beta))|}{|\Psi(\mathbf{r}'_p(0, \beta_0))|} \delta(\beta - \beta_0) \mathbf{A}(\mathbf{r}'_p(0, \beta)) d\beta, \quad (4.6)$$

where

$$\Psi(0, \beta_0) = \frac{\partial}{\partial \beta} \left[\frac{ct}{2|\mathbf{k}_r|} - x'_p(0, \beta) \right]_{\beta=\beta_0}, \quad (4.7)$$

and β_0 is the barycentric coordinate of the intersection point $\mathbf{r}'_p$ with $(\alpha, \beta) = (0, \beta_0)$.

Step 5: Using the sampling property of the Dirac delta function in (B.1), the closed-form expression of the EEC integral $s_n(t)$ at $(t, \mathbf{r}) = (t_0, \mathbf{r}'_p)$ can be obtained as:

$$\begin{aligned} s_n(t) &= \frac{|\mathbf{t}_n(\mathbf{r}'_p(0, \beta_0))|}{|\Psi(\mathbf{r}'_p(0, \beta_0))|} \left\{ \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \hat{\mathbf{t}}_n(\mathbf{r}'_p(0, \beta_0)) \right. \\ &\quad \left[D_{e,n}^I \hat{\mathbf{p}} \cdot \hat{\mathbf{t}}_n(\mathbf{r}'_p(0, \beta_0)) + D_{h,n}^I (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{t}}_n(\mathbf{r}'_p(0, \beta_0)) \right] \\ &\quad \left. + \hat{\mathbf{k}}_s \times \hat{\mathbf{t}}_n(\mathbf{r}'_p(0, \beta_0)) \left[D_{h,n}^M (\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{t}}_n(\mathbf{r}'_p(0, \beta_0)) \right] \right\} \end{aligned} \quad (4.8)$$

The parameters for $s_n(t)$ are determined using the intersection point $\mathbf{r}'_p$. It

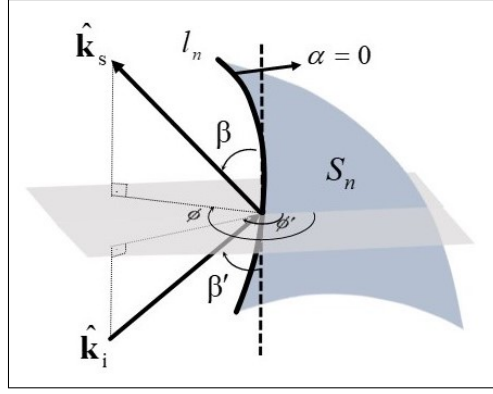


Figure 4.2: Parameters for the edge diffraction.

should be emphasized that $\hat{\mathbf{t}}_n(\mathbf{r})$, $D_{e,n}^I$, $D_{h,n}^I$, and $D_{h,n}^M$ are not constant through a quadratic edge and should be calculated for each time step and intersection point.

4.2. Numerical Examples

In this section, the evaluation of time domain EEC contributions to determine the diffraction from edges of the scatterers is considered. However, in the numerical examples the PO contributions are also included using the exact evaluation scheme presented in Chapter 3. In all examples presented in this section, the range-corrected scattered electric field of PO and EEC are denoted by $\mathbf{E}_{\text{rc}}^{\text{PO}}(\hat{\mathbf{k}}_s, t)$ and $\mathbf{E}_{\text{rc}}^{\text{EEC}}(\hat{\mathbf{k}}_s, t)$, and determined using (3.3) and (4.2), respectively. Hence, the total scattered electric field at far-zone can be written as

$$\begin{aligned} \mathbf{E}_{\text{tot}}^{\text{sca}}(\mathbf{r}, t) &= -\frac{\partial_t \delta(t - r/c)}{2\pi r c} * \mathbf{E}_{\text{rc}}^{\text{PO}}(\hat{\mathbf{k}}_s, t) \\ &\quad + \frac{\delta(t - r/c)}{4\pi r} * \mathbf{E}_{\text{rc}}^{\text{EEC}}(\hat{\mathbf{k}}_s, t) \\ &= -\frac{\partial_t \delta(t - r/c)}{2\pi r c} * \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t), \end{aligned} \quad (4.9)$$

where $\mathbf{E}_{\text{rc}}^{\text{sca}}(t)$ is the total range-corrected scattered electric field, which consists of the contributions from PO and EEC:

$$\mathbf{E}_{\text{rc}}^{\text{sca}}(t) = \mathbf{E}_{\text{rc}}^{\text{PO}}(\hat{\mathbf{k}}_s, t) - 2c^{-1} \partial_t^{-1} \mathbf{E}_{\text{rc}}^{\text{EEC}}(\hat{\mathbf{k}}_s, t), \quad (4.10)$$

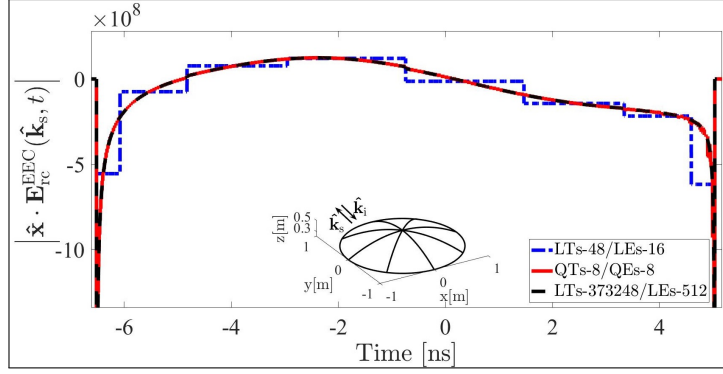


Figure 4.3: Comparison of the $\hat{x}$ -component of the range-corrected scattered electric field $\mathbf{E}_{rc}^{EEC}(\hat{\mathbf{k}}_s, t)$ for the parabolic scatterer.

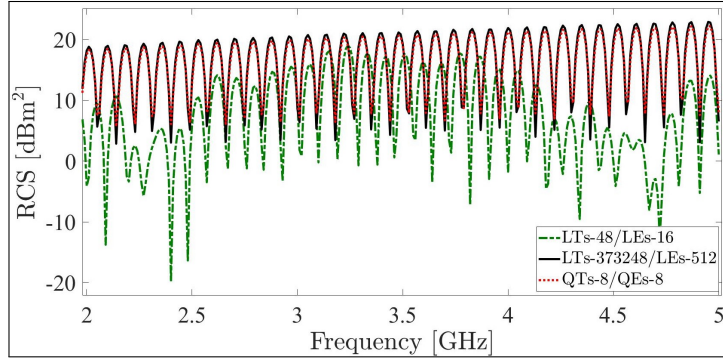


Figure 4.4: Comparison of RCS results for the parabolic scatterer.

where ∂_t^{-1} denotes the temporal integration. In all examples, it is assumed that the scatterers are residing in an unbounded space with permittivity ε_0 , permeability μ_0 , and the speed of wave is $c = 1/\sqrt{\varepsilon_0\mu_0}$, and a $\hat{\mathbf{p}}$ polarized plane wave travelling to $\hat{\mathbf{k}}_i$ direction, given in (2.1) and (2.2), is incident on the scatterers. In the figures presented in this section, quadratic and linear triangles abbreviated as QTs and LTs, while quadratic and linear edges abbreviated as QEs and LEs, respectively.

4.2.1. A Parabolic Scatterer

A parabolic scatterer with radius of 0.5 m and focal depth of 0.9 m (shown in Fig. 4.3), is analyzed to show the accuracy and validity of the proposed scheme. The scatterer is illuminated from incident angles $\theta = 120^\circ$ and $\varphi = 0^\circ$, and backscattered fields are observed ($\hat{\mathbf{k}}_s = -\hat{\mathbf{k}}_i$), the time step size $\Delta t = 0.999$ ps.

Fig. 4.3 plots the $\hat{x}$ -component of the range-corrected scattered electric fields due to the time domain EEC for QTs-8/QEs-8, LTs-48/LEs-16, and LTs-373248/LEs-512.

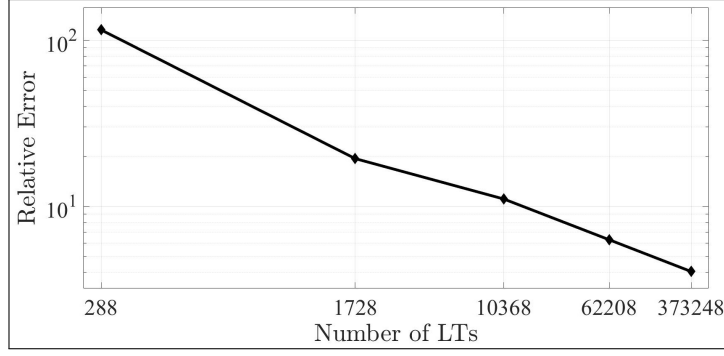


Figure 4.5: Error in RCS results obtained for LTs w.r.t. the RCS obtained for 8 QTs.

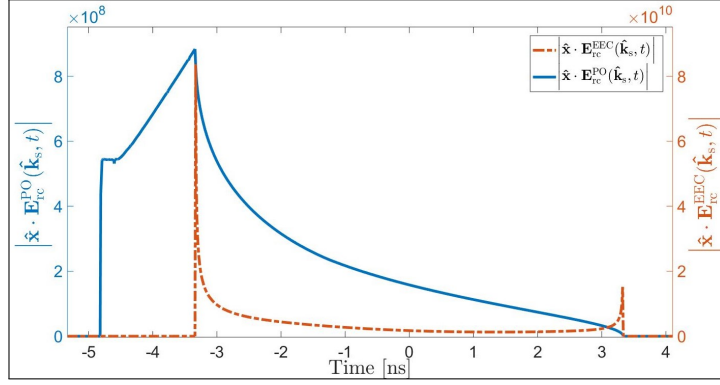


Figure 4.6: $\hat{x}$ -component of $\mathbf{E}_{rc}^{EEC}(\hat{\mathbf{k}}_s, t)$ and $\mathbf{E}_{rc}^{PO}(\hat{\mathbf{k}}_s, t)$ for QTs-1144/QEs-66 for the composite scatterer.

In this setup, $\hat{\mathbf{k}}_r = \hat{\mathbf{k}}_i$, the plane starts to intersect with the scatterer at $t = -7.445$ ns, and the intersection ends at $t = 5.036$ ns. Fig. 4.4 plots the RCS (including the both PO and EEC contributions) obtained by Fourier transforming the time domain PO and EEC results for QTs-8/QEs-8, LTs-48/LEs-16, and LTs-373248/LEs-512. It can be seen from Figs. 4.3 and 4.4 that the results for LEs/LTs converges to the results for QTs-8/QEs-8 as the number of LTs increases. Fig. 4.5 plots the relative error in the RCS results obtained for LTs-(288-373248)/LEs-(48-512) w.r.t. the RCS results obtained for QTs-8/QEs-8. The convergence of the results can be seen clearly.

4.2.2. A Composite Scatterer

A composite scatterer located in free space is analyzed to show the accuracy of the proposed scheme. The upper part of the scatterer is a parabolic surface with radius of 0.5 m and connected to a truncated circular cone with the height of 0.75 m, upper and lower radii of 0.5 m and 1 m. A $\hat{\mathbf{p}} = (-0.866, 0, -0.5)$ polarized plane wave

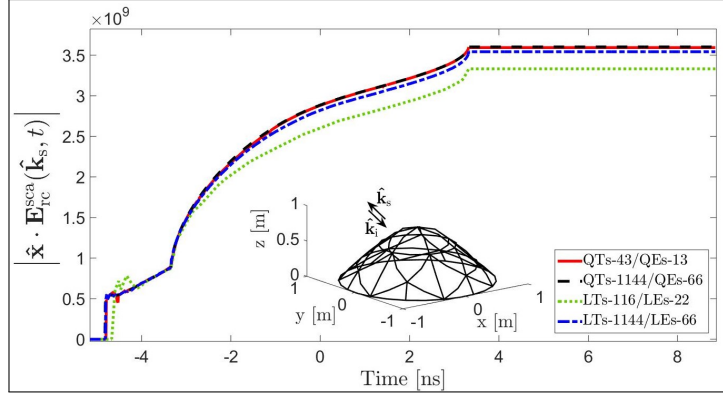


Figure 4.7: Comparison of the $\hat{x}$ -component of the total range-corrected scattered electric field $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ for the composite scatterer.

propagating to $\hat{\mathbf{k}}_i = (0.5, 0, -0.866)$ direction with the Dirac delta time signature, and the backscattered field is observed with the time steps size $\Delta t = 9.994$ ps. In this case, all scatterer's surface is illuminated and $\mathbf{k}_r = \hat{\mathbf{k}}_i$. The illuminated part of the scatterer is modeled by QTs-43/QEs-13 and QTs-1144/QEs-66, and for comparison it is modeled by LTs-116/LEs-22 and LTs-1144/LEs-66. 4-point GLQR is used to evaluate the PO integrals exactly.

Fig. 4.6 shows the $\hat{x}$ -component of the range-corrected scattered electric fields $\mathbf{E}_{rc}^{PO}(\hat{\mathbf{k}}_s, t)$ and $\mathbf{E}_{rc}^{EEC}(\hat{\mathbf{k}}_s, t)$ for QTs-1144/QEs-66, and Fig. 4.7 shows the comparison of the $\hat{x}$ -component of the range-corrected scattered electric field $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ as defined in (4.10) for different number of triangles/edges. It can be seen that the $\mathbf{k}_r$ -plane starts to intersect with the scatterer at $t = -4.826$ ns and intersection ends at $t = 3.336$ ns, and the results obtained using LTs/LEs converge to the results obtained using QTs/QEs.

4.3. Conclusions

In this chapter, the closed-form evaluation of the EEC contour integral on quadratic edges is presented. The closed-form expression is obtained by intersecting the plane, which is formed by the incidence and observation directions, and the quadratic edge segment. The numerical examples demonstrate the accuracy and efficiency of the developed closed-form expression, since accurate results can be obtained using less number of surface and edge elements. The modeling error of curved scatterers is reduced due to the use of quadratic triangle patches and the error in the calculation of the

radiation integrals is removed due to the closed-form evaluation of the EEC integral.

5. ANALYSIS OF IMPEDANCE SURFACES

In this chapter, the analysis of transient scattering from impedance surfaces using the PO approximation is presented. Specifically, the Leontovich impedance boundary condition is used as the surface impedance, whereas the formulation given in this chapter can be extended for higher order surface impedances [50, 51]. The time domain PO approximation for impedance surfaces has two difficulties: (i) the surface impedance approximations are usually formulated in frequency domain, which results in a frequency dependent surface impedance function [3, 13, 15, 16, 50–52]. For a time domain analysis, time domain counterpart of the surface impedance function is required [51–54]. However, it is not always possible to find a closed-form expression of the time domain surface impedance (or time dependent functions related with the PO approximation, see Section 5.1.1). To overcome this difficulty, a similar procedure given in [55] for time domain integral equations is followed. Particularly, a weighted rational function fit is employed to the frequency samples of the surface impedance (or related frequency dependent functions) using the fast relaxed vector fitting (FRVF) scheme [56–58]. Time domain functions are then obtained analytically by inverse Fourier transforming the rational function fit, which results in a weighted summation of the decaying exponential functions. Note that the rational function fit is usually employed to find the time domain response of the dispersive permittivity, e.g. [55, 59], or impedance, e.g. [51, 53, 54], whereas in this chapter it is employed to determine the time domain response of the functions involving reflection coefficients, incidence direction, surface orientation, and impedance function (see Section 5.1.1). (ii) Unlike the frequency domain counterpart, the scattered fields should be determined by the convolution of the time domain surface impedance and the PO integral. This convolution can be evaluated in several ways, e.g. using recursive convolution method [51, 53, 54, 59–61]. In this scheme, the time samples of the time domain surface impedance and the PO integral are calculated separately, then they are convolved numerically. It is also possible to evaluate this convolution analytically [55, 62], which can be preferred to eliminate the time samples of rational function fit. Note that very preliminary results of the developed scheme in this chapter are presented in [63], in which only planar scatterers with a constant (frequency independent) surface impedance can be analyzed.

The rest of the chapter is organized as follows: in Section 5.1, time domain PO formulation for surfaces that has Leontovich impedance boundary condition as the surface impedance, is presented. In Section 5.1.1, rational function fit to the time dependent surface impedance is explained. In Section 5.1.2, implementation details and special cases are discussed, and in Section 5.2, frequency domain formulation is briefly presented. In Section 5.3, numerical examples are presented and lastly, in Section 5.4 conclusions are drawn.

5.1. Time Domain Formulation

Let S^{imp} denote the surface of a scatterer with a time dependent surface impedance $Z(t)$ [13, 52], residing in an unbounded medium with permittivity ε_0 , permeability μ_0 , and intrinsic impedance η_0 . S^{imp} is excited by an incident plane wave with a Dirac delta time signature, the electric and magnetic field components of the incident wave is given in (2.1) and (2.2), respectively. Using the Leontovich impedance boundary condition in time domain, i.e. $\forall \mathbf{r} \in S^{\text{imp}}$ [52]

$$-\hat{\mathbf{n}}(\mathbf{r}) \times \hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{E}^{\text{tot}}(\mathbf{r}, t) = Z(t) * \hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{\text{tot}}(\mathbf{r}, t), \quad (5.1)$$

the PO approximated equivalent electric and magnetic current densities on the impedance surface S^{imp} can be given in terms of the total magnetic field $\mathbf{H}^{\text{tot}}(\mathbf{r}, t)$ as [3]

$$\mathbf{J}(\mathbf{r}, t) = \begin{cases} \hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{\text{tot}}(\mathbf{r}, t); & \mathbf{r} \in S_{\text{lit}}^{\text{imp}} \\ 0; & \mathbf{r} \in S_{\text{sha}}^{\text{imp}}, \end{cases} \quad (5.2)$$

$$\mathbf{M}(\mathbf{r}, t) = \begin{cases} -Z(t) * \hat{\mathbf{n}}(\mathbf{r}) \times \hat{\mathbf{n}}(\mathbf{r}) \times \mathbf{H}^{\text{tot}}(\mathbf{r}, t); & \mathbf{r} \in S_{\text{lit}}^{\text{imp}} \\ 0; & \mathbf{r} \in S_{\text{sha}}^{\text{imp}}, \end{cases} \quad (5.3)$$

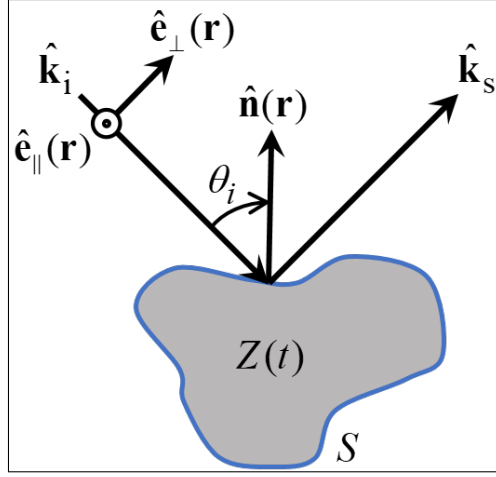


Figure 5.1: Definitions of the unit vectors on the impedance scatterer.

where $S_{\text{lit}}^{\text{imp}}$ and $S_{\text{sha}}^{\text{imp}}$ are illuminated and shadowed part of the impedance scatterer, and $\mathbf{H}^{\text{tot}}(\mathbf{r}, t)$ is approximated as [13]

$$\mathbf{H}^{\text{tot}}(\mathbf{r}, t) = \eta_0^{-1} \delta(t - \hat{\mathbf{k}}_i \cdot \mathbf{r}/c) * \mathbf{u}(\mathbf{r}, t). \quad (5.4)$$

Here $\mathbf{u}(\mathbf{r}, t)$ is a function of time, position, incidence direction, surface orientation and defined as

$$\mathbf{u}(\mathbf{r}, t) = \tilde{R}_{\parallel}(t) \left[(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) + \tilde{R}_{\perp}(t) \left[(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\perp}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\perp}(\mathbf{r}), \quad (5.5)$$

where $\hat{\mathbf{e}}_{\parallel}(\mathbf{r})$ and $\hat{\mathbf{e}}_{\perp}(\mathbf{r})$ are unit vectors in the parallel and perpendicular directions, respectively, and θ_i is the narrow angle between the propagation direction of the plane wave $\hat{\mathbf{k}}_i$ and the surface normal $\hat{\mathbf{n}}(\mathbf{r})$, as shown in Fig. 5.1. $\tilde{R}_{\perp}(t) = 1 - R_{\perp}(t)$ and $\tilde{R}_{\parallel}(t) = 1 - R_{\parallel}(t)$, where $R_{\perp}(t)$ and $R_{\parallel}(t)$ are functions of the surface impedance and their evaluation is explained in Section 5.1.1 in detail. Scattered fields have contributions from both the electric and magnetic currents, and the scattered electric field at the far-zone can be found as [46, 47]

$$\mathbf{E}^{\text{sca}}(\mathbf{r}, t) = \frac{\partial_t \delta(t - r/c)}{4\pi r c} * \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t), \quad (5.6)$$

where ∂_t denotes the temporal derivative and $\mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$ is the range-corrected scattered electric field [46, 47]:

$$\begin{aligned} \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = & \eta_0 \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \int_{S^{\text{imp}}} \mathbf{J}(\mathbf{r}', t) * \delta(t - \hat{\mathbf{k}}_s \cdot \mathbf{r}'/c) d\mathbf{r}' \\ & + \hat{\mathbf{k}}_s \times \int_{S^{\text{imp}}} \mathbf{M}(\mathbf{r}', t) * \delta(t - \hat{\mathbf{k}}_s \cdot \mathbf{r}'/c) d\mathbf{r}'. \end{aligned} \quad (5.7)$$

Substituting (5.2)-(5.4) in (5.7) yields

$$\begin{aligned} \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = & \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \int_{S_{\text{lit}}^{\text{imp}}} \left[\hat{\mathbf{n}}(\mathbf{r}') \times \mathbf{u}(\mathbf{r}', t) * \delta\left(t - 2\hat{\mathbf{k}}_r \cdot \mathbf{r}'/c\right) \right] d\mathbf{r}' \\ & - \hat{\mathbf{k}}_s \times \int_{S_{\text{lit}}^{\text{imp}}} \left[\hat{\mathbf{n}}(\mathbf{r}') \times \hat{\mathbf{n}}(\mathbf{r}') \times \frac{Z(t)}{\eta_0} * \mathbf{u}(\mathbf{r}', t) * \delta\left(t - 2\hat{\mathbf{k}}_r \cdot \mathbf{r}'/c\right) \right] d\mathbf{r}'. \end{aligned} \quad (5.8)$$

Assuming that the scatterer's surface is discretized with the linear triangular patches $S_{\text{lit}}^{\text{imp}} \simeq \bigcup_{n=1}^N S_n$, the integrals in (5.8) can be determined by the sum of the integrals over triangular patches. In addition due to the use of the linear triangles, the unit normal vector and parameters in $\mathbf{u}(\mathbf{r}, t)$ are constant throughout a patch, i.e. $\hat{\mathbf{n}}(\mathbf{r}) = \hat{\mathbf{n}}_n$ and $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_n(t)$ for $\mathbf{r} \in S_n$. By defining $\tilde{\mathbf{u}}_n(t) = Z(t) * \mathbf{u}_n(t)$, the range-corrected scattered electric field at the far-zone can be determined as

$$\begin{aligned} \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = & \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \sum_{n=1}^N \hat{\mathbf{n}}_n \times \mathbf{u}_n(t) * h_n(t) \\ & - \frac{1}{\eta_0} \hat{\mathbf{k}}_s \times \sum_{n=1}^N \hat{\mathbf{n}}_n \times \hat{\mathbf{n}}_n \times \tilde{\mathbf{u}}_n(t) * h_n(t). \end{aligned} \quad (5.9)$$

Here, as mentioned in Section 2, $h_n(t)$ is called the PO integral [17], the closed-form expressions can be found in (2.16) and (2.17) as well as in [17].

5.1.1. Evaluation of $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$

The impedance boundary conditions are usually frequency domain approximations, and their time domain counterpart can be obtained by inverse Fourier transforming the associated frequency domain approximation. However, it might not be always

possible to determine the time domain expressions of impedance boundary condition in closed-form. Therefore, in this thesis the samples of $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$ are determined by rational function fitting to their frequency domain samples and then inverse Fourier transforming them. For the rational function fit, fast relaxed vector fitting (FRVF) algorithm [56–58] is employed.

The frequency samples of the parallel and perpendicular reflection coefficients ($R_{\parallel}(\omega)$ and $R_{\perp}(\omega)$) can be determined using [13]

$$R_{\parallel}(\omega) = \frac{Z(\omega)/\eta_0 - \cos \theta_i}{Z(\omega)/\eta_0 + \cos \theta_i}, \quad R_{\perp}(\omega) = \frac{Z(\omega)/\eta_0 - \sec \theta_i}{Z(\omega)/\eta_0 + \sec \theta_i}, \quad (5.10)$$

respectively. Here $Z(\omega)$ is the surface impedance in frequency domain and $\omega = 2\pi f$ denotes the angular frequency. Note that $R_{\parallel}(\omega)$ and $R_{\perp}(\omega)$ also depend on θ_i , but this dependence is not explicitly shown for brevity. The frequency domain counterpart of $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$, i.e. $\mathbf{u}_n(\omega)$ and $\tilde{\mathbf{u}}_n(\omega)$, are given as follows [13]:

$$\begin{aligned} \mathbf{u}_n(\omega) = & \tilde{R}_{\parallel}(\omega) \left((\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \right) \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \\ & + \tilde{R}_{\perp}(\omega) \left((\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\perp}(\mathbf{r}) \right) \hat{\mathbf{e}}_{\perp}(\mathbf{r}), \end{aligned} \quad (5.11)$$

$$\begin{aligned} \tilde{\mathbf{u}}_n(\omega) = & Z(\omega) \tilde{R}_{\parallel}(\omega) \left((\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \right) \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \\ & + Z(\omega) \tilde{R}_{\perp}(\omega) \left((\hat{\mathbf{k}}_i \times \hat{\mathbf{p}}) \cdot \hat{\mathbf{e}}_{\perp}(\mathbf{r}) \right) \hat{\mathbf{e}}_{\perp}(\mathbf{r}). \end{aligned} \quad (5.12)$$

To determine the rational function fit to $\mathbf{u}_n(\omega)$ and $\tilde{\mathbf{u}}_n(\omega)$, FRVF is applied to four different frequency dependent functions:

$$\tilde{R}_{\parallel}(\omega) = \frac{2 \cos \theta_i}{Z(\omega)/\eta_0 + \cos \theta_i}, \quad (5.13)$$

$$\tilde{R}_{\perp}(\omega) = \frac{2 \sec \theta_i}{Z(\omega)/\eta_0 + \sec \theta_i}, \quad (5.14)$$

$$Z(\omega)\tilde{R}_{\parallel}(\omega) = \frac{2Z(\omega)\cos\theta_i}{Z(\omega)/\eta_0 + \cos\theta_i}, \quad (5.15)$$

$$Z(\omega)\tilde{R}_{\perp}(\omega) = \frac{2Z(\omega)\sec\theta_i}{Z(\omega)/\eta_0 + \sec\theta_i}, \quad (5.16)$$

using their samples at $\omega_i \in 2\pi[f_{\min}, f_{\max}]$, $i = 1, \dots, N_{\omega}^g$ [56–58]. As a result a weighted rational function fit to the functions, $g(\omega) \in \{\tilde{R}_{\parallel}(\omega), \tilde{R}_{\perp}(\omega), Z(\omega)\tilde{R}_{\parallel}(\omega), Z(\omega)\tilde{R}_{\perp}(\omega)\}$ can be determined as

$$F_g(\omega) = d^g + \sum_{i=1}^{N_p^g} \frac{b_i^g}{j\omega + a_i^g}, \quad (5.17)$$

where N_p^g denotes the number of poles, a_i^g and b_i^g are the pole and residue pairs, d^g is the constant term, j is the imaginary unit. Once a_i^g , b_i^g , and d^g are determined then (5.17) can be inverse Fourier transformed to find the time domain counterpart of (5.13)-(5.16):

$$F^g(t) = d^g\delta(t) + \sum_{i=1}^{N_p^g} b_i^g H(t) \exp[-a_i^g t]. \quad (5.18)$$

Here $H(\cdot)$ denotes the unit step function. Note that the rational function fit given in (5.17) can have a linear term with respect to ω but the surface impedance model used in this thesis does not require a linear contribution. When the rational function fit is applied to higher order impedance models as in [13, 50, 51], the linear term should be taken into account, this might also require modification of the PO formulation depending on the used impedance model. In addition, the causality of (5.18) is ensured by enforcing $a_i^g > 0$. Using the rational function fit in (5.18), $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$ can be determined as:

$$\begin{aligned} \mathbf{u}_n(t) \approx & F^{\tilde{R}_{\parallel}}(t) \left[\left(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}} \right) \cdot \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \\ & + F^{\tilde{R}_{\perp}}(t) \left[\left(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}} \right) \cdot \hat{\mathbf{e}}_{\perp}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\perp}(\mathbf{r}), \end{aligned} \quad (5.19)$$

$$\begin{aligned}\tilde{\mathbf{u}}_n(t) \approx & F^{Z\tilde{R}_{\parallel}}(t) \left[\left(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}} \right) \cdot \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\parallel}(\mathbf{r}) \\ & + F^{Z\tilde{R}_{\perp}}(t) \left[\left(\hat{\mathbf{k}}_i \times \hat{\mathbf{p}} \right) \cdot \hat{\mathbf{e}}_{\perp}(\mathbf{r}) \right] \hat{\mathbf{e}}_{\perp}(\mathbf{r}).\end{aligned}\quad (5.20)$$

5.1.2. Discussion

In this subsection some of the important formulation and implementation details are discussed:

- i) It can be seen in (5.9) that the temporal convolution of $h_n(t)$ and $\{\mathbf{u}_n(t), \tilde{\mathbf{u}}_n(t)\}$ is required to determine the range-corrected scattered electric field. Using the rational function fit in (5.18), this convolution can be written as

$$F^g(t) * h_n(t) = d^g h_n(t) + h_n(t) * \sum_{i=1}^{N_p^g} b_i^g H(t) \exp[-a_i^g t]. \quad (5.21)$$

In (5.21), the first term at the right hand side is obtained analytically using the properties of the Dirac delta function, whereas the second term can be evaluated in several ways. First one is evaluating the convolution $H(t) \exp[-a_i^g t] * h_n(t)$ analytically as in [55]. It is also possible to use recursive convolution method [51, 53, 54, 59–61], which reduces the size of temporal samples of the rational function fit, however using the recursive convolution might not be beneficial and effective, since the convolution $H(t) \exp[-a_i^g t] * h_n(t)$ can be determined analytically, where the temporal samples of $H(t) \exp[-a_i^g t]$ can be eliminated totally. Another way of determining the convolution at the second term in (5.21), which is chosen in this thesis, is sampling the functions $H(t) \exp[-a_i^g t]$ and $h_n(t)$ in time, then calculate the discrete convolution numerically. In this case, temporal samples of $H(t) \exp[-a_i^g t]$ can be limited to a preset accuracy level, which also reduces the number of samples account for the convolution, especially for fast decaying exponentials.

- ii) In specular reflection case, i.e. if the surface normal $\hat{\mathbf{n}}_n$ and $\mathbf{k}_r$ are parallel, then $t_1 = t_2 = t_3$ and $h_n(t) = A_n \delta(t - t_2)$. However the t_2 does not have to coincide with a temporal sampling point (e.g. $j\Delta t < t_2 < (j+1)\Delta t$) and the effect of the Dirac delta function cannot be directly added to the time samples of the scattered

field. In this case the intersection time $t_1 = t_2 = t_3$ and A_n are saved and taken into account separately. This case also affects (5.20), as

$$F^g(t) * A_n \delta(t - t_2) = d^g A_n \delta(t - t_2) + A_n \sum_{i=1}^{N_p^g} b_i^g H(t - t_2) \exp[-a_i^g(t - t_2)]. \quad (5.22)$$

- iii) As explained in Section 5.1.1, $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$ are functions of θ_i , which is determined by the surface normal of the patch $\hat{\mathbf{n}}_n$ and $\hat{\mathbf{k}}_i$ as shown in Fig. 5.1. For every patch, θ_i might be different, and $\{\mathbf{u}_n(t), \tilde{\mathbf{u}}_n(t)\}$ should be determined individually, hence the FRVF scheme should be applied to (5.13)-(5.16) for associated θ_i to determine $\{d^g, a_i^g, b_i^g\}$, $g \in \{\tilde{R}_{\parallel}, \tilde{R}_{\perp}, Z\tilde{R}_{\parallel}, Z\tilde{R}_{\perp}\}$. In this work, instead of applying the FRVF scheme for all surface patches, it is applied to a preset interval of θ_i in the beginning of the calculation. For example, $\{d^g, a_i^g, b_i^g\}$, $g \in \{\tilde{R}_{\parallel}, \tilde{R}_{\perp}, Z\tilde{R}_{\parallel}, Z\tilde{R}_{\perp}\}$ coefficients are determined for $\theta_i \in [0, 90]^\circ$ with $\Delta\theta_i = 1^\circ$ by applying FRVF scheme, then temporal samples of $\sum_{i=1}^{N_p^g} b_i^g H(t) \exp[-a_i^g t]$ are calculated and stored. For a particular value of θ_i (for a given triangular patch), these calculated and stored samples are interpolated.
- iv) The scattered fields are determined for an incident field with a Dirac delta time signature. Even though the Dirac delta function has an infinite bandwidth, the rational function fit is employed to a frequency band $[f_{\min}, f_{\max}]$, which restricts the validity of the solution to this band. In addition, the scattered fields for a specific time signature in $[f_{\min}, f_{\max}]$ can be obtained easily by convolving the desired signature $G(t)$ and the range-corrected scattered electric field in (5.8)

$$\tilde{\mathbf{E}}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t) = G(t) * \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t), \quad (5.23)$$

where $\tilde{\mathbf{E}}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, t)$ is the range-corrected scattered electric field due an incident plane wave with $G(t)$ time signature.

- v) If the frequency domain surface impedance is constant, i.e. $Z(\omega) = Z$, then there is no need to use FRVF scheme, since $b_i^g = 0$, $i = 1, \dots, N_p^g$ in (5.21). Specifically, $Z(\omega) = 0$ yields the PEC surface case.

5.2. Frequency Domain Formulation

The frequency domain analysis of scattering from impedance surfaces using PO approximation is relatively easier and straightforward than its time domain counterpart. In this section, $\exp[j\omega t]$ time dependence is assumed. Surface impedance of the scatterer S^{imp} in frequency domain is defined as $Z(\omega)$. The magnetic field component of the incident wave is given as

$$\mathbf{H}^{\text{inc}}(\mathbf{r}, \omega) = \frac{1}{\eta_0} \hat{\mathbf{k}}_i \times \hat{\mathbf{p}} E_i \exp[-jk \hat{\mathbf{k}}_i \cdot \mathbf{r}], \quad (5.24)$$

where $k = \omega/c$ is the wave number. Following the formulation in Section 5.1, the scattered electric far-field from a scatterer discretized with linear patches can be determine as

$$\mathbf{E}^{\text{sca}}(\mathbf{r}, \omega) = \frac{1}{4\pi} (jk) \frac{\exp[-jkr]}{r} \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, \omega), \quad (5.25)$$

where the range-corrected scattered electric field in frequency domain can be given as:

$$\begin{aligned} \mathbf{E}_{\text{rc}}^{\text{sca}}(\hat{\mathbf{k}}_s, \omega) &= \hat{\mathbf{k}}_s \times \hat{\mathbf{k}}_s \times \sum_{n=1}^N \hat{\mathbf{n}}_n \times \mathbf{u}_n(\omega) h_n(\omega) \\ &\quad - \frac{1}{\eta_0} \hat{\mathbf{k}}_s \times \sum_{n=1}^N \hat{\mathbf{n}}_n \times \hat{\mathbf{n}}_n \times \tilde{\mathbf{u}}_n(\omega) h_n(\omega). \end{aligned} \quad (5.26)$$

Here $\mathbf{u}_n(\omega)$ and $\tilde{\mathbf{u}}_n(\omega)$ are defined in (5.11) and (5.12), respectively. The closed-form expression of $h_n(\omega)$ can be determined by Fourier transforming $h_n(t)$ given in (2.16) and (2.17), also as shown in [17]

$$h_n(\omega) = \begin{cases} \frac{2A_n}{\omega^2} \left[\frac{\exp[-j\omega t_1]}{(t_2-t_1)(t_1-t_3)} + \frac{\exp[-j\omega t_2]}{(t_3-t_2)(t_2-t_1)} + \frac{\exp[-j\omega t_3]}{(t_2-t_3)(t_3-t_1)} \right]; & t_1 \neq t_2 \neq t_3 \\ \frac{2A_n}{\omega^2} \left[\frac{\exp[-j\omega t_2](j\omega t_2 - j\omega t_i + 1)}{(t_2-t_i)^2} - \frac{\exp[-j\omega t_i]}{(t_2-t_i)^2} \right]; & t_j = t_2 \neq t_i \\ A_n \exp[-j\omega t_2]; & t_1 = t_2 = t_3. \end{cases} \quad (5.27)$$

Note that in frequency domain, the specular reflection case does not have to be taken into account separately.

5.3. Numerical Examples

In this section, the accuracy and validity of the proposed scheme are demonstrated through the numerical examples. Unless otherwise specified, in all examples, a lossy scatterer S^{imp} with homogeneous permittivity ε , permeability μ , and conductivity σ resides in an unbounded free space is analyzed. The surface impedance of S^{imp} , i.e. $Z(\omega)$, is

$$Z(\omega) = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}, \quad (5.28)$$

under the assumption of $|N_R| \gg 1$ and $|\text{Im}(N_R)|\omega a\sqrt{\varepsilon_0\mu_0} \gg 1$, $\forall \omega \in 2\pi[f_{\min}, f_{\max}]$, where $N_R = c\mu/Z(\omega)$ is the complex refractive index, a is the minimum radius of curvature on S^{imp} , $\text{Im}(\cdot)$ operator yields the imaginary part of its argument, as explained in [50, 52]. Note that even it is possible to find the time domain counterpart of $Z(\omega)$, as shown in [52], inverse Fourier transform of (5.13)-(5.16) might not be determined analytically. In all examples, the scatterers are illuminated by a $\hat{\mathbf{p}}$ polarized plane wave with a Dirac delta time signature traveling to $\hat{\mathbf{k}}_i$ direction as given in (2.2).

In examples and figures, TD denotes the time domain results obtained using the proposed scheme in Section 5.1, while FD denotes the frequency domain results obtained using the formulation given in Section 5.2. RCS results are obtained as explained in Chapter 3.2 in detail and results for the PEC scatterers are obtained using the formulation in [17].

5.3.1. A Unit Square Plate

In this example, scattering from a unit square plate with a constant surface impedance of $Z(\omega) = \{0.6\eta_0, 1.6\eta_0\}$ is analyzed in 100 MHz-1.5 GHz frequency band. The square plate is centered on $x - y$ plane and modeled with 4 triangles. For this example, all triangles are always illuminated and the backscattered fields are observed, i.e. $\hat{\mathbf{k}}_i =$

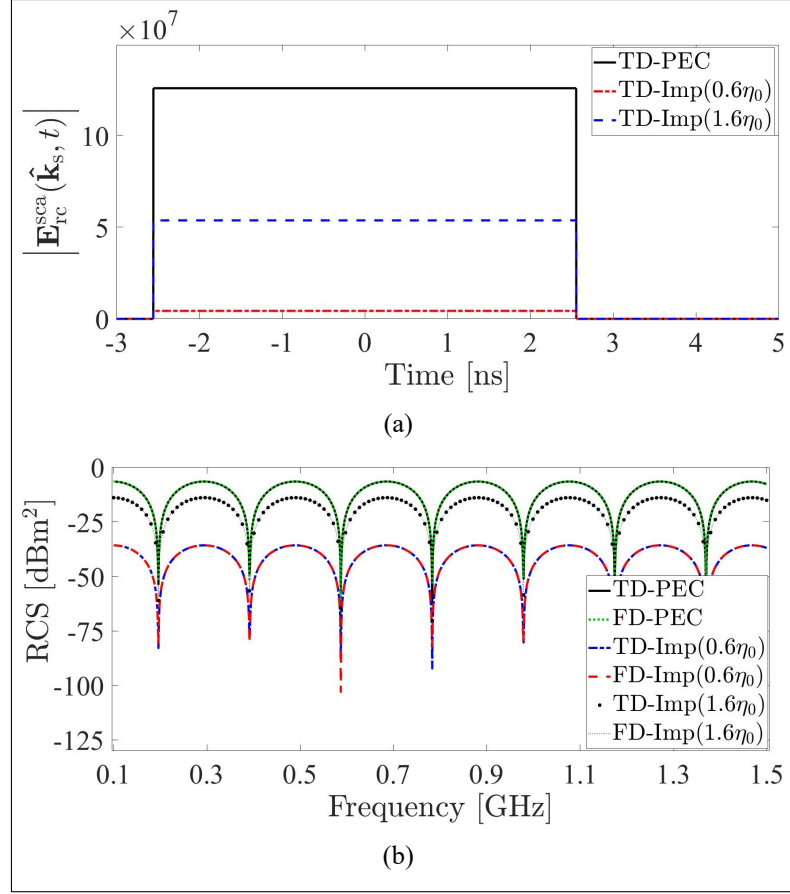


Figure 5.2: (a) Norm of the range-corrected scattered electric field and (b) backscattered RCS results for the PEC and impedance surfaces.

$-\hat{\mathbf{k}}_s$. In addition, there is no need to use the FRVF scheme since $Z(\omega)$ is constant. Time step size $\Delta t = 1.33$ ps.

Fig. 5.2(a) compares the norm of the range-corrected backscattered electric field $|\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)|$ for $Z(\omega) = 0.6\eta_0$ (TD-Imp(0.6 η_0)), $Z(\omega) = 1.6\eta_0$ (TD-Imp(1.6 η_0)), and PEC plate (TD-PEC). The properties of the incident wave are chosen as $\hat{\mathbf{k}}_i = (0.766, 0, 0.643)$ and $\hat{\mathbf{p}} = (0.643, 0, -0.766)$, which is equivalent to $\hat{\mathbf{p}} = \hat{\theta}$ (vertical) polarization at $\theta = 50^\circ$ and $\phi = 0^\circ$. It can be seen in Fig. 5.2(a) that the temporal behavior of $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ has a rectangular shape because of the symmetry of the problem to $x - z$ plane. Due to the constant impedance, the results for impedance surfaces are time limited between $t = -2.555$ ns and $t = 2.555$ ns, which is same for the PEC plate. Fig. 5.2(b) plots the backscattered RCS values in 100 MHz-1.5 GHz frequency band with frequency interval $\Delta f = 1$ MHz obtained using the time domain results and compares them with the frequency domain (FD) simulations. It can be seen from Fig. 5.2(b) that time and frequency domain results are coinciding very-well. In addition,

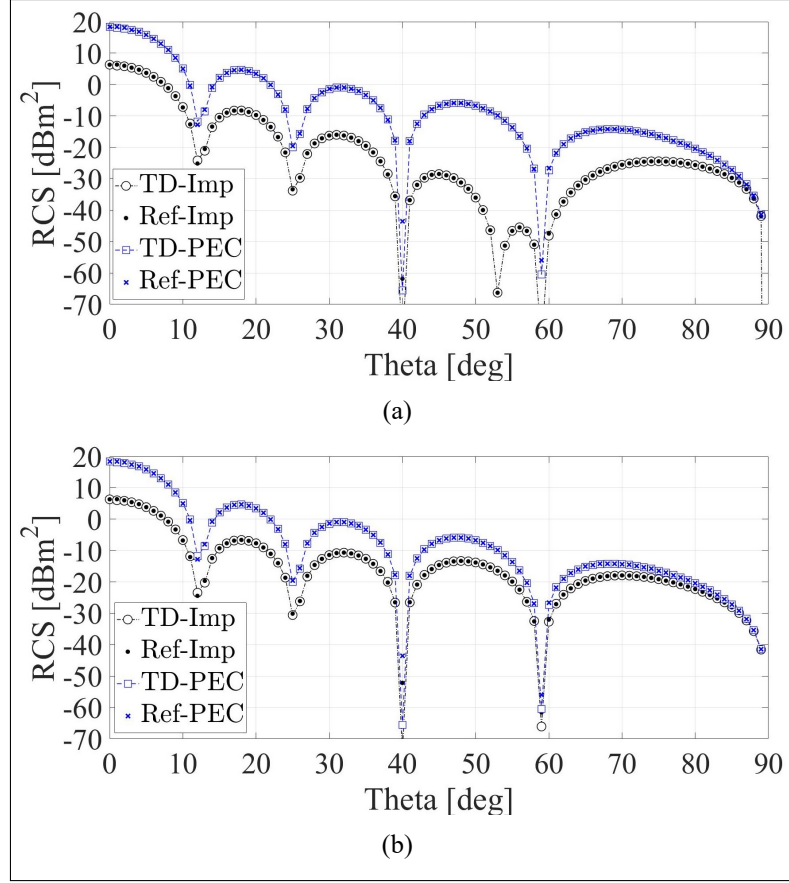


Figure 5.3: Monostatic RCS results at 700 MHz for $Z(\omega) = 0.6\eta_0$:
(a) horizontal polarization and (b) vertical polarization.

Figs. 5.3(a) and (b) plot the monostatic RCS values at 700 MHz for horizontal ($\hat{\mathbf{p}} = \hat{\phi}$) and vertical ($\hat{\mathbf{p}} = \hat{\theta}$) polarizations, respectively, and compares them with the RCS results given in [13] (“Ref-PEC” and “Ref-Imp”) for $Z(\omega) = 0.6\eta_0$ and PEC cases. In Fig. 5.3, $\hat{\mathbf{k}}_i$ direction is chosen as $\theta \in [0, 90]^\circ$ and $\phi = 0^\circ$.

5.3.2. A Unit Sphere

In this example, scattering from a unit sphere with $\epsilon = 10\epsilon_0$, $\mu = \mu_0$, and $\sigma = 0.04$ is analyzed in 100 MHz-10 GHz frequency band. The properties of the incident wave are chosen as $\hat{\mathbf{k}}_i = \hat{\mathbf{z}}$ and $\hat{\mathbf{p}} = \hat{\mathbf{x}}$, and the backscattered fields are observed ($\hat{\mathbf{k}}_s = -\hat{\mathbf{z}}$). The illuminated part of the sphere is discretized with 1555534 triangles and time step size $\Delta t = 1$ ps. Since $Z(\omega)$ in (5.28) is used as the surface impedance, the FRVF scheme is applied to (5.13)-(5.16) using $N_\omega^g = 991$ samples in the frequency band $f_{\min} = 100$ MHz and $f_{\max} = 10$ GHz, and $N_p^g = 4$ poles, which yield less than

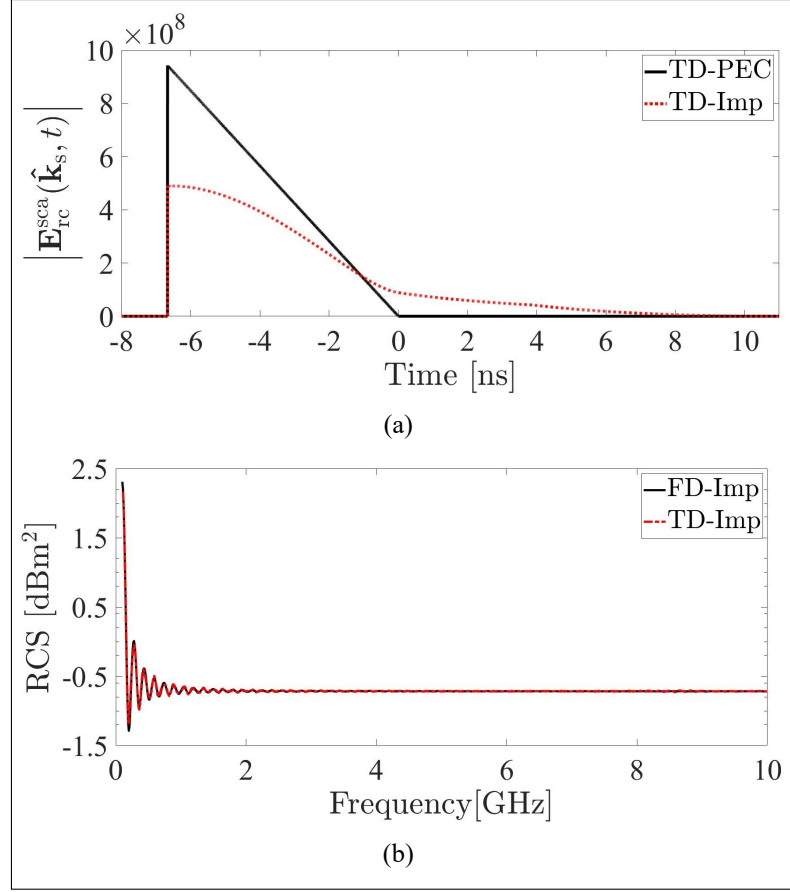


Figure 5.4: (a) Norm of the range-corrected scattered electric field and (b) backscattered RCS results for the unit sphere.

10^{-7} relative error $\forall \theta_i \in [0, 90]^\circ$ and $g \in \{\tilde{R}_{\parallel}, \tilde{R}_{\perp}, Z\tilde{R}_{\parallel}, Z\tilde{R}_{\perp}\}$.

Fig. 5.4(a) plots norm of the range-corrected electric field $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ for impedance and PEC surfaces (TD-Imp and TD-PEC). Note that the results for the PEC surface is time limited between $t = -6.67$ ns and $t = 0$ ns, however for the impedance surface $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ starts to change at $t = -6.67$ ns and decays as the time increases due to the convolution in (5.9). In Fig. 5.4(b), the backscattered RCS for the impedance surface obtained using the time domain results is plotted with the frequency step $\Delta f = 10$ MHz and compared with the frequency domain results (FD-Imp). Similar to the previous example, RCS obtained using time and frequency domain results agree very-well.

5.3.3. A Composite Scatterer

As the last example, scattering from a composite scatterer discretized with 74038 linear triangles is analyzed in 100 MHz-5 GHz frequency band. As shown in the inset

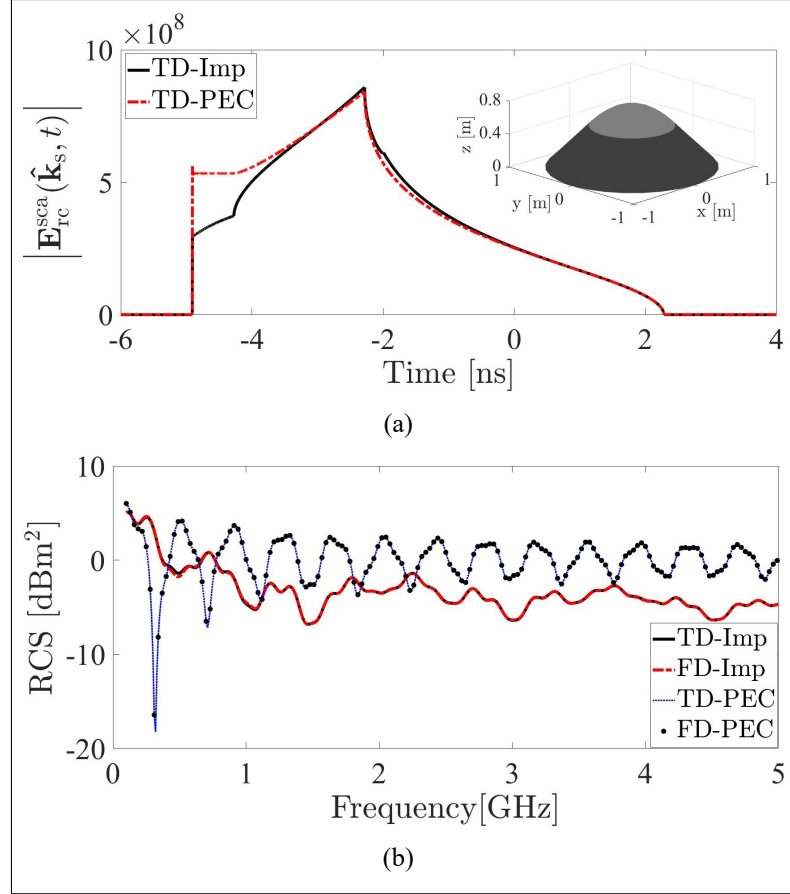


Figure 5.5: (a) Norm of the range-corrected scattered electric field and (b) backscattered RCS results for the PEC and composite scatterers.

of Fig. 5.5(a), the upper part of the scatterer is a parabolic surface with the focal length 0.25 m and radius 0.5 m, and is connected to a truncated circular cone with the height of 0.75 m, upper and lower radii of 1 m and 0.5 m. The upper part of the scatterer (light gray part in the inset of Fig. 5.5(a)) is an impedance surface with $\epsilon = 12\epsilon_0$, $\mu = \mu_0$, and $\sigma = 0.2$ and the lower part (dark gray part in the inset of Fig. 5.5(a)) is a PEC surface. A $\hat{\mathbf{p}} = (0.93, 0, -0.34)$ polarized ($\theta = 160^\circ$ and $\phi = 180^\circ$) incident field illuminates the scatterer from $\hat{\mathbf{k}}_i = (-0.34, 0, -0.93)$ direction and the time step size $\Delta t = 2$ ps. The backscattered fields are observed on $\hat{\mathbf{k}}_s = (0.34, 0, 0.93)$ direction. In this case scatterer's surface is completely illuminated. As in the unit sphere example, since $Z(\omega)$ in (5.28) is used as the upper surface's impedance, the FRVF scheme is applied to (5.13)-(5.16) using $N_\omega^g = 491$ frequency samples and $N_p^g = 6$ poles, which yield less than 10^{-6} relative error at $\forall \theta_i \in [0, 90]^\circ$.

Fig. 5.5(a) plots the norm of the range-corrected scattered electric field for the composite scatterer (TD-Imp) and compares it with the range-corrected scattered elec-

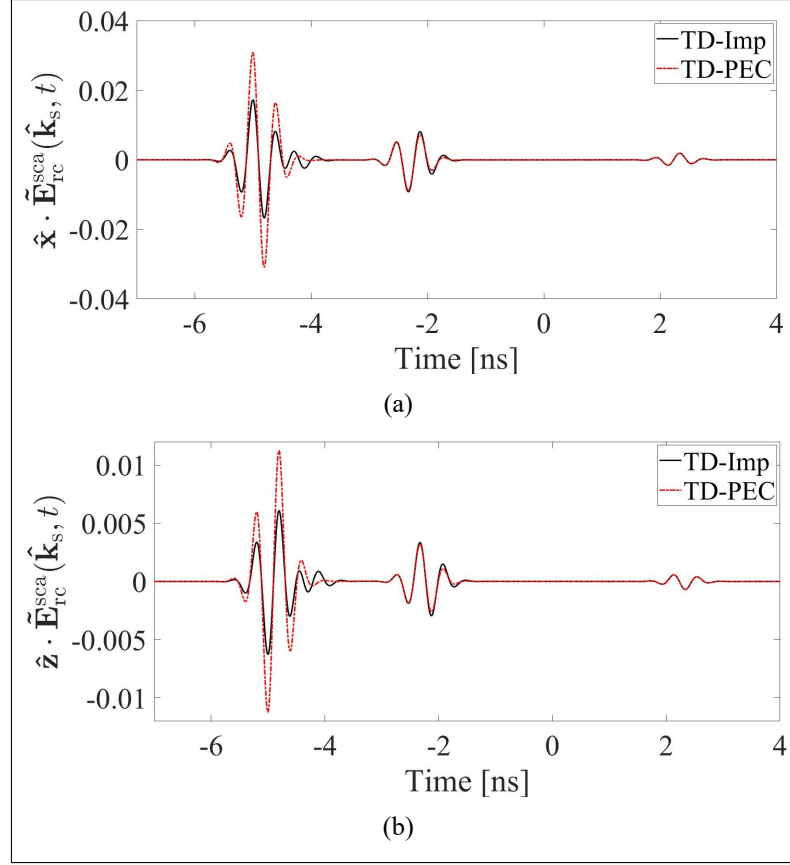


Figure 5.6: (a) $\hat{x}$ - and (b) $\hat{z}$ -components of the range-corrected scattered electric field with the modulated Gaussian signature.

tric field for the PEC case (TD-PEC), which assumes that all scatterer is a PEC surface. Fig. 5.5(b) compares the RCS for the impedance and PEC surfaces obtained using the time domain results (TD-Imp and TD-PEC) and the frequency domain results (FD-Imp and FD-PEC) with the frequency step $\Delta f = 10$ MHz. It can be seen from Fig. 5.5(b) that all frequency and time domain results agree very-well and RCS for the composite scatterer with impedance surface is smaller than RCS for the PEC case, especially at high-frequencies.

Fig. 5.6 plots the $\hat{x}$ - and $\hat{z}$ -components of the range-corrected scattered electric field with the modulated Gaussian signature $G(t)$ (i.e. $\tilde{\mathbf{E}}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ in (5.23)) for the composite (TD-Imp) and PEC (TD-PEC) scatterers. The modulated Gaussian signature $G(t) = \cos(2\pi f_0 t) \times \exp(-t^2/2\sigma_v^2)$, where the center frequency $f_0 = 2.55$ GHz, the duration $\sigma_v = 3/(2\pi f_{bw})$, and the effective bandwidth $f_{bw} = 1.8$ GHz. Note that due to the symmetry of the problem to the $x - z$ plane $\hat{y}$ -component of $\tilde{\mathbf{E}}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ is zero. $\tilde{\mathbf{E}}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$ is obtained by convolving the range-corrected scattered electric field $\mathbf{E}_{rc}^{sca}(\hat{\mathbf{k}}_s, t)$, given in Fig. 5.5(a), and the modulated Gaussian signature $G(t)$, as

explained in Section 5.1.2.

5.4. Conclusions

In this chapter, a scheme to analyze the transient scattering from impedance surfaces using PO approximation is presented. The scattered fields are determined by convolving the time dependent impedance and associated functions, which is determined by applying the FRVF scheme to their frequency domain counterpart to find a weighted rational function fit and applying inverse Fourier transform to the resulting fit, and the PO integral, which is determined using the closed-form expressions obtained by the Radon transform interpretation. Even the Leontovich impedance boundary condition is used in this chapter, the different impedance boundary conditions can be analyzed by modifying the formulation, since FRVF scheme is used for determining the time dependent impedance and associated functions. It is also assumed that the incident wave has a Dirac delta time signature, which has an infinite bandwidth, however due to FRVF scheme is applied in a certain band, validity of the solution is limited to this frequency band. The scheme presented in this section can be incorporated into a PO-time domain integral equation (TDIE) hybridized solver [64,65] by convolving the PO solution and the incident wave's time signature used in TDIE.

6. CONCLUSIONS AND FUTURE WORKS

In this thesis, schemes for exact evaluation of the PO and EEC integrals on linear and quadratic triangular patches and edges, respectively, are developed in time domain. The scatterers are assumed to be illuminated by a plane wave with Dirac delta time signature and the scattered fields are observed at far-zone. Using the Radon transform interpretation, it is shown that the radiation integrals can be reduced to an integral on the intersection of the triangular patch/edge and the $\mathbf{k}_r$ -plane, which is formed by the incidence and observation direction. Using this fact, the PO integral on the triangular surface is reduced to a line integral on the intersection of the patch and $\mathbf{k}_r$ -plane. Then the PO integral for linear triangular patches is evaluated in closed-form, since the intersection is a straight line. For quadratic triangular patches, the PO integral is evaluated exactly considering the intersecting curve in the barycentric coordinates of the quadratic triangle and using suitable coordinate transformation and suitable order Gauss-Legendre quadrature rule (GLQR). For the EEC integral, the intersection of the edge and the $\mathbf{k}_r$ -plane, in general, yields a point, and the EEC integral is evaluated in closed-form for quadratic edges. The accuracy and the validity of the closed-form expression of the PO integral on linear triangular patches, the developed scheme for exact evaluation of the PO integral on quadratic triangular patches, and the closed-form expression for the EEC integral on quadratic edges are demonstrated via numerical examples in Chapters 2, 3, and 4, respectively.

In addition, a formula for the USR case, which is the case if the scatterer fits into two consecutive time steps and the $\mathbf{k}_r$ -plane does not intersect with the scatterer at all, is suggested and its accuracy is demonstrated in Chapter 2.

At last, a scheme to analyze the transient scattering from impedance surfaces using PO approximation and linear triangular patches is presented in Chapter 5. The scattered fields are determined by convolving the PO integral, presented in Chapter 2, and the time dependent impedance and associated functions, which are determined by applying the FRVF scheme to their frequency domain counterpart to find a weighted rational function fit and applying inverse Fourier transform to the resulting fit. The accuracy and the validity of the developed scheme for the transient analysis of impedance surfaces using PO approximation is also demonstrated in Chapter 5.

The future works can be listed as follows:

- i) The developed exact evaluation scheme for the PO integral on quadratic triangular patches in Chapter 3 can be extended to higher order triangular patches, however the scheme developed in Chapter 3 requires the use of quadratic curve types and associated coordinate transformations to calculate the integral on the intersecting curve. This type of classification is not available in the literature for higher order curves. Therefore, a new formulation, which does not require this type of coordinate transformation should be developed.
- ii) In Chapter 4, the EEC integral on quadratic edges is evaluated in closed-form, since the intersection of the $\mathbf{k}_r$ -plane yields a point. This case is same for higher order edges, and the scheme presented in Chapter 4 can be easily extended to the closed-form evaluation of the EEC integral on higher order edges.
- iii) The scheme developed for the analysis of the transient scattering from impedance surfaces using PO approximation in Chapter 5 requires evaluation of the PO integral and the applying a rational function fit to the time dependent impedance and associated functions. Since the scatterer is modeled using linear triangular patches, the scattered fields can be determined by separate evaluation of the PO integral and the time dependent impedance (and associated functions), then convolving them. If the scatterer is modeled with higher order surfaces, then these evaluations cannot be performed separately, since $\mathbf{u}_n(t)$ and $\tilde{\mathbf{u}}_n(t)$ in (5.7) cannot be moved out of the integral and evaluation becomes extremely complicated. It can be concluded that for the transient analysis of impedance surfaces using PO approximation using linear triangular patches is straight-forward and has less mathematical complexity. Nevertheless, the scheme developed in Chapter 5 can be extended to the analysis of higher order impedance models.
- iv) The presented schemes in this thesis can be incorporated into a PO-time domain integral equation (TDIE) hybridized solver [64, 65] by convolving the PO solution and the incident wave's time signature used in TDIE.

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BIOGRAPHY

Aslihan Aktepe was born in 1993 in Zonguldak, Turkey. She took primary and secondary education in Ordu, and graduated from Ordu Anatolian High School in 2011. At the same year, she was admitted at the Gebze Technical University (GTU) a.k.a. Gebze Institute of Technology in Kocaeli under the Bachelor of Science program in Department of Electronics Engineering. She received the B.Sc. degree in Electronics Engineering from GTU in 2017. Since 2020, she has been a Research Assistant with the Electromagnetic Fields and Microwave Techniques Division, Department of Electronics Engineering, GTU, Kocaeli, Turkey, and is currently pursuing her M.Sc. degree.

Ms. Aktepe's main research interests include computational electromagnetics, especially high-frequency techniques, physical optics approximation, and diffraction theory. She is one of the recipients of the URSI Young Scientist Award (YSA) in 2020 and she is an organization committee member of 5th EMC Turkey Conference 2019 and X. URSI-Turkey National Meeting 2021.

APPENDICES

Appendix A: Publications of the Thesis

Aktepe A., Ülkü H. A., (2021), “On the closed-form evaluation of the PO integral using the Radon transform interpretation for linear triangles”, Turk J Elec Eng & Comp Sci, (accepted), DOI: 10.3906/elk-2009-2.

Aktepe A., Serim H. A., Ülkü H. A., (2020), “Transient analysis of scattering from impedance surfaces using physical optics approximation”, IET Microw Antenn Propag, 14 (15), 2020–2026.

Aktepe A., Ülkü H. A., (2020), “Exact evaluation of time domain physical optics integral on quadratic triangular surfaces”, IEEE T Antenn Propag, 68 (11), 7447–7456.

Aktepe A., Ülkü H. A., (2020), “Time domain physical optics integral evaluation on third order triangles”, in: 2020 XXXIIIrd Int Union of Radio Sci Gen Assy and Sci Symp, 1–3, Rome, Italy, 29 Aug.–5 Sep.

Aktepe A., Ülkü H. A., (2019), “Evaluation of PO and EEC radiation integrals on quadratic triangles and edges in time domain”, in: 2019 Fifth Int EMC Türkiye Conf, 1–3, Kocaeli, Turkey, 22–25 Sep.

Aktepe A., Ülkü H. A., (2019), “Evaluation of the contour integral on curved edges in time domain equivalent edge currents”, in: 2019 Int Appl Comput Electrom Soc Symp (ACES), 1–2, Miami, FL, USA, 14–19 Apr.

Aktepe A., Serim H. A., Ülkü H. A., (2018), “Kuatratik üçgenler ile modellenen saçıcıların fiziksel optik yaklaşımı ile zaman uzayında analizi”, in: 2018 URSI-Türkiye IX. Bilimsel Kongresi, KTO Karatay University, Konya, Türkiye, 6–8 Eyl.

Aktepe A., Serim H. A., Ülkü H. A., (2018), “Evaluation of time domain physical optics integral on curved surfaces”, in: 2018 2nd URSI Atlantic Radio Science Meeting (AT-RASC), 1–3, Meloneras, Spain, 28 May–1 June.

Aktepe A., Ülkü H. A., (2019), “İkinci dereceden üçgenler ile modellenen saçıcıların kırınımın fiziksel teorisi ile zaman uzayında analizi”, in: Gebze Teknik Üniversitesi Lisansüstü Araştırmalar Sempozyumu (LÜAS), GTÜ, Kocaeli, Türkiye, 17–18 Haz.

Aktepe A., Ülkü H. A., (2019), “İkinci dereceden üçgenler ile modellenen saçıcıların fiziksel optik yaklaşımı ile analizi için verimli gölgeleme yöntemi”, in: Gebze Teknik Üniversitesi Lisansüstü Araştırmalar Sempozyumu (LÜAS), GTÜ, Kocaeli, Türkiye, 17–18 Haz.

Aktepe A., Serim H. A., Ülkü H. A., (2018), “İkinci dereceden üçgenler ile modellenen saçıcıların fiziksel optik yaklaşımı ile zaman uzayında analizi”, in: Gebze Teknik Üniversitesi Lisansüstü Araştırmalar Sempozyumu (LÜAS), GTÜ, Kocaeli, Türkiye, 14–16 May.

Appendix B: Properties of Dirac Delta Function

In this section, the Dirac delta function's properties, which are used in this paper, are presented. These properties are either given in [66] or they can be derived by using the properties given there.

$$\delta(ax) = \frac{1}{|a|}\delta(x), a \neq 0 \quad (\text{B.1})$$

For an arbitrary $\Psi(x)$ function

$$\delta[\Psi(x)] = \sum_i \frac{\delta(x - x_i)}{|\Psi'(x_i)|} \quad (\text{B.2})$$

where x_i are the root of $\Psi(x)$ and the sum extends over all roots of $\Psi(x)$. $\Psi'(x)$ denotes the derivative w.r.t. its argument.

For an arbitrary $\Phi(x)$ function

$$\begin{aligned} \int \Phi(x)\delta(x - x_0)dx &= \Phi(x_0) \int \delta(x - x_0)dx \\ &= \Phi(x_0), \end{aligned} \quad (\text{B.3})$$

where it is tacitly assumed that the integration limits enclose the point at which the argument of the Dirac function vanishes.

Appendix C: Definitions for Quadratic Triangle

In this section, the definition of the quadratic triangle and determination of the intersection points of the plane with surface normal $\hat{\mathbf{k}}_r$ and the edges of the quadratic triangle are shown. Each quadratic triangle is defined by shape functions and the nodes $\mathbf{r}_i, i = 1, \dots, 6$. Thus, any point on a quadratic triangle's surface S_n can be represented using

$$\mathbf{r}(\alpha, \beta) = \sum_{i=1}^6 N_i(\alpha, \beta)\mathbf{r}_i, \quad (\text{C.1})$$

where α and β are the barycentric coordinates of the quadratic triangle. In (C.1), the shape functions $N_i(\alpha, \beta)$ are given [6] as

$$\begin{aligned} N_1 &= (2\gamma - 1)\gamma, \quad N_2 = (2\alpha - 1)\alpha, \quad N_3 = (2\beta - 1)\beta, \\ N_4 &= 4\gamma\alpha, \quad N_5 = 4\alpha\beta, \quad N_6 = 4\gamma\beta, \end{aligned} \quad (\text{C.2})$$

where $\gamma = 1 - \alpha - \beta$. The normal vector of the points on S_n can be determined as:

$$\mathbf{n}(\mathbf{r}(\alpha, \beta)) = \mathbf{v}_1(\alpha, \beta) \times \mathbf{v}_2(\alpha, \beta), \quad (\text{C.3})$$

where $\mathbf{v}_1(\alpha, \beta)$ and $\mathbf{v}_2(\alpha, \beta)$ are defined as

$$\mathbf{v}_1(\alpha, \beta) = \sum_{i=1}^6 \frac{\partial N_i(\alpha, \beta)}{\partial \alpha} \mathbf{r}_i, \quad \mathbf{v}_2(\alpha, \beta) = \sum_{i=1}^6 \frac{\partial N_i(\alpha, \beta)}{\partial \beta} \mathbf{r}_i. \quad (\text{C.4})$$

Assuming that $\mathbf{r}_i$ denote the rotated coordinates where $\hat{\mathbf{k}}_r = \hat{\mathbf{x}}_p$ and $x_i = \hat{\mathbf{x}}_p \cdot \mathbf{r}_i$, coefficients $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}, \tilde{e}, \tilde{f}$ in the quadratic equation (3.6) can be found as

$$\begin{aligned} \tilde{a} &= 2x_1 + 2x_2 - 4x_4, & \tilde{b} &= 2x_1 + 2x_3 - 4x_6, \\ \tilde{c} &= -3x_1 - x_2 + 4x_4, & \tilde{d} &= -3x_1 - x_3 + 4x_6, \\ \tilde{e} &= 4(x_1 - x_4 + x_5 - x_6), & \tilde{f} &= -x_1 + \frac{ct}{2|\mathbf{k}_r|}. \end{aligned} \quad (\text{C.5})$$

Independent from the form of the quadratic curve, the intersection of the plane and the edges of the S_n can be determined by solving (3.6) for $\beta = 0$, $\alpha = 0$, and $\gamma = 0$ (i.e. $\beta = 1 - \alpha$). For these cases, the intersection points can be found as follows:

$$\alpha_{1,2} = \frac{-(-3x_1 - x_2 + 4x_4) \pm \Delta_\alpha}{2(2x_1 + 2x_2 - 4x_4)}, \quad \beta_{1,2} = 0, \quad (\text{C.6})$$

$$\alpha_{3,4} = 0, \quad \beta_{3,4} = \frac{-(-3x_1 - x_3 + 4x_6) \pm \Delta_\beta}{2(2x_1 + 2x_2 - 4x_4)}, \quad (\text{C.7})$$

$$\alpha_{5,6} = \frac{x_2 + 3x_3 - 4x_5 \pm \Delta_\gamma}{4(x_2 + x_3 - 2x_5)}, \quad \beta_{5,6} = 1 - \alpha_{5,6}, \quad (\text{C.8})$$

Table D.1: Quadratic curve types.

Curve Type	Condition
Ellipse (Real)	$\Delta \neq 0, \Delta/I < 0, \zeta > 0$
Parallel Lines (Real)	$\Delta = 0, \zeta = 0, \kappa < 0$
Hyperbola	$\Delta \neq 0, \zeta < 0$
Intersecting Lines (Real)	$\Delta = 0, \zeta < 0$
Parabola	$\Delta \neq 0, \zeta = 0$
Coinciding Lines	$\Delta = 0, \zeta = 0, \kappa = 0$
Single Line	$\tilde{a} = 0, \tilde{b} = 0, \tilde{e} = 0$

respectively. In (C.6)-(C.8), Δ_α , Δ_β , and Δ_γ are given as

$$\Delta_\alpha = \left[-4 \left(-\frac{ct}{2|\mathbf{k}_r|} + x_1 \right) (2x_1 + 2x_2 - 4x_4) + (-3x_1 - x_2 + 4x_4)^2 \right]^{\frac{1}{2}}, \quad (\text{C.9})$$

$$\Delta_\beta = \left[-4 \left(-\frac{ct}{2|\mathbf{k}_r|} + x_1 \right) (2x_1 + 2x_3 - 4x_6) + (-3x_1 - x_3 + 4x_6)^2 \right]^{\frac{1}{2}}, \quad (\text{C.10})$$

$$\Delta_\gamma = \left[-4 \left(-\frac{ct}{2|\mathbf{k}_r|} + x_1 \right) (2x_2 + 2x_3 - 4x_5) + (-x_2 - 3x_3 + 4x_5)^2 \right]^{\frac{1}{2}}. \quad (\text{C.11})$$

If $\{\Delta_\alpha, \Delta_\beta, \Delta_\gamma\} < 0$, then there is no intersection with the associated edge. On the other hand, if $\{\Delta_\alpha, \Delta_\beta, \Delta_\gamma\} > 0$ then the intersection points can be determined as in (C.6)-(C.8). The points that satisfy the conditions, $0 \leq \{\alpha, \beta, \gamma\} \leq 1$, simultaneously, are the desired points of interest.

Appendix D: Quadratic Curve Types

In this section, quadratic curve types that can be represented by (3.6) are given. The type of the intersecting quadratic curve can be classified by the following quantities and Table D.1, which are given in the form of the general quadratic curve equation [48]. Note that only the interested types of curves are listed in Table D.1, whereas there are other, mainly imaginary, types of curves still can be represented by (3.6) [48].

$$\Delta = \begin{vmatrix} \tilde{a} & \tilde{e}/2 & \tilde{c}/2 \\ \tilde{e}/2 & \tilde{b} & \tilde{d}/2 \\ \tilde{c}/2 & \tilde{d}/2 & -\tilde{f} \end{vmatrix}, \quad \zeta = \begin{vmatrix} \tilde{a} & \tilde{e}/2 \\ \tilde{e}/2 & \tilde{b} \end{vmatrix}, \quad (\text{D.1})$$

$$\kappa = \begin{vmatrix} \tilde{a} & \tilde{c}/2 \\ \tilde{c}/2 & -\tilde{f} \end{vmatrix} + \begin{vmatrix} \tilde{b} & \tilde{d}/2 \\ \tilde{d}/2 & -\tilde{f} \end{vmatrix}, \quad I = \tilde{a} + \tilde{b}.$$

Appendix E: Diffraction Coefficients

In this section, diffraction coefficients, which are used in Section 4, are presented. These coefficients can be found in [11] and the parameters are given in Fig. 4.2.

$$D_{e,n}^I = \frac{-4 [\sin(\phi'/2) (\kappa - \sin \beta' \cos(\phi'/2))]}{\sin \beta' [\sin \beta' (\sin \beta \cos \phi + \sin \beta' \cos \phi') + \cos \beta' (\cos \beta - \cos \beta')]}, \quad (\text{E.1})$$

$$D_{h,n}^M = \frac{-2 [\sin \beta' \sin \phi' (1 - \sin \beta' \cos(\phi'/2) \kappa^{-1})]}{\sin \beta [\sin \beta' (\sin \beta \cos \phi + \sin \beta' \cos \phi') + \cos \beta' (\cos \beta - \cos \beta')]}, \quad (\text{E.2})$$

$$D_{h,n}^I = \frac{-2 [\sin \beta' (\cot \beta \cos \phi + \cot \beta' \cos \phi' + \cos(\phi'/2) \kappa^{-1} \zeta)]}{\sin \beta' [\sin \beta \cos \phi + \sin \beta' \cos \phi' + \cos \beta' (\cos \beta - \cos \beta')]}, \quad (\text{E.3})$$

$$\kappa = \sqrt{\sin \beta'} [\sin \beta' - \sin \beta \cos \phi + \cos \beta' (\cos \beta - \cos \beta')/2], \quad (\text{E.4})$$

$$\zeta = \cot \beta' [\sin \beta \cos \phi + \cot \beta' (\cos \beta - \cos \beta') - \sin \beta' \cot \beta \cos \phi], \quad (\text{E.5})$$

$$\phi = \tan^{-1} \left[\frac{\hat{\mathbf{n}}(\mathbf{r}'_p) \cdot \hat{\mathbf{k}}_s}{(-\hat{\mathbf{n}}(\mathbf{r}'_p) \times \hat{\mathbf{t}}_n(\mathbf{r}'_p)) \cdot \hat{\mathbf{k}}_s} \right], \quad (\text{E.6})$$

$$\phi' = \tan^{-1} \left[\frac{\hat{\mathbf{n}}(\mathbf{r}'_p) \cdot (-\hat{\mathbf{k}}_i)}{(-\hat{\mathbf{n}}(\mathbf{r}'_p) \times \hat{\mathbf{t}}_n(\mathbf{r}'_p)) \cdot (-\hat{\mathbf{k}}_i)} \right], \quad (\text{E.7})$$

$$\beta = \cos^{-1} \left[\hat{\mathbf{k}}_s \cdot \hat{\mathbf{t}}_n(\mathbf{r}'_p) \right], \quad (\text{E.8})$$

$$\beta' = \cos^{-1} \left[\hat{\mathbf{k}}_i \cdot \hat{\mathbf{t}}_n(\mathbf{r}'_p) \right]. \quad (\text{E.9})$$