

**T.C.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL**

**RECONFIGURABLE INTELLIGENT SURFACE BASED
COOPERATIVE MULTI-ANTENNA COMMUNICATIONS**



MUHAMMED FARUK KARABAY

**A THESIS OF MASTER OF SCIENCE
DEPARTMENT OF ELECTRONICS ENGINEERING**

ADVISOR: PROF. DR. OĞUZ KUCUR

JANUARY 2026

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LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

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*To my
Family...*

ABSTRACT

Over the recent years, reconfigurable intelligent surfaces (RIS) have been one of the main focuses to enhance the performance of communication systems in future generations. Due to the fact that this concept is very up-to-date, it has not yet been investigated in corporation with many multiple input multiple output (MIMO) systems. One of the main purposes of the study is to exploit the performance gain of RIS even further by the use of MIMO techniques. Here, RIS assisted MIMO systems are investigated over Rayleigh fading channels. Furthermore, we aimed to shed light on effect of physical impairments in RIS assisted MIMO systems. According to the existing literature, one of the main problems of RIS in real world scenarios is limited resolution phase shifting. This limited resolution causes a performance degrading effect on RIS, referred to as phase noise. This work analyzes the RIS assisted MIMO systems such as Joint Transmit and Receive Antenna Selection (JTRAS) and Transmit Antenna Selection-Maximal Ratio Combining (TAS-MRC) in the presence of phase noise to achieve more realistic and practical results. Statistical expressions are derived including outage probability, symbol error rate and asymptotic outage probability. The results are validated by Monte Carlo simulations.

Keywords: Reconfigurable Intelligent Surfaces, Phase Noise, Outage Probability, Symbol Error Rate, Asymptotic Analysis

ÖZET

Son yıllarda yeniden yapılandırılabilir akıllı yüzeyler (Reconfigurable Intelligent Surfaces, RIS), modern haberleşme sistemlerinin performansını artırmak için ana odak noktalarından biri olmuştur. Bu kavram oldukça güncel olmasına rağmen, birçok çoklu giriş çoklu çıkış (Multiple Input Multiple Output, MIMO) sistemleriyle birlikte henüz incelenmemiştir. Çalışmanın temel amaçlarından biri, MIMO tekniklerinin kullanımıyla RIS'in performans kazancını daha da artırmaktır. Bu çalışmada, Rayleigh sönmüleme kanallarında RIS destekli MIMO sistemleri incelenmiştir. Ayrıca, RIS destekli MIMO sistemlerinde fiziksel bozulmaların etkisine ışık tutmak amaçlanarak RIS'in ana donanım sıkıntılarından bazıları göz önünde bulundurulmuştur. Mevcut literatüre göre, gerçek dünya senaryolarında RIS'in ana sorunlarından biri sınırlı çözünürlüklü faz kaymasıdır. Bu sınırlı çözünürlük, faz gürültüsü olarak adlandırılmış olup RIS üzerinde performans düşürücü bir etkiye sebep olur. Bu çalışma, daha gerçekçi sonuçlar elde etmek için faz gürültüsü varlığında RIS destekli ortak alıcı ve verici anten seçimi (Joint Transmit and Receive Antenna Selection, JTRAS) ve Verici Anten Seçimi-Maksimum Oranlı Birleştirme (Transmit Antenna Selection-Maximal Ratio Combining, TAS-MRC) gibi MIMO sistemlerini analiz etmektedir. Kesinti olasılığı, sembol hata oranı ve asimptotik kesinti olasılığı dahil olmak üzere istatistiksel ifadeler türetilmiştir. Bu sonuçlar Monte Carlo simülasyonlarıyla doğrulanmıştır.

Anahtar Kelimeler: Yeniden Yapılandırılabilir Akıllı Yüzeyler, Faz Gürültüsü, Kesinti Olasılığı, Sembol Hata Oranı, Asimptotik Analiz

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LIST OF SYMBOLS AND ABBREVIATIONS

AWGN	: Additive White Gaussian Noise
BER	: Bit Error Rate
BPSK	: Binary Phase Shift Keying
CDF	: Cumulative Distribution Function
CSI	: Channel State Information
DO	: Diversity Order
i. i. d.	: independent and identically distributed
JTRAS	: Joint Transmit and Receive Antenna Selection
LoS	: Line-of-Sight
MGF	: Moment Generating Function
MIMO	: Multiple Input Multiple Output
M – PAM	: M-ary Pulse Amplitude Modulation
M – PSK	: M-ary Phase Shift Keying
MRC	: Maximal Ratio Combining
OP	: Outage Probability
PDF	: Probability Density Function
RF	: Radio Frequency
RIS	: Reconfigurable Intelligent Surfaces
SER	: Symbol Error Rate
SIMO	: Single Input Multiple Output
SISO	: Single Input Single Output
SNR	: Signal to Noise Ratio
TAS	: Transmit Antenna Selection
UE	: User Equipment

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1. INTRODUCTION

Future generations of mobile communications, specifically 6G and beyond, offer much higher speed, reliability and coverage compared to previous generations. These hard to achieve specifications require state-of-the-art solutions. One of these solutions, reconfigurable intelligent surfaces (RIS), can efficiently control phase of signals passively without needing to do complex decoding and encoding operations [1]. Since no radio frequency (RF) chains is used, complexity reduces greatly. Furthermore, compared to conventional related methods such as relaying, RIS offers more efficiency in terms of power and cost [2]. RIS placement is a key factor in system configurations. Many researches about RIS assisted systems consider a weak or blocked direct path [3]. Therefore, they can be deployed on the facades of the buildings, walls inside the buildings etc. to create a virtual strengthened direct path through transmitter to receiver. A typical system model is illustrated in Figure 1.1. Other than creating a virtual direct path and enhancing the signal quality, it is also possible to transmit differently polarized signals simultaneously. Therefore it can be said that natural wireless propagation environments can be controlled by RIS [1].

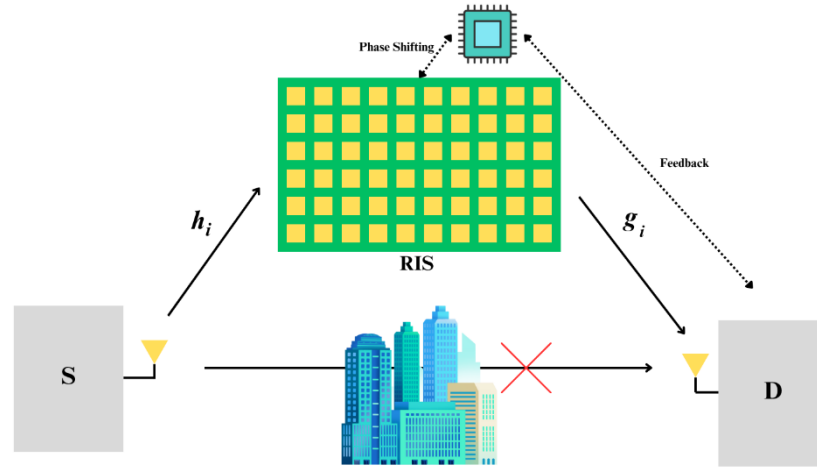


Figure 1.1: Standard RIS system.

Several studies such as [1], [4], [5], assume that phase shifting at RIS end is realized perfectly and that there is no phase component left in the reflected signal. However practical implementations of RIS are inherently affected by hardware impairments. Some of the main problems resulting from these impairments are phase estimation noise and phase quantization noise. Performance enhancement of RIS is heavily dependent on accuracy of applied phase shifts from each surface. In practice, these phase shifts are generally not perfect due to hardware limitations of varactor-based resonators and error margin of phase locked loops. These disruptions reduce the total gain of the RIS, thereby reducing the overall end-to-end signal to noise ratio (SNR) [6].

Phase quantization noise is also a very disruptive effect caused by limited resolution or 1-bit resolution phase shifters [7], [8]. Due to practical and operational limitations, any continuous phase can only be represented by several bits at most and be represented with L discrete levels. Therefore, optimal phase shift can only be realized by applying the closest discrete level of phase to the incoming signal. This occurrence is referred to as phase quantization noise and reduces the passive beamforming gain. It also decreases the diversity order which will be mentioned later again. To address this particular issue, several studies have investigated the impact of limited resolution or 1-bit phase shift on the overall system performance, indicating that limited resolution can still yield impressive gains, particularly for high number of reflecting surfaces [2], [8].

While earlier studies on RIS mainly focuses on single input single output (SISO) configurations, more recent studies switched their focus to multiple input multiple output (MIMO) configurations to further exploit the gain of RIS [5], [9], [10], [11], [12]. Authors of [5] investigate an RIS assisted transmit antenna selection (TAS) – maximal ratio combining (MRC) system configuration and show benefits of integrating this MIMO technique into an RIS system. They focus on physical layer security and analyze the system in terms of secure outage probability (OP) and secrecy capacity. Their conclusions confirm that integrating MIMO with RIS enhances security. In [10], authors investigate an RIS system that also employs TAS-MRC technique. They demonstrate that the performance improvements are achieved when an RIS, modeled as keyholes, is integrated with a MIMO system. Numerical and

analytical result indicate that as number of antennas on either transmitter or receiver increases, SNR gain also increases considerably for a desired OP level. In [11], a downlink system with simultaneously transmitting and reflecting RIS is examined where this RIS communicates with two users. MRC at the user side increases the diversity order. Once again, it is shown that MIMO enhances the OP and throughput results notably. Nevertheless, none of these works takes phase noise into account. Authors in [12] on the other hand, examine an RIS assisted Transmit Antenna Selection (TAS) system with physical impairments. They represent combination of I/Q imbalance, nonlinearity and phase noise with a single parameter.

To the best of authors' knowledge, some of the MIMO techniques remain untouched in RIS assisted communication systems. Especially joint transmit and receive antenna selection (JTRAS) technique has not yet been investigated in RIS assisted systems. JTRAS technique realizes a training period and decides in favor of the antenna pair that yields highest signal-to-noise ratio (SNR) [13]. After the selection is complete, system communicates over the dedicated antenna pair and therefore uses only single RF chain at both transmitter and receiver. This MIMO technique has the lowest complexity and provides full diversity. Therefore, integrating JTRAS to an RIS assisted communication system greatly enhances the diversity order. Goal of this study is to derive important performance metrics and examine the system with the help of these statistical expressions. To describe the system more clearly, we consider an RIS-assisted MIMO setup operating in the presence of phase noise.

There are many works in RIS literature which takes techniques such as MRC or beamforming into consideration such as [5], [9], [10], [11], [14]. However, effect of phase noise has not yet been thoroughly investigated to the best of our knowledge. One of the most comprehensive researches on effect of phase quantization noise is [8] but the considered setup is RIS assisted SISO system. Thus, its effect on various MIMO techniques carries a significance. Particularly in this work, we examine several cases for phase noise and discuss how to mitigate the effect of it with the help of MIMO. It is obviously possible to enhance the performance of a MIMO system by increasing the number of antennas. As the number of antennas increases, feedback complexity for the phase shift also increases. However, it is shown later in the numerical results section that, with less bits per shift and more antennas at transmitter balance can be maintained while enhancing performance. Finally, phase shift dependency issue for combining

techniques such as MRC and TAS-MRC in RIS assisted systems is discussed and a solution is proposed. This issue has not yet been discussed in literature to the best of our knowledge.



2. REVIEW OF METHODS AND STRUCTURES USED IN THE THESIS

MIMO systems are basis of many applications of modern communication. This concept has remained a fundamental component of wireless systems from 3G to 6G, owing to its ability to substantially enhance both capacity and reliability. The basic principle of MIMO is to employ multiple antennas at both the transmitter and the receiver, thereby incorporating the spatial dimension and achieving additional gains beyond those obtained in time and frequency. MIMO structures are able to yield higher spectral efficiency, diversity gain, and spatial multiplexing compared to SISO structures.

In SISO systems, Shannon capacity is solely reliant on SNR. In MIMO systems on the other hand, it is shown that capacity increases as number of antennas on either transmitter or receiver increases. By employing more than one antenna on either side, signal can be transmitted via multiple independent paths. This results in diversity gain and this is one of the main advantages of conventional MIMO systems. With the combination of multiple copies created by multiple antennas, error rates can be reduced significantly. Performance analysis of MIMO techniques are examined in many studies and it is observed that integrating receiver multiplexing with transmitter multiplexing provides full diversity in i.i.d. channels. Aforementioned channels are most commonly modeled as Rayleigh, Nakagami-m or Rician. In this work we consider Rayleigh fading channels.

2.1. Rayleigh Fading Channels

Transmitted signals in wireless communication environments encounter various obstacles such as buildings, walls, and vehicles. This multipath propagation alters the amplitude and phase of each propagation path independently. Therefore, the received signal exhibits random fluctuations. In environments such as classical urban settings, where the line of sight (LoS) component is absent or negligible, this statistical model best captures the characteristics of the received signal. Under this model, the multipath components are assumed to have uniformly distributed phase and circularly Gaussian

distributed in-phase and quadrature components. Therefore, envelope follows a Rayleigh distribution.

Rayleigh model is one of the most commonly used statistical fading channels because of the fact that its characterization of dense urban areas, indoor environments and high mobility non-LoS scenarios is very accurate. Furthermore, it provides tractability on mathematical analysis. In this model, the instantaneous channel power gain is exponentially distributed, which implies that the instantaneous SNR varies significantly over time. Consequently, the error performance of wireless systems operating under Rayleigh fading is highly sensitive to deep changes. This creates the need to employ MIMO techniques, which use diversity to mitigate sudden signal fades. By the help of MIMO techniques such as TAS, Receive Antenna Selection (RAS), JTRAS, it is possible to adverse the effects of random channel variations. Probability density function (PDF) and cumulative distribution function (CDF) of Rayleigh distributed random variable H are expressed, respectively as below.

$$f_H(h) = \frac{h}{\sigma^2} e^{-h^2/(2\sigma^2)}, \quad h \geq 0 \quad (2.1)$$

$$F_H(h) = 1 - e^{-h^2/(2\sigma^2)}, \quad h \geq 0 \quad (2.2)$$

Let us denote the received signal as $r = h s + n$. Here, h is fading coefficient, s is sent signal with unit power ($E_s = 1$) and n is additive white Gaussian noise (AWGN) with two-sided power spectral density of $N_0/2$. Therefore, we can express the received SNR as $\gamma = |h|^2 \frac{E_s}{N_0} = |h|^2 \bar{\gamma}$. Here, PDF, CDF and moment generating function (MGF) of γ can be expressed respectively as below.

$$f_\gamma(x) = \frac{1}{2\sigma^2} \exp\left(-\frac{x}{2\sigma^2}\right), \quad x \geq 0 \quad (2.3)$$

$$F_\gamma(x) = 1 - \exp\left(-\frac{x}{2\sigma^2}\right), \quad x \geq 0 \quad (2.4)$$

$$M_\gamma(s) = E[e^{s\gamma}] = \frac{1}{1 - 2\sigma^2 s}, \quad s < \frac{1}{2\sigma^2} \quad (2.5)$$

2.2. Obtaining SER Through CDF Method

This CDF based method computes the average symbol error rate (SER) by conditioning on the instantaneous SNR and averaging over its distribution. To be able to use this method, AWGN channel expression must be known. For any modulation type, SER in AWGN channel can be expressed as

$$P_s = E_Y[a Q(\sqrt{2bY})]. \quad (2.6)$$

Here, a and b parameters are given in Table 2.1 and $Q(\cdot)$ is the Gaussian Q-function.

Table 2.1: Modulation parameters of CDF method for various modulations.

Modulation	a	b	Explanation
BPSK	1	1	Exact
BFSK	1	0.5	Exact for coherent
M-PAM	$\frac{2(M-1)}{M}$	$\frac{3}{M^2-1}$	Exact for coherent
M-PSK	2	$\sin^2(\pi/M)$	Approximation
M-QAM	$4\left(1 - \frac{1}{\sqrt{M}}\right)$	$\frac{3}{2(M-1)}$	Approximation

Using these parameters, SER integral can be expressed using CDF [15] as

$$P_s = \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-bu}}{\sqrt{u}} F_Y(u) du. \quad (2.7)$$

The CDF based approach is very useful when $F_Y(u)$ has a simple analytical form. It is particularly most useful when CDF include exponentials and powers due to the fact that analytical integration gets easier in these cases. This makes the method rather appealing in antenna selection systems where order statistics get involved.

2.3. Obtaining SER Through MGF Method

The MGF based method is based on Craig's formula [16], leverages the fact that for a large class of modulation schemes, the average SER can be expressed using the

alternative representation of Gaussian Q function. It can be expressed as an integral involving MGF of the SNR as

$$P_S(E) = \sum_{i=1}^I \frac{\alpha_i}{\pi} \int_0^{\phi_i} \mathcal{M}_Y\left(-\frac{\beta_M}{\sin^2(\theta)}\right) d\theta, \quad (2.8)$$

where parameters α_i , ϕ_i and β_M are given in Table 2.2.

Table 2.2: SER parameters of MGF method for various modulation types.

Modulation	α_i	β_M	ϕ_i	Explanation
BPSK	1	1	$\pi/2$	I=1, Exact
Coherent BFSK	0.5	0.5		
M-PAM	$\frac{2(M-1)}{M}$	$\frac{3}{M^2-1}$		I=1, Approximation
M-PSK	2	$\sin^2(\pi/M)$		
M-PSK	1	$\sin^2(\pi/M)$	$\pi\left(1 - \frac{1}{M}\right)$	I=1, Exact
M-QAM	$4\left(1 - \frac{1}{\sqrt{M}}\right) - 4\left(1 - \frac{1}{\sqrt{M}}\right)^2$	$\frac{3}{2(M-1)}$	$\phi_1 = \pi/2$ $\phi_2 = \pi/4$	I=2, Exact for rectangular

Authors in [17], investigate a complex antenna selection scheme showing that in severe channel conditions, the technique is highly effective. More importantly, they derive analytical performance metrics with the help of this method. It is clear that, even with the complex mathematics of the system, it is possible to derive closed form expressions. In conclusion, the MGF method proves particularly effective for analyzing systems with high structural complexity.

2.4. TAS and RAS schemes

Antenna selection has emerged as an important technique in multi-antenna systems particularly in cases where RF chains or power consumptions are limited. Rather than activating all of the antennas at once, antenna selection schemes choose a subset of antennas that maximizes instantaneous SNR while maintaining low complexity as seen in Figure 2.1. This makes antenna selection rather attractive for practical MIMO systems and emerging frameworks such as RIS assisted communication systems.

2.4.1. TAS Scheme

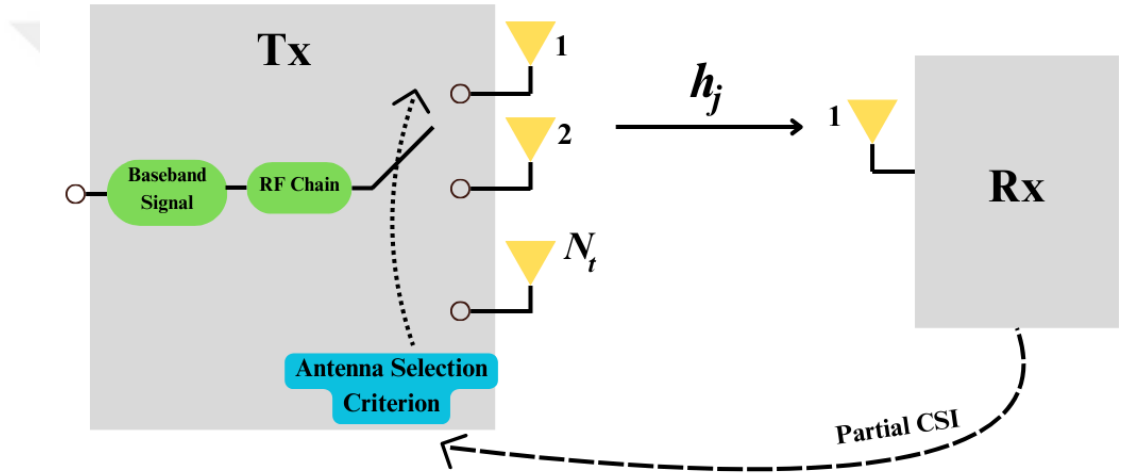


Figure 2.1: Transmit Antenna Selection architecture.

In TAS, the transmitter selects the antenna yielding the highest SNR. The selection is based on channel coefficient since all the average SNR values from different antennas are assumed to be equal. Expressing received signal sent from j^{th} transmitter antenna as $r_j = h_j s + n$, we can express SNR of single antenna as $\gamma_j = |h_j|^2 \frac{E_s}{N_0} = |h_j|^2 \bar{\gamma}$. Since average SNR values are the same among all of the antennas for i.i.d. case, Transmitter selects the best antenna with the following criterion:

$$j^* = \arg \max_j |h_j|^2. \quad (2.9)$$

Once j^* is selected RF chain connects to the j^* th antenna and communication proceeds through that antenna. This method provides full diversity while maintaining the use of

a single RF chain. Integrating this technique with a single input multiple output (SIMO) technique can further exploit diversity gain. The performance of TAS combined with MRC has been extensively analyzed, demonstrating that full diversity can be achieved with single RF chain at the transmitter [18].

2.4.2. RAS Scheme

In contrast, RAS performs the selection at the receiver by choosing the antenna that yields the highest post-processing SNR. This method is particularly efficient in downlink scenarios where user equipment (UE) has low complexity and needs effective power consumption. Similarly, expressing received signal at k^{th} receiver antenna as $r_k = h_k s + n$ we can write SNR through a single receiver antenna as $\gamma_k = |h_k|^2 \bar{\gamma}$. Therefore, receiver selects the antenna yielding the highest SNR as:

$$k^* = \arg \max_k |h_k|^2. \quad (2.10)$$

Once again, communication is realized through single RF chain, still yielding full diversity at receiver.

2.5. JTRAS Scheme

JTRAS is an efficient diversity technique in which both ends of the communication link participate in the antenna selection process. Unlike conventional TAS or RAS, which perform selection on only one side of the link, JTRAS jointly evaluates all possible transmit–receive antenna pairs and activates the one that provides the highest instantaneous SNR as seen in Figure 2.2. This results in a single active link and requires only one RF chain on both sides, yet still exploits the full spatial diversity offered by the antenna array. Antenna selection is made according to:

$$(j^*, k^*) = \arg \max_{j,k} |h_{jk}|^2. \quad (2.11)$$

Here, h_{jk} represents the channel coefficient between j^{th} transmitter and k^{th} receiver antenna. In [13], it is shown that the diversity order of a system that has N_t transmitter

and N_r receiver antennas is mN_tN_r , where m is the Nakagami-m fading parameter which equals 1 for the Rayleigh fading channel.

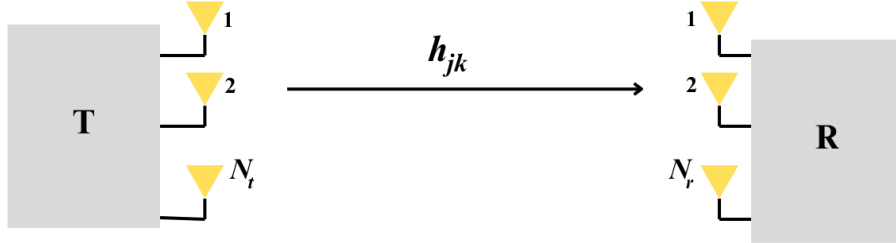


Figure 2.2: General JTRAS system model.

2.5.1. Statistical Expressions of JTRAS Scheme

Considering a MIMO system that has N_t transmit and N_r receive antennas, let us denote the signal received at the k^{th} receive antenna from the j^{th} transmit antenna as $r_{jk} = h_{jk}s + n$. Here, SNR of the signal through a single pair can be expressed as $\gamma_{jk} = |h_{jk}|^2 \bar{\gamma}$. Total SNR of the system is then maximum of the single pair SNRs which is expressed as $\gamma_{j^*k^*} = \gamma$. Writing the CDF of the SNR of a single pair as $F_{\gamma_{jk}}(u)$, with the help of higher order statistics [13], SNR of the JTRAS system becomes

$$F_{\gamma_{j^*k^*}}(u) = F_{\gamma}(u) = \left[F_{\gamma_{jk}}(u) \right]^{N_t N_r}. \quad (2.12)$$

Therefore PDF of the total SNR is written as

$$f_{\gamma}(u) = \frac{d}{du} F_{\gamma}(u) = N_t N_r \left[F_{\gamma_{jk}}(u) \right]^{N_t N_r - 1} f_{\gamma_{jk}}(u). \quad (2.13)$$

2.6. Performance of Antenna Selection Schemes

2.6.1. SER Derivation of RAS and TAS

Assume a RAS system with N_r receive antennas. Using the formulation in (2.13), PDF of SNR of RAS system under unit power Rayleigh fading can be written as

$$f_Y(x) = \frac{N_r}{\bar{Y}} e^{-x/\bar{Y}} (1 - e^{-x/\bar{Y}})^{N_r-1}. \quad (2.14)$$

Here, using SER formula of $P_S = \int_0^\infty Q(\sqrt{2x}) f_Y(x) dx$ and alternative representation of Q function [16],

$$P_S = \int_0^\infty \frac{1}{\pi} \int_0^{\pi/2} e^{-\frac{2x}{2 \sin^2(\theta)}} d\theta \frac{N_r}{\bar{Y}} e^{-x/\bar{Y}} (1 - e^{-x/\bar{Y}})^{N_r-1} dx. \quad (2.15)$$

By the help of binomial expansion and a simple integration it becomes

$$P_S = \frac{N_r}{\pi \bar{Y}} \sum_{k=0}^{N_r-1} (-1)^k \binom{N_r-1}{k} \int_0^{\pi/2} \frac{1}{\frac{1}{\sin^2(\theta)} + \frac{1+k}{\bar{Y}}} d\theta. \quad (2.16)$$

Integrals in the form of $\int_0^{\pi/2} \frac{1}{a+b \sin^2(\theta)} d\theta$ can be expressed as $\frac{\pi}{2\sqrt{a(a+b)}}$ with variable change of $\sin^2(\theta) = \frac{t}{1+t}$ and [19, eq. (3.241.2)]. Here, transforming inside of the integral and using this integral transition, we get

$$P_S = \frac{1}{2} \sum_{n=0}^{N_r} (-1)^n \binom{N_r}{n} \sqrt{\frac{\bar{Y}}{n + \bar{Y}}}. \quad (2.17)$$

Note that since statistical behavior of RAS and TAS are the same with only different antenna parameters, replacing N_r with N_t in (2.17) admits SER formula for TAS technique. Furthermore, replacing N_r with $N_t N_r$ here admits a SER formula for JTRAS technique under Rayleigh fading.

2.6.2. Numerical Results

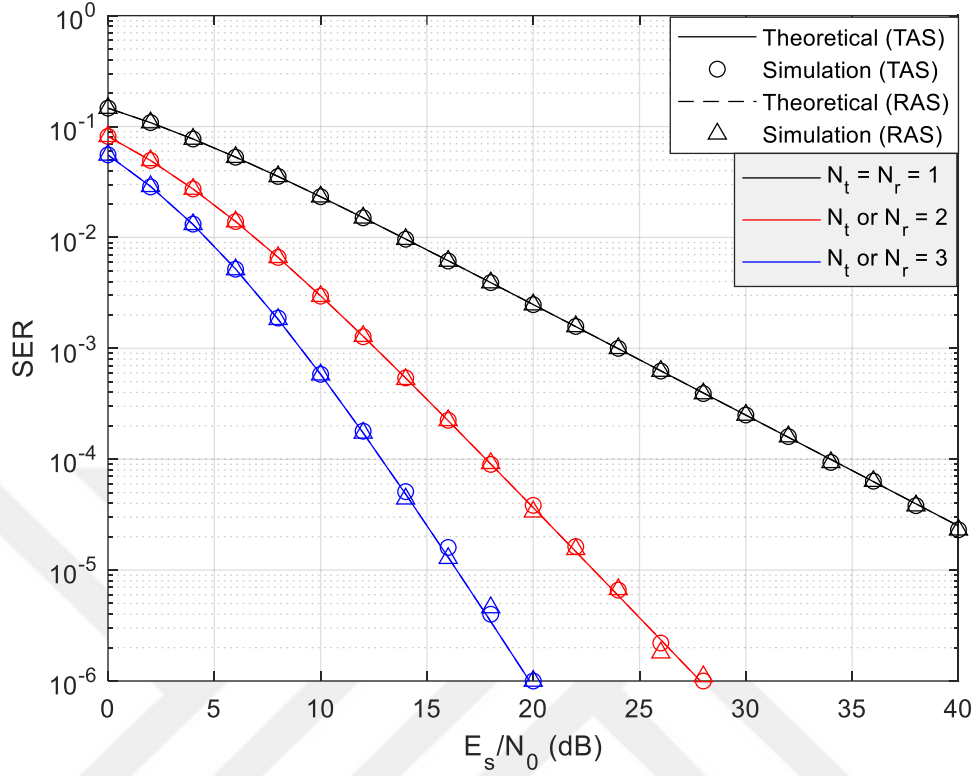


Figure 2.3: Performance comparison of RAS and TAS techniques.

Figure 2.3 shows that the SER curves for TAS and RAS nearly coincide for all antenna configurations, which is expected because both techniques select the branch with the highest instantaneous SNR, and under Rayleigh fading the channel statistics are symmetric with respect to the transmitter and receiver. As a result, the output SNR distributions of TAS and RAS are identical, and both the theoretical expressions and Monte Carlo simulations fall exactly on top of each other in the figure. Increasing the number of antennas yields a clear improvement in reliability: for TAS, moving from $N_t = 1$ to $N_t = 2$ shifts the SER curve by approximately 15.83 dB, and increasing from $N_t = 2$ to $N_t = 3$ provides an additional 5 dB gain at an SER of 10^{-4} . When the same increments of N_r are applied, SNR gains are also the same. These horizontal shifts directly reflect the increased diversity order obtained by selecting the best among certain amount of independent Rayleigh branches. Overall, the figure confirms that TAS and RAS offer identical performance in Rayleigh fading and that their diversity gain scales predictably with the number of available antennas.

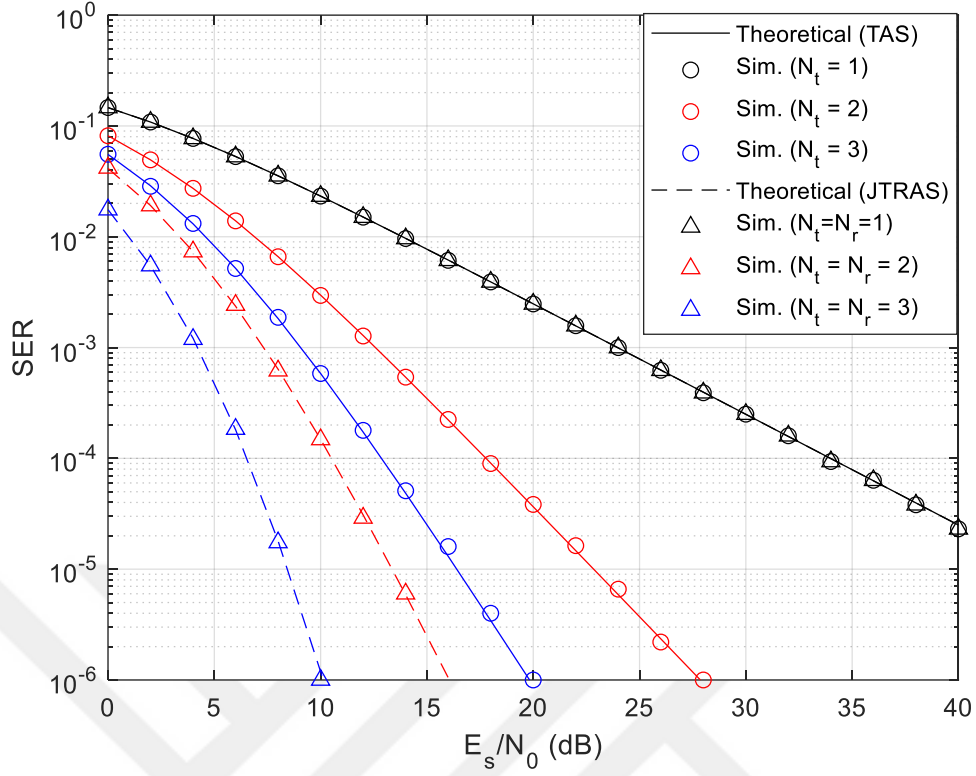


Figure 2.4: Performance comparison of JTRAS and TAS techniques.

Figure 2.4 compares the SER performance of TAS and JTRAS under Rayleigh fading for different antenna configurations, clearly illustrating the additional diversity benefit achieved when selection is performed jointly at both transmitter and receiver. As expected, increasing the number of transmit antennas in TAS from $N_t = 1$ to $N_t = 2$ yields a significant improvement of 15.83 dB at a SER of 10^{-4} . However, when both transmit and receive antenna count are incremented from 1 to 2, another SNR gain of approximately 7.47 dB is observed. Furthermore, using same number of antennas for both techniques really shows the superiority of JTRAS. Comparing JTRAS with $N_t = N_r = 2$ to TAS with $N_t = 3$ and $N_r = 1$, we observe an SNR gain of roughly 2.41 dB. Overall, the curves clearly show that while TAS benefits from increased transmitter diversity, the JTRAS leverages the full diversity potential of $N_t N_r$, achieving superior error performance with the same hardware complexity at each side.

From a broader system perspective, the operational principles of JTRAS are naturally aligned with the philosophy behind RIS. While JTRAS selects the strongest link among all transmit–receive antenna pairs, an RIS reshapes the propagation environment so that certain paths become inherently stronger. In this sense, an RIS

acts as a passive channel optimizer that enhances multiple candidate links simultaneously, whereas JTRAS functions as an active endpoint selector that chooses the best among them. When combined, the RIS improves the quality of several potential paths, and JTRAS subsequently identifies the optimal RIS-enhanced link. This synergy yields high diversity and strong link reliability with minimal RF-chain usage, making RIS-assisted JTRAS configurations particularly attractive for dense, obstructed, or coverage-limited environments where conventional MIMO strategies are less effective.

2.6.3. MRC and TAS-MRC Techniques

MRC is a linear diversity combining technique that achieves the maximum possible output SNR. The aim of this technique is to optimally combine the signals received over multiple receive antennas in order to mitigate the effects of multipath fading and additive noise. The basic principle of MRC is to weight each received signal by the complex conjugate of its corresponding channel coefficient and coherently combine them, resulting in the maximum achievable SNR at the receiver. While MRC yields the highest SNR among linear combining schemes, this performance comes at the cost of increased computational complexity. In particular, both the amplitude and phase components of the channel coefficients must be accurately known at the receiver. In contrast, antenna selection techniques require only partial channel state information (CSI), where a finite or limited number of feedback bits is sufficient for operation. Having the MRC technique's perks and disadvantages clearly stated, technical definition of the MRC system must be made. Assuming a system with N_r receive antenna and single transmit antenna, we can express the received signal from i^{th} receive antenna as

$$r_i = h_i s + n_i, \quad i = 1, 2, \dots, N_r. \quad (2.18)$$

After weighting the received signals, total signal becomes

$$\begin{aligned} y &= \sum_{i=1}^{N_r} h_i^* (h_i s + n_i). \\ &= (\sum_{i=1}^{N_r} |h_i|^2) s + \sum_{i=1}^{N_r} h_i^* n_i. \end{aligned} \quad (2.19)$$

Final MRC SNR can be expressed as

$$\gamma_{\text{MRC}} = \frac{E_s}{N_0} \sum_{i=1}^{N_r} |h_i|^2. \quad (2.20)$$

Here, E_s is the signal energy and N_0 is the power of noise. Clearly, MRC SNR is the summation of the single branch SNRs. Statistical expressions of MRC are derived in many works and SER over i.i.d. Rayleigh channels using binary phase shift keying (BPSK) in [20] is given as

$$P_S = p^{N_r} \sum_{k=0}^{N_r-1} (1-p)^k \binom{N_r-1+k}{k}, p = \frac{1}{2} - \frac{1}{2} \left(1 + \frac{1}{\gamma}\right)^{-1/2} \quad (2.21)$$

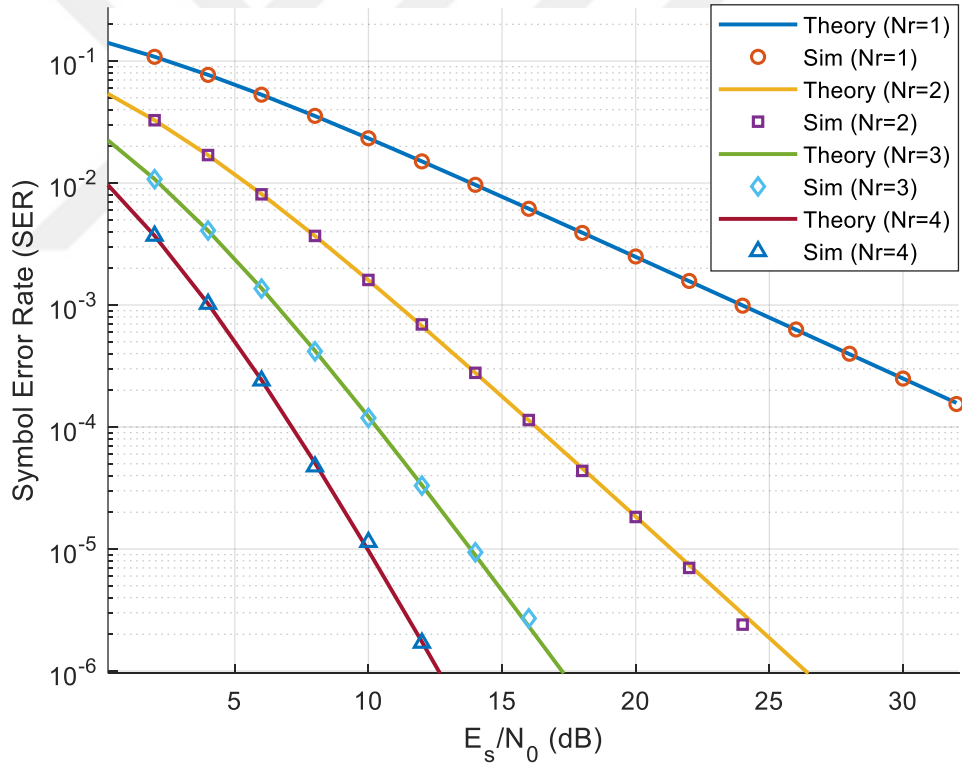


Figure 2.5: SER of BPSK over i.i.d. Rayleigh channels with MRC: Theory vs. Simulation.

Figure 2.5 presents the theoretical and simulated SER curves of an MRC system employing BPSK modulation over a Rayleigh fading channel. It is clearly observed that each increase in the number of receive antennas significantly enhances the overall system performance and increases the achievable diversity order. Specifically at a SER

level of 10^{-4} , a SISO system requires approximately an SNR of 33 dB, whereas an MRC system with $N_r = 2$ achieves the same performance at around an SNR of 16 dB. Further increases in the number of receive antennas continue to provide substantial SNR gains of approximately 6 dB and 3 dB respectively, demonstrating the effectiveness of MRC.

Although MRC provides optimal SNR performance at the receiver, the overall performance can further be enhanced by incorporation of MISO techniques. At this point, TAS-MRC is examined as a key MIMO technique. TAS-MRC is a very effective MIMO technique that exploits the coding gain of MRC and full diversity order provided by TAS, which results in higher performance with full system diversity. Assuming a system with N_t transmit and N_r receive antennas, the instantaneous SNR corresponding to each transmit antenna is given by (2.20). System then selects the transmit antenna that maximizes this SNR. Consequently, the overall system SNR is obtained as the maximum among the SNRs of all transmit antennas.

Performance analysis of TAS-MRC is made in [21] for various modulation types. Using M-ary phase shift keying (M-PSK) over Rayleigh channel, SER expression reduces to

$$\begin{aligned} \bar{P}_s(E) = & \frac{N_t}{\bar{\gamma}^{N_r} \Gamma(N_r)} \sum_{p=0}^{N_t-1} \sum_{k=0}^{p(N_r-1)} \binom{N_t-1}{p} (-1)^p \mu_k(p, N_r) \\ & \times \Gamma(k + N_r) \frac{1}{\pi} \int_0^{\frac{\pi(M-1)}{M}} \left[\frac{\sin^2\left(\frac{\pi}{M}\right)}{\sin^2(\theta)} + \frac{p+1}{\bar{\gamma}} \right]^{-(k+N_r)} d\theta \end{aligned} \quad (2.22)$$

Here, $\mu_q(t, 1) = \frac{1}{q} \sum_{\tau=1}^q (\tau t - q + \tau) \alpha_\tau \mu_{q-\tau}(t, 1)$ and $\alpha_\tau = \frac{\left(\frac{1}{\bar{\gamma}}\right)^\tau}{\tau!}$ and $\mu_k(\cdot, \cdot)$ is the multinomial coefficients [19].

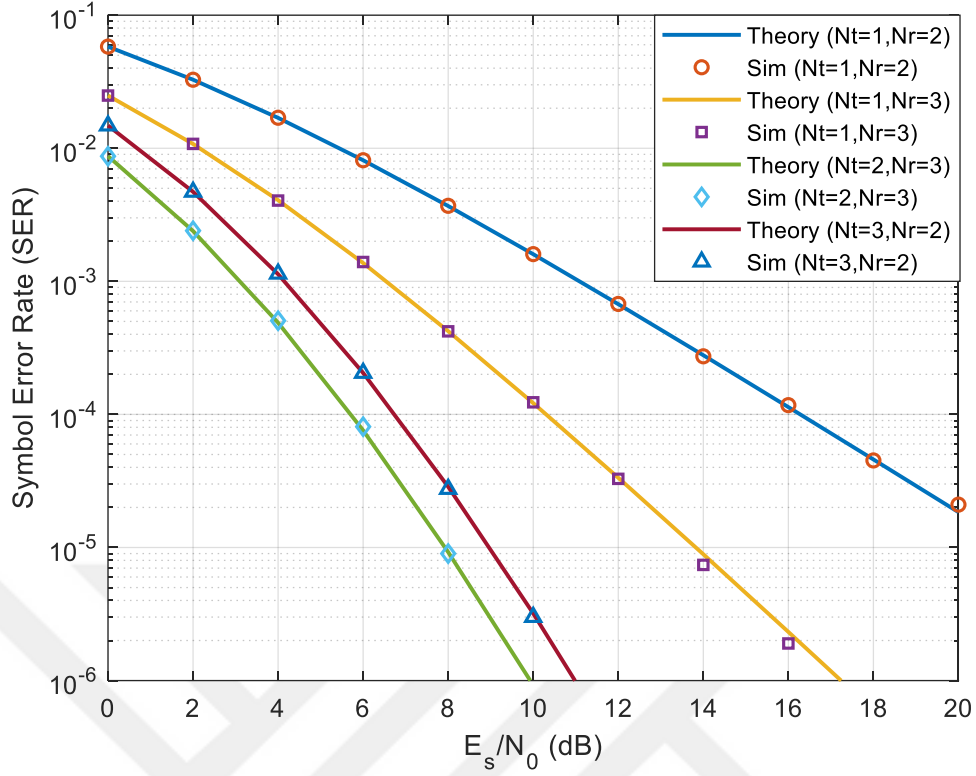


Figure 2.6: SER of BPSK over Rayleigh channel using TAS-MRC.

Figure 2.6 depicts SER curves of TAS-MRC technique over Rayleigh channel using BPSK. As a baseline, two results with single transmit antenna are plotted. For these cases TAS-MRC reduces to MRC. Considering the performance curve for $N_t = 1$ and $N_r = 3$, increasing number of transmit antennas by one provides an SNR gain of about 4.5 dB at a SER level of 10^{-4} . It is also clear that diversity order increases as the number of transmit antennas increases. Another key point here is system with $N_t = 2$ and $N_r = 3$ outperforms the system with $N_t = 3$ and $N_r = 2$ even though they have the same diversity order. That is because array gain provided by MRC is higher than that achieved by TAS. Consequently, when the majority of antennas are located at the receiver where MRC is employed, the system achieves an SNR gain of approximately 1 dB.

2.7. RIS Assisted Systems

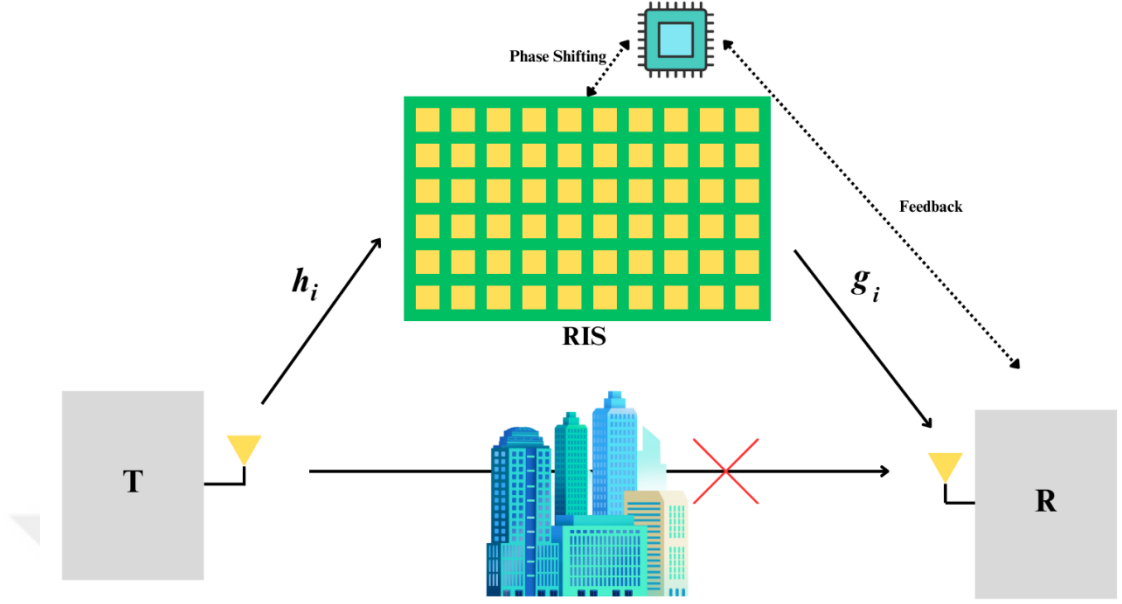


Figure 2.7: RIS assisted SISO system.

Let us consider a RIS-assisted SISO system with N surfaces as depicted in Figure 2.7. Let h_i and g_i be the fading coefficients between transmitter-RIS and RIS-receiver respectively. Direct link between transmitter and receiver is assumed to be either blocked or too weak because of urban elements such as buildings. Furthermore, $h_i = |h_i|e^{j\theta_i}$ and $g_i = |g_i|e^{j\varphi_i}$ follow complex gaussian distribution ($h_i, g_i \sim \mathcal{CN}(0,1)$) so that, their envelopes follow Rayleigh distribution with unit power. So the received signal can be expressed as:

$$r = \left[\sum_{i=1}^N h_i e^{j\varphi_i} g_i \right] s + n, \quad (2.23)$$

where φ_i is the applied phase shift of the i^{th} meta-surface of RIS and n is the AWGN. Here, $n \sim \mathcal{N}(0, N_0/2)$. Expressing (2.23) in the final form, we get:

$$r = \left[\sum_{i=1}^N |h_i| |g_i| e^{j\varphi_i} \right] s + n \quad (2.24)$$

Here, ϕ_i is the remaining phase component phase shift. To establish a baseline, we first consider the case with the perfect phase compensation. This case yields the maximum possible SNR [1]. The maximized SNR is then expressed as:

$$\gamma = \frac{\left| \sum_{i=1}^N |h_i| |g_i| \right|^2 E_s}{N_0} = \frac{(\sum_{i=1}^N |h_i| |g_i|)^2 E_s}{N_0} \quad (2.25)$$

Here, second equality arises from the fact that channel is purely real in perfect phase shift scenario. Denoting SNR as

$$\gamma = \frac{H^2 E_s}{N_0}, \quad (2.26)$$

mean and variance of H must be found to approximate the behavior of SNR. $|h_i|$ and $|g_i|$ are i.i.d. and follow unit power Rayleigh distribution. Hence, denoting $\alpha_i = |h_i| |g_i|$, mean and variance of the product can easily be found as $E[\alpha_i] = \pi/4$ and $var(\alpha_i) = 1 - \frac{\pi^2}{16}$, respectively. Therefore, γ is non-central chi square random variable with one degree of freedom and its MGF is as follows [1]:

$$M_Y(s) = \sqrt{\frac{1}{1 - \frac{sN(16 - \pi^2)E_s}{8N_0}}} \exp\left(\frac{\frac{sN^2\pi^2 E_s}{16N_0}}{1 - \frac{sN(16 - \pi^2)E_s}{8N_0}}\right) \quad (2.27)$$

By the help of MGF method, SER can be written as [17],

$$P_s = \frac{1}{\pi} \int_0^{(M-1)\pi/M} M_Y\left(\frac{-\sin^2(\pi/M)}{\sin^2(\eta)}\right) d\eta. \quad (2.28)$$

For BPSK, this integral is simply expressed as

$$P_s = \frac{1}{\pi} \int_0^{\pi/2} \left(\frac{1}{1 + \frac{N(16 - \pi^2)E_s}{8 \sin^2(\eta) N_0}} \right)^{1/2} \exp\left(\frac{-\frac{N^2\pi^2 E_s}{16 \sin^2(\eta) N_0}}{1 + \frac{N(16 - \pi^2)E_s}{8 \sin^2(\eta) N_0}}\right) d\eta \quad (2.29)$$

This approach in [1] leverages the Central Limit Theorem (CLT), since a RIS contains many reflecting elements. It gets more accurate as number of reflecting elements increases. However, this analysis does not stand accurate in high SNR regime and it must be avoided for asymptotic analysis [2]. This situation is particularly important in RIS assisted MIMO systems as their diversity order is proportionate to the product of number of antennas and surfaces, which is generally a quite high number.

2.7.1. Numerical Results of Ideal RIS

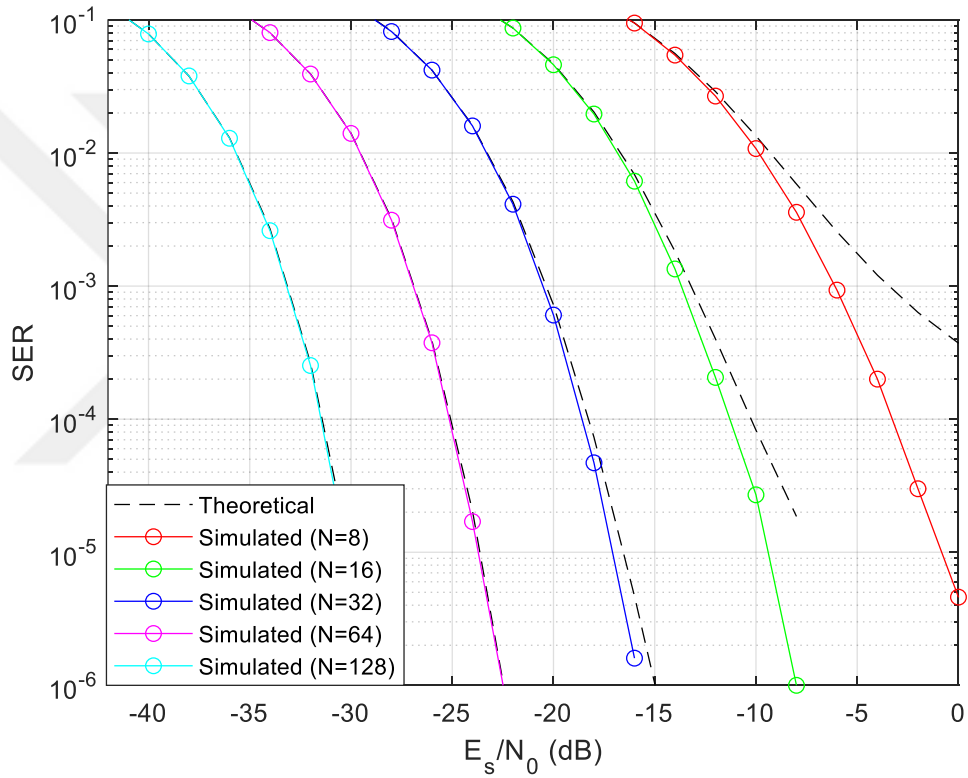


Figure 2.8: SER performance of RIS assisted SISO system with theoretical formula in (2.29) for varying number of reflecting elements.

Figure 2.8 illustrates the simulated and theoretical SER performance of BPSK modulation using RIS-assisted system. As expected, increasing the number of RIS elements significantly improves the error performance. This behavior is consistent with $\mathcal{O}(N^2)$ as mentioned in [1], where, $\mathcal{O}(\cdot)$ represents complexity operator. For relatively small number of surfaces, analysis does not stand accurate. It gets more precise in the visible area of error rate as number of surfaces increases. This confirms the behavior of CLT. Overall, these results verify that substantial performance gains are achievable with large scale RIS devices.

2.8. RIS Assisted Systems in the Presence of Phase Noise

Let us consider a RIS-assisted SISO system with N surfaces again. h_i and g_i are as given before. Direct link is assumed to be either blocked or too weak. So the received signal can be expressed as:

$$r = \left[\sum_{i=1}^N h_i e^{j\phi_i} g_i \right] s + n, \quad (2.30)$$

where ϕ_i is the applied phase shift of the i^{th} meta-surface of RIS and n is the AWGN. Here, $n \sim \mathcal{N}(0, N_0/2)$ and in this case we consider RIS systems with phase noise. So, due to practical phase shift constraints, RIS elements can only shift the phase of incident signal by the use of finite number of bits. Considering one bit phase shifting in our case (i.e. 0° or 180° phase shifting), the received signal is expressed as

$$r = \left[\sum_{i=1}^N |h_i| |g_i| e^{j\phi_i} \right] s + n. \quad (2.31)$$

Here, $\phi_i \sim \mathcal{U}[-\pi/2, \pi/2]$ is the remaining phase component after 1 bit quantized phase shift [8]. Then, SNR can be written as

$$\gamma = \frac{|\sum_{i=1}^N |h_i| |g_i| e^{j\phi_i}|^2 E_s}{N_0}. \quad (2.32)$$

2.8.1. Numerical Results of RIS in the Presence of Phase Noise

Figure 2.9 presents the SER performance of BPSK modulation when the RIS is affected by one bit phase noise, which is the most severe quantization noise. These results can be seen as the worst-case scenario. Even so, every increase of N yields substantial SNR gains. This shows that even under the harshest phase noise assumption, the RIS-induced array gain is not eliminated but only reduced. Hence, it can be concluded that even with hardware constraints, RIS is an affordable and resilient solution for harsh communication environments.

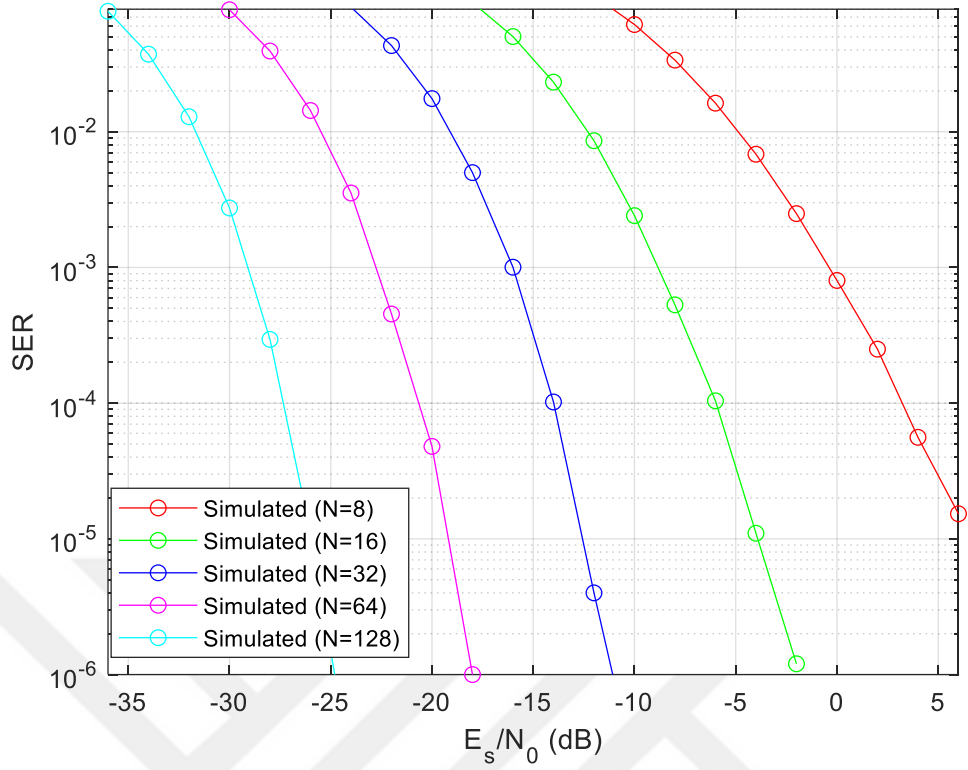


Figure 2.9: SER performance of RIS assisted SISO system with varying number of reflecting elements in the presence of phase noise.

2.9. Performance Comparison of RIS for Ideal and Phase Noise Cases

Figure 2.10 presents the SER performance of RIS-assisted BPSK transmission under both ideal phase adjustment and one bit quantization noise cases for different numbers of reflecting elements. As expected, the ideal case exhibits a consistent leftward shift in the SER curves as N increases, demonstrating the $\mathcal{O}(N^2)$ array-gain behavior of RIS [1]. Phase noise caused by one bit quantization shows a noticeable degradation relative to the ideal curves. The performance loss appears primarily as an SNR offset, confirming that coarse phase control limits the coherent combining capability of the RIS. Nevertheless, the general scaling trend with respect to N is preserved. Increasing the number of elements still yields considerable SNR gains. This illustrates that large-scale RIS deployments are inherently robust against phase noise, as the aggregation of many independently distorted phase contributions partially compensates for quantization errors. The results collectively demonstrate that phase noise reduces the effective RIS gain but does not alter the fundamental performance scaling, thereby

reinforcing the practicality of RIS designs even when high-precision phase control is not feasible.

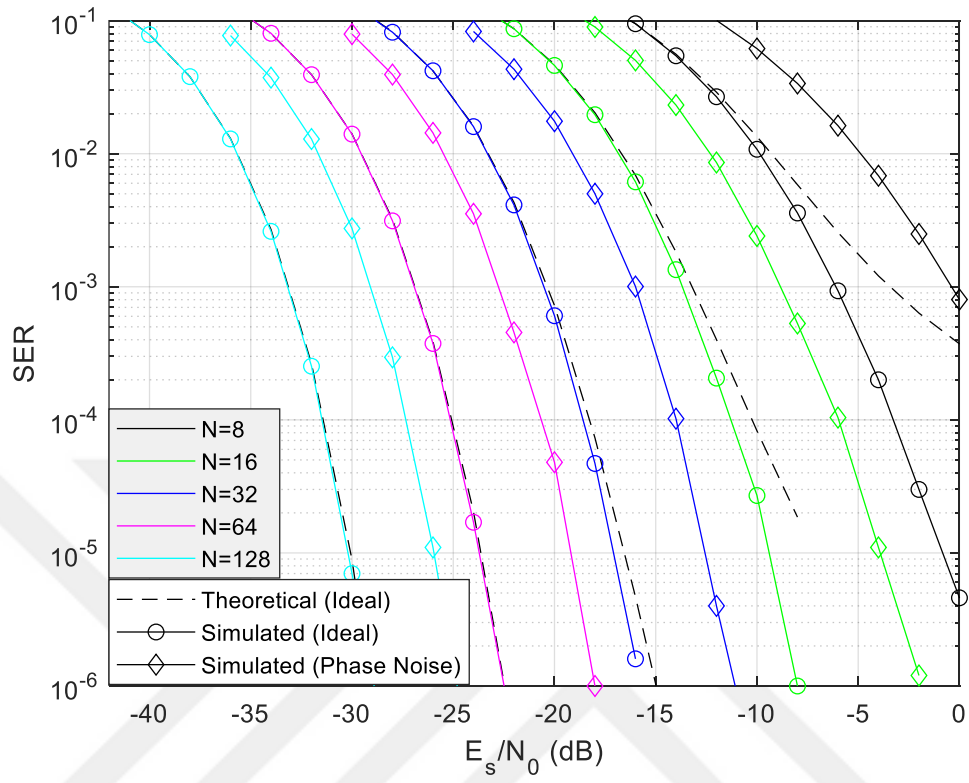


Figure 2.10: SER performance comparison of RIS with various number of reflecting surfaces in the absence and the presence of phase noise.

3. PERFORMANCE OF RIS ASSISTED MIMO SYSTEMS

Consider a MIMO system with a transmitter, an RIS device and a receiver. Transmitter and receiver have N_t and N_r antennas, respectively. RIS has N meta surfaces and it only modifies the phase of the incident signal. In other words, we employ a passive RIS. Channel from j^{th} transmit antenna to i^{th} meta surface of RIS is h_{ji} and channel from the i^{th} meta surface to k^{th} receive antenna is h_{ik} . Both links are i.i.d. Rayleigh channels with unit-scale parameter.

Here, JTRAS is employed over the transmitter-RIS-receiver link so training period is realized by activating single antenna pair at a time and receiver decides in favor of the strongest one. For each (j, k) antenna pair, phase shifts of RIS are determined as $\varphi_{jik} = -Q^{(1)}(\theta_{ji} + \psi_{ik})$, where $Q^{(1)}(\cdot)$ denotes 1 bit quantization operator. After quantization, ϕ_{jik} remained, which is caused by imperfect phase shift. Based on the effective channels, optimal antenna pair (j^*, k^*) is selected according to

$$(j^*, k^*) = \arg \max_{j, k} \left| \sum_{i=1}^N |h_{ji}| |g_{ik}| e^{j\phi_{jik}} \right|^2. \quad (3.1)$$

After selection, only the optimal pair remains active. Based on this, the total received signal including the direct link is

$$r = \left(\sum_{i=1}^N |h_{j^*i}| |g_{ik^*}| e^{j\phi_{j^*ik^*}} + \rho |h_d| e^{j\phi_d} \right) s + n, \quad (3.2)$$

where s is the unit power transmitted symbol and $n \sim \mathcal{N}(0, N_0/2)$ denotes AWGN and $h_d = |h_d| e^{j\phi_d}$ is the channel coefficient of direct link. $\rho = 0$ and 1 for no direct link and direct link, respectively. Without loss of generality, $\phi_{j^*ik^*} = \phi_i$ since only single antenna pair remains active after the selection. In this chapter, we first examine the system without and with phase noise and then finally analyze it with phase noise and direct link.

3.1. Performance Analysis of RIS-Assisted JTRAS System without Phase Noise

In this subsection, (3.2) is valid such that $\phi_i = 0$ and $\rho = 0$. Starting off by defining the OP as

$$P_{out} = Pr(\gamma < \gamma_{th}) = F_\gamma(\gamma_{th}). \quad (3.3)$$

Denoting $|h_i||g_i| = x_i$, we need to find PDF of x_i .

$$\begin{aligned} f_{x_i}(x) &= \int_0^\infty \frac{1}{y} f_{h_i}(y) f_{g_i}\left(\frac{x}{y}\right) dy, \quad x \geq 0 \\ &= \int_0^\infty \frac{x}{y\sigma^4} \exp\left(-\frac{y^2}{2\sigma^2} - \frac{x^2}{2\sigma^2 y^2}\right) dy. \end{aligned} \quad (3.4)$$

Here, applying $u = y^2$ and using [19, eq. (3.471.9)] in (3.4) we obtain $f_{x_i}(x) = x K_0(x)$, $x \geq 0$ where $K_\nu(\cdot)$ is the modified Bessel function of second kind with order of ν [19, eq. (8.432)]. For purpose of keeping the mathematic as simple and accurate as possible, we employ Gamma approximation. Authors of [2] show that Gamma approximation outperforms the CLT approach in high SNR regimes and for moderate number of surfaces. Moving on with this approach, first two moments must be matched to that of the Gamma distribution. Here, mean and the second moment of approximated distribution can easily be found as $E[x_i] = \pi/2$ and $E[x_i^2] = 4$ respectively. Expressing MGF and PDF of Gamma distribution as

$$\mathcal{M}_{x_i}(s) = \left(\frac{\beta}{\beta + s}\right)^\alpha \quad (3.5)$$

and

$$f_{x_i}(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, \quad x > 0. \quad (3.6)$$

Here, $\alpha = E[x_i]^2 / \text{var}(x_i) = \frac{\pi^2}{16-\pi^2}$ and $\beta = E[x_i] / \text{var}(x_i) = \frac{2\pi}{16-\pi^2}$. To clarify the received signal of single antenna, r can be written as

$$r = \left(\sum_{i=1}^N x_i \right) s + n = x_{tot} s + n . \quad (3.7)$$

Hence, MGF of x_{tot} is

$$\mathcal{M}_{x_{tot}}(s) = E[e^{sx_{tot}}] = E[e^{s \sum_i x_i}] = E[e^{sx_i}]^N = \mathcal{M}_{x_i}(s)^N \quad (3.8)$$

since x_i s are i.i.d.. MGF of x_{tot} is then

$$\mathcal{M}_{x_{tot}}(s) = \left(\frac{\beta}{\beta + s} \right)^{\alpha N} . \quad (3.9)$$

PDF and CDF is expressed as

$$f_{x_{tot}}(x) = \frac{\beta^{\alpha N}}{\Gamma(\alpha N)} x^{\alpha N - 1} e^{-\beta x} \quad (3.10)$$

and

$$F_{x_{tot}}(x) = \frac{1}{\Gamma(\alpha N)} \Upsilon(\alpha N, \beta x) \quad (3.11)$$

respectively. Here, $\Upsilon(\cdot, \cdot)$ is lower incomplete Gamma function [19, eq. (8.350)]. So, SNR of RIS assisted SISO system is $\gamma = |x_{tot}|^2 \frac{E_s}{N_0} = x_{tot}^2 \bar{\gamma}$. By the use of transformation of variables, PDF of SISO system can be written as

$$f_Y(\gamma) = \frac{\beta^{\alpha N}}{2\Gamma(\alpha N)\bar{\gamma}^{\alpha N/2}} \gamma^{\frac{\alpha N - 2}{2}} e^{-\beta\sqrt{\gamma/\bar{\gamma}}}, \gamma \geq 0 . \quad (3.12)$$

Hence, CDF is clearly

$$F_Y(\gamma) = \frac{1}{\Gamma(\alpha N)} \Upsilon(\alpha N, \beta\sqrt{\gamma/\bar{\gamma}}) . \quad (3.13)$$

Hereafter, CDF of MIMO system must be found. As previously discussed, the JTRAS scheme selects the transmit-receive antenna pair that yields the highest SNR, and the communication is subsequently performed over this optimum link. From an OP perspective, recalling that outage occurs when the instantaneous SNR falls below a specific threshold, the system is considered to be in outage only if all single antenna pair SNRs remain below this threshold. In other words, the maximum among these SNR values must also be smaller than the threshold for the outage event to occur. Accordingly, by employing higher-order statistics, the CDF of the RIS-assisted JTRAS scheme can be expressed as follows [13],

$$F_{Y_{jtras}}(\gamma) = \left(\frac{1}{\Gamma(\alpha N)} \right)^{N_t N_r} \gamma(\alpha N, \beta \sqrt{\gamma/\bar{\gamma}})^{N_t N_r}. \quad (3.14)$$

OP is acquired by substituting γ_{th} into (3.14). Continuing the analysis with SER, we chose to employ CDF method. As briefly recalled here, for any modulation scheme whose AWGN channel SER rate is expressed as $aQ(\sqrt{2b\bar{\gamma}})$, SER of the system can be obtained using (2.7) with CDF. This formula holds for the modulation type with different parameters given in Table 2.2. So, SER of the system can be expressed as

$$\begin{aligned} P_s &= \frac{a\sqrt{b}}{2\sqrt{\pi}} \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} \left(\frac{1}{\Gamma(\alpha N)} \gamma(\alpha N, \varsigma \sqrt{\gamma}) \right)^{N_t N_r} d\gamma \\ &= \frac{a\sqrt{b}}{\sqrt{\pi}} \int_0^\infty e^{-bt^2} \underbrace{\left(\frac{1}{\Gamma(\alpha N)} \gamma(\alpha N, \varsigma t) \right)^{N_t N_r}}_I dt \end{aligned} \quad (3.15)$$

Here, $\varsigma = \beta/\sqrt{\bar{\gamma}}$ and the transition is done by applying $\gamma = t^2$. To the best of the authors' knowledge, the integral of this expression does not admit a closed-form solution. Therefore, we employ a series expansion (check Appendix-A), i.e.,

$$\frac{\gamma(\alpha N, \varsigma t)}{\Gamma(\alpha N)} = (\varsigma t)^{\alpha N} e^{-\varsigma t} \sum_{m=0}^{\infty} \frac{(\varsigma t)^m}{\Gamma(\alpha N + m + 1)}. \quad (3.16)$$

Here, I can be expressed as

$$I = e^{-\zeta t N_t N_r} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \cdots \sum_{m_{N_t N_r}=0}^{\infty} \prod_{j=1}^{N_t N_r} \frac{(\zeta t)^{\alpha N + m_j}}{\Gamma(\alpha N + m_j + 1)}. \quad (3.17)$$

This expression is valid for a finite series truncated terms at K_{max} . Nevertheless, the computational efficiency is $\mathcal{O}((K_{max} + 1)^{N_t N_r})$. Therefore, the calculation difficulty grows greatly as number of antennas increase. Here, I can be re-expressed following the proof given in Appendix B, as

$$I = e^{-\zeta t N_t N_r} \sum_{k=0}^{\infty} \zeta_k t^{\alpha N N_t N_r + k} \quad (3.18)$$

Here, $k = m_1 + m_2 + \cdots + m_{N_t N_r}$ and the coefficient is

$$\zeta_k = \zeta^{\alpha N N_t N_r} \sum_{m_1 + \cdots + m_{N_t N_r} = k} \prod_{j=1}^{N_t N_r} \frac{\zeta^{m_j}}{\Gamma(\alpha N + m_j + 1)}. \quad (3.19)$$

With this new form, number of summations is reduced to a single one. In fact, the coefficient ζ_k can be calculated once and reused when needed via dynamic programming. Therefore, we avoid any redundant summations. That being said, the new form admits a complexity of $\mathcal{O}(K_{max} + 1) \approx \mathcal{O}(K_{max})$. Here, substituting I into (3.15) SER is written as

$$P_s = \frac{a\sqrt{b}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \zeta_k \int_0^{\infty} e^{-bt^2 - \zeta N_t N_r t} t^{\alpha N N_t N_r} dt. \quad (3.20)$$

To achieve a closed form, we employ [19, eq. (3.462.1)] and express SER as

$$P_s = \frac{a\sqrt{b}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \zeta_k (2b)^{-v/2} \Gamma(v) e^{\eta^2/(8b)} D_{-v}\left(\frac{\eta}{\sqrt{2b}}\right), \quad (3.21)$$

where $\eta = \zeta N_t N_r$, $v = N N_t N_r \alpha + k + 1$ and $D_p(z)$ is the parabolic cylinder function with integral representation of [19, eq. (9.241.2)]. Note that this function can be calculated via well-known software MATHEMATICA.

One of the key characteristics of RIS-assisted systems is their ability to achieve high diversity order (DO). Incorporating MIMO into such architectures further enhances the attainable diversity gain. Therefore, it becomes essential to analyze this combined effect as well. So, the DO is defined as

$$DO = \lim_{\bar{\gamma} \rightarrow \infty} -\frac{\log P_{\text{out}}}{\log \bar{\gamma}}. \quad (3.22)$$

Recalling the OP in terms of CDF as

$$P_{\text{out}}(\gamma_{\text{th}}) = F_{x_{\text{tot}}} \left(\sqrt{\frac{\gamma_{\text{th}}}{\bar{\gamma}}} \right), \quad (3.23)$$

we aim to find the behavior of the CDF for the small arguments. The reason for that is as $\bar{\gamma}$ goes infinity, argument of CDF tends to zero. Hence, we first derive the asymptotic CDF of SISO system for small arguments. Here, let us rewrite the CDF of x_i as

$$F_{x_i}(t) = P(x_i < t) = \int_0^t u K_0(u) du. \quad (3.24)$$

Employing the approximation $K_0(u) \sim -\ln(u)$ for small values of u [19, eq. (10.30.3)], we get the asymptotic CDF for x_i as

$$F_{x_i}(t) \approx \frac{t^2}{2} [2 - \ln(t)] \approx t^2. \quad (3.25)$$

Hereafter, MGF of x_i can easily be derived as $\mathcal{M}_{x_i}(-s) = 2/s^2$. Since $x_{\text{tot}} = \sum_{i=1}^N x_i$,

MGF of x_{tot} is

$$\mathcal{M}_{x_{\text{tot}}}(-s) = \left(\mathcal{M}_{x_i}(-s) \right)^N = (2/s^2)^N. \quad (3.26)$$

Applying [19, eq. (17.13.2)], PDF of x_{tot} is found to be

$$f_{x_{\text{tot}}}(t) = \frac{2^N}{\Gamma(2N)} t^{2N-1}. \quad (3.27)$$

Finally, the asymptotic CDF can be found as

$$F_{x_{\text{tot}}}(t) = \frac{2^N}{\Gamma(2N + 1)} t^{2N}. \quad (3.28)$$

Substituting $t = \sqrt{\gamma_{th}/\bar{\gamma}}$ and using (3.22) here, we get $DO_{\text{single}} = N$ for SISO system. This outcome is reasonable since multiple studies summarized in Table 1 in [3] reports the same result. Since asymptotic CDF of JTRAS system is obtained by factoring the power of SISO CDF by $N_t N_r$, it is clear to see that, DO of JTRAS system is $DO_{\text{jtras}} = NN_t N_r$.

3.2. Performance Analysis of RIS-Assisted JTRAS System in the Presence of Phase noise

In this part, we examine the system with phase noise. (3.2) is valid such that ϕ_i is uniformly distributed remaining phase component after the 1 bit phase quantization. Here, we define $e^{j\phi_i} := \cos(\phi_i)$ for one dimensional modulation types (i.e., BPSK and M-ary pulse amplitude modulation (M-PAM)) as in [23]. As a starting point, we derive the CDF for SISO configuration, i.e., ($j = k = 1$) in (3.2). Here, the PDF of cascaded channel with phase noise is considered, where $z_i = x_i \cos(\phi_i)$, $x_i = |h_{ji}| |g_{ik}|$ and the PDF of x_i is given by $f_{x_i}(x) = xK_0(x)$. It follows from [8] that under 1-bit quantization, ϕ_i is uniformly distributed over the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$. So, denoting $y_i = \cos(\phi_i)$, PDF of y_i can be easily derived as $f_{y_i}(y) = \frac{2}{\pi\sqrt{1-y^2}}$. Applying the transformation of random variables, we obtain

$$f_{z_i}(z) = \int_z^\infty K_0(x) \frac{2}{\pi\sqrt{1-(z/x)^2}} dx. \quad (3.29)$$

By the change of variable $u = \sqrt{x^2 - z^2}$, it becomes

$$f_{z_i}(z) = \frac{2}{\pi} \int_0^\infty K_0(\sqrt{u^2 + z^2}) du. \quad (3.30)$$

Applying [19, eq. (6.596.3)] and [19, eq. (10.39.2)], we obtain

$$f_{z_i}(z) = \sqrt{\frac{2z}{\pi}} K_{0.5}(z) = e^{-z}. \quad (3.31)$$

MGF of (3.31) can be easily derived as $\mathcal{M}_{z_i}(s) = \frac{1}{1-s}$. Let $z_{tot} = \sum_{i=1}^N z_i$. Since random variables z_i are i.i.d., MGF of z_{tot} becomes $\mathcal{M}_{z_{tot}}(s) = \left(\frac{1}{1-s}\right)^N$. Therefore, its PDF is $f_{z_{tot}}(z) = \frac{1}{\Gamma(N)} z^{N-1} e^{-z}$, $z \geq 0$. SNR can be written as $\gamma = |z_{tot}|^2 \frac{E_S}{N_0} = z_{tot}^2 \bar{\gamma}$. Employing the transformation of random variables, we express PDF of SNR as

$$f_\gamma(\gamma) = \frac{1}{2\Gamma(N)\bar{\gamma}^{N/2}} \gamma^{(N-2)/2} e^{-\sqrt{\gamma/\bar{\gamma}}}, \quad \gamma \geq 0. \quad (3.32)$$

Using the definition of CDF, it can be found as $F_\gamma(\gamma) = \frac{1}{\Gamma(N)} \Upsilon(N, \sqrt{\gamma/\bar{\gamma}})$. Applying [19, eq. (8.352)] here, the CDF of SNR can be written as

$$F_\gamma(\gamma) = 1 - e^{-\sqrt{\gamma/\bar{\gamma}}} \sum_{k=0}^{N-1} \frac{(\gamma/\bar{\gamma})^{k/2}}{k!}. \quad (3.33)$$

Recalling the derivation of OP of JTRAS system by the help of higher order statistics, CDF of JTRAS system is given by $F_{Y_{jtras}}(\gamma) = [F_\gamma(\gamma)]^{N_t N_r}$. Then, using the binomial expansion, CDF is obtained as

$$F_{Y_{jtras}}(\gamma) = \sum_{l=0}^{N_t N_r} \binom{N_t N_r}{l} (-1)^l e^{-l\sqrt{\gamma/\bar{\gamma}}} \left(\sum_{k=0}^{N-1} \frac{(\gamma/\bar{\gamma})^{k/2}}{k!} \right)^l. \quad (3.34)$$

Finally, CDF of SNR of JTRAS system is obtained as

$$F_{Y_{jtras}}(\gamma) = \sum_{l=0}^{N_t N_r} \sum_{m=0}^{l(N-1)} \binom{N_t N_r}{l} (-1)^l e^{-l\sqrt{\gamma/\bar{\gamma}}} \mu_m(l, N) (\gamma/\bar{\gamma})^{m/2} \quad (3.35)$$

where $\mu_m(l, N)$ is the multinomial coefficients defined in [19, eq. (0.314)]. Here, OP is $P_{\text{out}}(\gamma_{\text{th}}) = F_{Y_{jtras}}(\gamma_{\text{th}})$. Moving on with SER analysis, using the CDF method (2.7) SER can be expressed as

$$P_s = \frac{a\sqrt{b}}{2\sqrt{\pi}} \sum_{l=0}^{N_t N_r} \sum_{m=0}^{l(N-1)} \binom{N_t N_r}{l} (-1)^l \mu_m(l, N) \times \int_0^\infty \frac{e^{-b\gamma}}{\sqrt{\gamma}} e^{-l\sqrt{\gamma/\bar{\gamma}}} (\gamma/\bar{\gamma})^{m/2} d\gamma. \quad (3.36)$$

Substituting $t = \sqrt{\gamma}$ and applying [19, eq. (3.462.1)], we can obtain the closed form of SER as

$$P_s = \frac{a\sqrt{b}}{\sqrt{\pi}} \sum_{l=0}^{N_t N_r} \sum_{m=0}^{l(N-1)} \binom{N_t N_r}{l} (-1)^l \mu_m(l, N) \times \Gamma(m+1) \bar{\gamma}^{-m/2} (2b)^{-(m+1)/2} e^{c^2/(8b)} D_{-m-1} \left(\frac{c}{\sqrt{2b}} \right), \quad (3.37)$$

where $c = l/\sqrt{\bar{\gamma}}$. It is important to underline that the analysis is carried out under the one-dimensional modulation assumption. So that, it is valid for BPSK and M-PAM with relevant modulation parameters given before in Table 2.1.

Continuing with DO analysis, its definition is given before in (3.22). CDF of SISO RIS system in the presence of phase noise was derived as $P_{\text{out}} = F_Y(\gamma_{\text{th}}) = \frac{1}{\Gamma(N)} \Upsilon(N, \sqrt{\gamma_{\text{th}}/\bar{\gamma}})$. As $\bar{\gamma} \rightarrow \infty$ the second argument of incomplete Gamma function goes to zero. Therefore, by taking the first few terms of series expansion from [19, eq. (8.354.1)] into account, we obtain asymptotic expression of OP as

$$P_{\text{out}}^{\text{asym}} = \frac{A}{\bar{\gamma}^{N/2}} \left(1 - \frac{B}{\sqrt{\bar{\gamma}}} + \frac{C}{\bar{\gamma}} \right), \quad (3.38)$$

where $A = \frac{\gamma_{\text{th}}^{N/2}}{\Gamma(N+1)}$, $B = \frac{N\sqrt{\gamma_{\text{th}}}}{N+1}$, $C = \frac{N\gamma_{\text{th}}}{2(N+2)}$. Substituting (3.38) into (3.22) yields

$$DO = -\frac{\log(A)}{\log(\bar{\gamma})} - \frac{(-N/2)\log(\bar{\gamma})}{\log(\bar{\gamma})} - \frac{\log(1 - B\bar{\gamma}^{-1/2} + C\bar{\gamma}^{-1})}{\log(\bar{\gamma})}. \quad (3.39)$$

First and third terms both tend to zero as $\bar{\gamma} \rightarrow \infty$ and DO of SISO system is found as $DO_{\text{single}} = N/2$. Clearly, asymptotic OP of JTRAS system can be expressed as

$$P_{\text{out},j\text{tras}}^{\text{asym}} = (P_{\text{out}}^{\text{asym}})^{N_t N_r}. \quad (3.40)$$

The power term only scales the DO_{single} as DO is defined by logarithms. Finally, DO of RIS assisted JTRAS system with phase noise is $DO_{j\text{tras}} = \frac{NN_t N_r}{2}$.

The DO of SISO system is in perfect agreement with the upper bound derived in [8], where the DO for 1-bit quantization is shown to satisfy $DO < \frac{N+1}{2}$. Furthermore, authors of [13] demonstrated that, under Rayleigh fading, JTRAS system admits a DO of $N_t N_r$, equal to the product of numbers of transmit and receive antennas. Hence, the DO of RIS assisted JTRAS system is justified, since it is equal to $N_t N_r$ times the DO_{single} . Considering the previous subsection, it can be concluded that 1-bit phase quantization noise reduces the DO by half.

3.3. Performance Analysis of RIS-Assisted JTRAS System with Direct Link in the Presence of Phase Noise

Considering the system model in (3.2) with $\rho = 1$, we now analyze the system with direct link. JTRAS technique is applied solely to RIS link, rather than overall system. Denoting SNR of direct link by γ_d and considering MRC on the links at the receiver, the overall SNR can be expressed as $\gamma_{\text{tot}} = \gamma_{j\text{tras}} + \gamma_d$.

Assuming the channel coefficient of the direct link is $h_d = |h_d|e^{j\phi_d}$, $|h_d|$ follows Rayleigh distribution, while $e^{j\phi_d}$ represents the residual phase component after the one-bit quantized compensation is employed at the receiver. Here, ϕ_d is uniformly distributed over the interval $[-\frac{\pi}{2}, \frac{\pi}{2}]$ [8]. Definition of $e^{j\phi_i} := \cos(\phi_i)$ based on the fact that we employ one dimensional modulations still holds [23]. Since direct link is

clearly defined, we can derive the statistical expressions of SNR. In alignment with previous subsection, denoting $y_d = \cos(\phi_d)$, the PDF of y_d is $f_{y_d}(y) = \frac{2}{\pi\sqrt{1-y^2}}$. Here, let us denote $|h_d| = x_d$ and $z_d = x_d y_d$. We can easily obtain PDF of z_d as $f_{z_d}(z) = \frac{2z}{\pi} \int_0^1 e^{-z^2/(2y^2)} \frac{1}{y^2\sqrt{1-y^2}} dy$. Applying a few variable changes and using the properties of Gaussian PDF integral we obtain $f_{z_d}(z) = \sqrt{\frac{2}{\pi}} e^{-z^2/2}$. SNR of direct link is defined as $\gamma_d = |z_d|^2 \frac{Es}{N_0} = z_d^2 \bar{\gamma}$. By the transform of random variables, PDF of γ_d can easily be defined as $f_{\gamma_d}(\gamma) = \frac{1}{\sqrt{2\pi\bar{\gamma}}} \gamma^{-1/2} e^{-\gamma/2\bar{\gamma}}$.

Clearly, MGF of γ_d is $\mathcal{M}_{\gamma_d}(-s) = \sqrt{\frac{1}{1+2\bar{\gamma}s}}$. Since $\gamma_{tot} = \gamma_{jtras} + \gamma_d$, and the corresponding MGF is expressed as $\mathcal{M}_{\gamma_{tot}}(-s) = \mathcal{M}_{\gamma_{jtras}}(-s) \mathcal{M}_{\gamma_d}(-s)$. To find the MGF expression of γ_{jtras} , we use the CDF (3.35) and a Laplace transform property as $\mathcal{M}_{\gamma_{jtras}}(-s) = s \int_0^\infty e^{-s\gamma} F_{\gamma_{jtras}}(\gamma) d\gamma$. After applying $x = \sqrt{\gamma/\bar{\gamma}}$ and [19, eq. (3.462.1)], MGF of RIS assisted link is obtained as

$$\mathcal{M}_{\gamma_{jtras}}(-s) = \underbrace{\sum_{l=0}^{N_t N_r} \sum_{m=0}^{l(N-1)} \binom{N_t N_r}{l} (-1)^l \mu_m(l, N)}_{\mathcal{C}_{l,m}} \times (2\bar{\gamma})^{-m/2} \Gamma(m+2) s^{-m/2} e^{l^2/8s\bar{\gamma}} D_{-m-2} \left(\frac{l}{\sqrt{2s\bar{\gamma}}} \right). \quad (3.41)$$

Hereafter, we can obtain MGF of total SNR by multiplying the MGF of direct link and RIS link. Then we apply [19, eq. (9.240)] to $\mathcal{M}_{\gamma_{tot}}(-s)$ and get the final form of it as

$$\mathcal{M}_{\gamma_{tot}}(-s) = \mathcal{C}_{l,m} (s\bar{\gamma})^{-m/2} \Gamma(m+2) \frac{2 s^{-m/2}}{\sqrt{1+2\bar{\gamma}s}} \times \left[\frac{\sqrt{\pi}}{\Gamma\left(\frac{m+3}{2}\right)} {}_1F_1\left(\frac{m+2}{2}; \frac{1}{2}; \frac{l^2}{4s\bar{\gamma}}\right) - \frac{\sqrt{\pi} l}{\sqrt{s\bar{\gamma}} \Gamma\left(\frac{m+2}{2}\right)} {}_1F_1\left(\frac{m+3}{2}; \frac{3}{2}; \frac{l^2}{4s\bar{\gamma}}\right) \right] \quad (3.42)$$

where ${}_1F_1(\cdot; \cdot; \cdot)$ is the confluent hypergeometric function [19, eq. (3.462.1)]. Based on the derivation of Appendix C, we can obtain the CDF of total SNR with inverse Laplace transform,

$$\mathcal{L}^{-1} \left[\frac{\mathcal{M}_{\gamma_{tot}}(-s)}{s} \right] = F_{\gamma_{tot}}(t). \quad (3.43)$$

The notation in [24] is given below as

$$S_{C:D; D'}^{A:B; B'} \left(\begin{matrix} x \\ y \end{matrix} \right) = S_{C:D; D'}^{A:B; B'} \left(\begin{matrix} [(a): \theta, \Phi] & [(b): \Psi] & [(b'): \Psi'] \\ [(c): \delta, \epsilon] & [(d): \eta] & [(d'): \eta'] \end{matrix} \middle| x, y \right) =$$

$$\sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\prod_{j=1}^A \Gamma(a_j + m\theta_j + n\Phi_j) \prod_{j=1}^B \Gamma(b_j + m\psi_j) \prod_{j=1}^{B'} \Gamma(b'_j + n\psi'_j)}{\prod_{j=1}^C \Gamma(c_j + m\delta_j + n\epsilon_j) \prod_{j=1}^D \Gamma(d_j + m\eta_j) \prod_{j=1}^{D'} \Gamma(d'_j + n\eta'_j)} \frac{x^m y^n}{m! n!} \quad (3.44)$$

Following this notation, CDF of total SNR is expressed as

$$F_{\gamma_{tot}}(t) = \underbrace{\mathcal{C}_{\ell, m} 2^{-m-1} \bar{\gamma}^{-m/2} \Gamma(m+2) \sqrt{\pi}}_{\hat{\mathcal{C}}_{\ell, m}} \times$$

$$\left[\underbrace{\frac{\Gamma(\frac{1}{2}) e^{-\lambda t} t^{\frac{m+1}{2}}}{\sqrt{2\bar{\gamma}} \Gamma(\frac{m+3}{2}) \Gamma(\frac{m+2}{2})} S_{1:0;2}^{1:0;1} \left(\begin{matrix} [(1-\alpha_1): 1, 1] ; & - & ; [(p_1): 1] \\ [(\frac{3}{2}-\alpha_1): 1, 1] ; & - & ; [(q_1): 1], [(1-\alpha_1): 1] \end{matrix} \middle| x, y \right)}_{T_1(t)} \right. \quad (3.45)$$

$$\left. - \underbrace{\frac{1 \Gamma(\frac{3}{2}) e^{-\lambda t} t^{\frac{m+2}{2}}}{\bar{\gamma} \sqrt{2} \Gamma(\frac{m+3}{2}) \Gamma(\frac{m+2}{2})} S_{1:0;2}^{1:0;1} \left(\begin{matrix} [(1-\alpha_2): 1, 1] ; & - & ; [(p_2): 1] \\ [(\frac{3}{2}-\alpha_2): 1, 1] ; & - & ; [(q_2): 1], [(1-\alpha_2): 1] \end{matrix} \middle| x, y \right)}_{T_1(t)} \right]$$

Here, CDF is obtained by applying the inverse Laplace transform in Appendix C to MGF in (3.42). The parameters associated with the two terms separated by minus sign in the CDF are given by Table 3.1 as follows

Table 3.1: CDF parameters.

1 st	2 nd
$\alpha_1 = -\frac{m}{2}$	$\alpha_2 = -\frac{m+1}{2}$

Table 3.1: Continued.

$p_1 = \frac{m+2}{2}$	$p_2 = \frac{m+3}{2}$
$q_1 = \frac{1}{2}$	$q_2 = \frac{3}{2}$

Furthermore, $x = \beta t$ and $y = \lambda t$, where $\beta = \frac{l^2}{2\bar{y}}$ and $\lambda = \frac{1}{2\bar{y}}$. Finally, OP is expressed as $P_{out}(\gamma_{th}) = F_{\gamma_{tot}}(\gamma_{th})$. Proceeding with SER analysis, as mentioned before, SER is derived employing (2.7) based on CDF of total SNR. This integral is applicable to $F_{\gamma_{tot}}(t)$ when it is divided into two parts. For $r = 1, 2$ let us denote

$$P_{s_r} = \frac{a\sqrt{b}}{2\sqrt{\pi}} \tilde{C}_{l,m} \int_0^\infty \frac{e^{-bt}}{\sqrt{t}} T_r(t) dt. \quad (3.46)$$

So, SER can be expressed as $P_s = P_{s_1} - P_{s_2}$. The Srivastava-Daoust type double series admits a Fox-H function representation through the use of Mellin transform technique. By the Gamma function form, the Mellin-Barnes contour integral can be constructed. With the help of [19, eq. (3.478.1)], which is applied to (3.45), P_{s_r} are derived, from which the corresponding Fox-H function representations directly follow as given below

$$P_{s_r} = \frac{a\sqrt{b}}{2\sqrt{2}} \tilde{C}_{l,m} \frac{1}{\Gamma\left(\frac{m+3}{2}\right) \Gamma\left(\frac{m+2}{2}\right) \sqrt{\bar{y}} (b+\lambda)^{\frac{m+2}{2}} \delta_r} \times \quad (3.47)$$

$$H_{3,0:3,1;1,0}^{0,2;1,1;0,1} \left[\begin{matrix} (-\xi) \\ (-\eta) \end{matrix} \middle| \begin{matrix} (\alpha_r; 1,1), (\alpha_r; 1,1), \left(\frac{3}{2}\alpha_r; 1,1\right) ; & (1,1), (q_r, 1), (1-\alpha_r, 1) ; & (1,1) \\ - & (p_r, 1) & - \end{matrix} \right].$$

Here, $\delta_1 = 1$ and $\delta_2 = \frac{1}{2\sqrt{\bar{y}}(b+\lambda)}$. In conclusion, closed form SER expression P_s is obtained. Fox-H function can be efficiently calculated by standard software tools such as MATHEMATICA or MAPLE.

Finally, to determine the DO, we analyze the asymptotic OP of the system. DO is defined as in (3.22). For the purpose of deriving closed form expressions, we employ an alternative asymptotic CDF of SISO system. It can be written as

$$F_Y(u) = 1 - e^{-u} \sum_{k=0}^{N-1} \frac{u^k}{k!} \quad (3.48)$$

where $u = \sqrt{\frac{\gamma}{\bar{\gamma}}}$. This summation resembles the Taylor series expansion of e^u . Here, e^u can be expressed as

$$e^u = \sum_{k=0}^{N-1} \frac{u^k}{k!} + R_N(u), \quad R_N(u) = \sum_{m=0}^{\infty} \frac{u^{N+m}}{(N+m)!}. \quad (3.49)$$

Using the definition of $R_N(u)$, CDF can be expressed as

$$F_Y(u) = e^{-u} \left(e^u - \sum_{k=0}^{N-1} \frac{u^k}{k!} \right) = e^{-u} R_N(u). \quad (3.50)$$

Since we investigate the high SNR behavior of the system, asymptotic expression as $\bar{\gamma} \rightarrow \infty$ must be found. As $\bar{\gamma} \rightarrow \infty$, it follows that $u \rightarrow 0$, $e^{-u} = 1 + o(u)$ and $R_N(u) = \frac{u^N}{N!} + o(u^{N+1})$. Then, $F_Y(u) = e^{-u} R_N(u) = \frac{u^N}{N!} + o(u^{N+1})$. Hence, by substituting $u = \sqrt{\frac{\gamma}{\bar{\gamma}}}$, we obtain $F_Y^{asym}(\gamma) \approx (\gamma/\bar{\gamma})^{N/2}/N!$. Clearly, the asymptotic CDF of JTRAS becomes

$$F_{jtras}^{asym}(\gamma) = \frac{(\gamma/\bar{\gamma})^{(NN_t N_r/2)}}{(N!)^{N_t N_r}}. \quad (3.51)$$

Using the integral definition of the MGF given previously and applying [19, eq. (3.381.4)], we get

$$\mathcal{M}_{Y_{jtras}}^{asym}(-s) = \frac{\Gamma\left(\frac{NN_t N_r}{2} + 1\right)}{\bar{\gamma}^{NN_t N_r/2} (N!)^{N_t N_r} s^{\left(\frac{NN_t N_r}{2} + 1\right)}}. \quad (3.52)$$

The asymptotic MGF of the direct link can be written as

$$\mathcal{M}_{\gamma_d}^{\text{asym}}(-s) = \sqrt{\frac{1}{1+2\bar{\gamma}s}} \approx \frac{1}{\sqrt{2\bar{\gamma}s}}. \quad (3.53)$$

Then, asymptotic MGF of the total system is

$$\mathcal{M}_{\gamma_{\text{tot}}}^{\text{asym}}(-s) = \mathcal{M}_{\gamma_{\text{jtras}}}^{\text{asym}}(-s) \mathcal{M}_{\gamma_d}^{\text{asym}}(-s). \quad (3.54)$$

Finally, asymptotic CDF of the system SNR can be derived by the use of $\mathcal{L}^{-1}\left\{\frac{\mathcal{M}_{\gamma_{\text{tot}}}^{\text{asym}}(-s)}{s}\right\}$ and can be written as

$$F_{\gamma_{\text{tot}}}^{\text{asym}}(u) = \frac{\Gamma\left(\frac{NN_tN_r}{2} + 1\right) u^{\frac{NN_tN_r+1}{2}}}{\bar{\gamma}^{\frac{NN_tN_r+1}{2}} (N!)^{N_tN_r} \sqrt{2} \Gamma\left(\frac{NN_tN_r}{2} + 3\right)}. \quad (3.55)$$

Here, $P_{\text{out}}^{\text{asym}} = F_{\gamma_{\text{tot}}}^{\text{asym}}(\gamma_{\text{th}})$ and using the definition in, DO of the system is found to be $\frac{NN_tN_r+1}{2}$. This represents an increase of 1/2 compared to JTRAS system with no direct link, which is reasonable since there is an independent direct link with phase noise.

3.4. Performance of RIS Assisted TAS-MRC Systems in the Presence of Phase Noise

Considering a MIMO system with N_t transmit and N_r receive antennas and an RIS with N meta surfaces, we employ TAS-MRC. The technique is discussed in subsection 0. Let us denote the channel coefficient from j^{th} transmit antenna to i^{th} meta surface as h_{ji} and that from i^{th} meta surface to k^{th} receive antenna as g_{ik} . Here, $h_{ji} = |h_{ji}|e^{\theta_{ji}}$ and $g_{ik} = |g_{ik}|e^{\varphi_{ik}}$. Therefore, cascaded channel from each meta surface is $h_{ji} g_{ik} = |h_{ji}|e^{\theta_{ji}}|g_{ik}|e^{\varphi_{ik}}$ and after phase shifting effective channel of RIS for single antenna system is expressed as

$$\sum_{i=1}^N |h_{ji}| |g_{ik}| e^{\theta_{ji} + \varphi_{ik} + \psi_{jik}} = \sum_{i=1}^N |h_{ji}| |g_{ik}| e^{\phi_{jik}}. \quad (3.56)$$

Here, $\psi_{jik} = -\mathcal{Q}^{(n)}(\theta_{ji} + \phi_{ik})$ represents the phase shift of RIS with n bits of quantization where $\mathcal{Q}^{(n)}(\cdot)$ is the n bit quantization operator and ϕ_{jik} is the remaining phase. It is widely accepted in the literature that under Rayleigh fading, phase of the channel follows uniform distribution in the interval of $[0, 2\pi]$. After n bit quantized phase shifting, ϕ_{jik} becomes uniform in the interval of $[-\frac{\pi}{2^n}, \frac{\pi}{2^n}]$ [8].

It should also be noted that, under practical operating conditions, each meta surface element of an RIS can compensate the phase of the incident signal by referencing only a single receive antenna. In systems with independently fading channels, the signals observed at different receive antennas generally exhibit distinct phase responses, which cannot be simultaneously aligned by the RIS. Consequently, the RIS must select a reference receive antenna and perform phase compensation based on that antenna. While this limitation does not pose a significant issue for antenna selection schemes, it becomes a major challenge for signal combining techniques. In particular, when MRC is employed, signals received across multiple antennas are combined coherently, and the phase adjustment optimized for one antenna inevitably degrades the phase alignment at the remaining receive antennas.

Having this said, we examine 3 different scenarios for this technique. First, an ideal system is examined, which serves as a benchmark representing the best case. In addition, one bit and multi-bit phase shifting scenarios are examined where the aforementioned phase alignment limitation remains to be a problem. For the ideal case $\phi_{jik} = 0$ for every antenna pair through any surface. For the phase-quantized scenarios, the aforementioned uniform phase intervals are defined with respect to the reference antenna, while the remaining receive antennas experience uniformly distributed random phase offsets. It is worth noting that, phase shift with 3 or more bits can be referred to as continuous phase shift [8]. Finally, having the remaining phases clearly defined for all three scenarios, we can express the selection criterion based on MRC SNR. That is, selection at the transmitter is realized according to the coefficient of average SNR, given below.

$$j^* = \arg \max_j \left| \sum_{k=1}^{N_r} \left| \sum_{i=1}^N |h_{ji}| |g_{ik}| e^{j\phi_{jik}} \right|^2 \right|^2. \quad (3.57)$$

4. NUMERICAL RESULTS OF RIS ASSISTED MIMO SYSTEMS

In this section, we demonstrate analytical OP and SER and bit error rate (BER) results supported with Monte Carlo simulations. SNR threshold is set to $\gamma_{th} = 0$ dB. Note that we use relatively small values of N to demonstrate the validity of our analysis and to more clearly illustrate the DO.

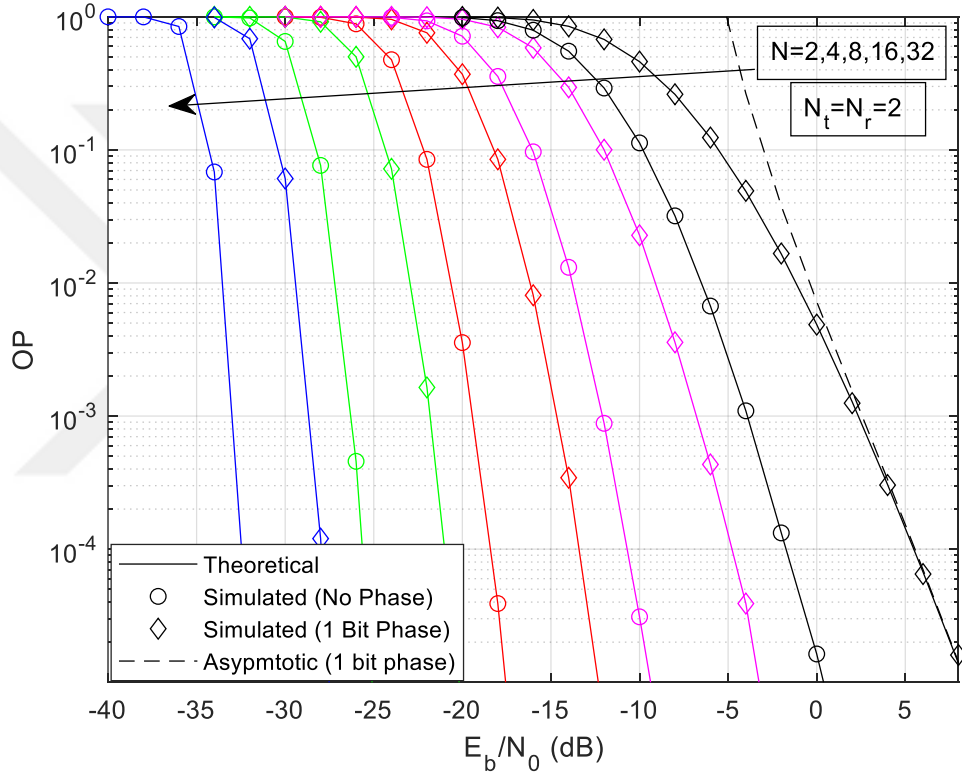


Figure 4.1: OP comparison of RIS assisted JTRAS system with no direct link in the absence and presence of phase noise.

Figure 4.1 shows OP of RIS-assisted JTRAS system for $N_t = N_r = 2$ and various number of meta surfaces with no phase error and with phase error caused by 1 bit quantization. Clearly, theoretical results match Monte Carlo simulations perfectly. Performance improves by approximately 7 – 10 dB for each successive increase in the number of surfaces. It is also evident that perfect phase shifting significantly improves performance by approximately 7 – 10 dB compared to the 1 bit phase quantization case. Furthermore, asymptotic OP for phase noise case, given by the $N_t N_r$

power of (3.38) is plotted for $N = 2$ and clearly overlaps with the exact results in high SNR regime.

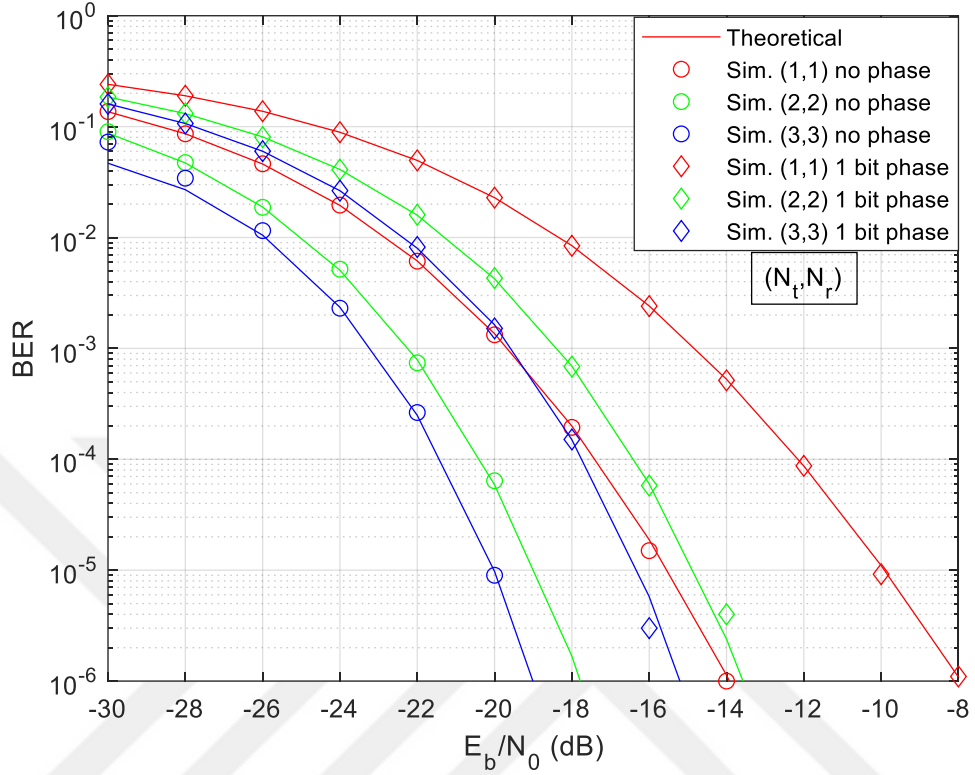


Figure 4.2: BER performance of JTRAS RIS system with no direct link using BPSK over Rayleigh channels.

Figure 4.2 is plotted to point out the performance improvement of the multi-antenna system compared to the single-antenna system for $N = 16$ and BPSK. It is clear to see that, in 1 bit phase case, adding extra 1 antenna to both the transmitter and receiver and using JTRAS technique enhances the performance by 4.8 dB at a BER of 10^{-5} . Adding yet another antenna pair still yields a substantial gain of about 1.5 dB under 1 bit quantization error case. As for the RIS-assisted system with no phase noise, adding extra 1 antenna to both nodes still yields considerable amount of SNR gain, about 3.4 dB. It can be inferred that, 1.4 dB less improvement of no phase case as compared to phase noise case at a BER of 10^{-5} , can be explained by the dominance of higher DO of no phase noise system. Consequently, it can be concluded that JTRAS is a quite effective technique at improving the performance of practical RIS systems.

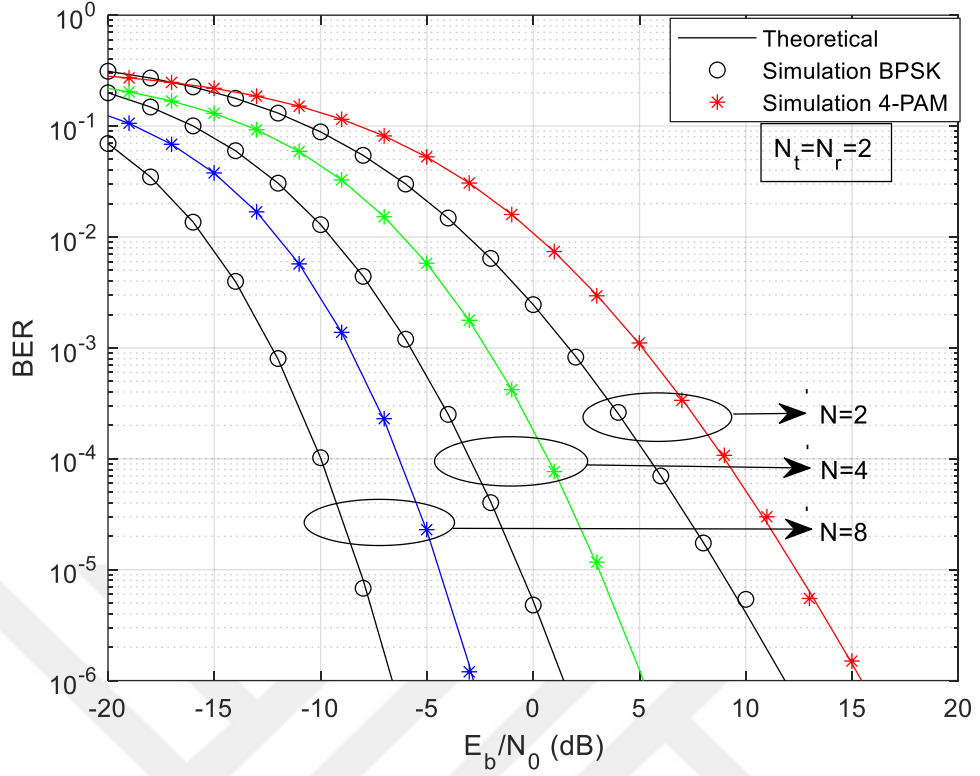


Figure 4.3: BER of the RIS-assisted JTRAS system without direct link for BPSK and 4-PAM.

Figure 4.3 shows the BER of the RIS-assisted JTRAS system without a direct link for $N_t = N_r = 2$ and different numbers of reflecting elements, using BPSK and 4-PAM with gray coding. Here, $E_b = E_s / \log_2 M$. As expected, BPSK outperforms 4-PAM in terms of BER. Here, the SNR gains of BPSK compared to 4-PAM at a BER of 10^{-5} for $N = 2, 4, 8$ are 3.51 dB , 3.7 dB and 3.72 dB , respectively. Furthermore, increasing the number of reflecting elements from 2 to 4 yields an SNR gain of 9.75 dB , while increasing it from 4 to 8 provides an additional 7.5 dB gain for BPSK at the same BER level. This reduction in incremental gain indicates that each additional surface contributes progressively less to the overall SNR improvement.

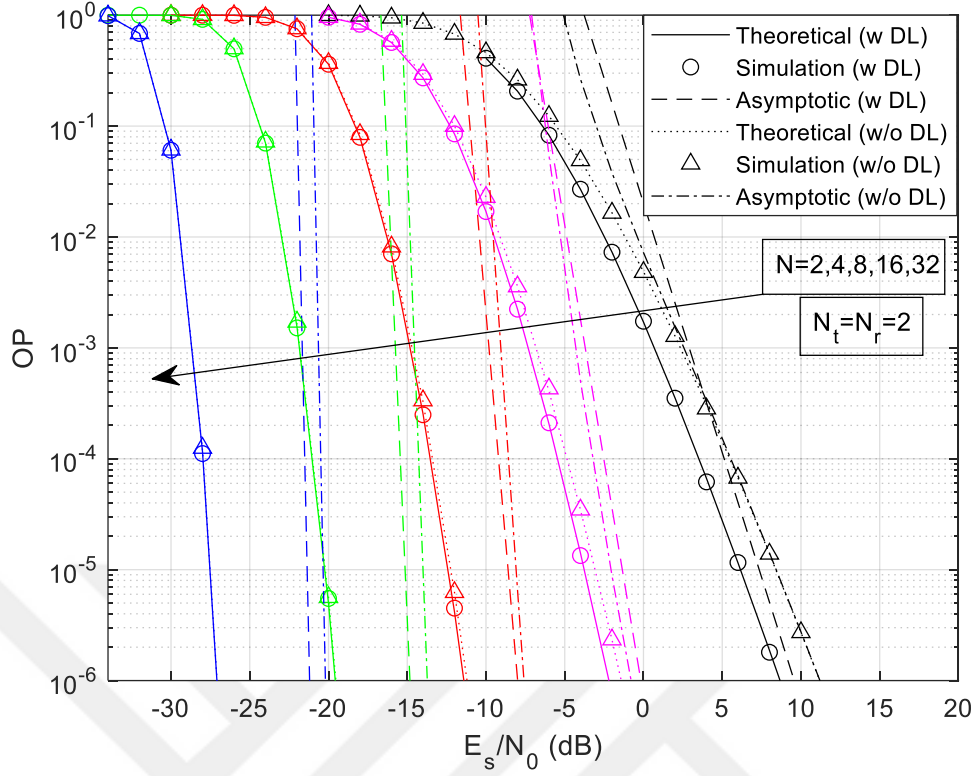


Figure 4.4: OP for the RIS-assisted JTRAS system with and without direct link for different numbers of N in the presence of phase noise.

Figure 4.4 illustrates the OP of the RIS-assisted JTRAS system for $N_t = N_r = 2$ and various numbers of reflecting elements, both with and without a direct link. Theoretical results are seen to perfectly match the Monte Carlo simulations, validating the analytical derivations. As observed, the impact of the direct link diminishes as N increases, and the RIS-assisted JTRAS link becomes dominant. Furthermore, the asymptotic OP expressions, i.e., the $N_t N_r$ -th power of (3.38), the direct plot of (3.55) are included to emphasize the DO. These asymptotic curves closely overlap with the exact results in the high-SNR region, confirming the theoretical DO analyses.

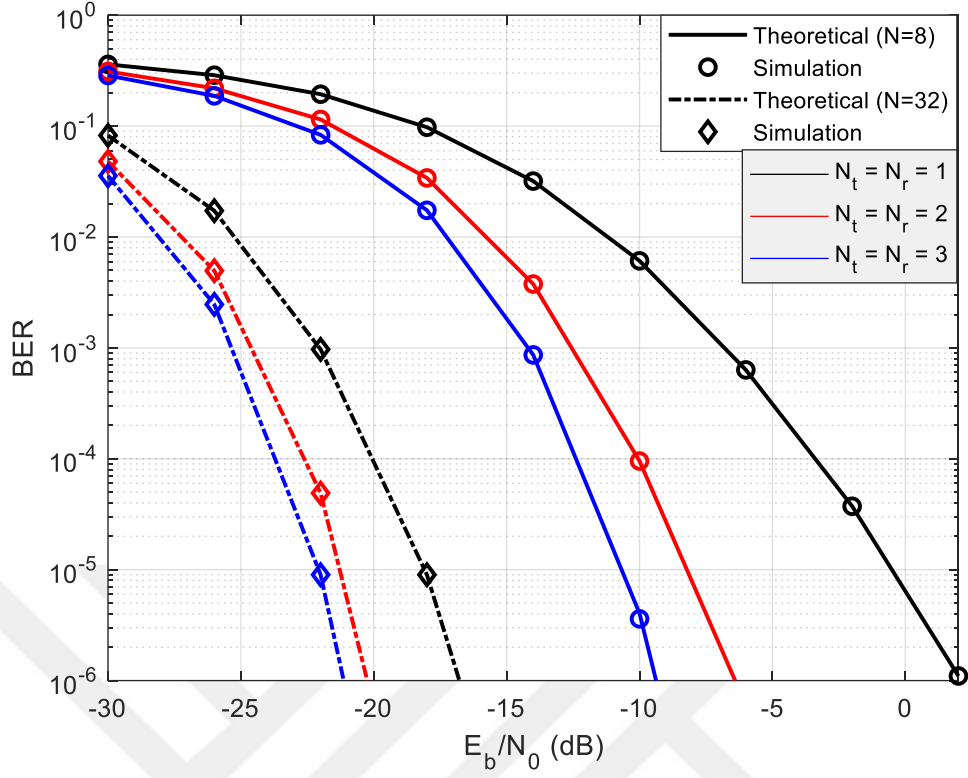


Figure 4.5: BER of the RIS-assisted single and multi antenna systems with direct link and phase noise for BPSK.

Figure 4.5 illustrates the performance improvement of the JTRAS system compared to the SISO system for different values of N using BPSK modulation. It is evident that increasing the number of antennas from 1 to 2 when $N = 8$ yields a gain of approximately 7.71 dB, while an increase from 2 to 3 provides about 2.57 dB at a BER of 10^{-5} . At the same BER level, when $N = 32$, these increments result in approximately 3.2 dB and 0.67 dB gains, respectively. As observed, the JTRAS technique leads to a noticeable improvement in system performance across various numbers of reflecting elements.

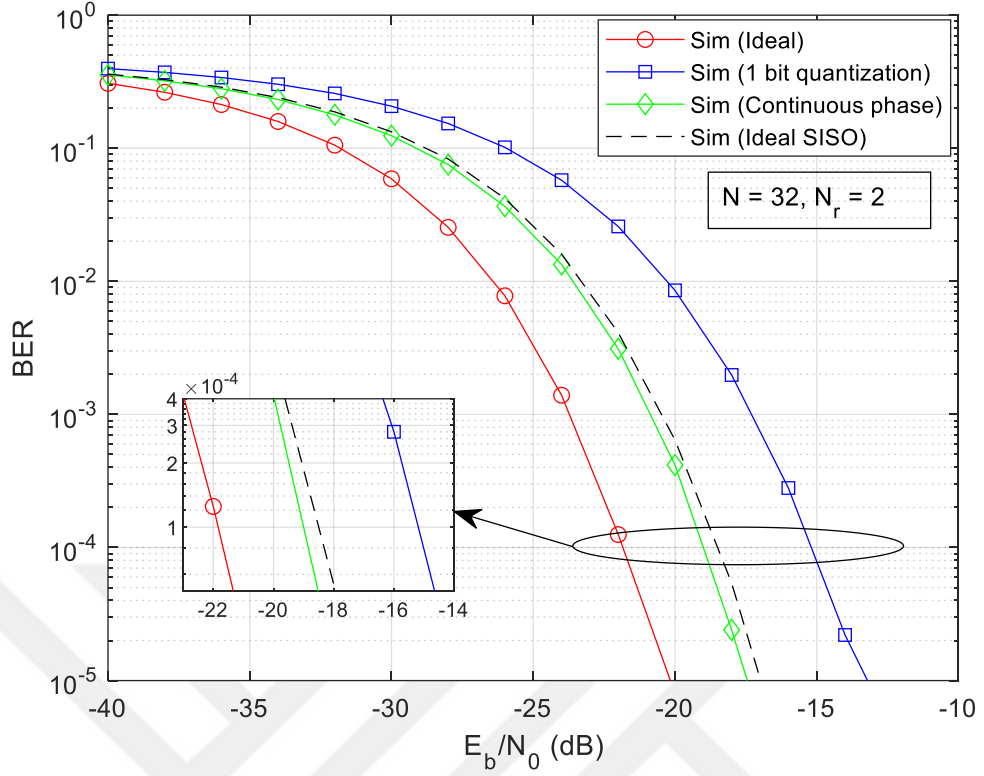


Figure 4.6: BER comparison of RIS assisted MRC system for every phase shift case and RIS assisted SISO system with ideal phase shift.

Figure 4.6 and Figure 4.7 depicts the Monte Carlo simulation results of BPSK over Rayleigh channel for RIS assisted MRC and TAS-MRC system respectively. For $N = 32$ and $N_r = 2$, continuous phase shift scenario of TAS-MRC barely outperforms ideal case of SISO RIS system. Nevertheless, despite the fact that RIS can only compensate based on single antenna, a gain of approximately 0.5 dB is achieved at the BER level of 10^{-4} compared to the ideal SISO benchmark. Furthermore, at the same BER level, ideal case has a great SNR gain of about 3 dB . As discussed before, this case is almost impossible to achieve. However, based on practical studies similar to [25], it is possible to transmit orthogonally polarized signals through RIS simultaneously. So, for this particular system setup where 2 receive antennas are used, horizontal and vertical polarized antennas can be employed. Therefore, with independent phase shifts, it is possible to achieve this case with an impressive SNR gain.

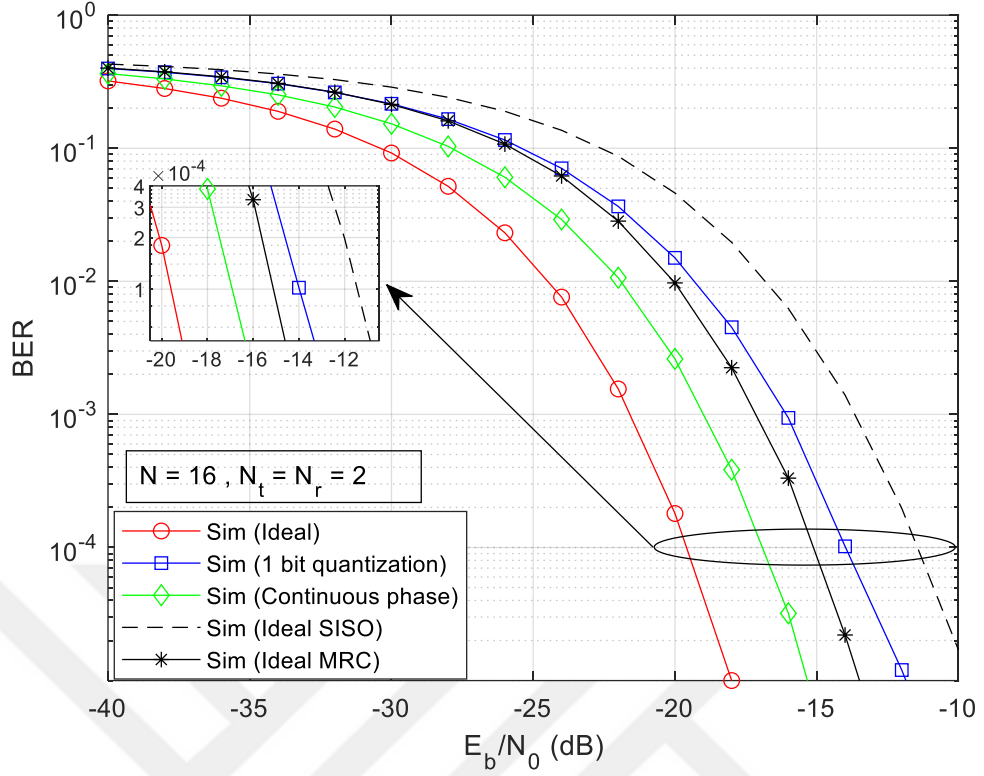


Figure 4.7: BER comparison of TAS-MRC, MRC and SISO RIS systems for various phase shift cases.

Figure 4.7 depicts BER curves of RIS-assisted TAS-MRC system using BPSK over Rayleigh channels. MRC and SISO scenarios are also plotted for comparison. It is observed that even the worst-case phase shifting scenario of TAS-MRC outperforms the ideal phase shifting SISO system. Moreover, the continuous phase TAS-MRC configuration achieves an additional SNR gain of approximately 2 dB at a BER level of 10^{-4} compared to the ideal MRC case. It is also worth noting that the ideal MRC scenario provides an SNR gain of about 1 dB over the 1 bit phase shift TAS-MRC case. These results indicate that TAS-MRC remains a viable and attractive option when reduced phase-shift complexity is desired, particularly when an additional antenna is available at the receiver.

5. CONCLUSIONS

In this thesis, the performance of RIS-assisted MIMO communication systems has been investigated with particular emphasis on practical phase-shifting constraints and diversity techniques. By considering both ideal and non-ideal phase configurations, the impact of realistic hardware limitations on system performance has been systematically analyzed.

The results demonstrate that RIS-assisted architectures can significantly enhance transmission reliability compared to conventional SISO systems, even under practical phase-shifting scenarios. JTRAS is examined analytically for ideal phase shift and 1 bit phase shift with and without direct link. It is observed that total diversity order magnifies by multiplication of transmit and receive antenna. In other words, full diversity is achieved in RIS assisted JTRAS system, providing exponential growths in high SNR regime. In addition, the incorporation of diversity techniques such as MRC and TAS-MRC provides noticeable performance gains, especially at moderate SNR levels.

Furthermore, the analysis reveals that although ideal phase alignment offers the best achievable performance, practical phase-shifting schemes still provide meaningful improvements with reduced system complexity. In this context, TAS-MRC emerges as an attractive alternative when lower phase-shift resolution is desired, as it can partially compensate for phase imperfections through the use of additional antennas while maintaining a favorable performance–complexity trade-off.

Another important observation is related to the phase compensation limitation inherent to RIS-assisted MIMO systems. Since practical RIS implementations can typically reference only a single receive antenna, perfect phase alignment across multiple antennas is not feasible. While this limitation has a limited impact on antenna selection schemes, it poses a more significant challenge for coherent combining techniques such as MRC. However, this can be mitigated by polarization techniques for specific antenna configurations.

In conclusion, this study highlights that RIS-assisted MIMO systems offer substantial performance benefits even under practical phase-shifting constraints. The findings

suggest that system designers should carefully balance the number of RIS elements, phase resolution, and receive-side diversity techniques to achieve optimal performance with reasonable complexity. Future work may extend this analysis to include the phase dependency issue for combining techniques or more generalized channels.



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BIOGRAPHY

Muhammed Faruk Karabay received the B.Sc. degree from Yıldız Technical University in 2023. He is currently pursuing M.Sc. degree in Gebze Technical University where he is also working as a research assistant since 2024. His research interests include wireless communications, with a particular focus on RIS, MIMO systems, and real-time communication systems.



PUBLICATIONS AND PRESENTATIONS FROM THE THESIS

M. F. Karabay; O. Kucur. “Yeniden Yapılandırılabilir Akıllı Yüzeylerin Çok Antenli Sistemlerde Faz Gürültüsü Varlığında Performansı” presented at the national conference 33. IEEE Sinyal İşleme ve İletişim Uygulamaları Kurultayı, İstanbul, Türkiye, 2025.

M. F. Karabay; O. Kucur. “Performance Analysis of RIS-Assisted Joint Transmit and Receive Antenna Selection”, submitted to International Journal of Electronics and Communications (AEÜ), January 2026.



APPENDICES

APPENDIX-A: SERIES EXPANSION OF LOWER INCOMPLETE GAMMA FUNCTION

From [19], lower incomplete Gamma can be expressed as

$$\frac{\Upsilon(s, x)}{\Gamma(s)} = \frac{1}{\Gamma(s)} \int_0^x t^{s-1} e^{-t} dt. \quad (A.1)$$

Here, let us replace $t = au$ and $dt = a du$, hence get

$$\frac{\Upsilon(s, x)}{\Gamma(s)} = \frac{a^s}{\Gamma(s)} \int_0^1 u^{s-1} e^{-au} du. \quad (A.2)$$

Note that,

$$e^{-au} = e^{-a(1-(1-u))} = e^{-a} \sum_{m=0}^{\infty} \frac{a^m (1-u)^m}{m!}. \quad (A.3)$$

Substituting (A.3) into (A.2)

$$\frac{\Upsilon(s, x)}{\Gamma(s)} = \frac{a^s}{\Gamma(s)} e^{-a} \sum_{m=0}^{\infty} \frac{a^m}{m!} \int_0^1 u^{s-1} (1-u)^m du \quad (A.4)$$

Finally, using the integral representation of Beta function [19], we get

$$\frac{\Upsilon(s, x)}{\Gamma(s)} = x^s e^{-x} \sum_{m=0}^{\infty} \frac{x^m}{\Gamma(s+m+1)}. \quad (A.5)$$

APPENDIX-B: ALTERNATIVE FORM OF PRODUCT OF SUMMATIONS

Consider a summation with the following format:

$$S(a) = \sum_{m=0}^{\infty} c_m a^m. \quad (B.1)$$

Multiplying this summation to itself yields

$$\begin{aligned} S(a)^2 &= \left(\sum_{m_1=0}^{\infty} c_{m_1} a^{m_1} \right) \left(\sum_{m_2=0}^{\infty} c_{m_2} a^{m_2} \right) \\ &= \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} c_{m_1} c_{m_2} a^{m_1+m_2}. \end{aligned} \quad (B.2)$$

Here, denoting $k = m_1 + m_2$, we can express the product of sums as

$$S(a)^2 = \sum_{k=0}^{\infty} a^k \sum_{m_1+m_2=k} c_{m_1} c_{m_2}. \quad (B.3)$$

The last transition arises from the fact that for a fixed value of $m_1 + m_2 = k$, the term a^k can be factored out of the summation. This logic is applicable to arbitrary number of parameters of m . So, let us express this as

$$\begin{aligned} S(a)^N &= \left(\sum_{m=0}^{\infty} c_m a^m \right)^N \\ &= \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} \cdots \sum_{m_N=0}^{\infty} \left(\prod_{i=1}^N c_{m_i} \right) a^{m_1+m_2+\cdots+m_N}. \end{aligned} \quad (B.4)$$

Here, denoting $k = m_1 + m_2 + \cdots + m_N$, the product of the sums can finally be written as

$$S(a)^N = \sum_{k=0}^{\infty} a^k \sum_{m_1+\dots+m_N=k} \prod_{i=1}^N c_{m_i}. \quad (B.5)$$

APPENDIX-C: INVERSE LAPLACE TRANSFORM OF CERTAIN EXPRESSIONS

Inverse Laplace transform of expressions of the form

$$f(t) = \mathcal{L}^{-1} \left[\frac{s^\alpha {}_1F_1 \left(a; b; \frac{\beta}{s} \right)}{\sqrt{1+2\bar{\gamma}s}} \right] \quad (C.1)$$

can be derived by splitting this expression into two parts and convolving them. First, inverse Laplace transform of $s^\alpha {}_1F_1 \left(a; b; \frac{\beta}{s} \right)$ can easily be derived as

$$f_0(t) = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \frac{\beta^n}{n!} \frac{t^{n-\alpha-1}}{\Gamma(n-\alpha)} \quad (C.2)$$

by the help of [19, eq. (9.210)]. Here, $(\cdot)_n$ is Pochhammer symbol. Second, convolving $f_0(t)$ with the inverse Laplace transform of the remaining part, we get

$$f(t) = \frac{e^{-\lambda t}}{\sqrt{2\bar{\gamma}\pi}} \frac{\Gamma(b)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n) \beta^n}{\Gamma(b+n) \Gamma(n-\alpha) n!} \times \underbrace{\int_0^t (t-\tau)^{-1/2} e^{\lambda\tau} \tau^{n-\alpha-1} d\tau}_{I_1}. \quad (C.3)$$

Here, by employing the Taylor expansion of $e^{\lambda\tau}$ and using [19, eq. (3.191.1)] in I_1 , we obtain

$$I_1 = \sum_{m=0}^{\infty} \frac{\lambda^m}{m!} t^{n+m-\alpha-1/2} \frac{\Gamma\left(\frac{1}{2}\right) \Gamma(n+m-\alpha)}{\Gamma\left(n+m+\frac{1}{2}-\alpha\right)}. \quad (C.4)$$

Combining all terms derived so far, we obtain $f(t)$ in the form of Srivastava-Daoust type function [24] as

$$f(t) = \frac{\Gamma(b)}{\Gamma(a)} \frac{e^{-\lambda t}}{\sqrt{2\gamma}} t^{-\alpha-\frac{1}{2}} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\Gamma(a+n) \Gamma(n+m-\alpha)}{\Gamma(b+n) \Gamma(n-\alpha) \Gamma\left(n+m+\frac{1}{2}-\alpha\right)} \frac{(\beta t)^n}{n!} \frac{(\lambda t)^m}{m!} \quad (C.5)$$