

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**A NUMERICAL INVESTIGATION OF POWDER SPREADING
IN ADDITIVE MANUFACTURING**

MUHAMMET FURKAN ÇOŞKUN
A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING

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YÜKSEK LİSANS JÜRİ ONAY FORMU

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SUMMARY

Powder-Bed-Based additive manufacturing processes such as laser powder bed fusion, binder jetting and electron beam melting are commonly utilized in various critical areas for example, medical, aviation and energy. In these processes, metals, ceramic or polymer powders can be used to create complex geometries. Common to all these operations, the powders are first spread onto the built platform layer by layer fashion and selectively fused or bound with a suitable method. The quality of the process depends on several process parameters. Many of the process parameters have been studied extensively. It was shown that the spreading parameters can affect the process outcomes, but there is still a need to extend the current understanding on how various process parameters can influence the powder bulk density.

In this study, a discrete element method (DEM) based numerical approach was used to investigate the relationship between process parameters and powder layer bulk density. The discrete element method was used to determine the effects of the layer thickness, rotational and transitional velocities selected as parameters on the production quality during or after the process in the spreading process with a powder spreader roller. To increase the accuracy of the current numerical model, the 'Hertz-Mindlin and Johnson Kendall's' contact model, which reveals the interactions between particle-particle and particle-wall, has been added to the numerical model. For the main powder material, Ti6Al4V (Titanium Alloy), which is widely used in additive manufacturing, was used to develop a common interpretation for comparison. The obtained powder mass densities were analyzed using analysis of variance (ANOVA) and the results obtained were similar to those found in the literature.

Key Words: Additive Manufacturing (AM), Powder Spreading (PS), Electron Beam Melting (EBM), Laser Bed Fusion (LBF), Binder Jetting (BJ), Discrete Element Method (DEM)

ÖZET

Lazer toz yatağı füzyonu, bağlayıcı püskürtme ve elektron ışını ergitme gibi toz yatağı bazlı eklemeli üretim süreçleri, tıp, havacılık ve enerji gibi çeşitli kritik alanlarda yaygın olarak kullanılmaktadır. Bu işlemlerde karmaşık geometriler oluşturmak için metal, seramik veya polimer tozlar kullanılabilir. Tüm bu işlemlerde ortak olan, tozlar önce inşa edilmiş platform üzerine katmanlar halinde yayılır ve uygun bir yöntemle seçici olarak ergitilir veya bağlanır. Prosesin kalitesi bir dizi proses parametresine bağlıdır. Proses parametrelerinin işlemler üzerine etkileri kapsamlı bir şekilde incelenmiştir. Yayılma parametrelerinin işlem sonuçlarını etkileyebileceği gösterilmiş, fakat çeşitli işlem parametrelerinin toz kütle yoğunluğunu nasıl etkileyebileceği konusundaki anlayışın geliştirilmesine ihtiyaç duyulmaktadır.

Bu çalışmada, proses parametreleri ile toz katman kütle yoğunluğu arasındaki ilişkiyi araştırmak için ayırık eleman yöntemi (AEY) tabanlı sayısal bir yaklaşım kullanılmıştır. Toz serpmeye merdanesiyle yapılan serpmeye işleminde parametre olarak seçilen tabaka kalınlığı, dönme ve geçiş hızlarının proses sırasında veya sonrasında üretim kalitesi üzerindeki etkilerini belirlemek ayırık eleman yöntemi kullanılmıştır. Mevcut sayısal modelin doğruluğunu arttırmak amacıyla da parçacık-parçacık ve parçacık-duvar arasındaki etkileşimlerini ortaya çıkaran 'Hertz-Mindlin ile Johnson Kendall's' kontak modeli sayısal modele eklenmiştir. Karşılaştırmalı olarak ortak bir yorum geliştirmek için ana toz malzemesi içinse, eklemeli imalatla yaygın olarak kullanılan Ti6Al4V (Titanyum Alaşımı) kullanılmıştır. Elde edilen toz kütle yoğunlukları, varyans analizi (ANOVA) kullanılarak analiz edilmiş ve elde edilen sonuçların literatürde bulunan sonuçlara benzer ortaya konulmuştur.

Anahtar Kelimeler: Eklemeli İmalat (Eİ), Toz Serpmeye (TS), Elektron Işını Ergitme (EIE), Lazer Toz Yatağı Füzyonu (LZYF), Bağlayıcı Püskürtme (BP), Ayırık Eleman Yöntemi (AEY)

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LIST OF ABBREVIATIONS AND ACRONYMS

<u>Abbreviations</u>	<u>Explanations</u>
<u>and Acronyms</u>	
AM	: Additive Manufacturing
DEM	: Discrete Element Method
PS	: Powder Spreading
EBM	: Electron Beam Melting
LBF	: Laser Bed Fusion
SEBM	: Selective Electron Beam Melting
SLM	: Selective Laser Melting
AOR	: Angle of Repose
BJ	: Binder Jetting
MJF	: Multi Jet Fusion
DMT	: The Derjaguin, Muller, and Toporov
PSD	: Particle Size Distribution
JRK	: Johnson–Kendall–Roberts

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1. INTRODUCTION

The term 'Additive Manufacturing' states an extensive family of Manufacturing Technologies [1]. Additive manufacturing, referred to in short as AM, is an advanced manufacturing process, where parts are created by adding additive materials which is metal, polymeric or ceramic via layer by layer [2]. Additive Manufacturing provides production that is easily performed by employing CAD design. Feedstock materials are being built layer-by-layer via direct material deposition onto the substrate until the desired geometry is created. Compared to the traditional subtractive and formative manufacturing applications, such as machining operations that produce the parts via removing the material from rough shapes to obtain parts with precision tolerance, AM applications built 3D parts by adding and stacking layers of material onto each other [3]. Thanks to the different features compared to traditional methods, it has huge potential in revolutionizing the manufacturing industry by allowing rapid design. This manufacturing process allows us to create complex parts and provides simplified supply chain prototyping, customized and tailored production, flexible and design freedom [4]. This technology can also generate less waste product than traditional processes [5].

Additive manufacturing consists of 8 steps in general, and these steps also describe how the process works. Generally, most additive manufacturing methods follow these steps. These steps, also can be seen in Figure 1.1, are, respectively [6]:

- Functional CAD Design
- Converting CAD Design to file extensions such as STL/AMF that can be read
- Importing STL to Machine
- Set-up arrangements
- Building operation
- Part removal from Machine
- Post-processing approach of component
- Applications

In functional CAD design, which is the first of these eight steps, the part to be produced is designed in a functional way suitable for what it looks like and its intended

use. The second of the eight steps, converting to STL or AMF formats, is generally necessary and valid for all AM machines. In STL format type, only the geometry of the CAD model is provided with the surface data. These surface data are made up of triangles. In this way, the remaining modeling history and construction data is extracted from the CAD model. In the third step, the STL or AMF type files prepared in the second step are directly imported to the AM machine. In the fourth step, different specific features are determined according to the type of AM machine. Apart from these, in fact, general features such as layer thickness and process velocity, which are valid for most AM, are determined especially at this step and defined by the machine. The fourth step in the building requires considerable manual control during the first few stages of this step, in fact most of the process is automatically managed by the computer. After a few layers are performed semi-automatically at the beginning, it continues and is terminated completely under computer control. In the sixth step, sometimes the component may be ready for use after this process. However, in general, components must be post-processed before use. But in both cases, the component is produced from the table with the power of the operator or with an additional tool. The seventh step, post-Processing, is usually done manually and can be very diverse and specific depending on the components produced. This step may include abrasive finishing, polishing, and sandpapering or coating processes, as well as chemical or thermal treatments in some applications. In the eighth step, the part is ready for use and assembly.

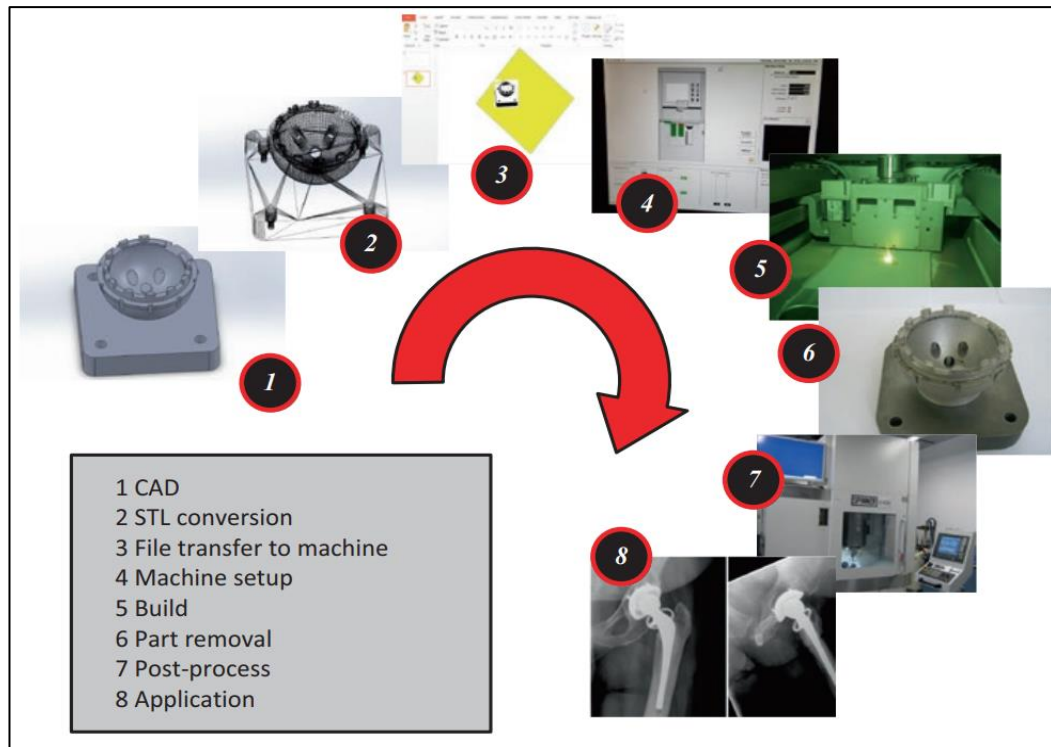


Figure 1.1: The eight stages of the AM.

Material science is a key for deep understanding of developments in AM technologies. In the beginning, AM technologies were produced with materials that were available and developed to match other processes. However, AM processes are different from other production methods, and these existing materials were poorly capable and far from ideal for these new applications. Due to their layered microstructure, the mechanical properties of AM parts might be considerably different from conventionally manufactured parts [7]. For example, due to the weaknesses of the material in the system, early photo-curable resins were quite brittle and easily deformed models. Powders employed in Laser-Based Powder Bed Fusion (LB-PBF) processes degraded rapidly during the process, and most of the materials resulted in unusable parts. As time progressed, AM technology became better understood and with the experience gained in this regard, the materials were developed to be suitable for AM processes. The materials have been adjusted in such a way that more suitable and better-quality parts of different processes are produced according to the operating parameters. As a result of these, parts can be created much more resisting, are able to produce longer-lasting parts. Most importantly, the part can be produced in the desired geometry as near-net-shape. These new advanced materials have caused the processes to be adjusted to produce higher temperature materials (including metals), smaller

property sizes and faster throughput [6]. Although there are many plastic/polymer components suitable for AM, some special materials are attracting more industry attention, as shown in Figure 1.2.

Especially in recent years, the ability to produce designs inspired by nature has increased in parallel with the development of the additive manufacturing method. In the past, especially in the years of the industrial revolution, functional design and machine element geometries were generally created from simple rectangles or rounds. The main reason behind this was the manufacturability concern. Contrary to these rough and dysfunctional geometries, living things in nature or committees formed by these creatures offer us more effective and functional designs. For example, designing the Shinkansen Bullet Train in Japan inspired by the Kingfisher's beak, and producing aerodynamic clothing inspired by the skin of a shark [8]. This type of nature inspired situation is called Biomimetic or Biomimicry, and these applications are increasing with the increase of developments in additive manufacturing. And in the future, designs will be embodied by the additive manufacturing method by imitating the superior features of different living things [9].

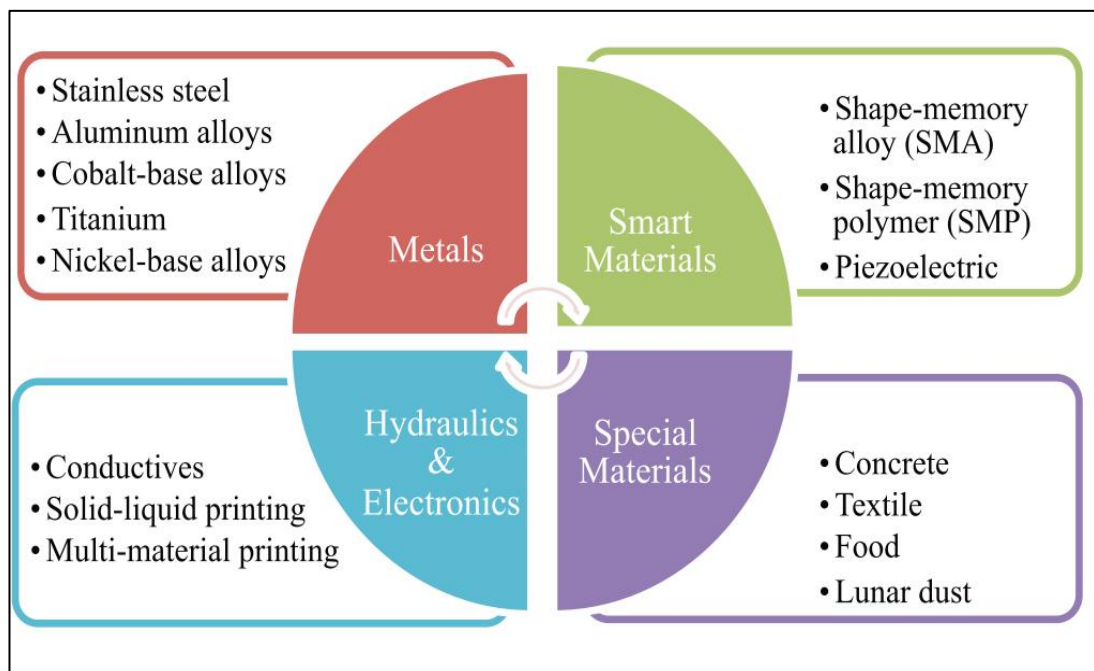


Figure 1.2: Current research materials for AM in the forthcoming era.

Fundamentally, based on most processes in the capture of Additive Manufacturing, it is categorized according to its material feed stock, build volume and energy source etc. According to in Figure 1.3 [10], AM process is usually divided and categorized into seven main categories. In the seventh category of AM, Powder Bed Fusion (PBF) technology provides very different and numerous production possibilities. Powder Bed Fusion, which uses laser or electron beam as an energy source to fuse these particle materials to form parts by sequencing powdered materials layer by layer, is gaining more and more importance in AM technology [11, 12]. Recently, the most growing manufacturing techniques in AM applications is the PBC which is also known as powder bed-based techniques, where a thin layer of particles whose diameters can be changed from a few microns (typically 10–50 μ m) to several dozen , is spread by a blade or roller and then sintered or melted by either an electron beam or laser; the process is then repeated until the desired part to be produced is fully built [1, 13]. Complicated shapes could be easily produced and give chance the production of topology optimized components, which reduces the total weight and material consumption and simplify the manufacturing of shape-customized objects.

Additive manufacturing technology is very useful compared to other traditional production methods, especially for prototype production. For complex parts that need to be manufactured using traditional production methods, the prototype causes a lot of cost, time, and effort during the production phase. AM process provides near-net-shape parts with good mechanical properties, and it has low surface roughness values in terms of tribological properties. In Additive Manufacturing, especially considering 3D printers, it is appropriate to carry out the production at a time and place of our request depending on the virtual model. [14].

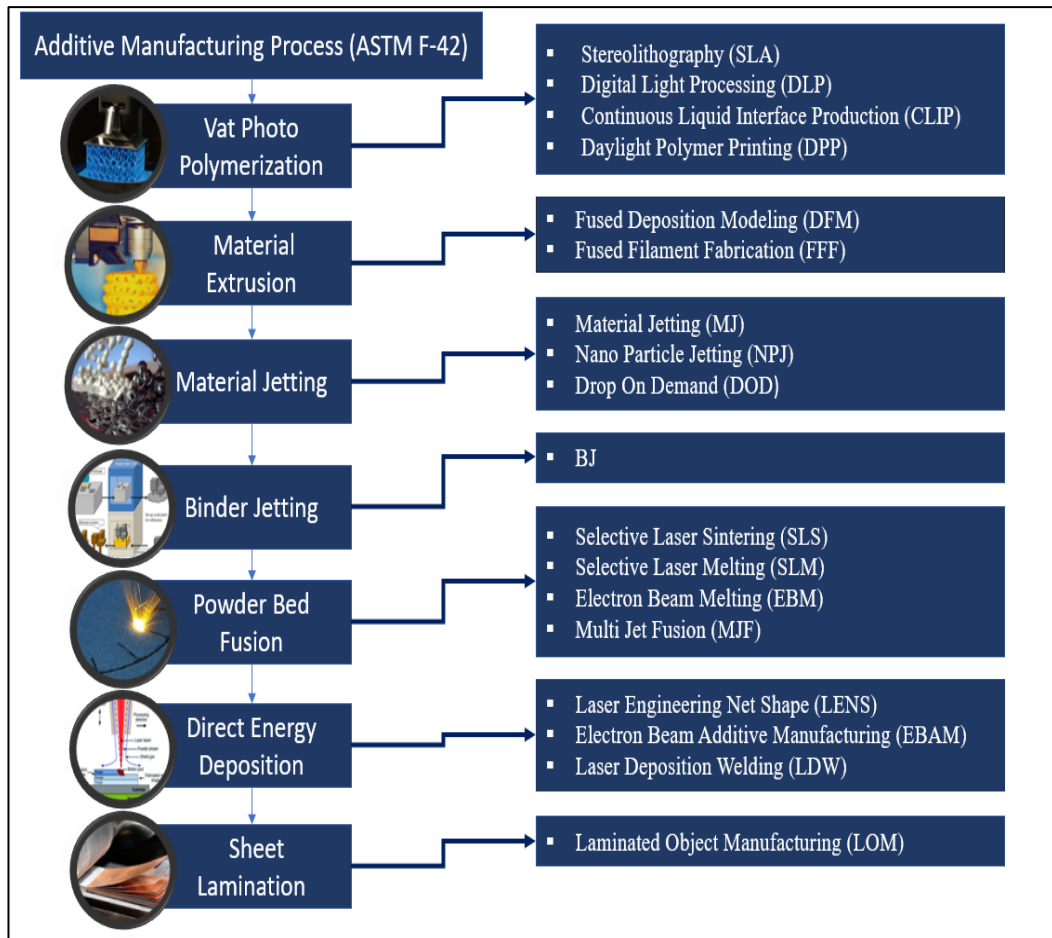


Figure 1.3: Classification of AM process according to ASTM F-42.

2. POWDER BED-BASED ADDITIVE MANUFACTURING

Powder Bed-Based additive manufacturing methods, which have existed since the first use of additive manufacturing technology, still maintain their importance as an advanced manufacturing technology and continue its development. Polymers and metals are very suitable for use as particulate materials. On the other hand, ceramic and composite materials are also used to a lesser extent. In this method, many different energy sources are used to fuse the particles. Particle Bed-Based additive manufacturing methods are grouped as the material used or the fusion source. Laser or electron beam can be used as the melting source, or interparticle bonding can be achieved by using a binder selectively [6]. Powder Bed-Based additive manufacturing has a wide range of applications in many different sectors such as medicine, aviation, construction, biology and architecture [15].

In the different types of processes mentioned above, powders are created by spreading through a spreader, which might be roller or blade, throughout the working area to create a layer. The energy source is programmed and applied to the surface of the powder bed selectively, after the powder bed is properly spread by the spreader in such a way as to provide the desired geometry. Additional powder is spread throughout the work area, and the process is repeated to build a solid three-dimensional part [16].

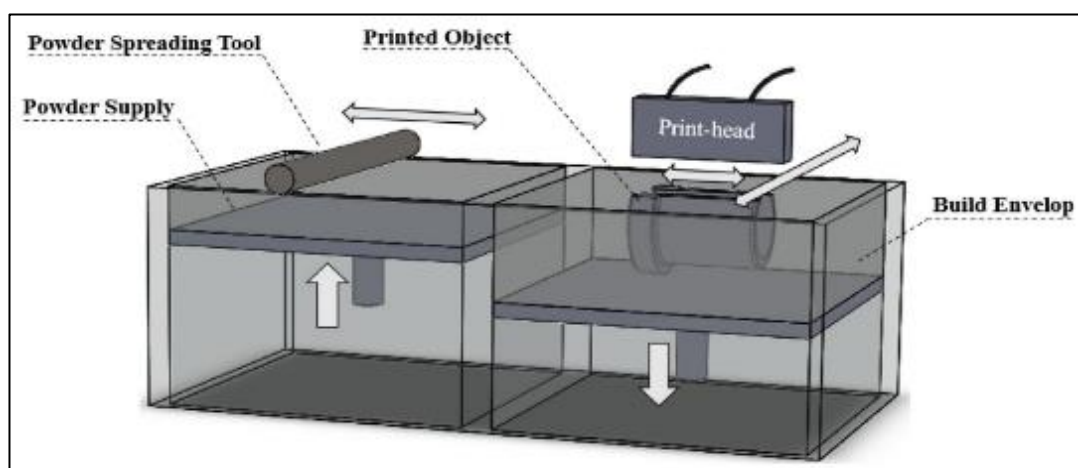


Figure 2.1: Illustration of binder jetting.

2.1 Multi Jet Fusion

Multi Jet Fusion technology is an advanced manufacturing method patented and developed by HP company based on powder bed [17]. Process is performed by JP Jet Fusion 3D 4200 printer. Generally, PA12 polymer is used as the particle material, and in the process, these particles are first deposited onto the platform as seen in Figure 2.2. The special fusing agent (surfactant), which consists of a distinguishable black ink and contains an infrared absorbing material, is carefully applied to the powder deposited according to the component to be given desired layer. By adding detailing agents in addition to fusing agents, fusion of particles among themselves is prevented and this improvement has a positive effect on resolution. As a finishing touch for the first layer, a heat source is provided by an infrared lamp on top of the build plate. Here an infrared lamp is used to sinter and fuse the building material. PA12 material is used as particle material, polymer material, in heating process, fusing agents convert IR radiations applied by infrared lamps into thermal energy and provide sintering and fusing process. The fused and sintered particulate material turns into layers [18].

The processes mentioned above for a single layer are applied repetitively until the desired component is fully built. Compared to other powder-bed-based processes, the Multi Jet Fusion process is much faster via use of planar IR radiation treatment source it uses [19].

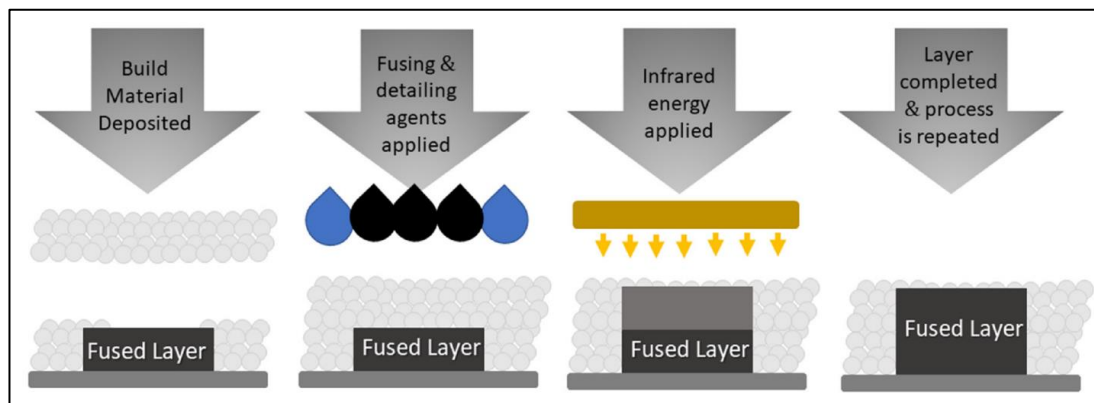


Figure 2.2: Schematic of MJF process steps.

2.2 Binder Jetting

Binder Jetting (BJ) manufacturing technology was first developed via works at Massachusetts Institute of Technology (MIT). Subsequently, it was patented by Emanuel Sachs and its provinces were presented to humanity as a manufacturing technology [20]. The Binder Jetting process is applied by following a total of four steps as seen in Figure 2.3. These are Printing, Curing and Powdering, Sintering and Infiltration and finally Finishing steps, respectively. In the printing process, the desired component geometry is first designed or modeled virtually, and it is then saved as an STL extension. The 3D STL file is transferred to the device that will perform other physical operations. First physical operation starts with spreading operation, the particles are distributed on the build plate with a roller that rotates counterclockwise and can move back and forth, to create a thin layer. After spreading, a liquid binder, a polymer solution in solvent, is applied from the printhead onto the layer based on 3D data. Some manufacturing errors may be encountered in the Binder Jetting process. These are usually caused by the binder not showing enough effect. The main reason why it does not show this effect is that the binder does not dry out. If the binder is not dried, various cracks may appear or it may result in the binder remaining on the spreader during the formation of the second layer, resulting in an error in the process. To eliminate such errors and to ensure quality production, the electric heater is passed over the layer and curing is provided. In some applications, curing and depowdering operations are applied after the printing process is completed to obtain sufficient green strength. For this process, the powder bed is taken out of the printer and placed in a furnace with a temperature of 180-200 ° degrees to ensure sufficient drying, where the desired shape is separated manually from the powder bed. After curing and depowdering operations, the bulk density of the component generally reaches 50-60% compared to the density of the material used as particles [20, 21]. It is possible to increase this bulk density value with an advanced process, sintering or infiltration. The surface roughness of a part produced by the binder jet method is generally in the range of 6-8 microns. Some finishing operations can be applied to reduce the surface roughness and to obtain the desired surface roughness. Commonly used Finishing processes are bead blasting and tumble polishing. In the case of bead blasting, the surface roughness can be reduced to 7 microns. On the other hand, in the case of tumble polishing, the average surface roughness can be reduced to 1.25 microns [22].

Materials with high production rates, in which polymers, metals and ceramics are used as particles, are successfully applied by Binder Jetting process. A wide variety of techniques are used in the BJ process, including printing techniques, dynamic binder/powder interaction and postprocessing method [23]. Compared to other Powder-bed-based production methods, the variety of materials used in the binder jetting manufacturing method is very high. Such as bio-materials, ceramics, metals, polymers are suitable for use as particulate materials. This wide use of materials allows to have a very large manufacturing area, and even optimum manufacturing conditions are carried out by studies and trials, and the usage areas are increasing day by day [24, 25].

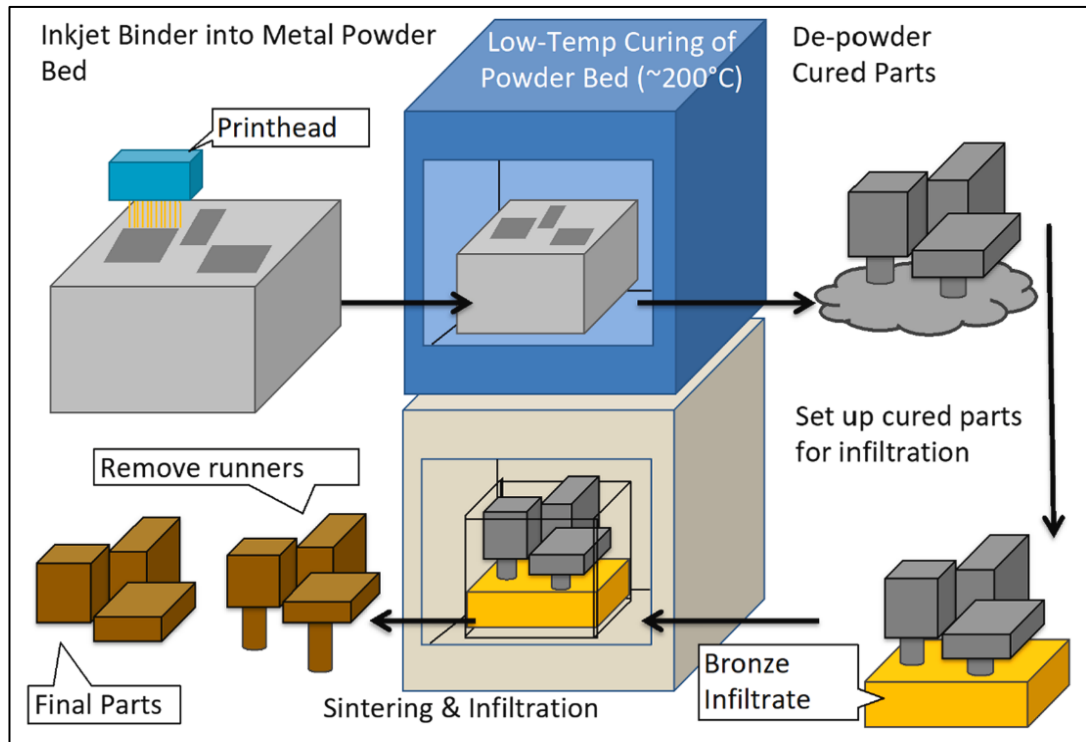


Figure 2.3: General illustration of binder jetting process steps.

2.3 Selective Electron Beam Melting

In the electron melting system, which is very similar to the laser melting system, electron is used as the melting source instead of laser light, and the process takes place in a vacuum chamber. Although SEBM and SLM processes are similar to each other, there are great differences in these two production methods due to the types of energy sources they use. In particular, the energy provided by electrons and photons has a nature and therefore has different disadvantages and advantages. As an advantage, the electron beam can be focused on a point by means of lenses, thus allowing very fast movement. However, as a disadvantage, vacuum pressure is needed and limited to the use of conductive materials only.

In the SEBM process, which has a mechanism that can move precisely at the micron level, first, a layer of 40-140 microns thick is on a metal plate called a stage. The base plate, which is another plate seen in Figure 2.4a, is placed on top of the layer. Vacuum pressure value above about 10^{-3} Pa is gained to the system. After falling to the desired vacuum level, the system reaches the required temperature value by scanning the electron beam. At these temperature values, particles spread on the base plate simultaneously and a layer is formed. After this layer, as seen in figure 2.4d, the particles are prevented from being charged by scanning the electron beam again. As a finishing touch to the first layer, the electron beam selectively scans across the 2D slice model to give the desired geometry 2.4e. After the process of the first layer is finished, the system is lowered down by the next layer thickness by means of a mechanism capable of up-down movement, as can be seen in Figure 2.4f [26, 27].

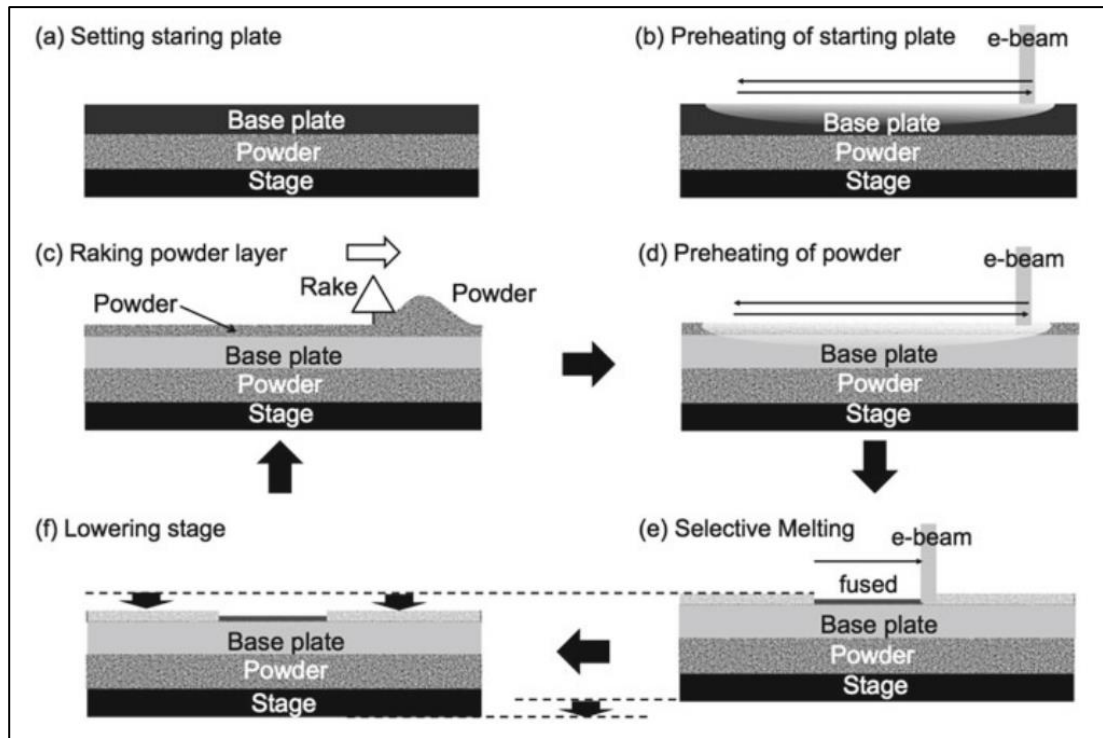


Figure 2. 4 :Process chain of selective melting.

2.4 Selective Laser Melting

Laser powder bed fusion provides the opportunity to produce metal parts, especially structures with complex geometries, via layer-by-layer. Due to these characteristics, it provides flexible, less waste material and very fast prototype part production [12]. Due to the development of aviation, bio-implantation, automotive and many other industries, the use and development of LPBF applications are increasing every day. The LPBF process first of all, particles with small diameters are deposited by a spreader, then a laser beam is used to scan and melt and combine the powder particles according to the desired geometry to be created [28]. These process continue by repeating until the final desired geometry is formed [2, 29].

Selective laser melting of metals (SLM) is very effective in the production of parts with geometry that are too complex to be produced by conventional manufacturing methods. Control of mechanical properties is possible at very small scales, e.g. micro scale, and this feature attracts a lot of attention [30]. Despite these superior features, the process is very difficult to implement due to features such as various physical mechanisms and the necessity of using various parameters at the optimum point. If these features or parameters are not selected correctly, it may cause

deterioration in the part desired to be produced. For these reasons, many tests and experiments have been carried out in recent years to determine the optimum points. With these studies, the ability and importance of the method increase day by day [31].

Although it varies according to the component produced, SLM generally consists of two basic steps. These steps are powder-spreading and powder-melting, respectively. The first step is the same as for other power-bed-based methods. As seen in Figure 2.5, there is a delivery chamber that can move up and down. Before the roller or blade used as a spreader moves, this chamber raises the required powders upwards to form the layer thickness. The rollers distribute the raised powders towards the building chamber. Thus, the first layer is formed. The same is true for other layers and repeats this process for each. It has the same ability to move up and down in the building chamber. In this chamber, the desired piece finds its final form. The powders transferred by the delivery chamber with the spreader are collected here and the high-energy laser as the melting source is applied according to the CAD data whose geometry is desired to be given. After the completed laser processes, the building chamber moves down and sets the required layer space for the next layer. These operations are repeated until the desired net-geometry is reached [32, 33].

Academic and industrial research activities related to AM usually focus more on the fusion and melting process, ignoring the spread of powders to form the powder bed. In general, industrial uses seem to take into account basic and relatively easy parameters to change at first. These are the scanning progress, the particulate material used, and the heat source. [34]. Different from the basic parameters, the parameters that reveal the dynamic behavior and static state of the particles with numerical simulations also have important effects on increasing the production quality. Production parameters such as powder layer thickness, force-exerted formation during solidification, change of angle of repose and layer surface roughness, which is of great importance between layers, need to be understood very well with numerical simulations [35]. In order to better see the powder layer structures and forces during the spreading process, it is necessary to determine and understand the dynamics of the powder during the spreading. [36].

Experimental approaches can be challenging due to the lack of standardized procedures to improve the quality of the spreading process. Usually, powder motions and behaviors during the spreading process by in-situ experimental measurement is

difficult. The current morphology of the powder heap and powders relation with each other or between the wall, roller during spreading process can be obtained, which can indicate the flowability of powder by measuring the angle of repose, volume fraction and the probable particle cluster by high-speed camera [37]. In addition to the information obtained with high-speed cameras, it is very important to understand some of the micro-scale dynamics of the particles in the system. Observing the movements of the particles in the system individually and offering optimum parameter selections is an essential requirement for the development of existing production methods. In order to meet these essential needs, DEM based numerical simulations should be used [38].

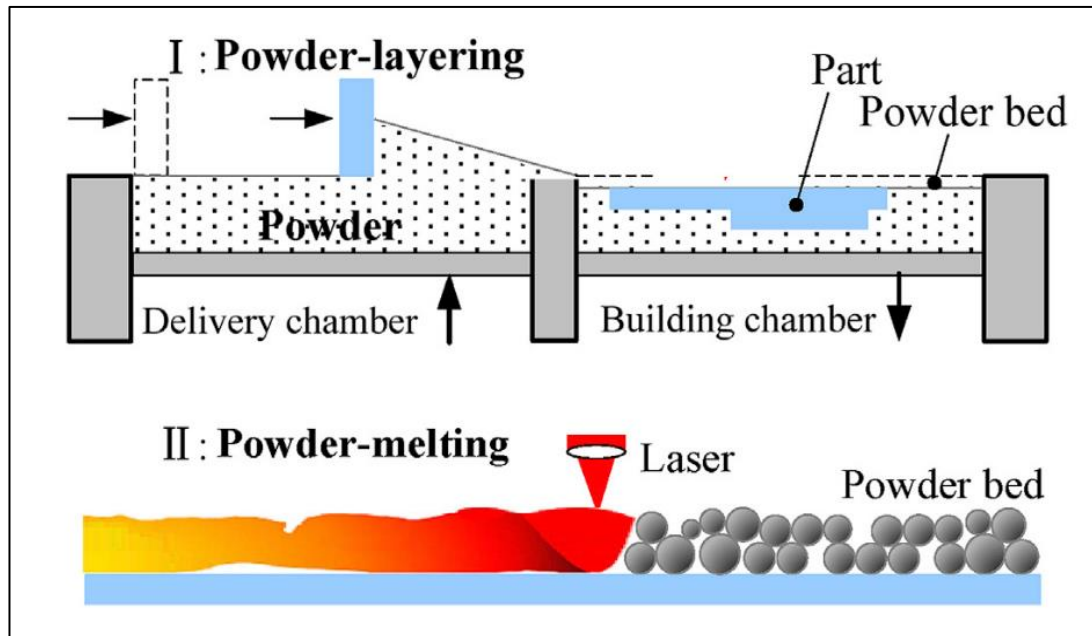


Figure 2.5: Basic processing steps of SLM.

3. DISCRETE ELEMENT METHOD

Discrete Element Method was first proposed by Cundall and Strack [38] for use in particle analysis, then widely implemented in simulations of some systems [39]. The DEM model proved that it has the ability to accurately predict the individual particle motion and some dynamics related to these motions in its numerical solutions. In order to understand the parameter selection of variable coating speeds and variable particle sizes and their effects on surface roughness, Parteli and Paschel developed a numerical workbench that simultaneously solves Newton's equations of translation and motion for all individual particles of powder [40]. Otherwise, Mindt et al. [41] focused on the spreading process of commercial titanium alloy powder and monitored the load on the partially built part. Subsequently, compared the results with literature reports. In the studies of Haeri and his colleagues, especially the speed parameter on coating and the displacement of plate were focused on, and their effects on the powder packaging density were investigated [2]. In addition, the performances of two different spreaders, both blade and roller types, were compared to understand the effect of spreader geometries. As a result of this comparison, the superiority of the cylinder's effect on spreading compared to the blade-type spreader was revealed. These numerical results are validated by experiments. Chen et al. [42] a numerical model based on the DEM and using a blade as a spreader in this numerical model, investigated the particle's dynamic behaviors and the contact forces and cohesive forces, and proposed to investigate the flow behavior of powder in layers. The model was compared and verified with the experimental results in terms of profiles, which is macro-scale, of the powder during layering process.

Similar evidences by Meier et al. [43] employed a model, which is computational, to determine the important the powder recoating process and verified with experimental result. Haeri [44] developed and used a new technique, the multi-sphere approach (MSA), instead of using particles with super spherical geometry in simulations to increase the convergence of the solution. Then he revealed the effects of these realistic particles on the powder bed. Nan et al. [45] studied the effects of friction-induced jamming inherent in powders. They experimentally revealed the properties of 316L powders using various methods and employed the necessary experimental values on their simulation.

Similar findings can be observed in the studies of Fouda et al. [46], In Fouda's study, he chose the particle material used as a coating as a spherical titanium alloy (Ti-6Al-4V), and a blade-type ideal spreader was also used as a spreader. Fouda's study found that the packing density of the final layer was affected by the inertia of the layers previously deposited by melting and the initial expansion of the spreader. In addition, the dual interactions of the parameters of the process and the mechanisms within the system were emphasized. Han et al. [47] focused on providing a systematic approach to determine the most optimal layer thickness by considering the packaging density. In the analysis of this optimal layer thickness, they developed a numerical model based on the DEM model and used the optimal parameters obtained here in his study, where he experimentally examined the microstructure and tensile strength. Phua et al. [48] analyzed the complex mechanisms between the titanium alloy they chose as the particle material and the blade they chose as the spreader in their analysis with a model based on the DEM model. In addition, they calibrated the particle properties with a logical and coherent powder model by matching the size, shape, and forces between particles with experimental measurements. Gurnon et al. [49] observed analyzes available in the literature and found that these studies generally examined a single-layer and their small number of particles. Gurnon studied a multi-layer numerical model, arguing that these studies were not suitable for some multi-layer models. The model was verified by employing static Angle of Repose measurements experimentally. Chen et al. [50] used a counterclockwise rotating roller in their DEM-based numerical analysis, in which the velocity of the powder particles, the contact forces of the powder particles with each other and with the spreader, and the coordination numbers of the particles were selected as parameters. The physical mechanisms with the parameters used here gave a new perspective. Valerio et al. [1] worked on the spreading operations and explained their physical circumstance with a numerical model based on DEM.

These investigations show that DEM is helpful for any family of numerical methods for simulating and computing powder mechanical behavior to come through the experimental challenges in AM processes.

3.1 Governing Equations

A “super-spherical approach” approach was used as the particle type because the powders have a very dense particle flow while a spreader well layering process [38, 51]. During the layering, Newton’s second law describes the motion of powder particles individually. As shown in Figure 3.2, The equations for each particle in the system due to the translational and rotational motion at time t due to the interactions of the particles i present in the system with each other j or with other parts in the system can be written as follows:

$$m_i \frac{du_i}{dt} = G_i + \sum_j (F_{ij}^n + F_{ij}^r) \quad (1)$$

$$I_i \frac{dw_i}{dt} = \sum_j (T_{f,ij} + T_{r,ij}) \quad (2)$$

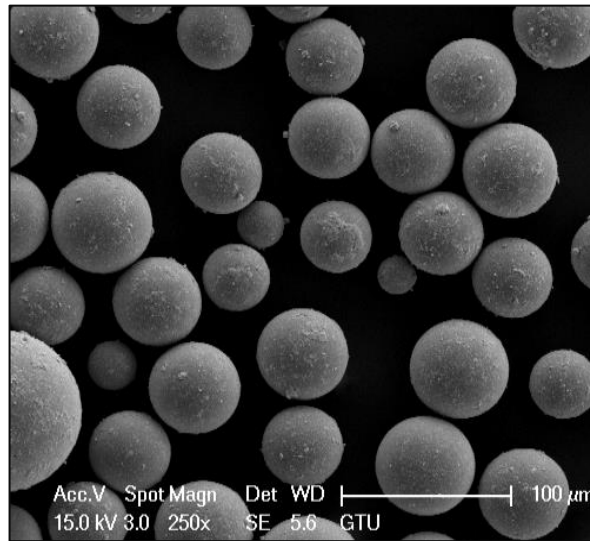


Figure 3.1:SEM image of 316 L stainless steel.

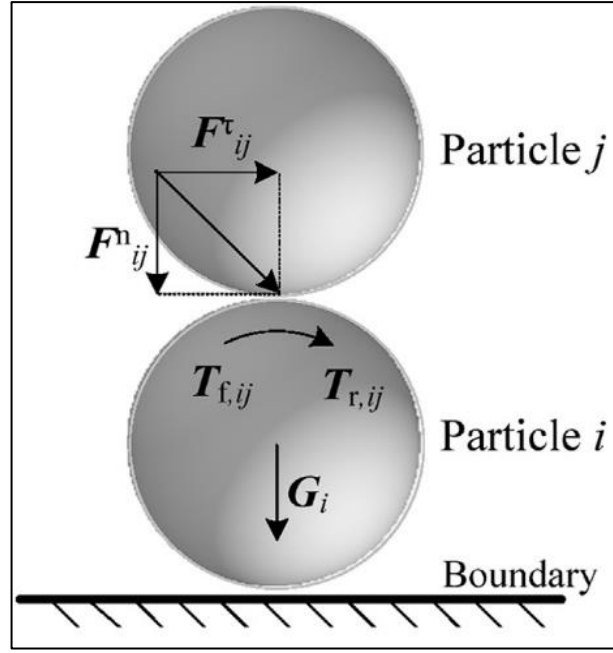


Figure 3.2: Illustration of force and torque for two interacting super-spherical particles.

Where u_i , I_i , m_i and ω_i are, respectively, translational velocity, moment of inertia, the mass, and rotational velocity of particle i ; F^n_{ij} and F^τ_{ij} are, respectively, the normal and tangential interaction forces between particle i and j . G_i is the gravitational force of particle i . The normal and tangential forces inherent in the system can be derived directly from the contact and damping force formulas, i.e., $F^n_{ij} = F^n_{c,ij} + F^n_{d,ij}$ and $F^\tau_{ij} = F^\tau_{c,ij} + F^\tau_{d,ij}$. Then, the model proposed by Tsuji et al [52], this model is also based on the Hert-Mindlin contact force model, simplified for ease of use, which makes an acceptable accuracy demonstrate of the particle dynamic behavior during the particle contacts [53], is applied to calculate the damping forces ($F^n_{d,ij}$ and $F^\tau_{d,ij}$) and tangential contact force forces $F^\tau_{c,ij}$ as a result of the deformation of the particles. $T_{f,ij}$ is the torque which is formed by the contact forces driving the other particle which cause particle i to rotate, and $T_{r,ij}$ is the rolling friction torque that opposes to the rotation of particle i . All details related to forces and moments, including translational and rotational motion, are summarized in Figure 3.3. Note that the normal contact force $F^n_{c,ij}$, considering the component of cohesion force.

The cohesion forces between particles cause the collision and which makes great effects on moving behaviors of particles. At micron scale especially for dry metal powder, there is unnecessary to pay regard to liquid bridge force and electrostatic

force, which are described as cohesion forces. Especially in the case of using particles smaller than 100 microns in diameter, Van der Walls force forces begin to dominate and this situation disrupts the homogeneity in the layer and reduces the production quality [54–56].

Following the JKR theory [57], the additional attractive force $F_{c,ij}^n$ between particles was explained by the Van der Walls force, resulting in changing the Hertz-Mindlind contact model [52] by placing the normal contact force $F_{c,ij}^n$. As a result, the surface can be attributed to the surface energy density as follows:

$$F_{c,ij}^n = -4(\pi E^* \gamma)^{\frac{1}{2}} r^{3/2} + \frac{4E^*}{3R^*} r^3 \quad (3)$$

Where the r is the contact spot radius [58]

$$r^4 - 2R'\delta^n r^2 - \frac{4\pi\gamma}{E^*} R^{*2} r + R^{*2} \delta^{n2} = 0 \quad (4)$$

It should be noted that there is still an attractive force in Equations (3) and (4) even if the particles are not in contact (i.e., $\delta^n < 0$). For the normal contact force, a critical value is needed to calculate the normal contact force $F_{c,ij}^n$, from the reference [59]. This is expressed as:

$$\delta_c^n = \frac{r_c^2}{R^1} - \sqrt{\frac{4\pi\gamma r_c}{E^*}}, r_c = \frac{9(3\sqrt{2} - 4)\pi\gamma R^{*2}}{8\sqrt{2}E^*} \quad (5)$$

It should be noted that the δ_c value in Equation 5 is always less than zero. In case of $\delta_n < \delta_c$, in other words, the zero value returns to the normal contact force $F_{c,ij}^n$, and may be omitted in the DEM model to reduce the computational cost as a result.

In case of $\delta_c < \delta_n < 0$, the maximum cohesion force, also known as the “pull-out force [57, 59]”, $F_{pullout}$, can be obtained by maximizing the exact value of $F_{c,ij}^n$ in Eq. (3), which is given by:

$$F_{pullout} = \max|F_{c,ij}^n| = \frac{3}{2}\pi R^*\gamma \quad (6)$$

To simplify the solution and show the effect of cohesive force dimensionless in a single equation, the parameter K can be expressed as:

$$K = \frac{F_{pull-out}}{G_i} = \frac{3\pi R\gamma}{2m_i g} = \frac{9\gamma}{8\rho_i g R^2} \quad (7)$$

Here, parameter K represents the number of bonds in a way, and depending on the height of this parameter, it is understood how strong the bond is between particles.

Forces and torques		Symbols	Equations
Normal direction	Interaction force	$\mathbf{F}_{ij}^n$	$\mathbf{F}_{ij}^n = \mathbf{F}_{c,ij}^n + \mathbf{F}_{d,ij}^n$
	Contact force	$\mathbf{F}_{c,ij}^n$	$F_{c,ij}^n = -4\sqrt{\pi\gamma E^*} r^{\frac{3}{2}} + \frac{4E^*}{3R^*} r^3,$ $r^4 - 2R^* \delta^n r^2 - \frac{4\pi\gamma}{E^*} R^{*2} r + R^{*2} \delta^{n2} = 0$
	Damping force	$\mathbf{F}_{d,ij}^n$	$F_{d,ij}^n = -\sqrt{\frac{20}{3}}\beta \left[E^* m^* (R^* \delta^n)^{\frac{1}{2}} \right]^{\frac{1}{2}} u_{ij}^n$
Tangential direction	Interaction force	$\mathbf{F}_{ij}^t$	$\mathbf{F}_{ij}^t = \mathbf{F}_{c,ij}^t + \mathbf{F}_{d,ij}^t$
	Contact force	$\mathbf{F}_{c,ij}^t$	$F_{c,ij}^t = -\min\{F_t, 8G^* \sqrt{R^* \delta^n} m^* \}$
	Damping force	$\mathbf{F}_{d,ij}^t$	$F_{d,ij}^t = \sqrt{\frac{80}{3}}\beta (G^* \sqrt{R^* \delta^n} m^*)^{\frac{1}{2}} u_{ij}^t$
Friction force		$\mathbf{F}_f$	$F_f = \mu_s \mathbf{F}_{c,ij}^n $
Friction torque		$\mathbf{T}_{f,ij}$	$\mathbf{T}_{f,ij} = \mathbf{R}_i \times \mathbf{F}_{c,ij}^t$
Rolling torque		$\mathbf{T}_{r,ij}$	$\mathbf{T}_{r,ij} = -\mu_r R_i \mathbf{F}_{c,ij}^n \hat{\omega}_i$
Gravitational force		$\mathbf{G}_i$	$\mathbf{G}_i = m_i \mathbf{g} = \frac{4}{3} \pi R_i^3 \rho_i \mathbf{g}$

Where the equivalent properties in terms of the properties of particle i and j are respectively $R^* = R_i R_j / (R_i + R_j)$, $m^* = m_i m_j / (m_i + m_j)$, $E^* = E_i E_j / [(1 - \xi_i^2) E_i + (1 - \xi_j^2) E_j]$, and $G^* = G_i G_j / [(1 - \xi_i) G_i + (1 - \xi_j) G_j]$; the transform coefficient is $\beta = \ln e / (\ln^2 e + \pi^2)^{0.5}$; the unit angular velocity is $\hat{\omega}_i = \omega_i / |\omega_i|$; R_i , E_i , G_i , ξ_i are respectively the radius, Young's modulus, shear modulus, and Poisson ratio of particle i and j ; γ , μ_s , μ_r , and e are respectively the surface energy, sliding friction coefficient, rolling friction coefficient, and restitution coefficient between particle i and j when a collision occurs (where: $\mathbf{n} = \mathbf{R}_i / |\mathbf{R}_i|$, $u_{ij}^n = (\mathbf{u}_i - \mathbf{u}_j) \cdot \mathbf{n}$, $u_{ij}^t = (\mathbf{u}_i - \mathbf{u}_j + R_i \times \omega_i - R_j \times \omega_j) \times \mathbf{n}$); then δ^n and δ^t signify, i the normal and tangential overlaps between particle i and j during the collision and can be obtained through the integration of u_{ij}^n and u_{ij}^t ; ρ_i is the density of particle i ; and $\mathbf{g}$ is the gravitational acceleration.

Figure 3.3: Forces and torque acting on particle i .

The frictional elasticity theory, which is suitable for a system in which a wall and sphere geometry are in contact, was first proposed by Hertz [60] (also reported in Johnson [61]). By developing the frictional elasticity theory, Mandolin and Deresiewicz were also able to add tangential directions to this normal contact state [62]. Mindlin and Deresiewicz developed a synthesis that explains the tangential contact and even the oblique impact situation with a progressive procedure from a simple normal contact case and proposed this synthesis with rules considered different types of conditions. In their studies, it was observed that the whole loading history and instantaneous changes in normal and tangential forces affected the general force-displacement relationship. The rules that Mindlin and Deresiewicz described in order

to obtain more convergent results for certain load-displacement curves were revised and developed by Vu-Quoc and Zhang [63].

In the absence of tangential displacement, the effects must be linear, and the variation of the system is relatively simpler than in other cases. For these reasons, there is no harm in presenting it with a non-linear relationship. In solving an elastic problem describing the general state of two spheres in contact with different mechanical properties, the maximum pressure, radius of the contact area, and total displacement can be calculated by Johnson's approach [61].

$$p_{max} = \frac{3f_n}{2\pi a^2} \quad (8)$$

$$a = \left(\frac{3R_{eq}}{4E_{eq}} f_n \right)^{1/3} \quad (9)$$

$$\delta_n = \frac{a^2}{R_{eq}} \quad (10)$$

Normal force f_n is:

$$f_n = -K_n \cdot \delta_n^{3/2} \quad (11)$$

Normal elastic constant:

$$K_n = \frac{4}{3} E_{eq} \sqrt{R_{eq}} \quad (12)$$

The equivalent Young's modulus E_{eq} and the equivalent radius R_{eq} , are reported in the appendix A.

As a result, it can be said that the tangential forces generated during the cycle are very close to reality, considering the condition of defining curves such as initial loading, unloading, or reloading. In addition, considering the case where the normal and tangential displacements change simultaneously, an incremental procedure must now be followed to accurately reveal the tangential displacements. In the calculation of the real tangential force f_t , the previous tangential force f_{t0} and the tangential displacement $\delta_t - \delta_{t0}$ can be calculated by an incremental rigidity K_t as a basis. To express this:

$$f_t = f_{t0} + K_t(\delta_t - \delta_{t0}) \quad (13)$$

The tangential force can also be found correctly if the tangential elastic constant K_t is correctly calculated using a stepwise approach, since information on the loading history is essentially retained at previous tangential forces and displacements. If the normal displacements are held constant and tangential displacements are applied, the tangential stiffness can simply be calculated based on the true normal displacement.

$$K_t = K_{t0} \left(1 - \frac{\frac{2}{3} K_{t0} \delta_t}{\mu f_n} \right)^{1/2} \quad (14)$$

μ , K_{t0} are the friction coefficient and initial stiffness constant, respectively.

$$K_{t0} = 8G_{eq} \sqrt{R_{eq} \cdot \delta_n} \quad (15)$$

In terms of the actual normal force:

$$K_t = K_{t0} \left(1 - \frac{f_t}{\mu f_n} \right)^{1/3} \quad (16)$$

In the case of a full load that has reached the regime, the force-displacement path is formed in successive gradual steps. It forms the shape of the tangential stiffness constant due to the micro-scale slip state that occurs on the contacting surfaces. Based

on Columb's law of friction, it can also be demonstrated that there is a limit on the maximum tangential stress here.

$$|t| \leq \mu |p| \quad (17)$$

Eq. (17) applies essentially to all contacting surfaces. If a general constant is to talk about normal displacement, it must be equal to Coloumb's law of friction, as mentioned above. This situation can be briefly called micro-slip. To explain the case of gross slip, here the slip area covers the entire contact area. This is why adhering parts cannot be mentioned here, and the law of friction must be on an integral scale. As a result, the value of the stiffness constant tends to be zero.

$$|f_t| \leq \mu |f_n| \quad (18)$$

On the other hand, in the unloading path, the micro-slips present in the contacting surfaces differ in the loading path and display the behavior in a hysterical fashion. In a regime where tangential displacements begin to decrease, the stresses and displacements at the turning point must be factored in. Otherwise, a complete convergence for the solution cannot exist. Mindlin and Deresiewicz suggested that turning force $f_t^{(TP)}$ should be used to calculate the stiffness constant. Considering this, it makes perfect sense to use it with the elastic constant Eq. (12) on an unloading curve:

$$K_t = K_{t0} \left(1 - \frac{f_t^{(TP)} - f_t}{2\mu f_n} \right)^{1/3} \quad (19)$$

It is possible to construct a complete unloading curve using successively increasing constant normal displacement steps, decreasing the tangential displacement until $-\delta_t^{(TP)}$. If we consider the stress and deformation distributions at this point, it is seen that the positive loadings, which had a high effect before, no longer have an effect and the distributions are now equal to the negative initial loading condition.

The corresponding stiffness constant after the second turning $f_t^{(TTP)}$ point is:

$$K_t = K_{t0} \left(1 - \frac{f_t - f_t^{(TTP)}}{2\mu f_n} \right)^{1/3} \quad (20)$$

A representation of the general initial loading-unloading-reloading curves with the tangential elastic K_t constant values corresponding to each state is shown in the Figure 3.3.

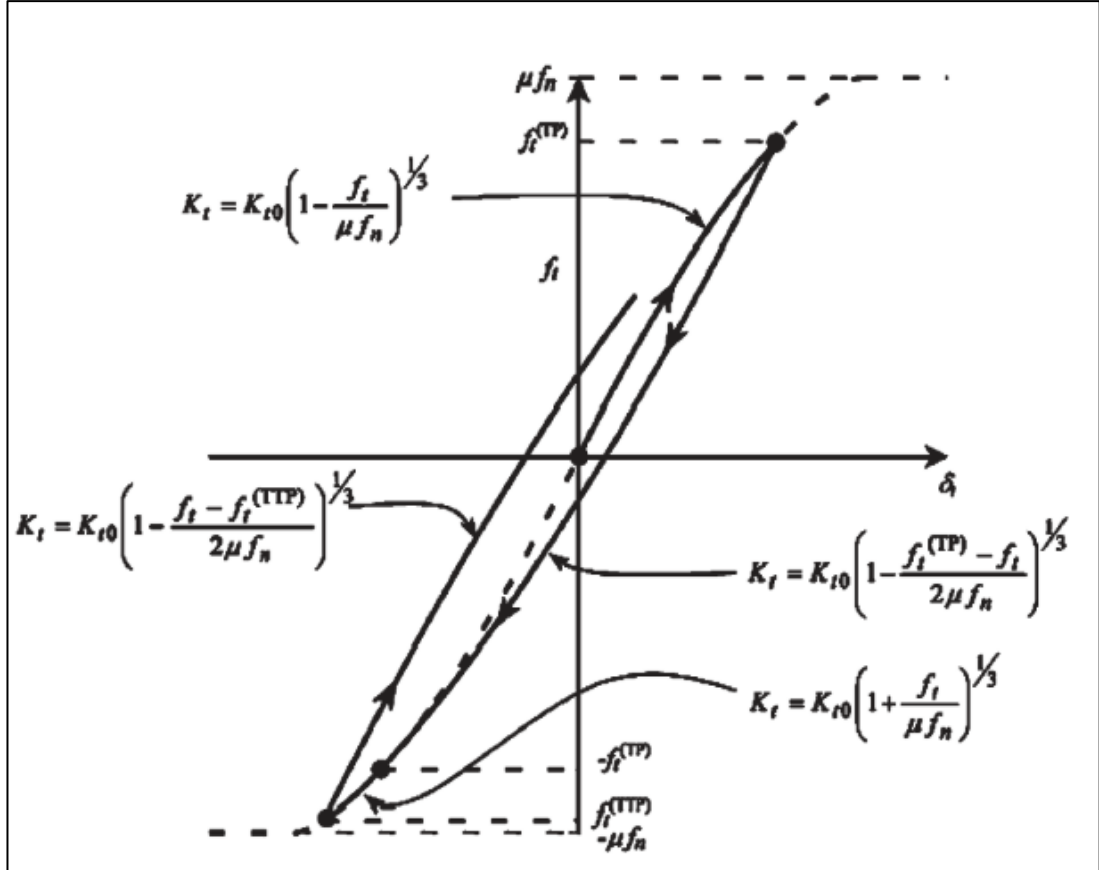


Figure 3.4: $f_t - \delta_t$ curve for constant normal displacement.

The underlying reason for this is to move to the next curve in a few basic steps by providing the conditions of constant normal displacement and actual normal displacements in the previous curve. The convergence of the calculation can be increased if a suitable assumption is developed according to the changing conditions. But hypotheses themselves often do not draw satisfactory conclusions. Even if it is close to the real solution, calculations should be made approximately. The basic assumptions are:

- Contact area which is circular and pressure distribution
- The alteration in normal displacement and tangential displacement are not connected to each other, i.e. If we talk about two separate effects, the problem can easily be solved by superimposing them.
- Contact area which is circular and pressure distribution
- Torsion or rolling is not included in the calculation.

Definition 1.

The state of the basic loading history condition system $(\delta_n, \delta_t, f_n, f_t)$ might be explained by normal displacement (corresponding to δ_n) and tangential loading curve $f_t - \delta_t$ (through a first loading/unloading/reloading curve). This can be done regardless of the loading path.

If we consider both the normal displacement and the tangential displacement case, which are increasing. According to the first assumption, the new normal force can be used directly as described in Section 2.1.1. As a result, tangential problems can be easily solved with the help of tangent curves $f_t - \delta_t$. These tangent curves correspond to previous and new constant normal displacement δ_{n0} and δ_{n1} , respectively.

Therefore, the tangential problem can be solved with the help of the tangential $f_t - \delta_t$ curves corresponding to the previous and the new constant normal displacements δ_{n0} and δ_{n1} , respectively (in Figure 3.4).

Figure 3.4 clearly shows that state 1 is found when viewed on the tangential curve corresponding to the new constant displacement δ_{n1} . As a result, it defines the minimum tangential displacement change $\Delta\delta_t$ to validate the assumption 3 that will be valid in the next step, that is, the basic loading history assumption. If we are to develop an expression in terms of variation of normal force in terms of variation of tangential force, it should be known here that the variation of tangential force is equal to $\mu\Delta f_n$, based on according to third assumption.

$$|\Delta f_t| \geq \mu |\Delta f_n| \quad (21)$$

If it is necessary to offer an alternative, this time the displacement changes:

$$|\delta_{t1}| \geq \Delta\delta_t^{(SL)} = \frac{\mu|\Delta f_n|}{K_{t0}} \quad (22)$$

What is expressed here with $\Delta\delta_t^{(SL)}$ is actually the minimum tangential displacement change for the purpose of ensuring the loading history condition. As a result of this, it now provides the possibility of evaluating $\Delta\delta_t^{(SL)} = \delta_{t1} - \delta_{t0} \cdot K_{t0}$ at the new normal force.

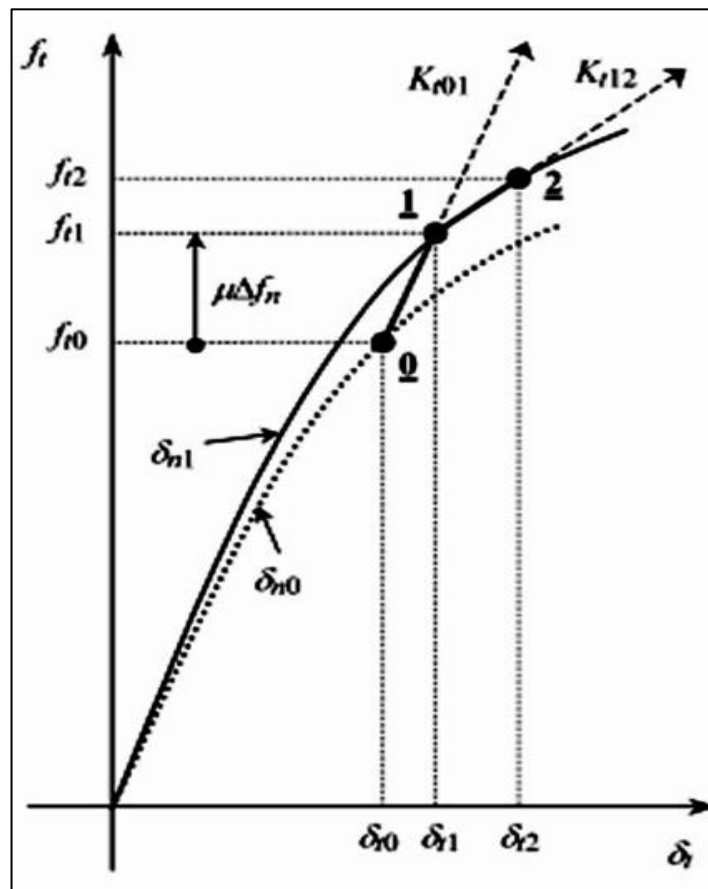


Figure 3.5: Tangential loading curve for the case: Increasing δ_n , δ_t .

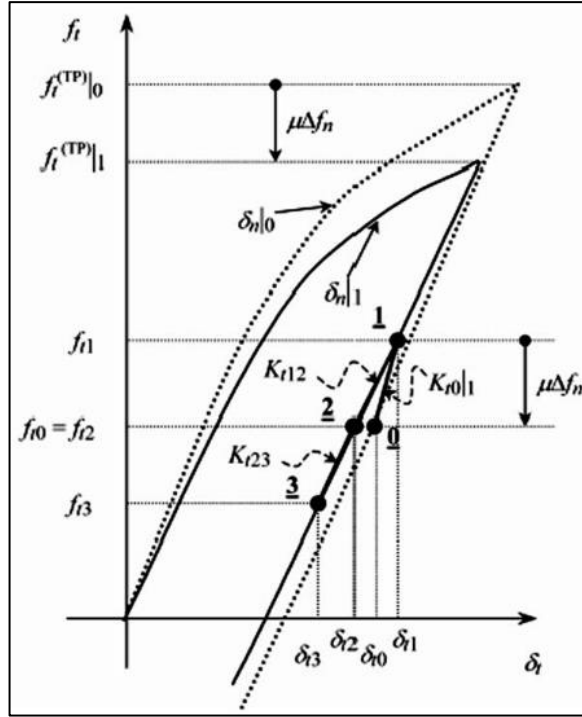


Figure 3.6: Tangential loading curve for the case: decreasing δ_n , δ_t .

The state change from the initial state 0 to the actual state determines whether ($\delta_t \geq \delta_{t1}$) or not ($\delta_t < \delta_{t1}$) the subsequent steps will correspond to Assumption 3.

If we are to be evaluated from a computational point of view, state 1 is characterized by the following tangential force and displacement:

$$f_{t1} = f_{t0} + \mu \Delta f_n \quad (23)$$

$$\delta_{t0} = \delta_{t0} + \Delta \delta_t^{(SL)} \quad (24)$$

If δ_t is less than δ_{t1} (no basic loading history following) the new force f_t can be calculated as

$$f_t = f_{t0} + K_{t0-1} \cdot (\delta_t - \delta_{t0}) \quad (25)$$

State 1 is first obtained with help of Eq. (23) and then the second step (to state 2 in Figure 3.4) is performed along the new constant normal displacement curve, through a stiffness constant K_{t12} calculated at state 1:

$$K_{t12} = K_{t-0} \left(1 - \frac{f_{t1}}{\mu f_{n1}} \right)^{1/3} \quad (26)$$

$$f_{t2} = f_{t1} + K_{t12} \cdot (\delta_{t2} - \delta_{t1}) \quad (27)$$

As it is explained, this derivation is suitable for a first loading case (in Figure 3.4). On the other hand, here we can mention that the procedure can also be generalized to unloading and reloading situations for normal displacement. In order to define the basic concepts underlying the procedure, the case of a decrease in the normal displacement and a decrease in the tangential displacement will be briefly described (in Figure 3.5).

When we consider the situations in which the normal displacement decreases, there is no harm in using the simple loading history hypothesis. On the other hand, in the last case in the mechanical transformation, it is clearly seen that it depends on the new constant normal substitution curve. Therefore, the basic loading history condition is met, on condition that in all the previous steps basic loading paths were followed.

The complete transformation is divided into basic steps. State 1 is reached along a straight line of slope $K_{t0|1}$, so that the corresponding force increases ($\Delta f_n < 0$):

$$f_{t1} = f_{t0} - \mu \Delta f_n \quad (28)$$

Because the tangential displacement decreases in the unloading curve, the constant normal displacement remains below the unloading section in the final case. Here, to go further from state 1 to the new state (e.g., state 3 in Figure 3.5), which is also called the next necessary step, the stiffness constant is also necessary. To calculate the stiffness constant, Eq. (19), which corresponds to the state 1, can be employed directly.

$$K_t = K_{t0|1} \left(1 - \frac{f_t^{(TP)}|1 - f_{t1}}{2\mu f_n} \right)^{1/3} \quad (29)$$

Where $f_t^{(TP)}|1$ is the turning point force:

$$f_t^{(TP)}|1 = f_t^{(TP)}|0 + \mu \Delta f_n \quad (30)$$

The final force at state 3 can then be calculated as,

$$f_{t3} = f_{t1} + K_{t13} \cdot (\delta_{t3} - \delta_{t1}) \quad (31)$$

Because of the large change in tangential displacement, it would be appropriate to specify an alternative. This alternative is to evaluate the stiffness constant according to case 1 and use it to calculate the displacement δ_{t2} . Then another stiffness constant can be evaluated at state 2 where the tangential force is equal to the previous step ($f_{t2} = f_{t0}$) and used to perform the last step from states 2 to 3.

In cases other than the cases described above, the stiffness constants corresponding to the basic steps and dividing steps can be taken into account and used in a similar way. It can even be evaluated this way in all situations close to turning points. Many discussions about Mindlin and Deresiewicz practices are available in the literature. The most comprehensive discussion of these is covered extensively in the work of Vu-Quoc and Zhang [63]. In addition, the procedure stages that are trying to describe are based directly on displacement, according to the investigations on the solution of the tangential elastic-friction problem. The main difference is that the choice of cases (normal and tangential increases or decreases) is based not on forces, but on displacements. Selections should be based on the displacement itself to avoid inconsistency of the procedure. Otherwise, it will result in a presser error.

3.2 The JKR Approximation

When the Hertzian case is compared with the JKR approximation, the surface energy is added to the calculations in the JKR approximation. Therefore, surface energy indirectly changes penetration by expanding the contact area. An increase in the surface area and a change in penetration also cause a change in the stored elastic energy. Johnson et al. [64], who want to use this situation, first apply a pressure P_1 based on the Hertzian state to find the contact radius, and then a constant contact radius that has reached the regime is calculated. Then it reduces the pressure P_1 to P in the (radius) system that has reached the regime. To find the equilibrium radius of contact properly, they define the surface energy $-\pi a^2 w$ of the system and use a balance between surface, potential, and elastic energy.

Unfortunately, the effectiveness of adhesion forces in these calculations is not clearly evident. Since it is not clear where and how the adhesion force affects the derivatives of the JKR approach, its compatibility with the general contact mechanics theory cannot be mentioned precisely. To gain more insight on these points [65], the Sneddon approach has been used to determine the situations that will create this situation that does not have a definite truth.

Sneddon [66] obtained a solution to improve Boussinesq problem, from provided the basic formulas for the penetration depth of the tip of the punch, whose profile is given by $f(r/a)$ (with $f(0) = 0$), for the total load P that must be applied to the punch in order to achieve this penetration, distribution of the pressure correspondent to $\sigma_y(r, 0)$ under the punch, and for the displacement $u_y(r, 0)$ of the surface of the indented plane:

$$\delta = \int_0^1 \frac{f'(x)dx}{\sqrt{1-x^2}} + \frac{\pi}{2} x(1) \quad (32)$$

$$= \frac{3\pi a K}{4} \int_0^1 x(t)dt \quad (33. a)$$

$$= \frac{3aK}{4} \left[\delta - \int_0^1 \frac{f'(x)dx}{\sqrt{1-x^2}} \right] \quad (34.b)$$

$$\sigma_y(r, 0) = -\frac{3K}{8a} \times \left[\frac{x(1)}{\sqrt{1-\frac{r^2}{a^2}}} - \int_{r/a}^1 \frac{x(t)dt}{\sqrt{t^2-\frac{r^2}{a^2}}} \right] \quad (35)$$

$$\sigma_y(r, 0) = -\frac{3K}{8a} \times \left[\frac{x(1)}{\sqrt{1-\frac{r^2}{a^2}}} - \int_{r/a}^1 \frac{x(t)dt}{\sqrt{t^2-\frac{r^2}{a^2}}} \right] \quad (36)$$

$$u_y(r, 0) = \int_0^1 \frac{x(t)dt}{\sqrt{\frac{r^2}{a^2}-t^2}} \quad (37)$$

Where:

$$x(t) = \frac{2}{\pi} \left[\delta - t \int_0^1 \frac{f'(x)dx}{\sqrt{t^2-x^2}} \right] \quad (38)$$

And

$$\frac{1}{K} = \frac{3}{4} \frac{1-v^2}{E} \quad (39)$$

For punches with continuous profile, Sneddon provided the displacement $x(1)$ of an arbitrary rigid body to be equal to zero so that it has finite stresses at the edge of the contact and used this criterion to determine δ .

$$K_I = \frac{-3K}{8} x(1) \sqrt{\frac{\pi}{a}} \quad (38)$$

the stress σ_y and the discontinuity of displacement δ

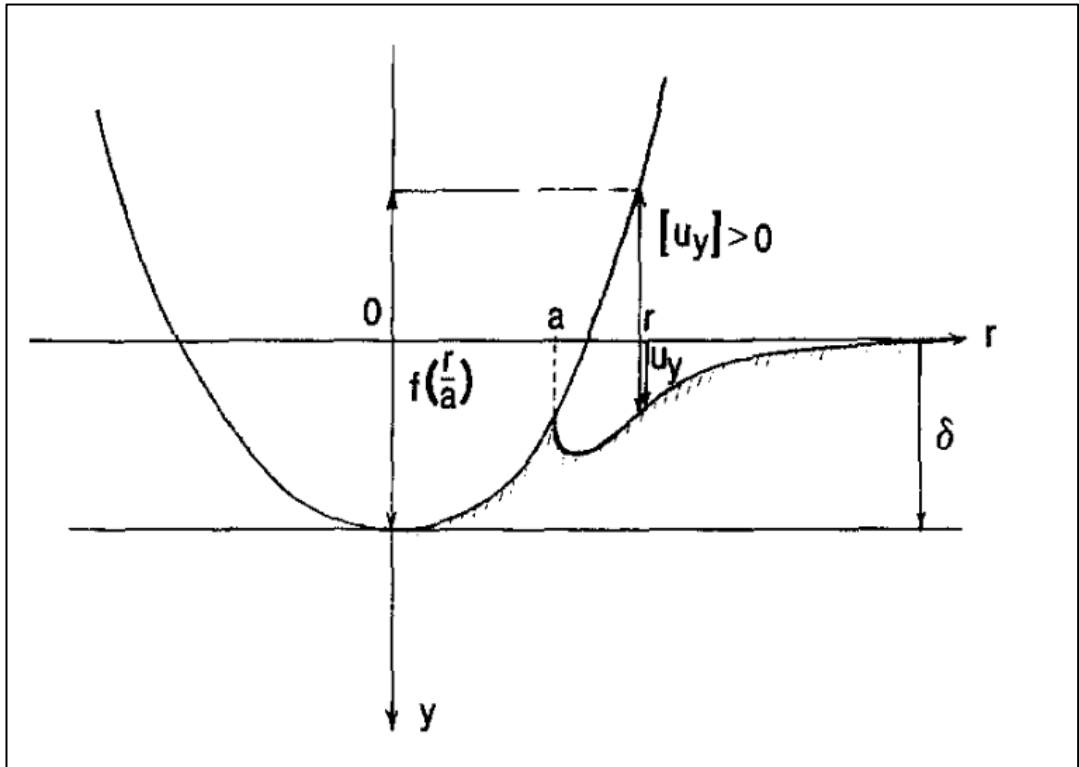


Figure 3.7: Displacement of elastic contact.

$$[u_y] = f\left(\frac{r}{a}\right) - \delta + u_y(r, 0) \quad (39)$$

at a distance ρ from the edge of the contact can be written

$$\sigma_y(a - \rho, 0) \approx \frac{K_I}{\sqrt{2\pi\rho}} \quad (40)$$

$$[u_y](a + \rho, 0) \approx \frac{4(1 - \nu^2)}{E} K_I \sqrt{\frac{\rho}{2\pi}} \quad (41)$$

These are the fracture mechanics formulas for plane strain (within a factor 2 for $[u_y]$), where K_I is the stress intensity factor. In the linear elastic fracture mechanics approach, the energy release rate is thus:

$$G = \frac{1}{2} \frac{(1 - \nu^2)}{E} K_I^2 \quad (42)$$

Let P_1 ($P_1 > P$) be the apparent load that, if $x(1) = 0$, will give the same contact radius observed with $x(1) \neq 0$ under the load P .

$$P_1 = \frac{3aK}{2} \left[\int_0^1 \frac{f'(x)dx}{\sqrt{1-x^2}} - t \int_0^1 \frac{xf(x)dx}{\sqrt{1-x^2}} \right] \quad (43)$$

(For a flat punch $P_1 = 0$).

From Eqs. [32.a] and [32.b], equation can be got:

$$P_1 - P = \frac{-3aK}{4} x(1) = (4\pi a^3)^{1/2} K_I \quad (44)$$

i.e.,

$$G = \frac{(P_1 - P)^2}{6\pi a^3 K} \quad (45)$$

The equilibrium can be given by the Griffith relation:

$$G = w$$

and the stability of this equilibrium by $(\delta G / \delta a) > 0$, the result is usually different at fixed load P and at fixed grips δ .

The derivation is straightforward for a fiat punch ($f(x) = 0$). Assuming a radius of contact a (this means a negative load to have a crack), $P_1 = 0$,

$$x(t) = x(1) = \frac{2\delta}{\pi}$$

And

$$\delta = \frac{2P}{3aK} \quad (46)$$

$$\sigma_y(r, 0) = -\frac{P}{2\pi a^2} \frac{1}{\sqrt{1 - \frac{r^2}{a^2}}} \quad (47)$$

$$u_y(r, 0) = \frac{1 - \nu^2}{\pi E} \frac{P}{a} \sin^{-1} \frac{a}{r} \quad (48)$$

$$G = \frac{P^2}{6\pi a^3 K} = \frac{3K\delta^2}{8\pi a} \quad (49)$$

Eqs. [45] to [47] were first provided by Boussinesq. The equation given by $G = w$ is always unstable either at fixed load or at fixed grips, and the adherence force is:

$$P_{min} = -\sqrt{6\pi a^3 K w} \quad (50)$$

A result given by Kendall [67].

For a spherical punch, with

$$f(x) = \frac{a^2}{2R} x^2, \quad x = r/a \quad (51)$$

Get

$$x(t) = \frac{2}{\pi} \left(\delta - \frac{a^2}{R} t^2 \right) \quad (52)$$

$$x(1) = \frac{2}{\pi} \left(\delta - \frac{a^2}{R} \right) \quad (53)$$

If we take $x(1) = 0$ to have finite stresses, we have:

$$\delta = \frac{a^2}{R} \quad (54)$$

$$p = \frac{a^3 K}{R} \quad (55)$$

$$\sigma_y(r, 0) = -\frac{3}{2} \frac{P}{\pi a^2} \sqrt{1 - \frac{r^2}{a^2}}, \quad r < a \quad (56)$$

$$u_y(r, 0) = \frac{a^2}{\pi R} \left[\sqrt{\frac{r^2}{a^2} - 1} + \left(2 - \frac{r^2}{a^2} \right) \sin^{-1} \frac{a}{R} \right], \quad r > a \quad (57)$$

$$[u_y] = \frac{a^2}{\pi R} \sqrt{\frac{r^2}{a^2} - 1} + \frac{r^2 - 2a^2}{\pi R} \cos^{-1} \frac{a}{R} \quad (58)$$

which are the Hertz and Boussinesq (Ibis) results. On the other hand, if we take $x(1) \neq 0$, we get:

$$\delta = \frac{a^2}{3R} + \frac{2P}{3aK} \quad (59)$$

$$\sigma_y(r, 0) = -\frac{K_1}{\sqrt{\pi a}} \frac{1}{\sqrt{1 - \frac{r^2}{a^2}}} - \frac{3aK}{2\pi R} \sqrt{1 - \frac{r^2}{a^2}} \quad (60)$$

$$u_y(r, 0) = \frac{2(1 - \nu^2)}{\pi E} K_1 \sqrt{\pi a} \sin^{-1} \frac{a}{r} + \frac{a^2}{\pi R} \left[\sqrt{\frac{r^2}{a^2} - 1} + \left(2 - \frac{r^2}{a^2} \right) \sin^{-1} \frac{a}{R} \right] \quad (61)$$

$$[u_y] = \frac{2(1-\nu^2)}{\pi E} K_1 \sqrt{\pi a} \cos^{-1} \frac{a}{r} + \frac{a^2}{\pi R} \sqrt{\frac{r^2}{a^2} - 1} + \frac{r^2 - 2a^2}{\pi R} \cos^{-1} \frac{a}{R} \quad (62)$$

And

$$P_1 = \frac{a^3 K}{R} \quad (63)$$

which is the Hertzian load giving the radius of contact a . The energy release rate is:

$$G = \frac{(P_1 - P)^2}{6\pi a^3 K} = \frac{3K}{8\pi a} \left(\delta - \frac{a^2}{R} \right)^2 \quad (64)$$

and the Griffith criterion gives the equilibrium relations:

$$\frac{a^3 K}{R} = P + 3\pi w R + \sqrt{6\pi w R P + (3\pi w R)^2} \quad (65)$$

$$\delta = \frac{a^2}{R} - \left(\frac{8\pi w a}{3K} \right)^{1/2} \quad (66)$$

which are the JKR results.

Under zero load, the radius of contact is given by:

$$a_0^3 = \frac{6\pi w R^2}{K} \quad (67)$$

At fixed load, instability appears for:

$$\left(\frac{\partial G}{\partial a} \right)_P = \frac{P_1^2 - P^2}{2\pi a^4 K} = 0 \quad (68)$$

i.e.,

$$P_{min} = -P_1 = -\frac{3}{2}\pi wR \quad (69)$$

$$a = \left(\frac{3}{2}\frac{\pi wR^2}{K}\right)^{1/3} = 0.63a_0 \quad (70)$$

$$\delta = -\frac{a^2}{3R} = -\left(\frac{\pi^2 w^2 R}{12K^2}\right)^{\frac{1}{3}} \quad (71)$$

instability is given by:

$$\left(\frac{\partial G}{\partial a}\right)_\delta = -\frac{3a^2}{R} = -\left(\frac{3}{4}\frac{\pi^2 w^2 R}{K^2}\right)^{1/3} \quad (72)$$

$$a = \left(\frac{\pi wR^2}{6K}\right)^{1/3} = 0.30a_0 \quad (73)$$

$$P = -5P_1 = -\frac{5}{6}\pi wk \quad (74)$$

Now we see that the JKR theory is indeed included in the general theory of elasticity, but the a priori hypothesis of zero stress at the edge of the contact failed to reveal it.

3.3 The DMT Approximation

Unlike the JKR approach, the DMT theory offers a broader solution by taking into account the interaction forces outside the contact area. However, due to the inability to obtain analytical results other than the calculation with the consistent numbers provided by Muller *et al.* [68], it was assumed that these interaction forces do not deform the profile that remains Hertzian. The only helpful consequences are that the pull-out force $2\pi wR$ is reached at zero contact radius [69] and the contact radius under zero load is given by:

$$a_0^3 = \frac{2\pi w R^2}{K} \quad (75)$$

Considering the thermodynamic method [68], external attraction forces that are not in the contact area add a virtual charge that affects $2\pi w R$ as it approaches the contact area, while in the force method this virtual load constantly increases from $2\pi w R$ (see in Figure 3.7). Note that if this fictitious load remained equal to $2\pi w R$, one would have:

$$\frac{a^3 K}{R} = P + 2\pi w R \quad (76)$$

i.e.,

$$G = \frac{P_1 - P}{2\pi R} \quad (77)$$

This accurately gives us the pull-out force in the case $a=0$ and the contact radius under zero load. This is the summary of the conclusion we will draw below.

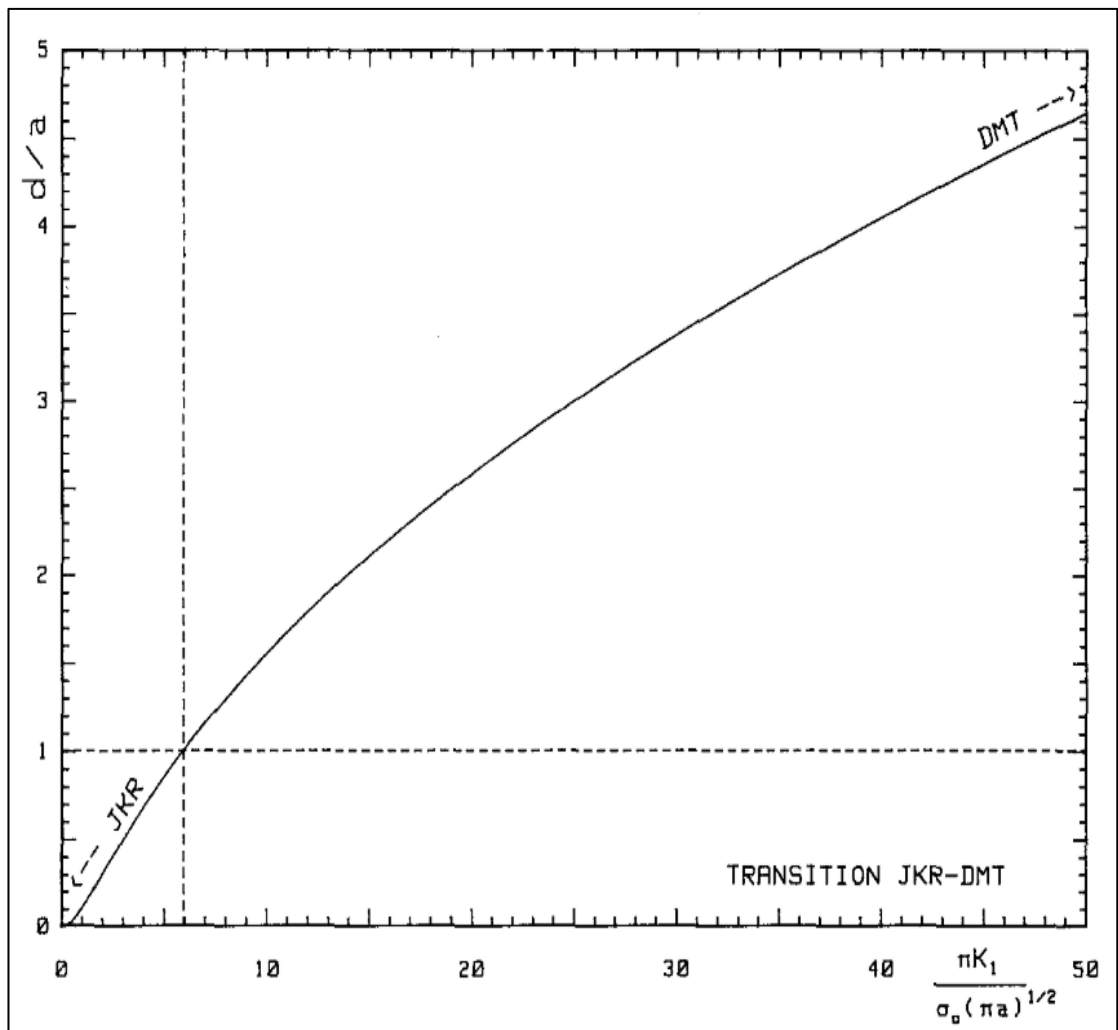


Figure 3.8: Variation of d/a with $\pi K_1/\sigma\sqrt{\pi a}$.

4. METHODOLOGY

As we discussed in previous sections, DEM is numerical method and is suitable to predict the individual powder particles motion and particle-based dynamics in powder spreading process accurately. Thanks to these features, DEM has been used for almost all simulations as a tool. In this paper, the varying simulations were carried out via DEM software EDEM[®]; provided by Altair. In order to obtain more accurate information, The ‘Hertz-Mindlin and Johnson Kendall's’ contact model [70] was employed to determine the interactions between particle-particle and particle-walls.

4.1 Material

In order to simulate particle dynamic behaviors and their fundamentals via DEM; the material, which is used in simulation, properties and conditions must be described and accurately in the program as input parameters and boundaries. Although some of these parameters are difficult to measure [71], they are often obtained from literature directly [45, 72, 73].

In this study, Ti6Al4V (Titanium Alloy), which is widely used in Additive Manufacturing, were employed to develop a common interpretation in a comparative way. Physical and mechanical properties of the particulate material required for use in simulations are given in Table 4.1. It is characterized by characteristic applications and have been defined in previous works. Commercial steel is selected as a building plate material.

Table 4.1: Properties and input parameters of Ti6Al4V (Titanium Alloy) material.

Particle material density (kg/m ³)	4300
Walls material density (kg/m ³)	8500
Particle shear modulus (MPa)	1
Particle Poisson ratio	0.3
Wall shear modulus (MPa)	1
Walls Poisson ratio	0.3
Particle–particle coefficient of restitution	0.5
Particle–wall coefficient of restitution	0.5
Particle–particle coefficient of sliding friction	0.5
Particle–wall coefficient of sliding friction Particle–particle	0.5
Particle–particle coefficient of rolling friction	0.01
Particle–wall coefficient of rolling friction	0.1

4.2 Particle Size Distribution and Layer Thickness

Particle size distribution is an important parameter for spreading operation and it has direct negative and positive effect on packing fraction. Therefore, there are many different types of particle distribution methods used in literature such as random, half-normal, log-normal, linear-normal etc. Among these methods, the log-normal size distribution is the one that seems closer to real circumstance than the others. Therefore, the distribution of particle diameter was chosen according to log-normal size distribution. A log-normal particle size distribution (PSD) is one of the most used in literature and it is fitted to the particle size distribution of all materials that were used in this study. Here, the particle diameter was employed as 50 microns and the log-normal size distribution was defined as 0.05 in standard deviation and its mean was zero value for all materials. As seen in Figure 4.1, which is taken over from GTU Test Laboratory, particles in the form of satellites increase the number of particles used in the simulation too much and accordingly increase the solution time. In addition, the effect of satellite-shaped particles on density variation in simulations is negligible. For this reason, super-spherical-shaped particles are modeled here, like literature.

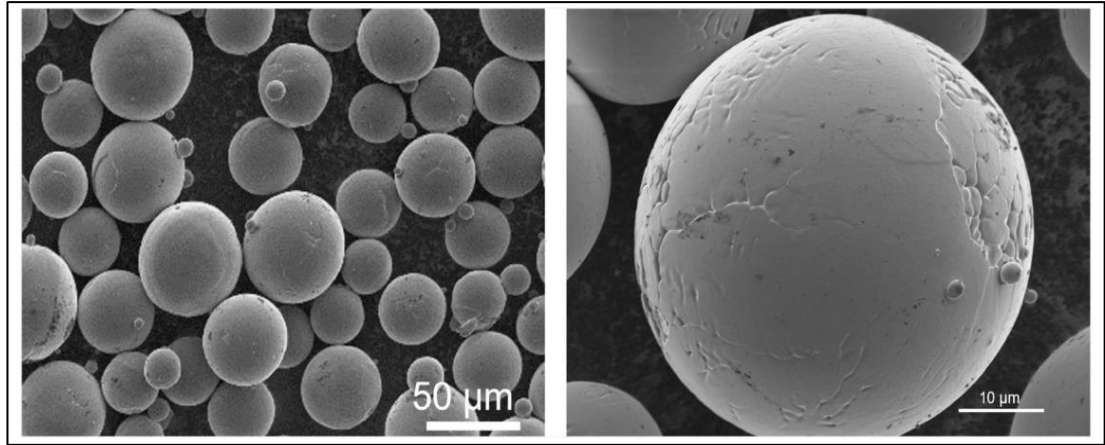


Figure 4.1: Sem images of Ti-Alloy particles.

In order to determine the influence of the layer thickness on volume fraction/bulk density and angle of repose, four widely used thickness values in the previous works were employed here. They are 100, 120, 140 and 160 microns, respectively. This base plane and its layer thickness were created virtually via a 3D CAT program, Catia V5 Student Version.

4.3 Geometry

A powder spreading system with a spreader and a building plate, which is commonly used in production methods such as Selective Laser Melting (SLM), Binder Jetting (BJ) and Selective Electron Beam Melting (SEBM), is modeled as seen in Figure 4.2. The system consists of three main components. They are roller (1), powder heap (2) and a building plate (3) in which the desired depth is provided to give the layer geometry, respectively. Here, powders are produced in ten thousand units per second by means of a virtual plate placed in a suitable area in the system. These produced powders are stored in front of the roller with the effect of gravity and form a heap (2). Another important physical parameter, the vertical gap, is between the roller and the building plate, and this is one of the main parameters that directly affect the system. Throughout the simulations performed in this paper, this gap was constant and about 50 microns, which is used most of the typical electron beam melting (EBM) machine. Gap thickness also can be employed as parameter for analysis. It can be used for further applications.

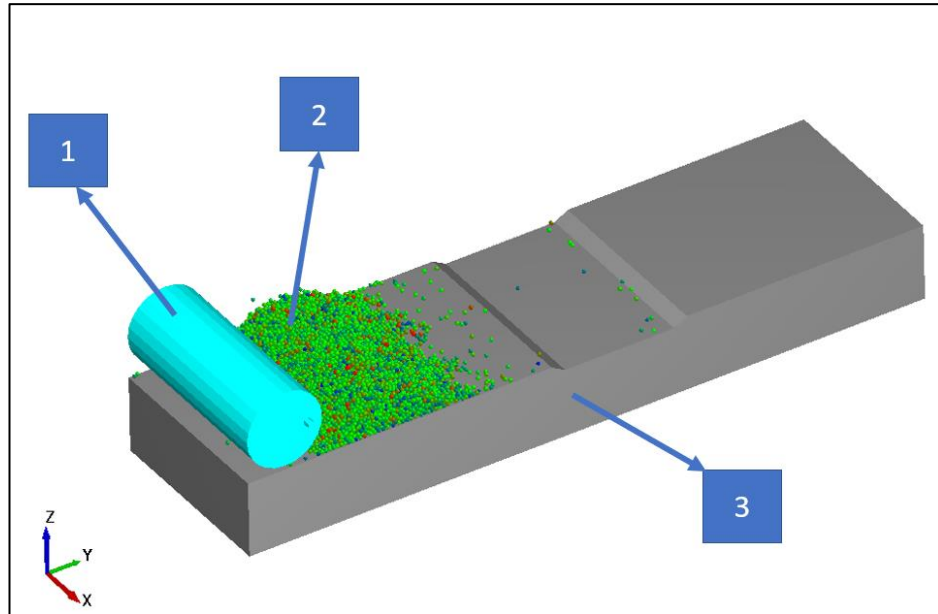


Figure 4.2: DEM simulation set-up.

In the build plate designed for different layer thicknesses, as can be seen in Figure 4.3, radius has been added at the edges of the modeled space for the layer thickness to prevent jamming.

In case cannot be provided radius to the corner parts, jamming event occurs. As indicated by the green circle in figure 4.3, the particles are trapped and accumulate energy in this region between the edge of the build plate and the rollers. This energy causes the particles to jump after a while. As a result of this jump, large gaps are formed in the layer.

Considering the production quality in the formation of the first layer, a homogeneous particle distribution within the layer is desired. Due to the mentioned gaps and irregularity in the layer, the density of the first layer to be produced cannot reach the desired level.

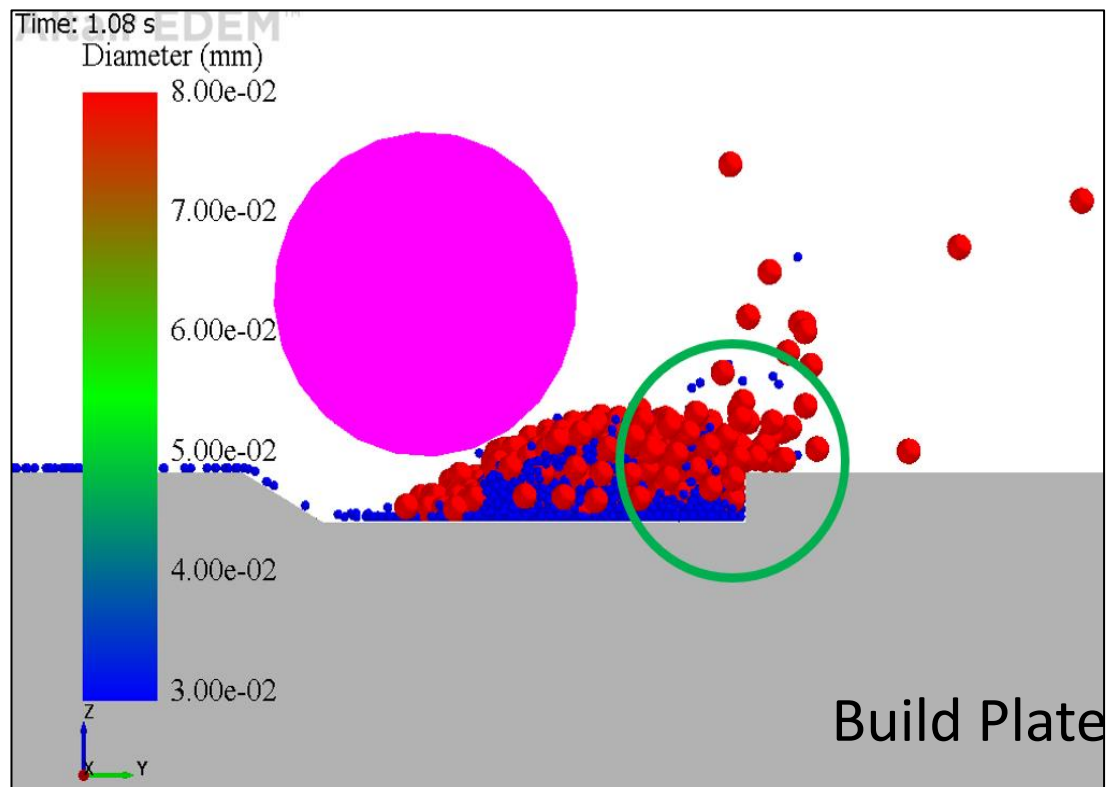


Figure 4.3: Image of accumulated particle at the edge of the build plate.

5. RESULTS

Here, the simulations are grouped into four main groups according to the layer thickness. The groups gathered in these four main groups are divided into three subgroups according to their rotational movements. The rotational motion, which is divided into three subgroups, was finally divided into three subgroups according to the translational movements, and a total of 36 simulations were performed to determine the angle of repose and bulk density/volume fraction and their relationships each other.

5.1 Results for Dynamic Angle of Repose

The dynamic angle of repose corresponding to the results of the rotational and translational velocities in each layer thickness were tried to be measured by taking screen shots of different times so that the rollers were in the same position. To determine the dynamic angle of repose at an accurate time, all time steps of the simulation were examined and the time to reach the regime was determined as the moment when the powders first filled the space modeled for the layer thickness, as shown in Figure 5.1. This time, which corresponds to 1.3 seconds in a 10 mm/s translation movement, was determined as 1.15 seconds and 1.1 seconds for other translation movements, for example 20 and 30 mm/s, respectively.

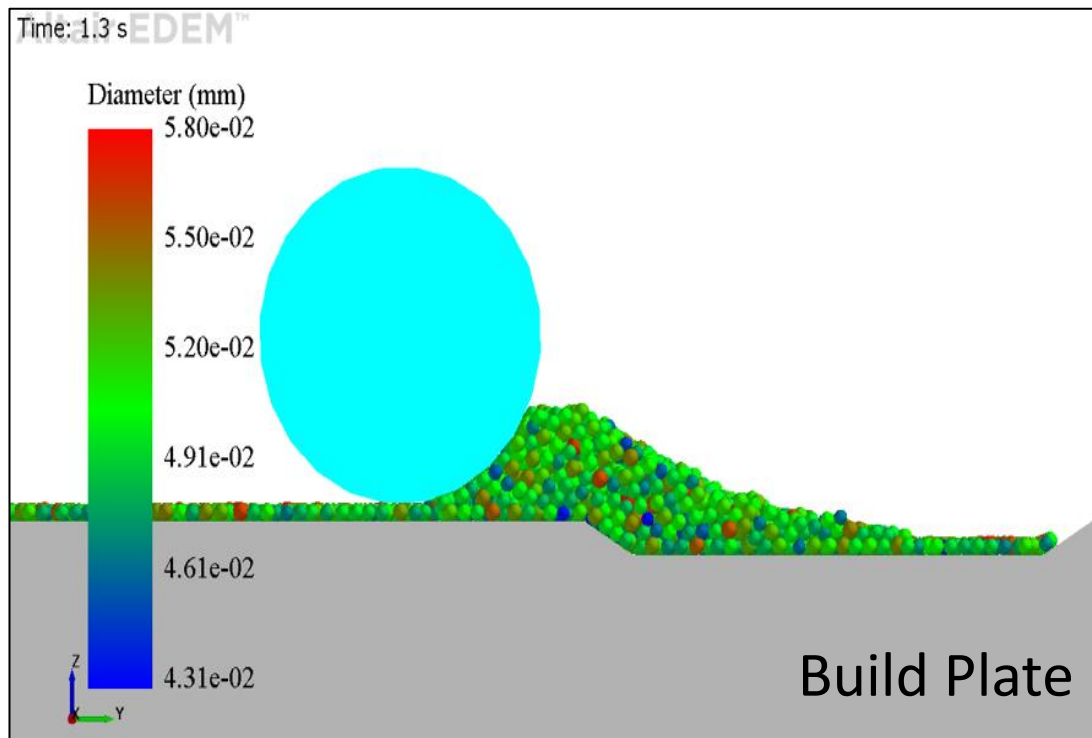


Figure 5.1: Screen shot at $t=1.3$ second, rotational velocity:150 rpm, transitional velocity:10 mm/s.

The Sketch Tracer, interface of the Catia V5 Student Version, program was used to virtually determine the dynamic angle of repose corresponding to the times specified above. First, screen shots for all simulations were imported into the program, and then a line corresponding to the agglomeration, which is created by the powders in front of the roller, and another line from the roller's contact point with the particles, parallel to the Y axis, was drawn, as can be seen in the Figure 5.2. This was performed sequentially to find the dynamic angle of repose of all simulations.

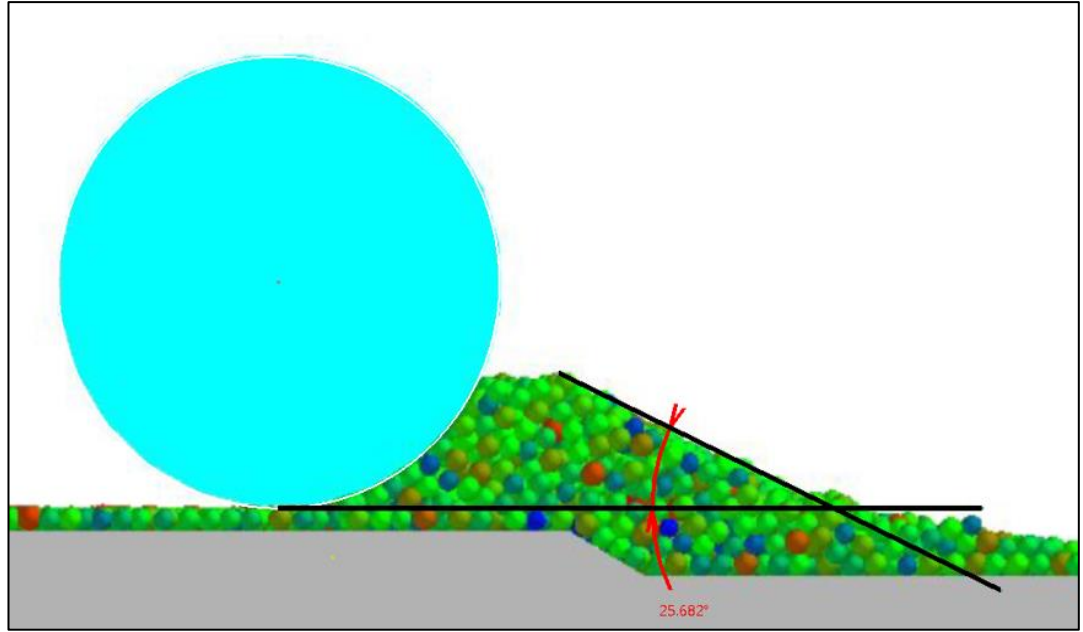


Figure 5.2: Virtual measurement dynamic of angle of repose [degree °].

The Minitab program was used to examine the dynamic angle of repose calculated virtually for all simulations, according to the layer thickness, rotational and transitional velocities selected as parameters. As a general observation, the three main parameters are summarized as shown in Figure 5.3 to show the effect on the angle of repose. It was measured to be between 24 ° and 32 ° degrees for all simulations.

First, dynamic angle of repose increased with increasing layer thickness between 100 and 160 microns. However, the same situation cannot be said for rotational and transitional velocities. When we look at the rotational velocity values, it is observed that it is approximately 26.5 degrees at 150 rpm and 250 rpm, while this value is approximately 26.6 degrees for 200 rpm with a slight difference. On the other hand, when we look at angle of repose values according to the transitional velocity, there is no ever-increasing trend compared to the layer thickness effects. In addition, the differences between the angles of repose are high compared to the rotational velocities effect on dynamic angle of repose. According to Transitional Velocity effects on angle of repose, 10 mm/s and 20 mm/s velocities are approximately the same. However, 20 mm/s velocity's effect on the dynamic angle of repose is the lowest compared to the other two velocities.

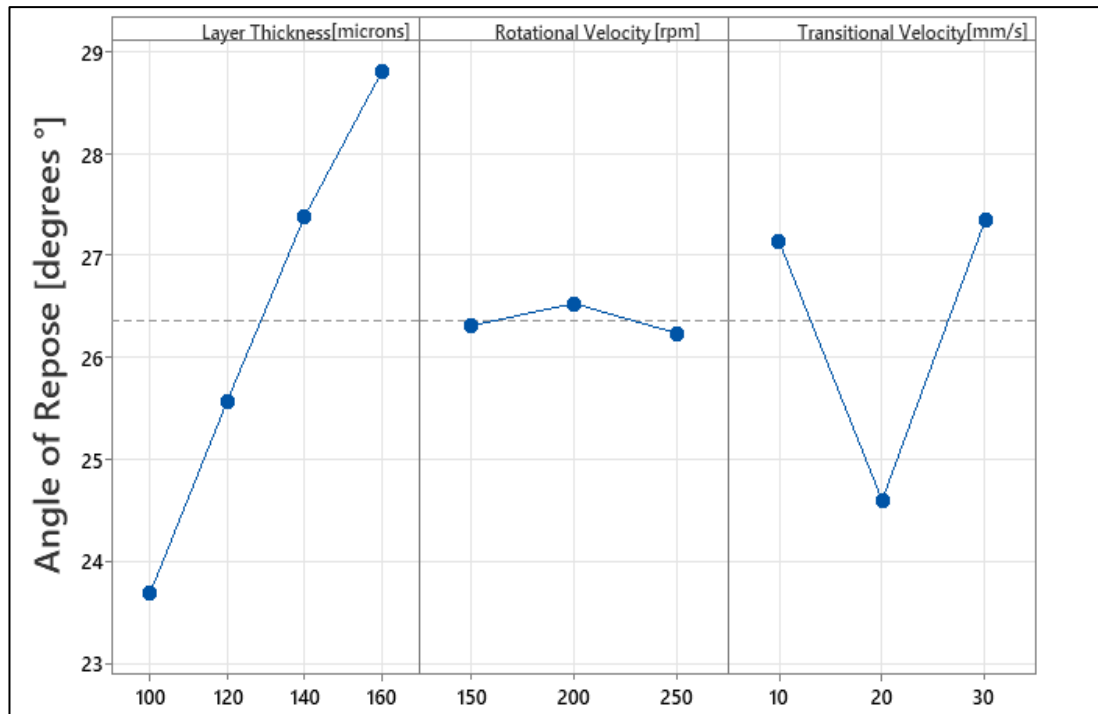


Figure 5.3: Parameters effect on dynamic angle of repose [degree °].

In order to validate the results as determined before, the Analysis of Variance, which can be seen in Table 5.1, can reveal the effects of the three main parameters on the dynamic angle of repose. It is clearly seen that the layer thickness and transitional velocity values have a much greater effect on the dynamic angle of repose than the rotational velocity values according to P-Value and Contribution of Source. On the other hand, it is seen in the same table that the error percentage is too high. This may be due to the inconsistent effect of transitional velocity values on the angle of repose.

Table 5.1: Analysis of variance for dynamic angle of repose.

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Layer Thickness	9	133.149	56.14%	133.15	44.383	26.23	0
Rotational Velocity	2	0.551	0.23%	0.511	0.2755	0.16	0.851
Transitional Velocity	2	56.107	23.65%	56.107	28.0536	16.58	0
Error	28	47.386	19.98%	47.386	1.6924		
Total	35	237.193	100%				

5.2 Results for Bulk Density

The bulk density that means quality of the spreading process, corresponding to the results of the rotational and translational velocities in each layer thickness were tried to be measured by a virtual box which is provided by the EDEM as seen in Figure 5.4. Here, the density sensor box is dimensioned to fully cover the layer thickness. Since the layer thickness will be used for density determination, it is located properly to cover the entire layer thickness.

The density calculation that the program provides us with is relatively simple. The density sensor box, which is highlighted as green, determines all the particle numbers in the volume it occupies, and then the program obtains a coefficient by adding the volumes they have and dividing them by the volume of the density sensor box. This coefficient also represents the volume fraction. Finally, the bulk density is calculated by multiplying this coefficient with the density of the particle material which is used for all simulations.

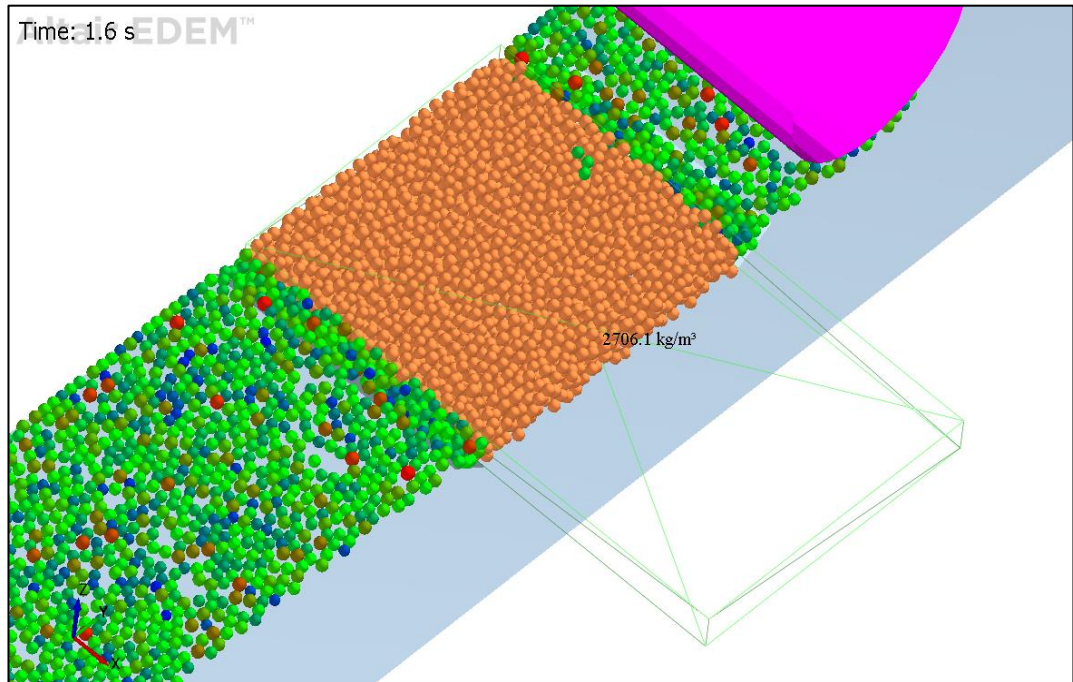


Figure 5.4: Virtual box employed for measurement of bulk density [kg/m^3].

To examine the solutions of bulk density according to some important parameters, the statistical analysis of bulk density was followed in a similar way to the statistical analysis of the results of dynamic angle of repose. Transitional, rotational

velocities and layer thickness were selected as parameters and the results were added to Minitab. Then, the results were summarized as shown in the Figure 5.5. Looking at the Figure 5.5, it was seen that the upper value of bulk density is 2800 kg/m³ and the lower value is 2600 kg/m³.

When an overview is made according to the parameters, dramatic decreases stand out. Considering the layer thickness and transitional velocity, it is seen that the bulk density values decrease depending on the increase of these parameters. On the other hand, a dramatic change in rotational velocity values cannot be mentioned relatively. However, unlike other parameters, it is seen that the bulk density value slightly increases with the increase of rotational velocity value.

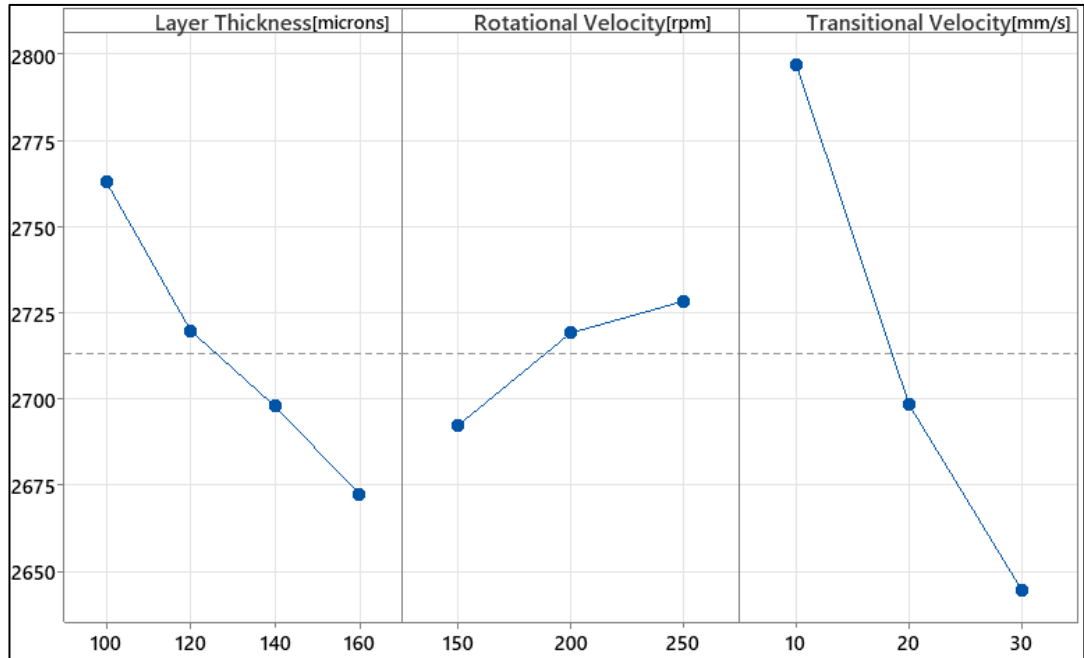


Figure 5.5: Parameters effect on bulk density [kg/m³].

In order to validate the results as determined before, the Analysis of Variance table can reveal the effects of the three main parameters on the bulk density. It is clearly seen that the layer thickness and transitional velocity values have a much greater effect on the bulk density than the rotational velocity values according to P-Value and Contribution of Source. Here, statistical examinations on bulk density give more consistent results than dynamic angle of repose examination, especially based on P-Value.

Table 5.2: Analysis of variance for bulk density.

Source	DF	Seq SS	Contribution	Adj SS	Adj MS	F-Value	P-Value
Layer Thickness	9	39744	18.75%	39744	13247.9	18.48	0
Rotational Velocity	2	8452	3.99%	8452	4226.0	5.90	0.007
Transitional Velocity	2	143730	67.8%	143730	71864.8	100.25	0
Error	28	20071	9.47%	20071	716.8		
Total	35	211996	100%				

5.3 Bimodal Particle Size Distribution

In the analysis mentioned above, the particles are made with a certain standard deviation and the maximum minimum particle sizes are not too far from each other. Bimodal size distribution analyzes are seen in the literature. The results of these analyzes give much better results than non-bimodal ones. However, if bimodal size distribution is to be made, different parameters should be considered this time. Especially in the case of using particles that are very different in diameter from each other, bad results can be obtained for the layer where high quality is desired. If these bad results are mentioned here, as can be seen in Figure 5.6, it is not possible to talk about a homogeneous particle mixture within the layer. In this analysis, in which 30-micron and 70-micron diameter particles were used, respectively. Small diameter particles were collected at the bottom of the layer, while those with larger diameters were collected at the top. This is not a desired situation in terms of production quality. Obtaining a homogeneous mixture, which is the ideal situation, cannot be achieved with particle groups that differ in diameter from each other. This situation should be avoided and particle groups with similar diameters should be used.

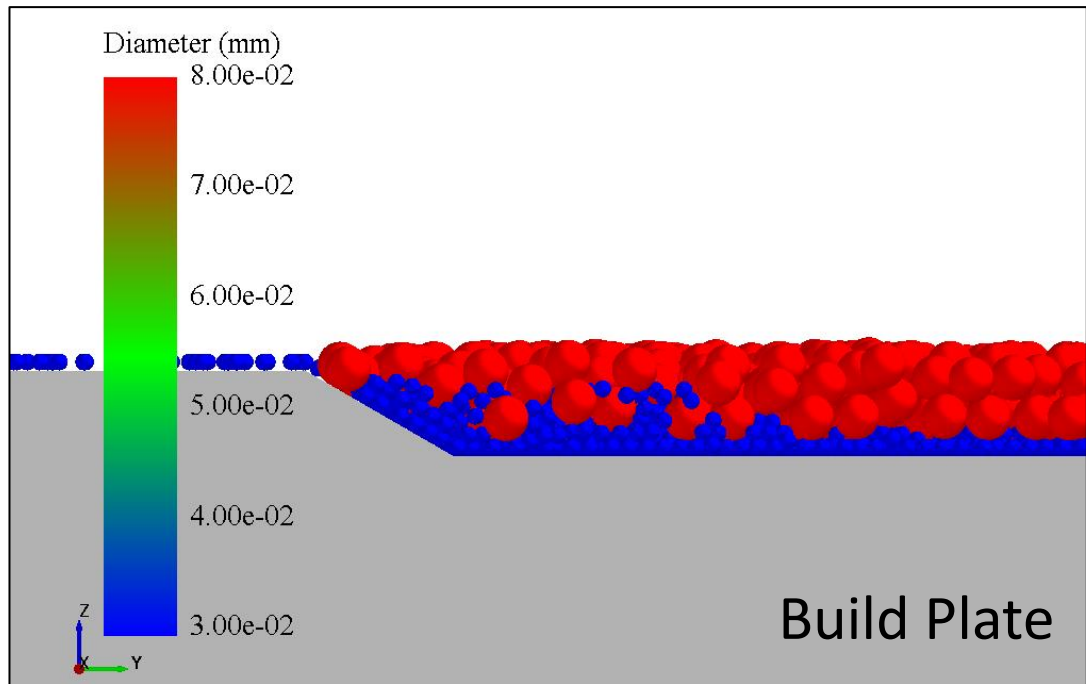


Figure 5.6: Non-homogeneous layer.

When the binary bimodal groups formed by particles of 50 microns together with particles with a diameter of 20, 25 and 30 microns, which are not very different from each other in terms of diameter, are examined, the above-mentioned situations do not occur. The main reason underlying this event is that small particles in binary groups formed with particles very different in diameter from each other are relatively more concentrated at the bottom of the layer. In the case of using particles with 20-micron diameter lognormal diameter distribution varying between 1-5% by mass of the total layer, a more homogeneous distribution is achieved as seen in Figure 5.7. Although a homogenous distribution was observed through the layer, the bulk density values were lower than the non-bimodal.

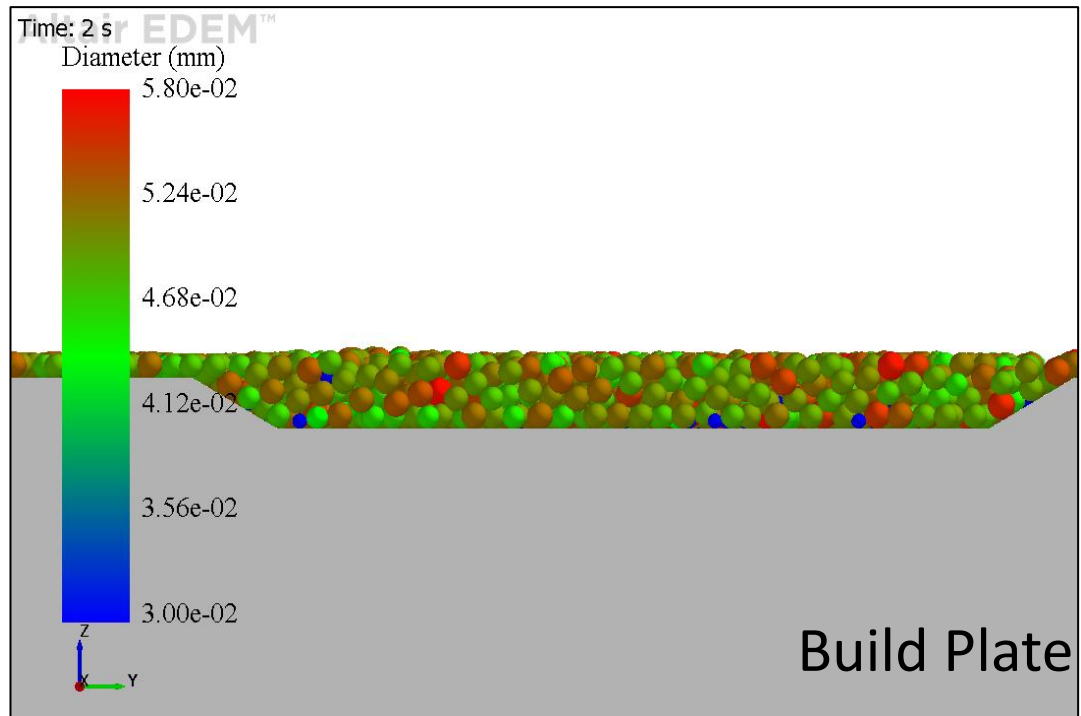


Figure 5.7: Homogenous layer with bimodal binary groups.

Although a homogenous distribution was observed through the layer no significant improvement was observed in the bulk density values for 100-micron layer thickness at 10 mm/s transitional and 150 rpm rotational roller speeds compared to non-bimodal simulation result, as seen in Table 5.3.

Table 5.3: Bulk density value for 100-micron layer thickness at 100 mm/s transitional and 150 rpm rotational roller velocities.

Bimodal Binary Groups [micron]	Bulk Density [kg/m3]
50-30	2745.04
50-25	2832.16
50-20	2814.97

5.4 Surface Roughness and Heaps Effects

Another factor affecting quality in powder-bed-based additive manufacturing methods is surface roughness. Although the bulk density values are at desired levels, the roughness of the surface is also very important. Although simulated as a single layer DEM model, in fact Powder-bed-based additive manufacturing methods are multi-layered and the situations between layers are also very important. If the desired conditions between the layers, the absence of clear particle boundaries of the two layers, high bonding to each other, cannot be achieved, it is inevitable to experience poor quality in the production. To achieve the desired manufacturing quality, it is necessary to understand the surface's roughness circumstances. The results that were performed here from DEM model show that local heaps were formed on the surface as you see in the yellow circle indicated in Figure 5.8.

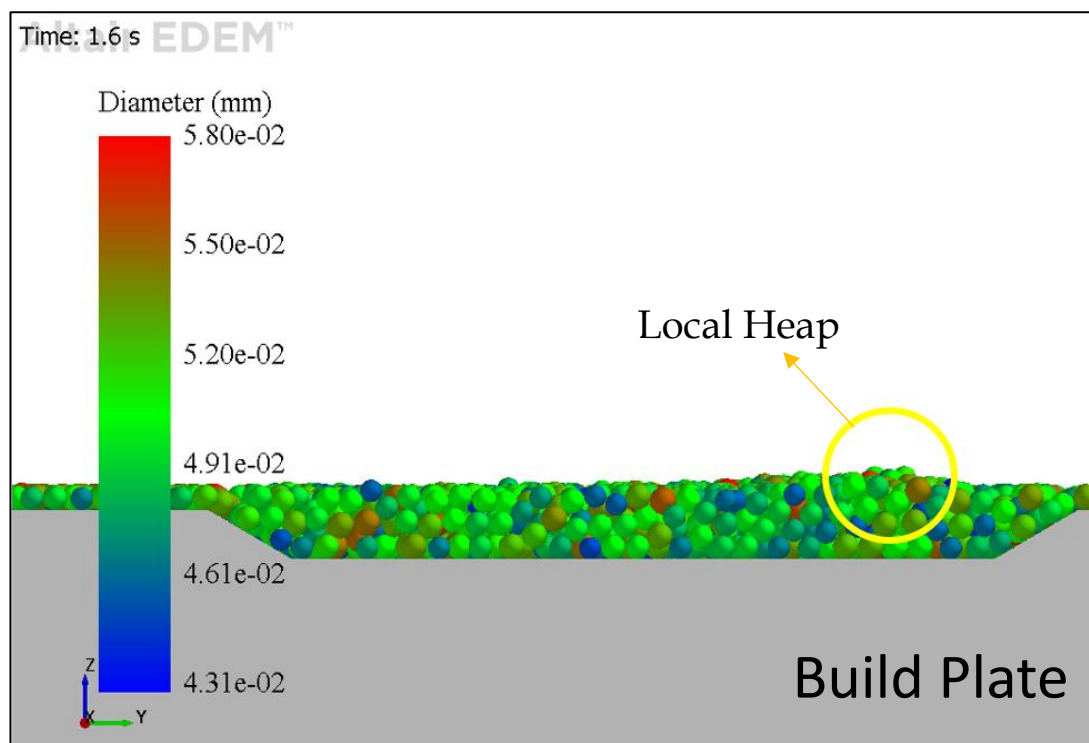


Figure 5.8: Surface condition in the used of single roller.

After the first spreading process, the particles are bound together, either by a fusing source, laser or electron beam, or a binder, before passing on to the second layer. The local heaps mentioned above gain a more rigid feature after these processes. As a requirement of the process, in the second spreading operation performed to form

the second layer, the particles can no longer be distributed homogeneously here and cause voids in the region corresponding to the heaps. This situation results in the weakening of its mechanical properties by causing voids in the components to be produced. If the components produced with weak mechanical properties are used under a load or functionally, great stresses occur in the voids inside and the components start to crack from there due to these stresses. These cracks then reach large sizes and cause the piece to be discarded.

In order to eliminate the negative effects arising from the results of local heaps, a method not encountered in literature has been developed here. As can be seen in the figure 5.9, after the first layer completes the roller process on the left with gap of 50 microns between the build plate, the roller with gap of 70 microns from the build plate performs the final surface correction process in the opposite direction.

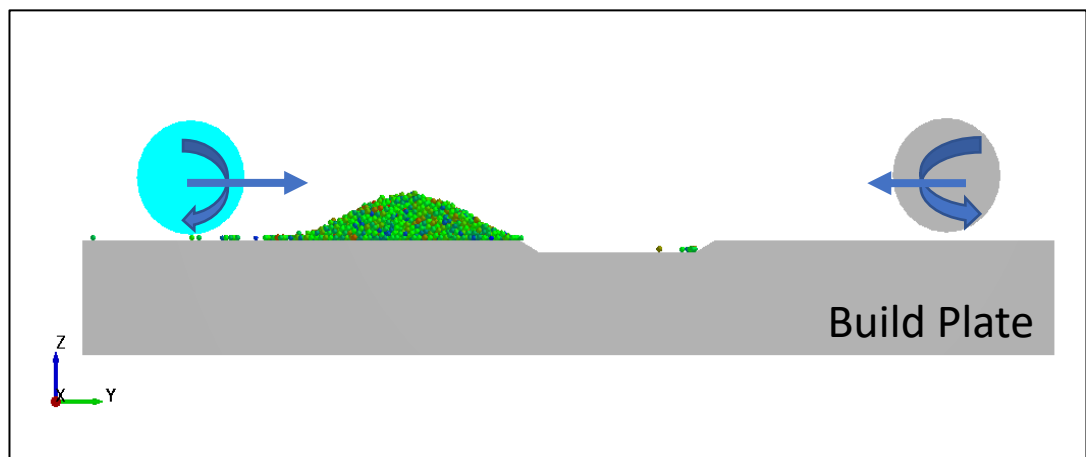


Figure 5.9: Dem simulation set-up for back and forward rollers used.

In this way, no local heaps appear on the layer surface compared to figure 5.8. As seen in Figure 5.9, the bulk density was also measured as 2810 kg/m^3 and the bulk density value in the layer was preserved and even increased relatively. Thanks to this improvement, as described above, voids will not form in the produced part and will provide relatively more stable mechanical properties.

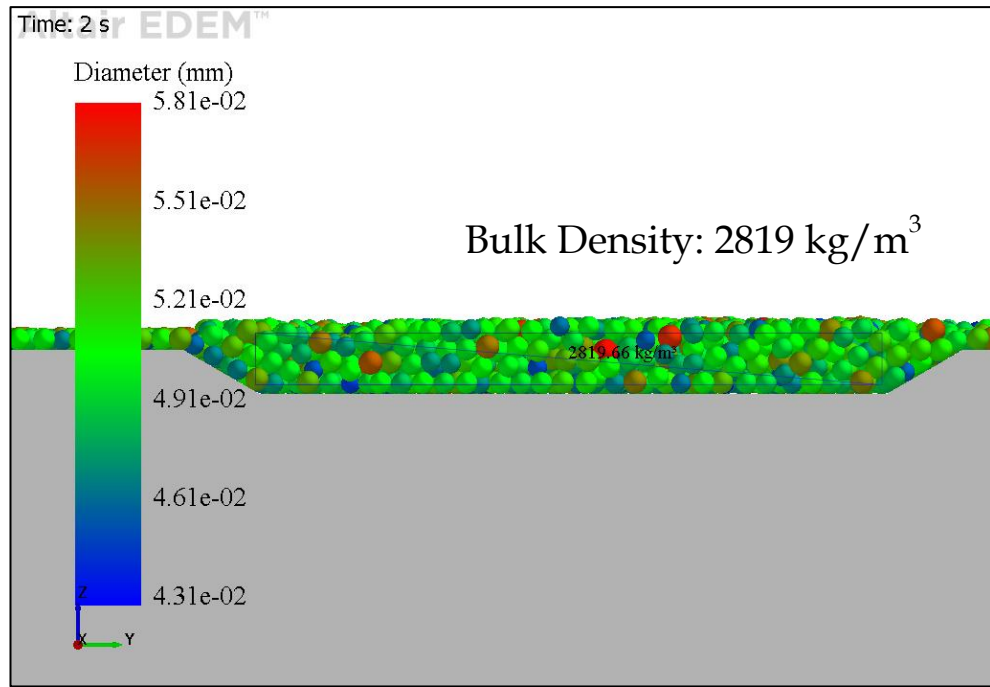


Figure 5.10: Surface condition in the use of back and forward roller.

After seeing the improvement result obtained in the double roller-driven model made for 10 mm/s transitional velocity. It became necessary to run simulations with different transitional velocity values under the same conditions. As mentioned previously, the production speed of Powder Bed-Based additive manufacturing methods is also an important capability. Increasing or maintaining this ability by taking into account the production quality is one of the main goals. The use of two or more spreaders is one of the weaknesses of manufacturing in terms of duration. But on the other hand, when using a single spreader, the bulk density value is very negatively affected, and the production quality decreases at the desired speeds. In order to eliminate these negative situations, the system using double spreaders is proposed here. It meets the time corresponding to the total spreading process in the use of a single spreader, this time with a two-spreader system with higher spreader speeds. In addition, significant improvement has been achieved in both bulk density values and surface condition.

It is necessary to verify the mentioned improvement at several different transitional speeds in order to develop a common interpretation. For this reason, the existing dual-use roller model was run at transitional speeds of 10, 20 and 30 mm/s, respectively, and the results show the same improvement at different speeds corresponding to bulk density, as can be seen in Figure 5.11. When the bulk density

values were examined, it is clearly seen that with the increase in the transitional velocity in the double spreader model, dramatic decreases were not observed compared to the single spreader model. The effects of the decrease in production quality due to the increase in transitional velocities were also prevented, thus providing a higher quality production opportunity.

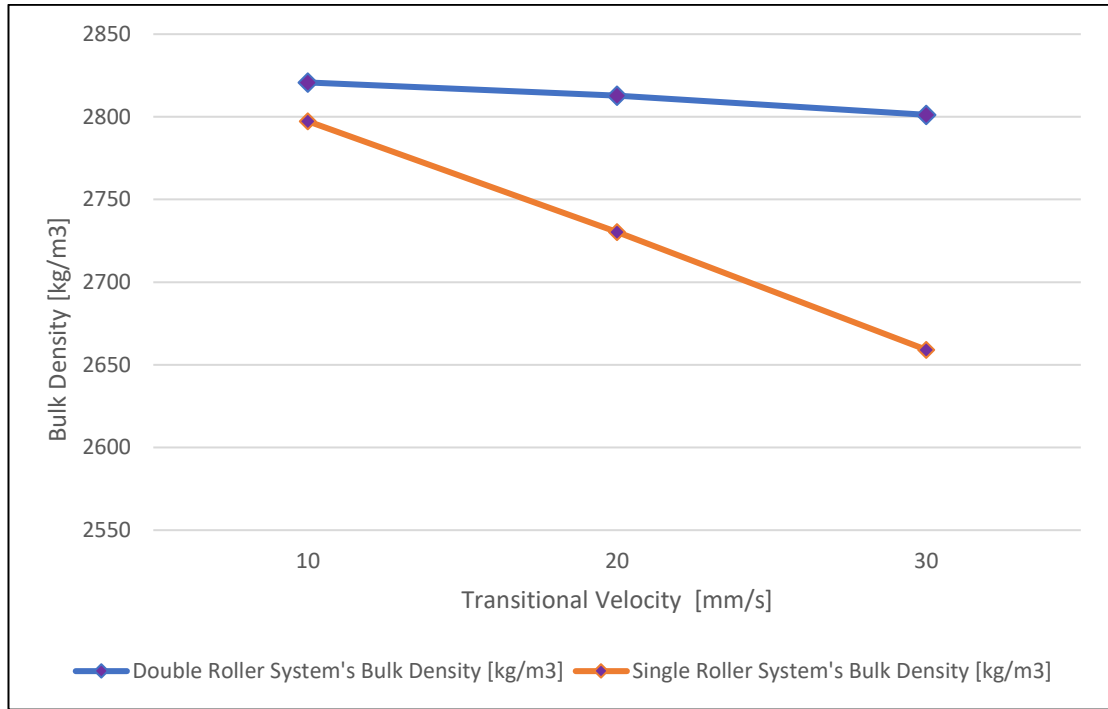


Figure 5.11: Bulk density values for single and double used roller system.

5.5 Particle Size effect on Static Angle of Repose

Similar to Dynamic Angle of Repose, the static angle of repose was measured virtually in the Catia V5 Student Version program by taking a screen shot after the powder was produced virtually and formed a full pile on the build plane. Here, after a line is thrown parallel to the slope created by the stack, another line is thrown parallel to the build plane as seen in Figure 5.12. Then, as a capability of the program, the angle between these two lines is measured and the static angle of repose is determined.

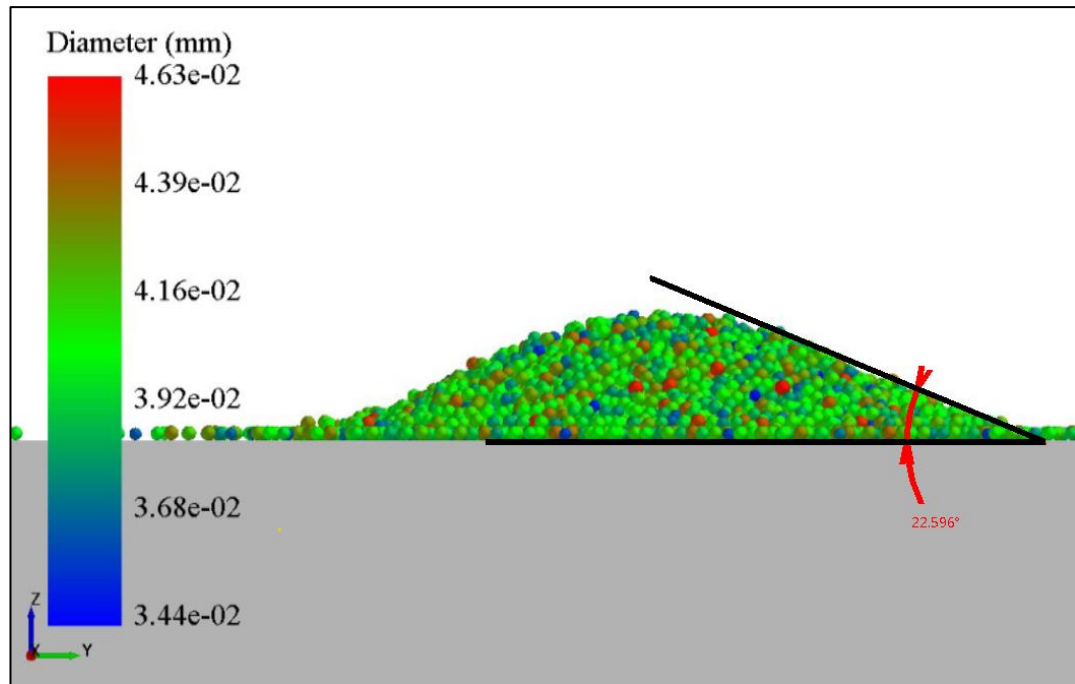


Figure 5.12: Static angle of repose measurement.

In order to investigate the effect of particle size on the static angle of repose, powder piles were formed using log-normal size distribution particles with a diameter of 40, 50, 60, 70 and 80 microns, respectively. When the analysis results are examined, as seen in figure 5.13, the static angle of repose angle increases in parallel with the increase in particle size.

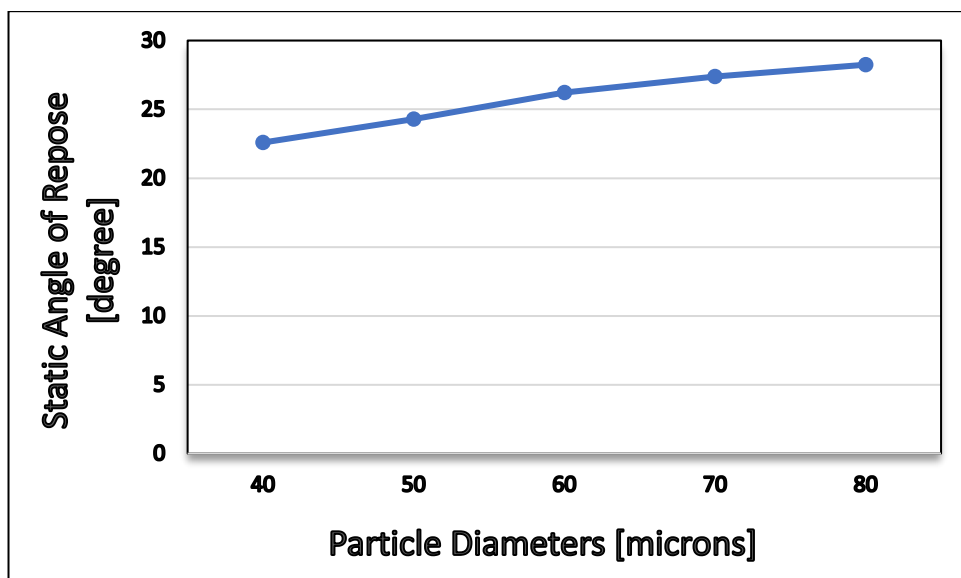


Figure 5.13: Effect of particle size on the static angle of repose.

6. DISCUSSION

Studies in literature, the angle of repose generally increases with the increase of the transitional velocity. As a result of this, the particle flow has become difficult, and this increases the volume fraction and makes the production poor quality [22, 42]. Contrary to the literature, no clear statistical results were found for angle of repose depending on the main parameters in this study. Here, only the effect of the layer thickness on the angle of repose is clearly demonstrated. The main reason for this is that the angle of repose, which is also seen in Figure 5.2, fills the gap opened for the layer thickness, and the screenshot is taken at the first moment. Here, it is directly caused by the increase in the angle of repose with the increase in thickness. However, despite this finding, a clear relationship with bulk density could not be determined.

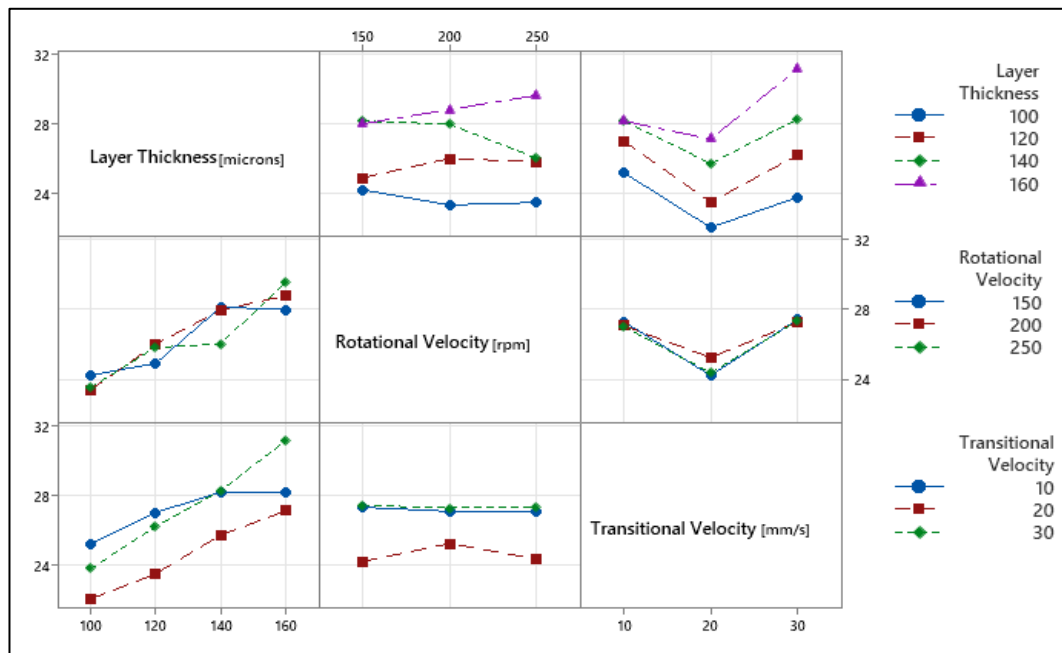


Figure 6.1: Interaction plots for dynamic angle of repose [degree °].

Contrary to the statistical analysis made according to the Angle of repose, it is possible to say certain determinations depending on the parameters in the bulk density analysis. According to the three main parameters used, the changes are clear, and it is possible to suggest meaningful relationships as mentioned before. As summarized in Figure 6.2, bulk density was more affected by transitional velocity than others. This is in line with the information that is obtained in most studies in literature.

It is very important that the bulk density, which is seen as a quality parameter in particle-based additive manufacturing methods, or the pack fraction, which is a similar discourse, is low. This study was mainly carried out to reveal the optimum points of the parameters and suitable parameter selections for particle-based additive manufacturing were presented. Unfortunately, production time, which is one of the biggest weaknesses of additive manufacturing, appears here again. Because, as mentioned before, increasing the transitional velocity negatively affects the bulk density in the layer, and if we want to make a fast production, it is inevitable to obtain low quality layer thickness. This situation can be eliminated up to a certain level by low layer thickness and high rotational movement.

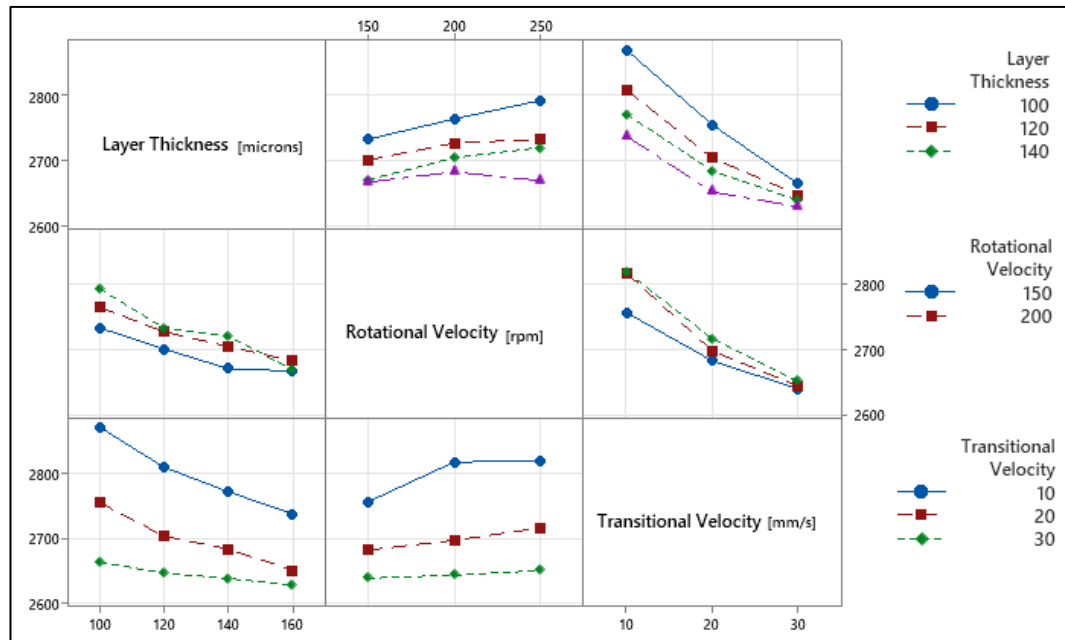


Figure 6.2: Interaction plots for bulk density [kg/m³].

It is useful to examine the residual plots in addition to the bulk density and angular repose interaction plots to demonstrate the consistency of the results of the analyzes. Looking at Bulk Density and Dynamic Angle of Repose Residual Plots here as can be seen in Figure 6.3 and Figure 6.4, there are four main plots that differ from each other in shape but support each other in meaning. These four plots, which contain various upper and lower limits, mainly show deviations. Excessive increase or decrease shows the consistency of the results of the analysis made relatively.

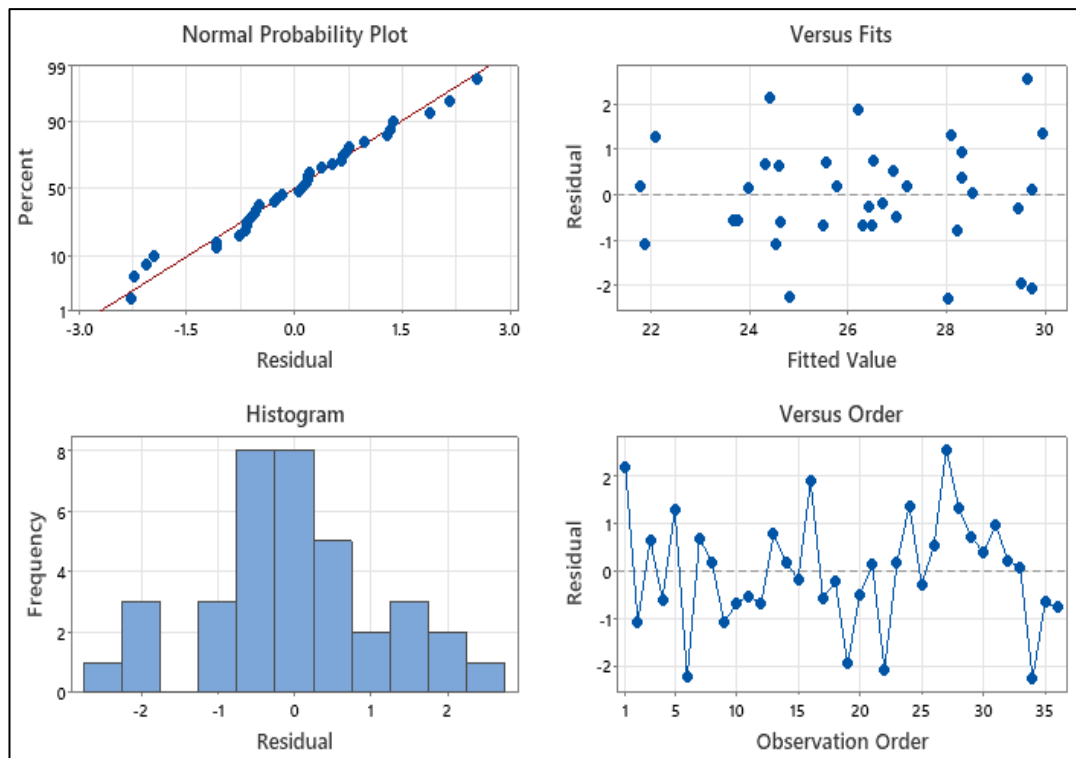


Figure 6.3: Residual plots for dynamic angle of repose [degree °].

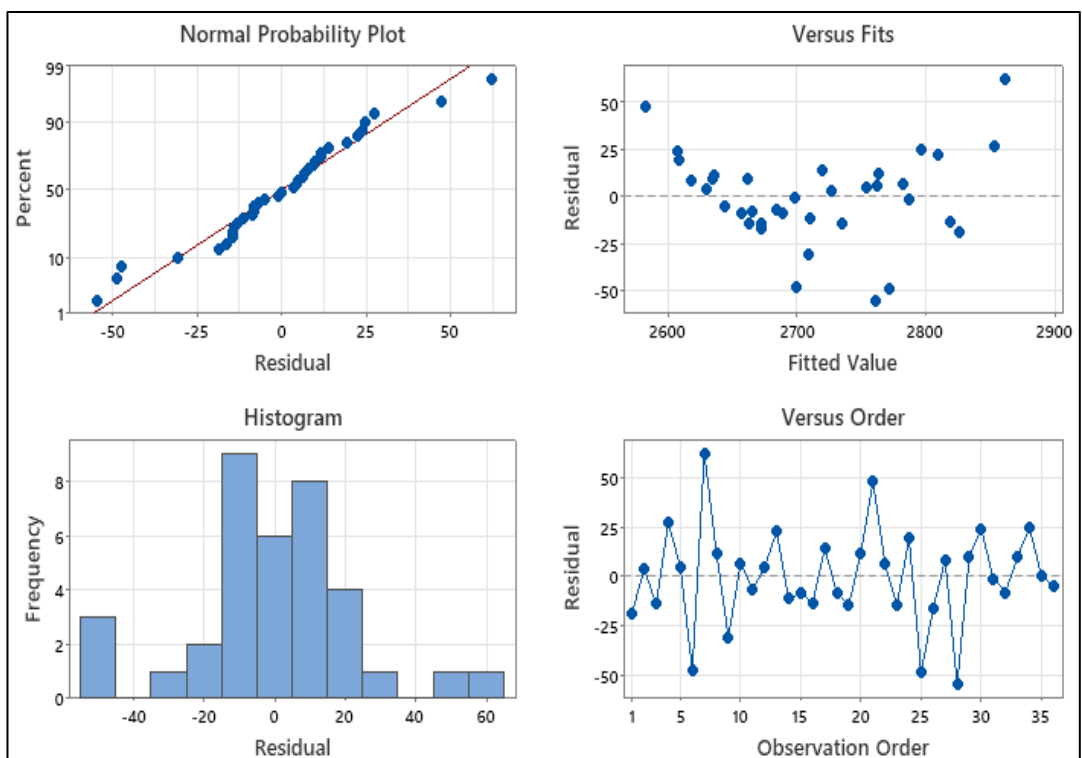


Figure 6.4: Residual plots for bulk density [kg/m³].

In addition to Figure 5.5, look at figure 6.4 to observe the effect of parameters on the layer packing fraction. Although it has the same curves to 5.5. Percentage observation is more acceptable for a comparative attitude.

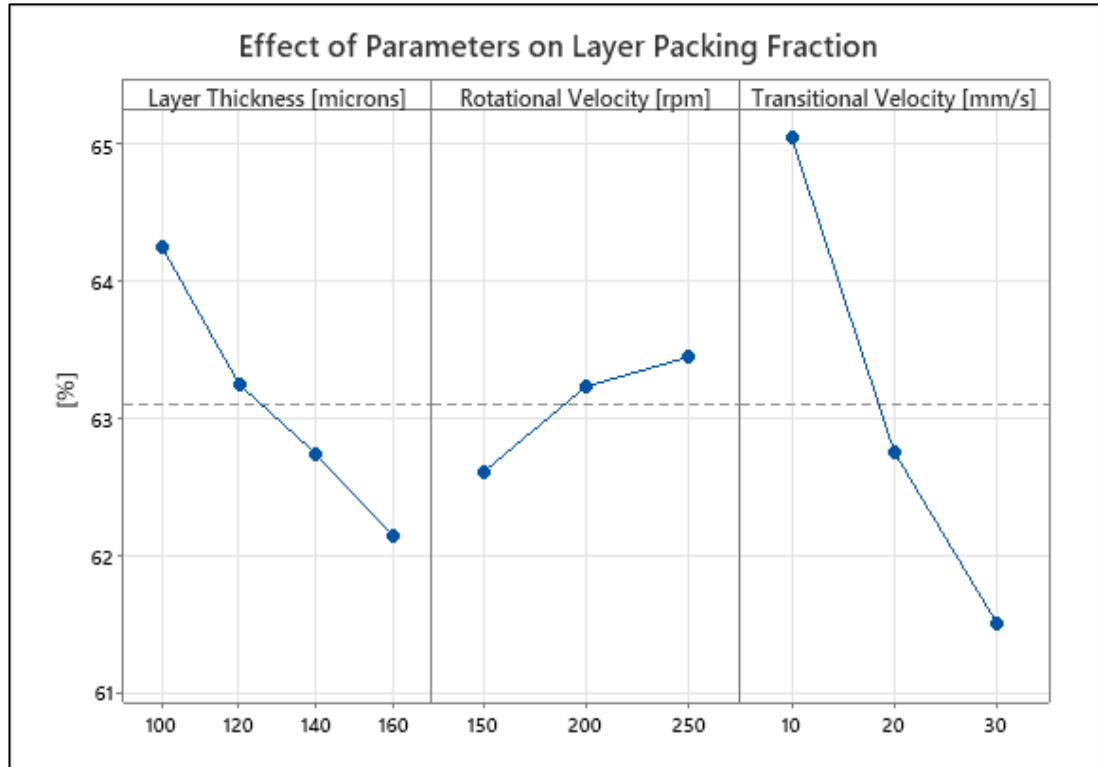


Figure 6.5: Effect of parameters on layer packing fraction.

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BIOGRAPHY

Muhammet Furkan ořkun graduated from Karadeniz Technical University with a bachelor’s degree in mechanical engineering in 2020, which he started in 2015. He has continued his education with master’s degree at the Department of Mechanical Engineering, Graduate School of Natural and Applied Sciences, Gebze Technical University since 2021. He started his career as an Engine Test and Calibration Engineer at TUMOSAN Engine and Tractor Company in the last quarter of 2021. After a 6-month working period, he moved to FEV Trkiye as a Powertrain Integration and Design Engineer and is currently working there.

APPENDICES

Appendix A:

For a collision of two spheres of different materials, the equivalent properties (radius R_{eq} , mass M_{eq} , Young's modulus E_{eq} and shear modulus G_{eq}) in terms of the properties of the single particles i and j are, respectively,

$$R_{eq} = \left(\frac{1}{R_i} + \frac{1}{R_j} \right)^{-1}, \quad M_{eq} = \left(\frac{1}{M_i} + \frac{1}{M_j} \right)^{-1},$$

$$E_{eq} = \left(\frac{1-v_i^2}{E_i} + \frac{1-v_j^2}{E_j} \right)^{-1},$$

$$G_{eq} = \left(\frac{1-v_i}{G_i} + \frac{1-v_j}{G_j} \right)^{-1},$$

For a collision of a sphere i with a wall j , the same relations apply for E_{eq} and G_{eq} , whereas $R_{eq} = R_i$ and $M_{eq} = M_i$. The non-dimensional parameters used in the development of the approximated analytical solution are:

$$\psi = \frac{\kappa}{\mu} \frac{v_t}{v_{n0}}, \quad \chi = \frac{\kappa(1+1/R_g^2)}{2},$$

$$K = \frac{K_{t0}}{K_n} = \frac{(1-v_i)/G_i + (1-v_j)/G_j}{(1-0.5v_i)/G_i + (1-0.5v_j)/G_j}.$$

The definition of the non-dimensional variables

$$\tau^* = \frac{\tau}{\tau_c}, \quad f_n^* = \frac{f_n}{f_n^{(\max)}}, \quad f_t^* = \frac{f_t}{\mu f_n^{(\max)}}, \quad v_n^* = \frac{v_n}{v_{n0}}$$

$$v_t^* = \psi = \frac{\kappa}{\mu} \frac{v_t}{v_{n0}}, \quad \delta_n^* = \frac{\delta_n}{C^2 R_{eq}}, \quad \delta_t^* = \frac{\kappa}{\mu} \frac{\delta_t}{C^2 R_{eq}}$$