

T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL OF NATURAL AND APPLIED SCIENCES

**MULTI-OBJECTIVE OPTIMIZATION OF AN AXIAL
COMPRESSOR BLADE**

EMRE ERDEMİR
A THESIS SUBMITTED FOR THE DEGREE OF
MASTER OF SCIENCE
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FEN BİLİMLERİ ENSTİTÜSÜ

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KANATÇIĞININ ÇOK AMAÇLI
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DANIŞMANI
PROF. DR. MEHMET ALİ ARSLAN

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2020

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SUMMARY

Critical coincidences between natural frequencies of the blade and excitation orders of an aeroengine in critical operating speed zones, give rise to resonant vibrations and high alternating stress concurrently. The dynamic forces or vibrational alternating stresses acting on the blades can result in high cycle fatigue failure. Avoiding resonance conditions is essential for operational life of the blade consequently.

The aims of the study are to state methodology of vibrational behaviour optimization of the blade to avoid from resonance conditions and to expedite the process by comparing bladed disk and blade-only model modal analysis results to show the blade-only results are sufficient to reduce computational time and by evaluating a relationship between strain energy density regions of the blade and results of parameter correlation to eliminate parameter correlation step. Another aim of the study is to validate optimum and initial geometries in terms of structural integrity by performing experimental vibration tests.

In this thesis, the blade is parameterized by using ANSYS BladeModeler. Modal analysis results of the bladed disk and blade-only models are compared. Campbell and SAFE Diagrams are drawn to detect resonance conditions. Parameter correlation is performed to find dominant parameters. Strain energy density regions and dominant parameter locations are compared to show the parallelism between each other. Optimization using multi-objective genetic algorithms is conducted to find the optimum design. Finally, the optimized and initial blade geometries are manufactured, and experimental vibration tests are performed to obtain natural frequencies of the geometries to compare FEA and test results.

Key Words: Axial Compressor Blade, Modal Analysis, Strain Energy, Parameter Correlation, Multi-Objective Genetic Algorithm

ÖZET

Kritik çalışma hızı bölgelerinde kanatçık doğal frekansları ile bir hava aracı motorunun uyarım kademeleri arasındaki kritik çakışmalar, eşzamanlı olarak tınlayıcı titreşimlerin ve yüksek değişken gerilmelerin artmasına sebep olur. Kanatçıklara etkiyen dinamik kuvvetler ya da titreşim kaynaklı değişken gerilmeler yüksek çevrimli yorulma hasarı ile neticelenebilir. Kanatçık çalışma ömrü için kritik çalışma aralığındaki rezonans durumlarından sakınmak önem arz eder.

Çalışmanın amaçları, rezonans durumlarından kaçınmak için kanatçığının titreşimsel davranış optimizasyonu yöntemini belirlemektir ve hesaplama süresini düşürmek için sadece kanatçık sonuçlarının yeterli olduğunu göstermek maksadıyla kanatçık-disk ile sadece kanatçık titreşim analizi sonuçlarının farklarını karşılaştırıp parameter korelasyonu adımı elemek maksadıyla parameter korelasyonu sonuçları ile kanatçık gerinme enerjisi yoğunluk bölgeleri arasındaki ilişkiyi değerlendirerek optimizasyon sürecini hızlandırmaktır. Çalışmanın bir diğer amacı ise deneysel titreşim testleri yaparak yapısal bütünlük açısından başlangıç ve eniyi geometrileri doğrulamaktır.

Bu tezde, kanatçık ANSYS BladeModeler kullanılarak parametrik hale getirilmiştir. Kanatçık-disk ile sadece kanatçık modellerinin modal analiz sonuçları karşılaştırılmıştır. Rezonans durumlarını tespit etmek için Campbell ve SAFE diyagramları çizilmiştir. Baskın parameterleri bulmak için parameter korelasyonu yapılmıştır. Gerinme enerjisi yoğunluk bölgeleri ile baskın parametrelerin konumları, birbirleri arasındaki paralelliği göstermek için karşılaştırılmıştır. Optimum tasarımı bulmak için çok amaçlı genetik algoritmalar kullanılarak optimizasyon yapılmıştır. Son olarak, eniyilenmiş ve ilk kanatçık geometrileri imal edilmiştir ve geometrilerin sonlu elemanlar analiz ile test sonuçlarını karşılaştırmak için doğal frekanslarını elde etmek üzere deneysel titreşim testleri gerçekleştirilmiştir.

Anahtar Kelimeler: Eksenel Kompresör Kanatçığı, Modal Analiz, Gerinme Enerjisi, Parameter Korelasyonu, Çok Amaçlı Genetik Algoritma

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LIST of ABBREVIATIONS and ACRONYMS

<u>Abbreviations</u>	<u>Explanations</u>
<u>and Acronyms</u>	
HCF	: High Cycle Fatigue
FEA	: Finite Element Analysis
E	: Modulus of Elasticity
ρ	: Density
ν	: Poisson's Ratio
Hz	: Hertz
RPM	: Revolution per Minute
LE	: Leading Edge
TE	: Trailing Edge
C	: Chord
M	: Meridional Plane
CP	: Control Point
1F	: 1 st Flexural/Flap
2F	: 2 nd Flexural/Flap
3F	: 3 rd Flexural/Flap
1T	: 1 st Torsion
2T	: 2 nd Torsion
3T	: 3 rd Torsion
1E	: 1 st Edgewise
1-2S	: 1 st - 2 stripe
1-3S	: 1 st - 3 stripe
EO	: Excitation Orders or Engine Order
N_v	: Number of Vane or Static Parts
ND	: Nodal Diameter
N	: Number of the Blades
WB	: ANSYS Workbench
CAD	: Computer Aided Design
L	: Attenuation Length
R	: Inner Radius

t	: Thickness
DP	: Design Point
BC	: Boundary Condition
BD	: Blisk
BO	: Blade-Only
$[k]_e$	: Stiffness Matrix
$[m]_e$	: Material Matrix
Z	: Shape Function
w	: Natural Frequency
SED	: Strain Energy Density
GDO	: Goal Driven Optimization
RSO	: Response Surface Optimization
DO	: Direct Optimization
DOE	: Design of Experiment
DX	: DesignXplorer
CCD	: Central Composite Design
Y	: Number of Required Design Points
MOGA	: Multi-Objective Genetic Algorithms
NLPQL	: Nonlinear Programming by Quadratic Lagrangian
MISQP	: Mixed-Integer Sequential Quadratic Programming
f_0	: Stress-Free Natural Frequency
f_s	: Prestressed Natural Frequency
f_{EO}	: Engine Order Frequency
FFT	: Fast Fourier Transform
FRF	: Frequency Response Function
Seqv	: Equivalent Stress

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1. INTRODUCTION

1.1. General

A typical gas turbine engine is mainly comprised of compressor, combustion chamber, turbine and exhaust system or nozzle modules which are shown in Figure 1.1 [1]. Also, gas turbine engine generates power to the aircraft in terms of thrust or mechanical work by means of rotating the shaft(s). When the power turns into; jet thrust, shaft, propeller or fan rotation, this kind of gas turbines is respectively called as turbojet, turboshaft, turboprop or turbofan engine. Compressor module is responsible for drawing air in from intake, which is located forefront of the engine, compressing and providing the air into the combustion chamber with a high-pressure ratio of compressor outlet pressure to compressor inlet pressure.

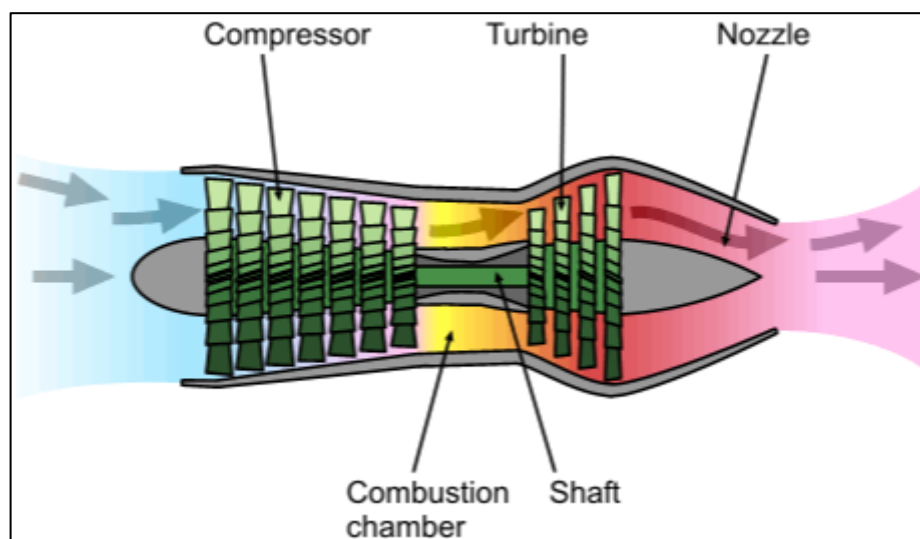


Figure 1.1: Turbojet engine.

While the compressor stages rotating, the blades become deformed under aerodynamic excitation caused by perturbations of pressure, mechanical and thermal loads. These combined loads can create mean and alternating stresses on the blades, and this can cause High Cycle Fatigue (HCF) failure. One quarter of gas turbine failures occurs because of HCF [2]. Almost half of gas turbine failures (%42) occurs because of the blades [3]. Thus, avoiding from critical coincidences between natural frequencies of the blades and excitation orders of the engine in critical operating speed

zones or possible resonance conditions is one of the way to lower HCF failures which ensures accomplish having more competitive aero-engine design goal. In this thesis, a detailed methodology of avoiding from possible resonances of an axial transonic compressor blade is stated.

1.2. Literature Review

The design process of the blades generally starts with aerodynamic design and continues with mechanical design which both are related with blade angles and thickness distributions on layers or sections from hub region to tip region. Many researchers have studied on blade design and effects of design parameters on requirements by means of parametric blade design and optimizing the design. In following paragraphs, a summary of literature with respect to the topics in this thesis is given concisely.

Reid et al. studied both aerodynamic and mechanical design of highly loaded high-pressure-ratio compressor and defined the blade shape comparison between low and high aspect ratio blades and defined basic parameters of compressor blade design, blade shape is specified by mean-line angles at inlet, transition, and outlet and by maximum thickness, leading- and trailing-edge thicknesses, and the locations of maximum thickness and transition points [4]. G. D. Burger et al. studied on low aspect ratio compressor blade aerodynamic and mechanical design and defined airfoil sections by specifying total and front chord, total and front camber, maximum thickness and its location chordwise, and leading and trailing edge radii and used mean camber line method to define profile [5]. Pugachev et al. divided design into four main variable groups such as profile thickness distribution, profile chord, profile offset and profile stagger angle. They controlled these main variables with nine parameters for each section of the blade independently by using cubic spline curves. Also, design variables are defined, thickness distributions are controlled at leading edge, trailing edge and maximum thickness location, chord is controlled by locations of leading and trailing edges, offset and stagger angle variables are used to add sweep, lean and to adjust stacking axis of the blade [6]. Myoren et al. presented a study about designing method of an axial transonic compressor blade in order to perform multi-objective optimization of aerodynamic performance. The blade is defined by sweep, stagger,

incidence angles, three radial profiles, which have suction and pressure surfaces consisting of two circular arcs and called as hub, mean, and tip sections or layers, and a radial stacking line, which is defined as a 4th order non-uniform B-spline curve including four control points and two slopes at the blade hub and tip [7]. Piollet et al. generated a blade geometry in order to evaluate parameter modification effects on nonlinear vibration simulation results. In order to control the parameters, four reference or control points such as inlet, maximum thickness point, transition point and outlet/exit by using cubic polynomial curves [8].

A parametric blade design is needed to perform optimization for the design, so several researchers used different methods to create a parametric blade design. Hecker et al. generated a parametric airfoil design by using third order Bezier curves with four control points in order to get a smooth airfoil geometry and sufficient design freedom [9]. Kiran implemented a parametric approach for the blade geometry design which is defined by set of geometric and aerodynamic parameters by using splines with control points [10]. Li et al. modelled a parametric geometry in parallel sections from hub to tip by using B-spline curves for pressure & suction sides of the airfoil and arcs for leading & trailing edges [11]. Aziz et al. used fourth order Bezier curve with five control points on mean camber line by dividing chord length five equally spaced locations for each three sections of the blade. Also, first and last control points are fixed in order not to affect the work transferred to the fluid [12]. Gohari et al. performed parameterization of a blade by utilizing Bezier curve with seven control points and fixed control points on leading and trailing edge to fix blade spacing [13].

It is also important to know vibrational behaviour of blades such as natural frequencies and mode shapes, then to evaluate design in terms of possible resonance conditions in order to meet design requirements. This topic has also been a research area for engineers. Armstrong et al. published a paper about practical aspects of compressor blade vibration, defined fundamental blade modes as flexural, torsional and edgewise modes and stated that 90 percent of the blade vibrational failures caused by these modes [14]. Singh developed a vibrational behaviour evaluation tool, called as SAFE or Interference Diagram, for blades to get a failure free and reliable blade design. Information about nodal lines-nodal diameters of bladed disks, and also possible-true resonance conditions by describing differences between Campbell and SAFE diagrams are given in the paper [15]. Holloway et al. prepared aerodynamic and mechanical design report of a high-pressure compressor and determined design criteria

for airfoils in terms of vibrational characteristics in order to avoid possible resonance conditions on Campbell diagrams [16]. Homji also defined fundamental blade vibration modes, blade-disk vibration definitions and design evaluation tools such as Campbell, SAFE(Interference) and Goodman Diagrams in the paper [3].

Most of researchers used finite element analysis tools to analyse the vibrational behaviour of the blades. There are two most common modelling methods for bladed disk designs; one is called as cyclic symmetry modelling and the other one is called as blade-only. Inoyama prepared a thesis about vibration analysis of a compressor blade by using Bernoulli-Euler, Rayleigh, Rayleigh-Ritz, two-dimensional plate and Timoshenko beam analytical methods. Also, Inoyama performed analysis by using NASTRAN for three models, straight plate, tapered solid and blade models in order to show FE model is valid accurately [17]. Zhao et al. implemented cyclic symmetric modelling to an aero-engine compressor blade-disc assembly FE model in order to get dynamic characteristics of bladed disc design and reduce computational effort. ANSYS software is used to model sector or cyclic symmetric FE model and a Campbell Diagram is drawn by performing stress-free and prestressed modal analyses in the paper [18]. Beirow et al. modelled a high pressure compressor blisk model with using sector modelling method by applying cyclic symmetric boundary condition in order to get mistuning behaviour of blades, described blade mode families by plotting nodal diameter vs. normalized frequency same as SAFE diagram without RPM lines and compared FEA results with experimental results. [19]. Kumar et al. published a paper about the model characteristics of a transonic axial compressor blade by using ANSYS and compared cyclic symmetry and blade-only modelling both numerically and experimentally [20]. Samrat et al. carried out modal analysis of an axial flow compressor blade by using ANSYS and compared numerical results with experimental result of free-free condition by using MATLAB [21]. Xuemin Ye et al. performed a study on blade tip grooving effects on performance and dynamic behaviour of a blade by performing blade-only analysis on ANSYS [22].

Sensitivity or parameter correlation analyses are performed in order to identify correlation between input and output parameters to make optimization process easier and eliminate relatively unrelated input parameters. Most researchers studied about sensitivity analysis of the blade parameters. Berger et al. performed a parameter correlation analysis to find dependence between geometrical & material parameters and natural frequencies & maximum amplitude by using Spearman rank correlation

method [23]. Heinze et al. performed a study on inhomogeneity of the blade material, is accounted and detected influences of geometric parameters on high cycle fatigue strength and natural frequencies of the blades by using Spearman rank correlation method [24]. Rajasekharan et al. compared sensitivity analysis methods, such as regression based Pearson and Spearman correlation, variance based Fourier amplitude and Sobol indices, and used Sobol indices in order to find the effect of the blade material of anisotropy angles on the blade displacement [25]. Backhaus et al. performed sensitivity analysis to find relation levels between aerodynamic & mechanical input parameters and natural frequencies of the blade by showing correlation matrix and using Spearman method [26].

The effect of strain energy density regions for each mode shape of a structure on dynamic behaviour has been studied by researchers. Rao published a paper and stated that the strain energy regions can be considered as a measure for the stiffness of the structure [27]. Zhou et al. studied on a method for designing dynamically of plate-like structures depending on modal strain, compared the contribution of the elements with high and low modal strain for natural frequencies, and point out that higher modal strain regions of a particular mode shape much more affected the natural frequency of the mode [28]. Choi et al. showed the relation between element stiffness, modal strain energy and modal parameters, also, used an algorithm based on modal strain energy to improve damage quantification of a plate structure [29]. Chang et al. performed mode optimization of a car body by improving high strain energy regions which are the main cause of the structure's dynamic weakness [30].

Extensive studies have been conducted on multiobjective optimization of the blades by using genetic algorithms. Myoren et al. performed on a transonic axial compressor blade design to ensure desired performance points by using multi-objective genetic algorithms (MOGA) and compared the optimized blade with the original blade [7]. Pierret et al. defined a method to optimize a compressor blade design by using design of experiment(DOE) procedure to build a database with central composite design technique, by utilizing response surface optimization to create a metamodel and by applying genetic algorithms on created metamodel to accelerate the optimization problem [31]. Samad et al. performed shape optimization for an axial compressor blade by using DOE to create a design space in order to use it into response surface optimization method [32]. Shabliy et al. implemented ANSYS Design Exploration(DX) tools, such as parameter correlation to find dependence of

parameters, design of experiment with central composite design(CCD) method, response surface to get metamodel and multi-objective optimization of an axial compressor blade with maximum von Mises stress constraint and aerodynamic objectives such as pressure rate and adiabatic efficiency in order to search candidate design points on the response surface by using genetic algorithms [33]. Yao et al. also used design of experiment with central composite design(CCD) method, combined with the response surfaces by using genetic aggregation method and performed multi-objective genetic algorithms to find optimum design points on the response surfaces by using ANSYS [34].

1.3. Purpose and Structure of Thesis

In this thesis, the aim is to evaluate a relationship between strain energy density regions for each mode shape of an axial bladed disk(blisk) compressor low-aspect-ratio blade and to state a quick methodology for optimization of the blades. In order to accomplish these goals, structure of the thesis is built as follows respectively. Blade parameterization is executed by using Bezier curves to have smoother profiles. Blade thicknesses and angles are controlled by control points for each of the blade sections. Results of sector modelling method with cyclic symmetry boundary condition and blade-only modelling methods is compared by using ANSYS software as Finite Element (FE) tool. Campbell and SAFE diagrams are used as tools for design vibrational evaluation. Parameter correlation is performed by using Spearman rank correlation method to find dependence between input and output parameters. Results of parameter correlation and strain energy density regions of the blade are compared to expedite optimization process for vibrational behaviour of the blade. Design of experiment with using central composite design method is carried out to build a design space. Response surface method is performed by using genetic aggregation method to create metamodels. Multi-objective optimization with using genetic algorithms is conducted on response surfaces to find optimum design points. Finally, optimized & initial blade geometries are manufactured, and vibration tests are performed to obtain natural frequencies of the blades utilizing a hammer and microphone.

2. METHODOLOGY

A flow chart, which indicates the path to create the methodology, is shown in Figure 2.1.

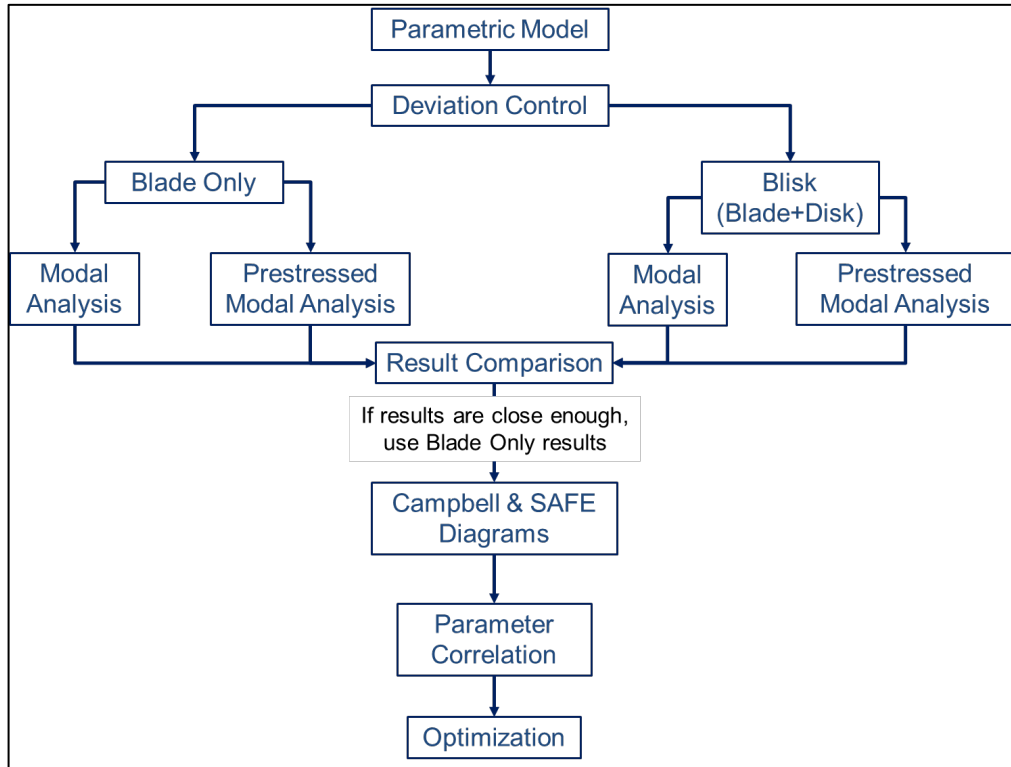


Figure 2.1: Flow Chart.

2.1. Bezier Curves

Parametric curves are used in many applications such as geometry design, numerical analyses, computer graphics, fonts and animations. Bezier Curve is one of the parametric curves and this curve type is used frequently for modelling of smooth curves. The Bezier Curve was developed in 1960s by a French Engineer, Pierre Bezier in order to use for designing of car bodies.

Form of the curve is specified by the position of the control points and moving a control point can affect whole curve. While using this curve type, if it is desired not to change specific location of the design, control points around this specific location can be fixed. First and last control points define beginning and end of the curve, other control points are used to change the shape of the curve. Bernstein polynomial

mathematically underlies for the Bezier curve. Bernstein basis function for degree of n and Bezier curve function are expressed as follows:

Bernstein Basis Function:

$$B_{i,n}(u) = \binom{n}{i} \cdot u^i \cdot (1 - u)^{(n-i)} \quad (2.1)$$

Bezier Curve Function:

$$C(u) = \sum_{i=0}^n P_i \cdot B_{i,n}(u) \quad (2.2)$$

P represents control point, u is any location on the Bezier curve $0 \leq u \leq 1$ and for $n=3$, cubic Bezier Curve Function is expressed as follow:

$$C(u) = P_0(1 - u)^3 + P_1 3u(1 - u)^2 + P_2 3(1 - u)u^2 + P_3 u^3 \quad (2.3)$$

An example of cubic Bezier curve is shown in Figure 2.2, blue line represents first derivative of Bezier curve function and green line shows second derivative of the curve [41].

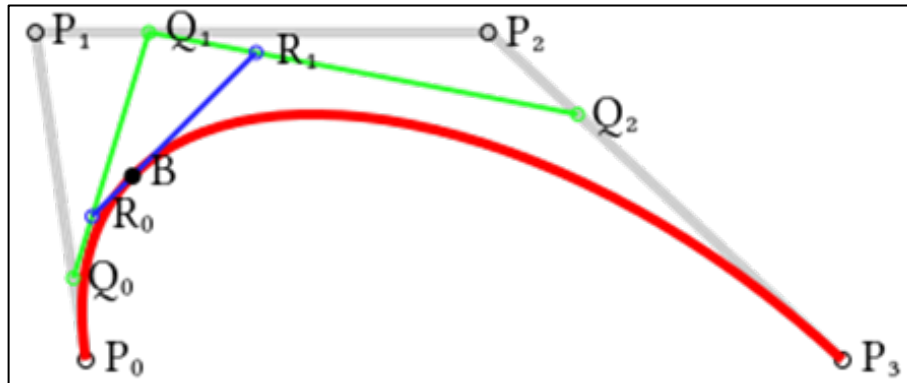


Figure 2.2: Cubic Bezier Curve.

In this current study, Bezier curves are used for defining blade thicknesses and angles of each of the blade sections in order to have a smooth parametric model. Also, first and last two control points of both thickness and angle distribution curves at each blade section are fixed. By having smooth parametric model and fixing some control

points, it is aimed to avoid manufacturing of a geometry, which is not aerodynamically friendly, and not to change the aerodynamic parameters rapidly.

2.2. Airfoil Definitions

Before parametric modelling of the blades, the terminology related to the airfoil and blade is better to be known. Airfoil means a cross-section of the blade which is able to convert the energy of the fluid into mechanical work while moving through the air. An airfoil geometry is shown in Figure 2.3 [5].

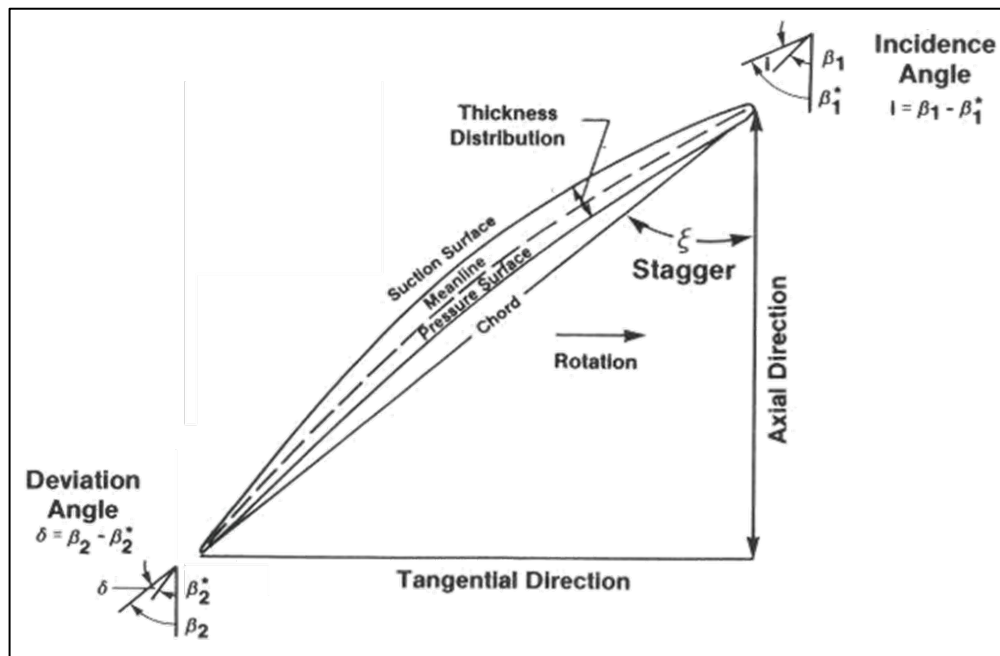


Figure 2.3: Airfoil Terminology.

The front and rear edge of airfoil are respectively called as leading edge (LE) and trailing edge (TE). Chord(C) can be described as the distance between LE and TE of the airfoil. Meanline is a line which bisects the suction (convex) side (SS) and pressure (concave) side (PS) of the airfoil. Chamber is the curvature of the meanline. Thickness distribution specifies the thickness at each point on the meanline and thickness is applied perpendicular to the meanline. Incidence angle is the difference between the inlet angle of the air and the leading edge meanline angle. Deviation angle is the difference between the exit angle of the air and the trailing edge meanline angle. Metal angle/theta(θ) is the angle between the meanline and the axial direction. Aspect

ratio is defined as ratio of the height and chord of the blade. If the ratio is lower than 2, it is low aspect ratio or higher than 2, it is high aspect ratio blade. Hub and tip or shroud section of the blade locates at the bottom and top of the blade, respectively. Hub, tip, height, leading edge and trailing edge are shown in Figure 2.4 with meridional plane view.

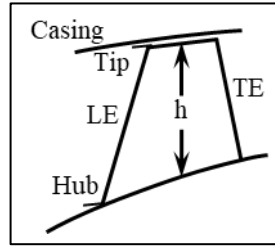


Figure 2.4: Meridional(M) Plane View.

In this thesis, parametric geometry of a 1st stage transonic low aspect ratio axial compressor blade is generated. For the transonic working conditions ($0.72 < \text{Mach number} < 1$), maximum thickness and chamber of the blade suggested to be located at 50% of the chord in order to make the air follow the surfaces of the blade more efficiently because the lesser chamber means the more efficiency for a transonic blade [35]. These considerations are taken into account in order to avoid changing aerodynamic behaviour rapidly while improving mechanical or vibrational behaviour of the blade.

2.3. Parametric Geometry

In the first step, a parametric geometry is needed to perform any sensitivity analysis or optimization. After aerodynamic design of the blade geometry is finished, the geometry is imported with its layers and profiles using ANSYS BladeModeler module in order to generate parametric blade model as shown in Figure 2.5. When importing the blade geometry, number of layers and control points can be selected to have same geometry with the original geometry but layer numbers and control points must be reduced by using Bezier curve conversion option, which has 16 control point number limitation, in order to have advantage of less required analysis numbers and computational time by using curve convert option as shown in Figure 2.6.

Angle and thickness distribution curve types can be changed by using “Segment Type” option between Spline and Bezier. Bezier curve is chosen for both angle and thickness distributions in order to protect smoothness of the blade. Angle Definition Type is chosen as metal or theta angle because this option enables fixing inlet and outlet angles. By using “Insert Point” and “Delete Point” options, control points can be added or deleted at any location in meridional line or plane (%M). The coordinates of the control points are expressed by using percentage of meridional line and starts from LE with 0 and ends at the TE with 100. If the angle and thickness definitions is chosen as “interpolated”, program interpolates parameters of the layer by means of upper and lower layers. These options are shown in Figure 2.5 and Figure 2.6.

Finally, control points are also selected as input parameters by using “Use as Input Parameter” option which is shown in Figure 2.6. If one control point (CP) is not chosen as input parameters, this means that this CP is fixed.

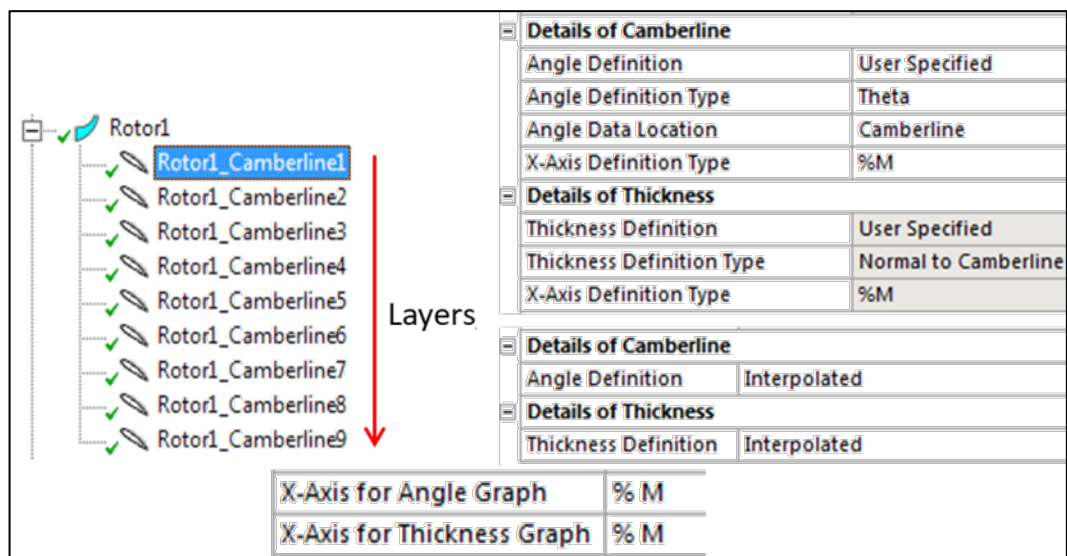


Figure 2.5: Blade Layers from Hub to Tip and Camberline Options.

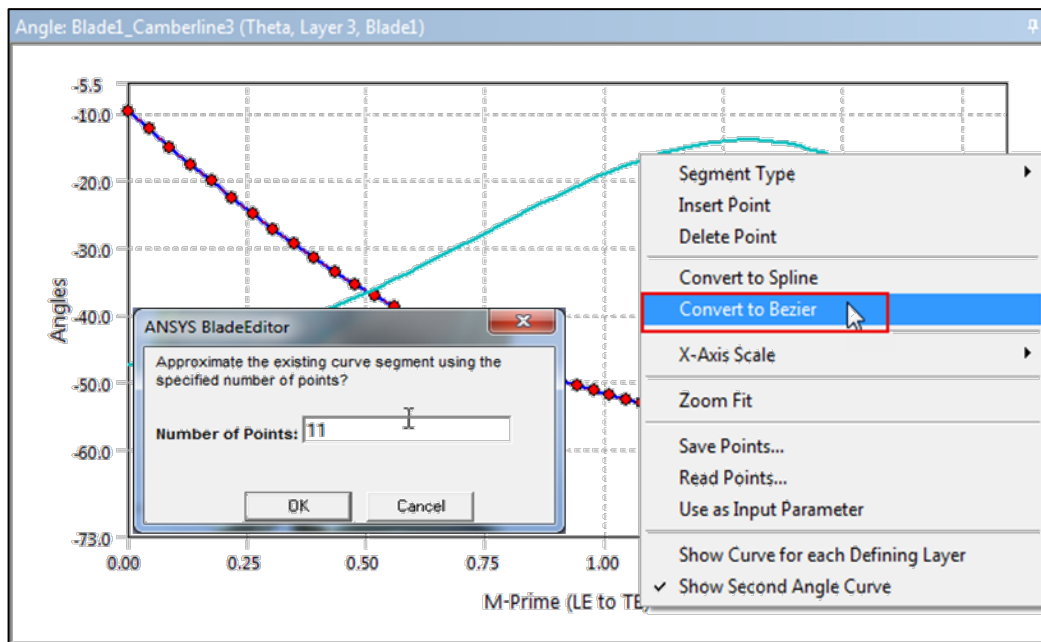


Figure 2.6: Metal(theta) Angle Distribution of a Layer, Convert Options.

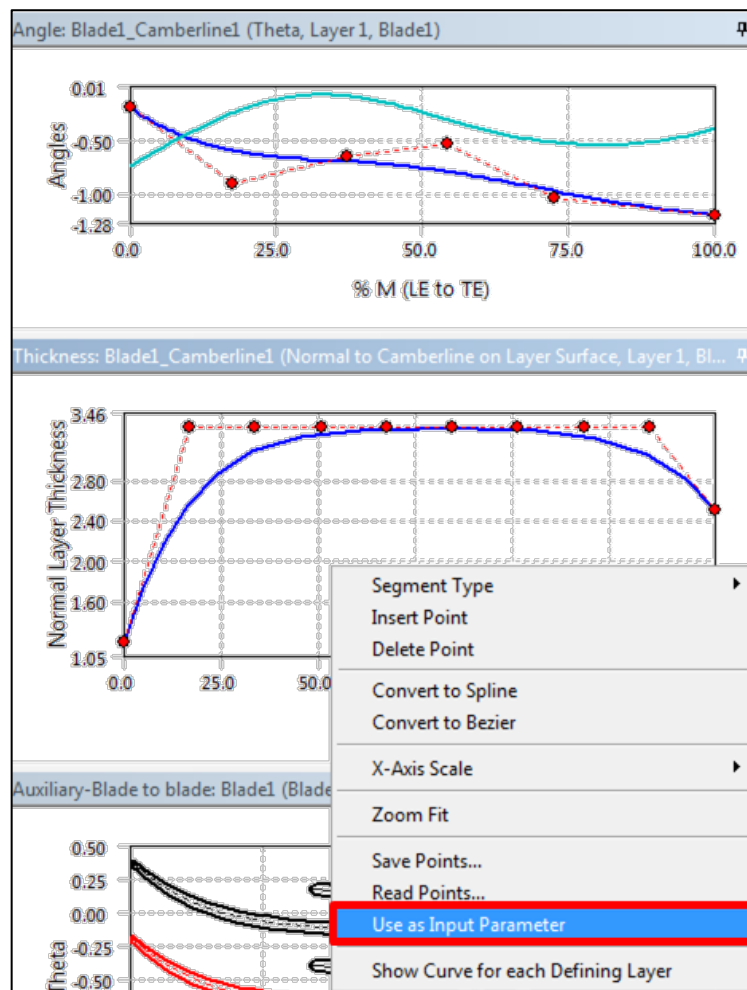


Figure 2.7: The “Use as Input Parameter” Option.

2.3.1. Parametric Geometry Assumptions

In this study, after the import operation of the initial geometry is performed, 9 layers/sections from hub to tip and splines with 50 control points for angles and thickness distribution at each layer are used to have same geometry with initial design, but, to reduce computational time, it is necessary to reduce layer and control point numbers. So, the number of layers and control point numbers are chosen as follows:

For metal angle distribution:

- Layers numbers are reduced to 3, which are Hub(L1), Mean(L5) and Shroud(L9), other layers are obtained interpolating by the software.
- Control point numbers are reduced to 7 for these layers; two for both end are fixed in order to avoid changing inlet and exit angles, others are located at %33, %50, %67 of meridional lines(%M), as shown in Figure 2.10.

For thickness distribution:

- Layer numbers are reduced to 5, which are Hub, L3, Mean, L5 and Shroud(L9), other layers are obtained interpolating by the software.
- Control point numbers are reduced to 9 for these layers; two for both end are fixed in order to avoid changing inlet and exit angles, others are located at %25, %37.5, %50, %67.5 and %75 of %M, shown in Figure 2.11.

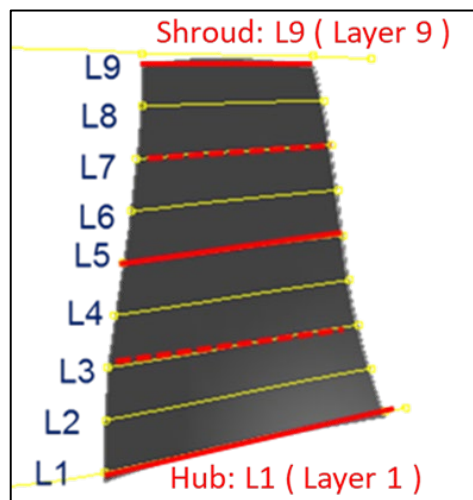


Figure 2.8: Layers of the Blade.

In Figure 2.8, for the layers shown with red lines, thicknesses are taken as input and for the layers shown with dashed red lines, theta angles are interpolated by the software.

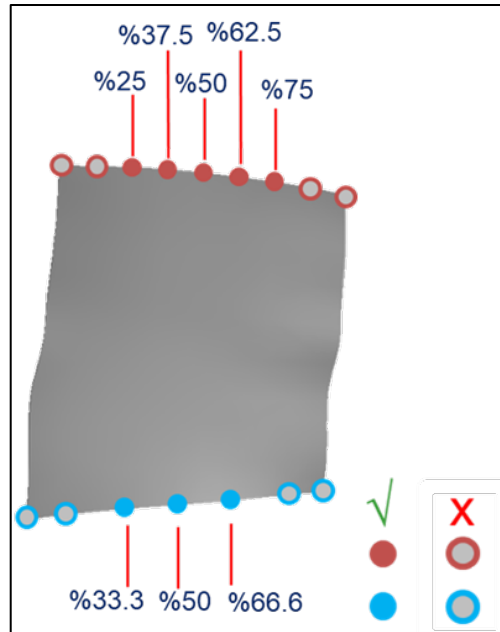


Figure 2.9: Angle & Thickness CP Locations in %M.

In Figure 2.9, blue and red points represent metal angles and thicknesses respectively. Points with grey fill mean that the CP is fixed.

Grey dots represent the parameters as input and blue lines are the real distribution of metal angle and thickness of the blade for a layer in following figures.

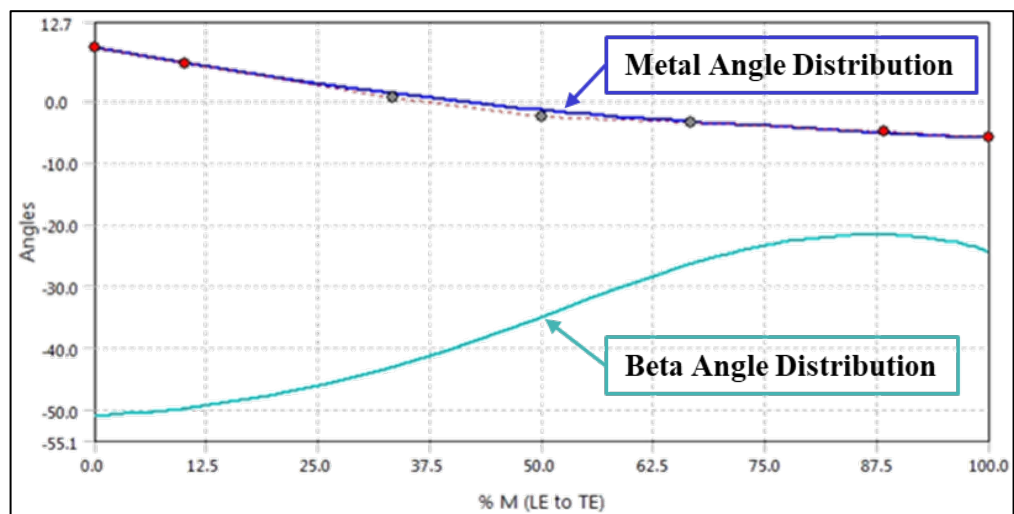


Figure 2.10: Angle Distribution CP.

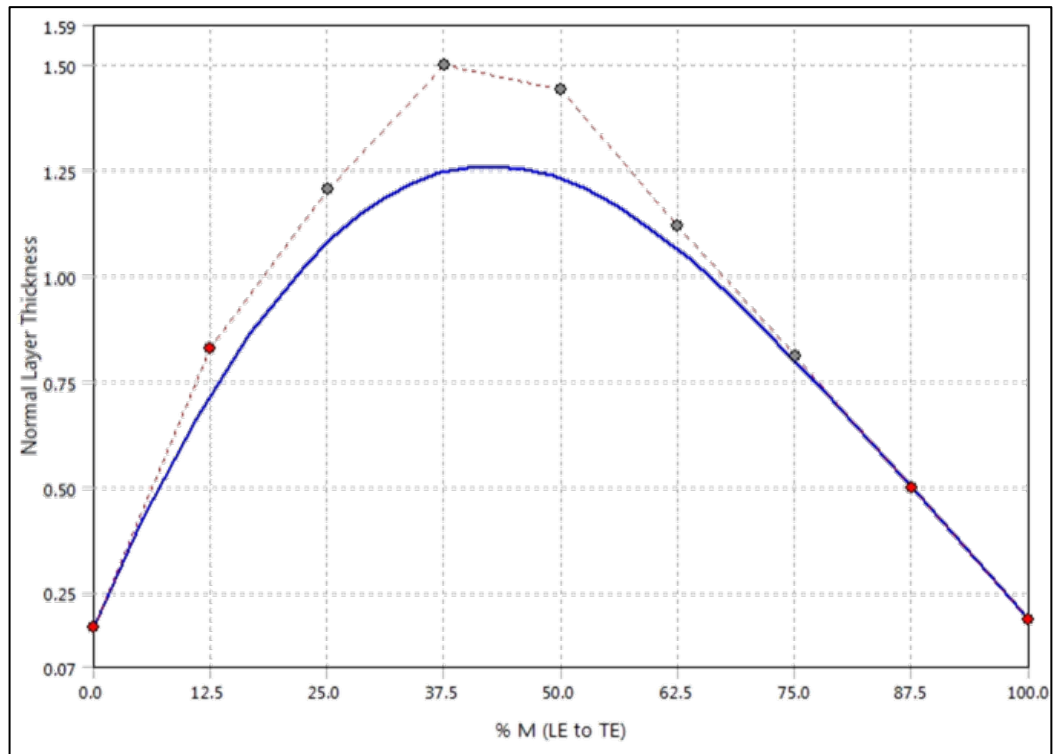


Figure 2.11: Thickness Distribution CP.

In order to perform bladed disk(blink) modal analyses and to draw SAFE diagram, sector model and disk geometry are needed. By using “SectorCut” option of the BladeModeler, 1D disk geometry in meridional plane can be imported and 3D blink sector model can be created, which are both shown in Figure 2.12.

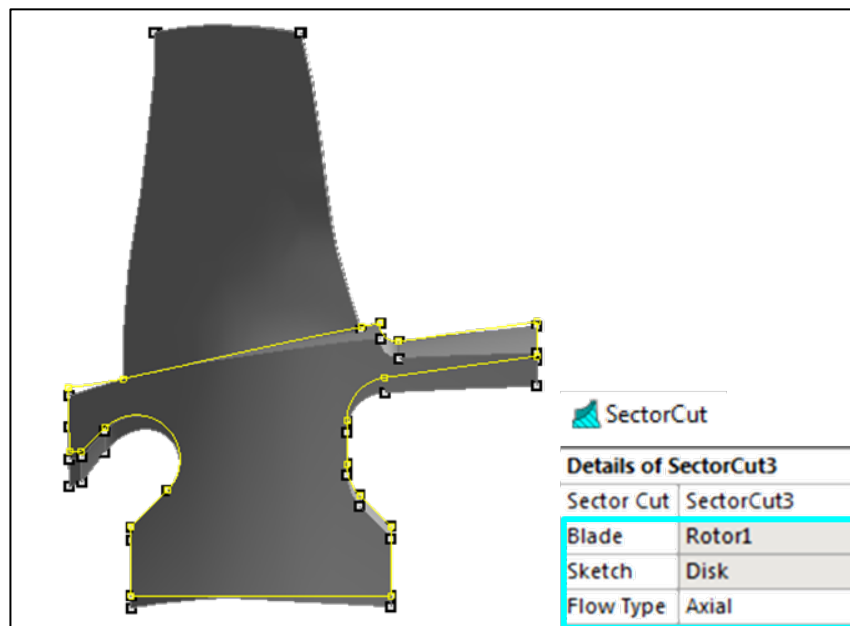


Figure 2.12: 1D Disk(Yellow), 3D Blisk Sector Model & Sector Cut Option.

2.3.2. Changing of Parameters

The change in the thickness or angle curves require much effort than the change of the control points because of the formulation of the Bezier curves. While increasing a CP two times, max thickness is changing much less as shown in Figure 2.13. Blue line is the new thickness distribution, black line is the original thickness distribution and red dots are control points.

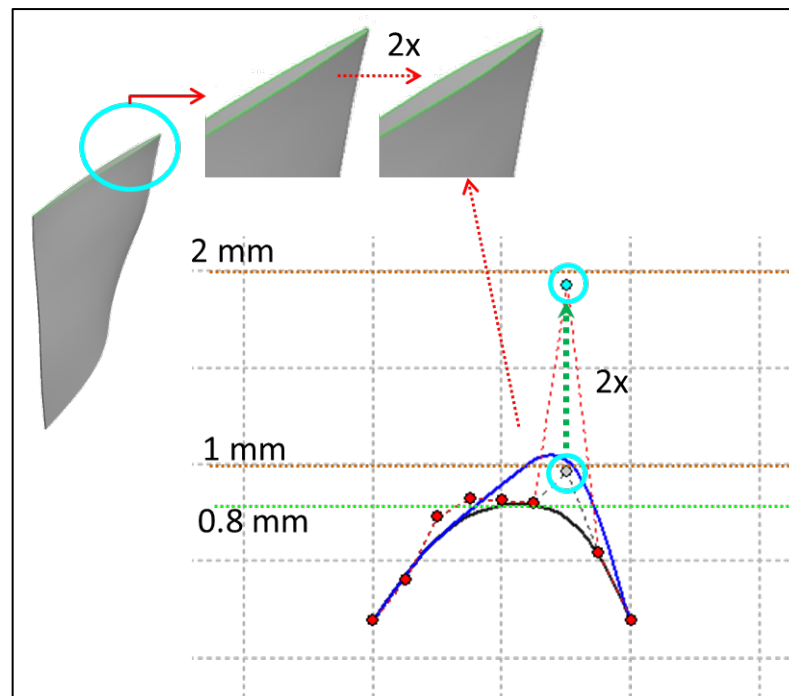


Figure 2.13: Changing differences between parameter curves and CPs.

Effects of changing thickness and angle parameters are shown in Figure 2.14 and green are is changed profile and pink area is the original profile.

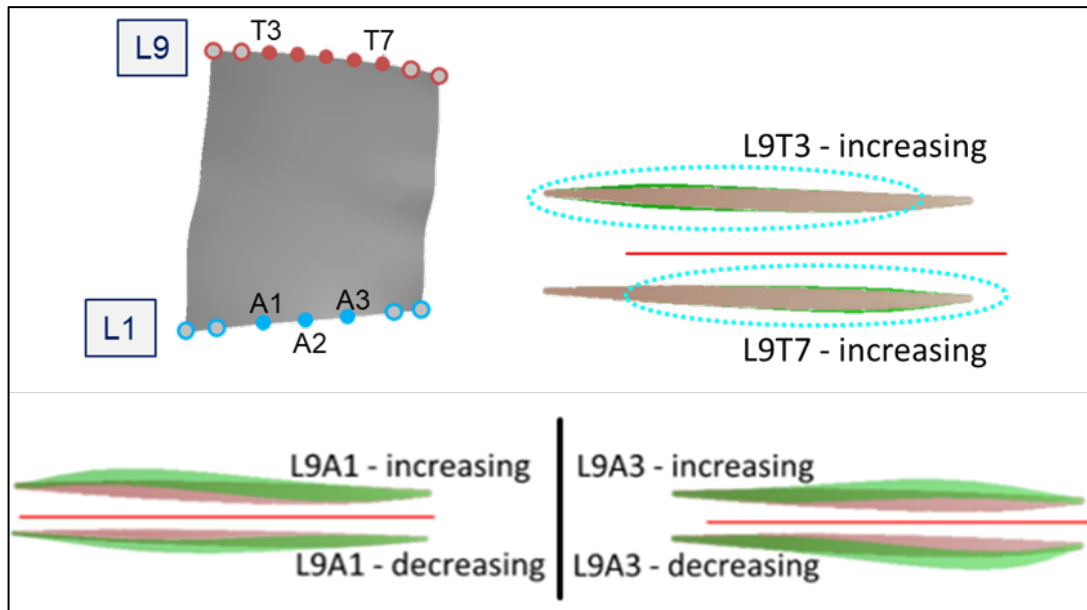


Figure 2.14: Effects of Changing Parameters.

2.3.3. Geometry Deviation Check

While reducing the number of the control points, the parametric geometry deviates automatically. If the deviations of the parametric geometry are within manufacturing tolerances, the geometry is acceptable because it is likely to encounter any geometries within the tolerances in practice. It is a control check in order to avoid having completely different geometry from the original geometry. The geometry deviation is checked using ANSYS SpaceClaim. Tolerance is selected to show deviations.

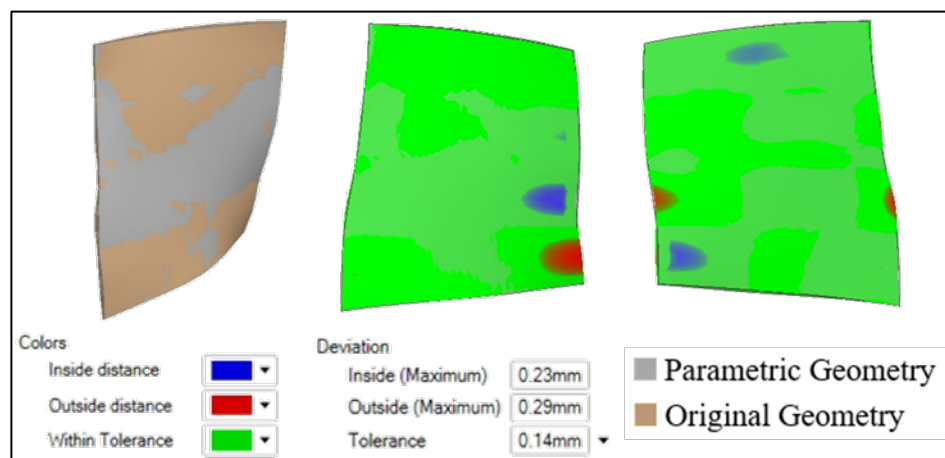


Figure 2.15: Geometry Deviation Check.

2.4. Vibrational Evaluation Tools for Design

Vibrations caused by dynamic forces acting on the blade may cause resonance as a result of coincidences between natural frequencies of the blades and excitation orders in the critical operating range.

Campbell and SAFE diagrams are used in order to identify these possible and true resonance conditions. True resonance condition occurs when the following two scenarios occur:

- Coincidence between exciting force frequency and natural frequency, which can be checked with drawing Campbell Diagram,
- Coincidence between exciting force shape and mode shape of the natural frequencies which can be checked with drawing SAFE Diagram [15].

Avoiding these coincidences in critical operating range is essential in terms of the blade operational life. Nevertheless, it may not be possible to avoid from every coincidence. In such cases, Goodman Diagram is used to evaluate the condition of the design, but the Goodman Diagrams are not in the scope of this study.

2.4.1. Fundamental Blade Modes

Fundamental blade modes consist of flexural or flap, torsion, edgewise and stripe modes as shown in Figure 2.16 and Figure 2.17, red lines represents nodal lines where deformations are equal to zero and signs show the direction of deformation [3]. Coincidences between low natural frequency modes (flap, torsion, and edgewise fundamental modes) and low excitation orders (1-4 EO), can cause loss of all or significant part of the blade. Also, it is easier to excite lower modes than higher modes, and coincidences of higher modes can cause loss of small parts of the blade. So, the complex or higher mode coincidences are less significant than the lower mode coincidences.

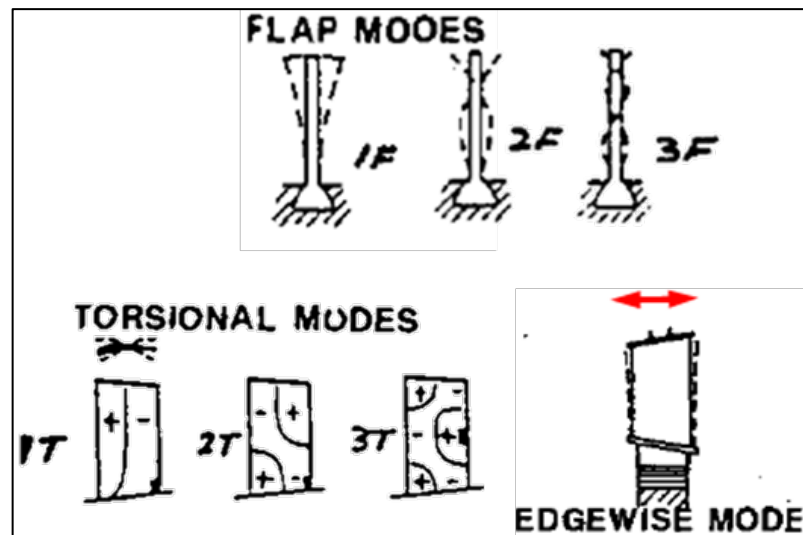


Figure 2.16: Fundamental Modes of the Blades.

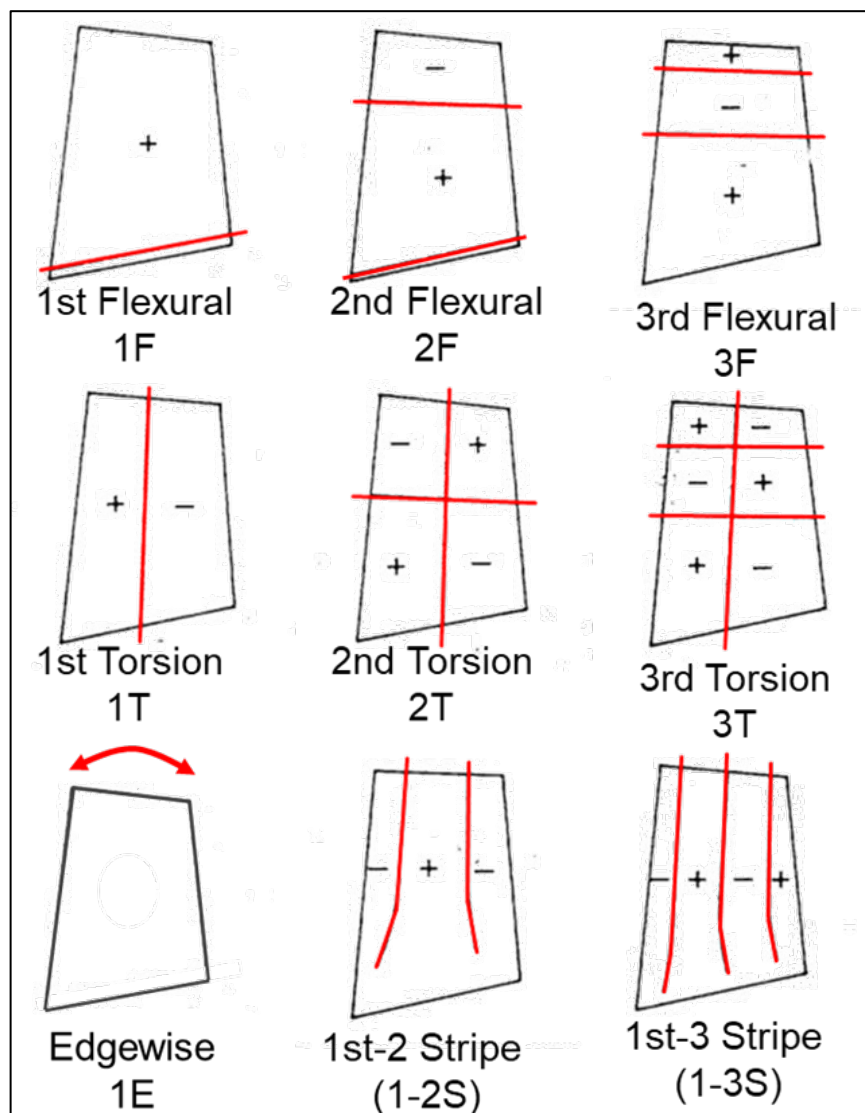


Figure 2.17: Fundamental Modes, Nodal Lines and Deformation Directions.

2.4.2. Excitation Sources

There are two excitation types of sources; first type disturbs the flow and causes varying pressure distributions because of static parts such as stator vanes and struts. Second type occurs because of stalls and blade rubbings [37]. Some root causes of excitation orders (EO) are as follows:

- Inlet distortion of the flow, causes first four low engine orders,
- Inlet Struts,
- Frame Struts,
- Bleed Ducts,
- Stator Vanes.

The flow passing through vanes is a direct excitation source for neighbour rotating blades. Also, the difference between forward and backward vane numbers is recommended to control with Campbell Diagram. EOs are drawn for a rotating blade in Campbell using a linear line equation shown below [16].

$$f_{EO} = \frac{N_v \times RPM}{60} [Hz] \text{ \& } f_{EO} = \frac{|N_{v1} - N_{v2}| \times RPM}{60} [Hz] \quad (2.4)$$

Where f_{EO} is frequency of the excitation/engine order, N_v is number of vane or static parts, N_{v1} and N_{v2} are forward and backward static part numbers, RPM is rotational speed, respectively. In this thesis, the rotating blades locate at the first stage between 6 inlet struts at forward and 40 stator vanes at backward. First four EOs, 6th EO, 34th EO as difference of static parts and 40th EO are drawn in Campbell Diagram, which is shown in Figure 2.37. For instance, if the maximum rotational speed is 15000 RPM and the EO number is 4, the frequency line of the 4th EO is plotted in Figure 2.19 and the frequency of the EO at 15000 RPM is calculated as follows:

$$f_{EO} = \frac{4 \times 15000}{60} = 1000 \text{ Hz at } 15000 \text{ RPM} \quad (2.5)$$

2.4.3. Campbell Diagram

Campbell Diagram shows the information of frequency versus RPM. Campbell diagram examples are shown in Figure 2.18 and Figure 2.19. Figures show; operating range, where the blade spends most of its life, between idle/minimum cruise and redline/max transient speed, natural frequencies of the blade between stress-free condition (0 RPM) and prestressed condition (at any RPM), excitation orders and coincidences which are possible resonances.

It is suggested to have a blade design that its 1F mode should have at least %15 margin above the 2nd EO and coincidences should be away from the operating range and Figure 2.19 shows the margin higher than %15 between 1F and 2nd EO [16]. But, avoiding from all coincidences may not be possible. Also, in order to take any probability such as manufacturing scatter, it is suggested to apply %10 margin to natural frequency curves and shown in Figure 2.20 [38].

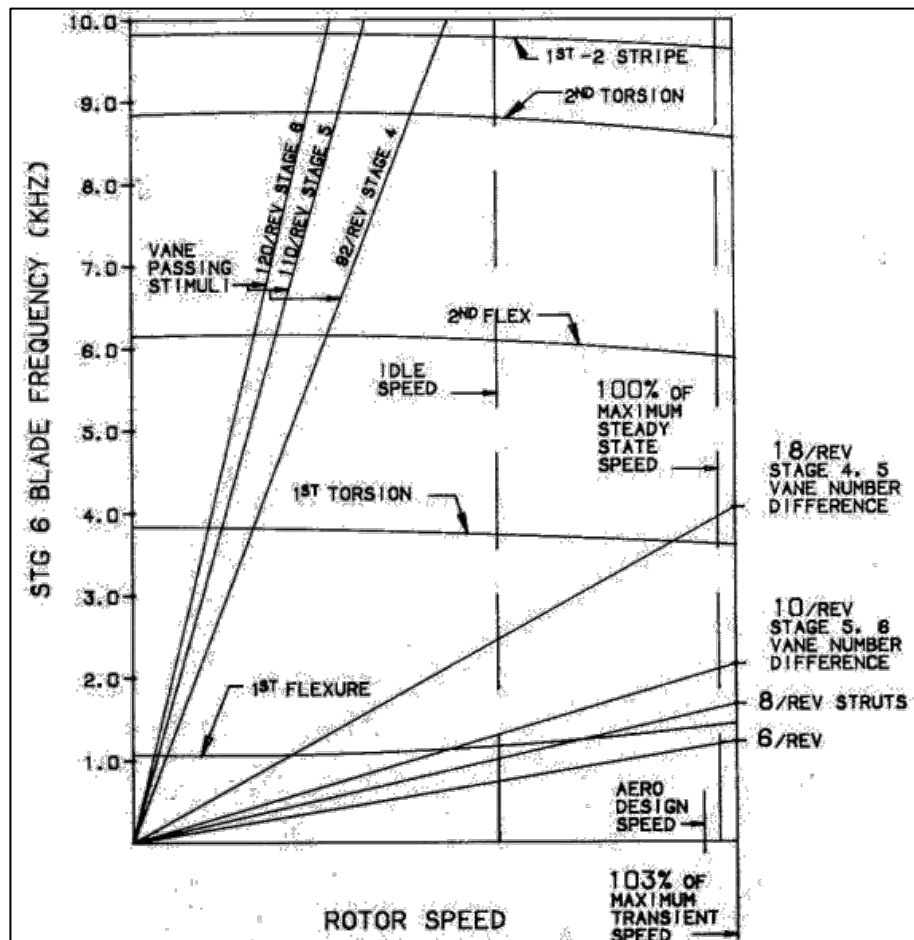


Figure 2.18: Campbell Diagram Example – 1.

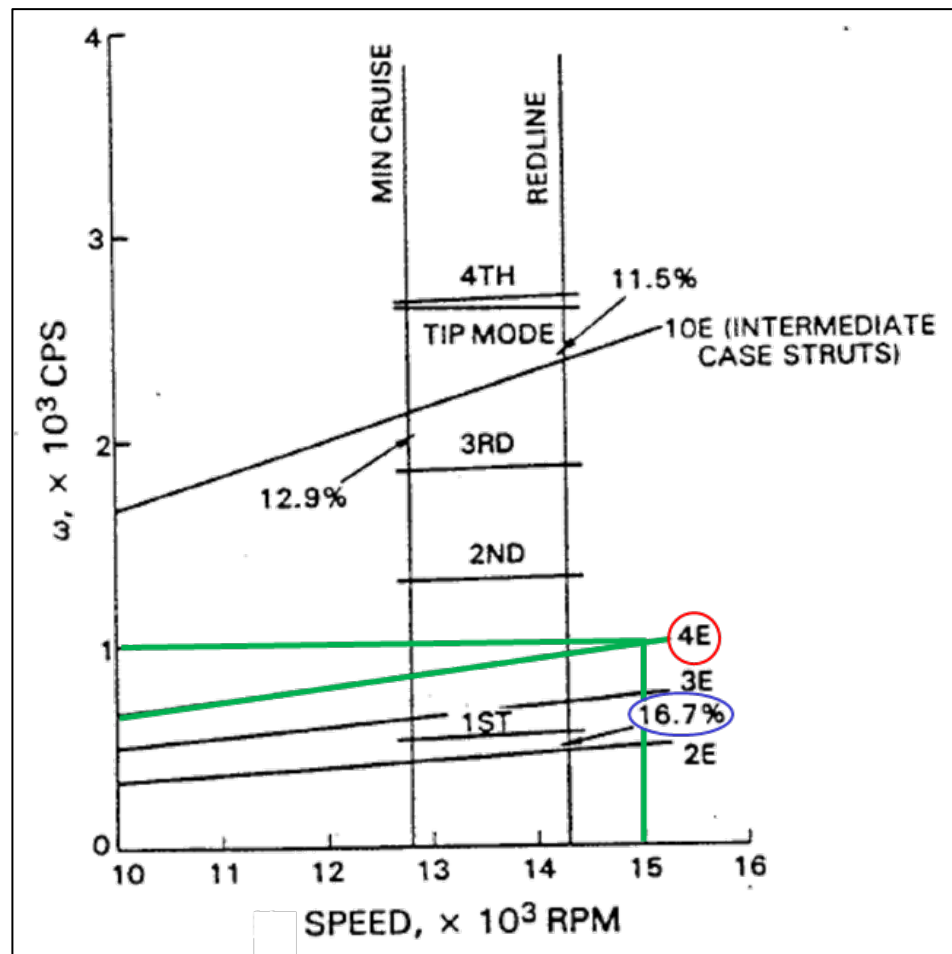


Figure 2.19: Campbell Diagram Example – 2.

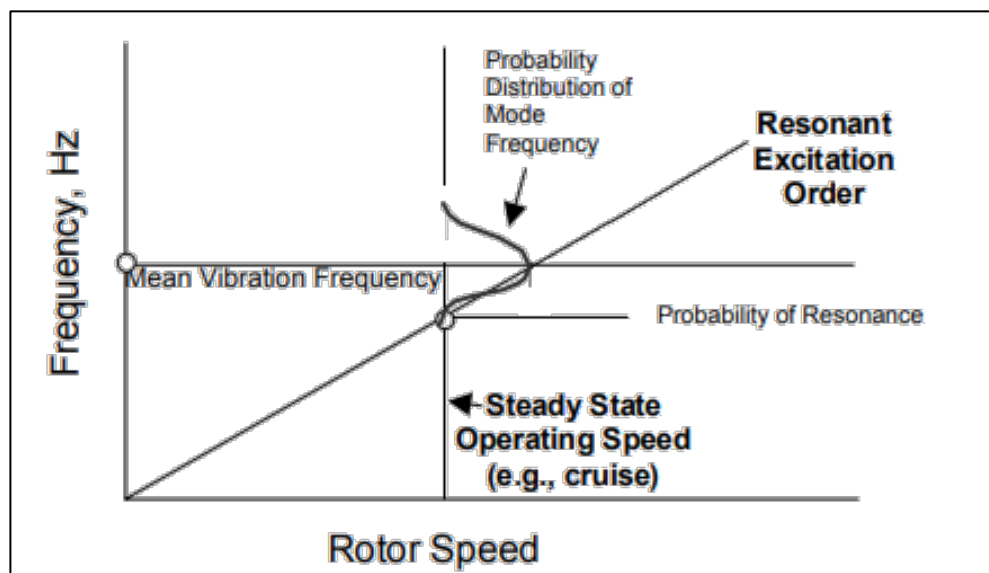


Figure 2.20: The Frequency Probability Margin.

2.4.4. SAFE Diagram

Partial or possible resonance conditions can be found by drawing Campbell diagram, but it is also necessary to draw SAFE diagram in order to find true or full resonance conditions. This diagram shows information of natural frequencies versus nodal diameter which represents number of zero displacement lines where the displacement direction of the structure changing. Zero nodal diameter modes also called as umbrella modes which do not have any zero displacement lines in the geometry. Nodal diameter numbers of the blisk geometry is calculated as follows:

If N, number of the blades at the stage is, odd:

$$ND = \frac{N-1}{2} \quad (2.6)$$

If N, number of the blades at the stage, is even:

$$ND = \frac{N}{2} \quad (2.7)$$

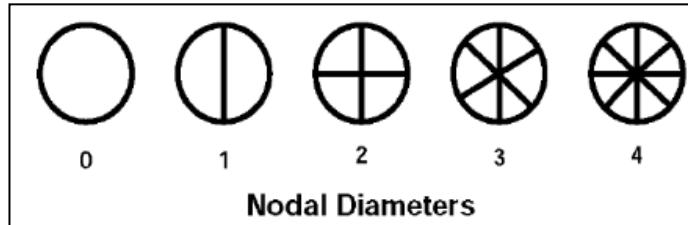


Figure 2.21: Nodal Diameters(displacement direction changes in every line).

For blisk geometry, all modes show nodal diameter characteristic, but some mode shapes are blade dominant and others are disk dominant. Also, natural frequencies of the blade dominant modes mostly do not change with nodal diameters, for this reason blade modes tend to be flat in SAFE diagram. SAFE diagram is also related with Campbell diagram, for each nodal diameter a Campbell diagram can be drawn and this relation is shown in Figure 2.23 [15]. Also, a SAFE diagram example is shown in Figure 2.22, the area between black lines represents operating range, the black lines are drawn by using following equation:

$$f[\text{Hz}] = \frac{ND \times \text{RPM}}{60} \quad (2.8)$$

For instance, when the blade number is 150, the nodal diameter is calculated using Equation 2.8 and equals to 75 because the blade number is even. Then, the frequency line is drawn at the 75th ND for 3790 RPM in Figure 2.22 [15]. The frequency is calculated as follows:

$$f = \frac{75 \times 3790}{60} = 4737.5 \text{ Hz} \quad (2.9)$$

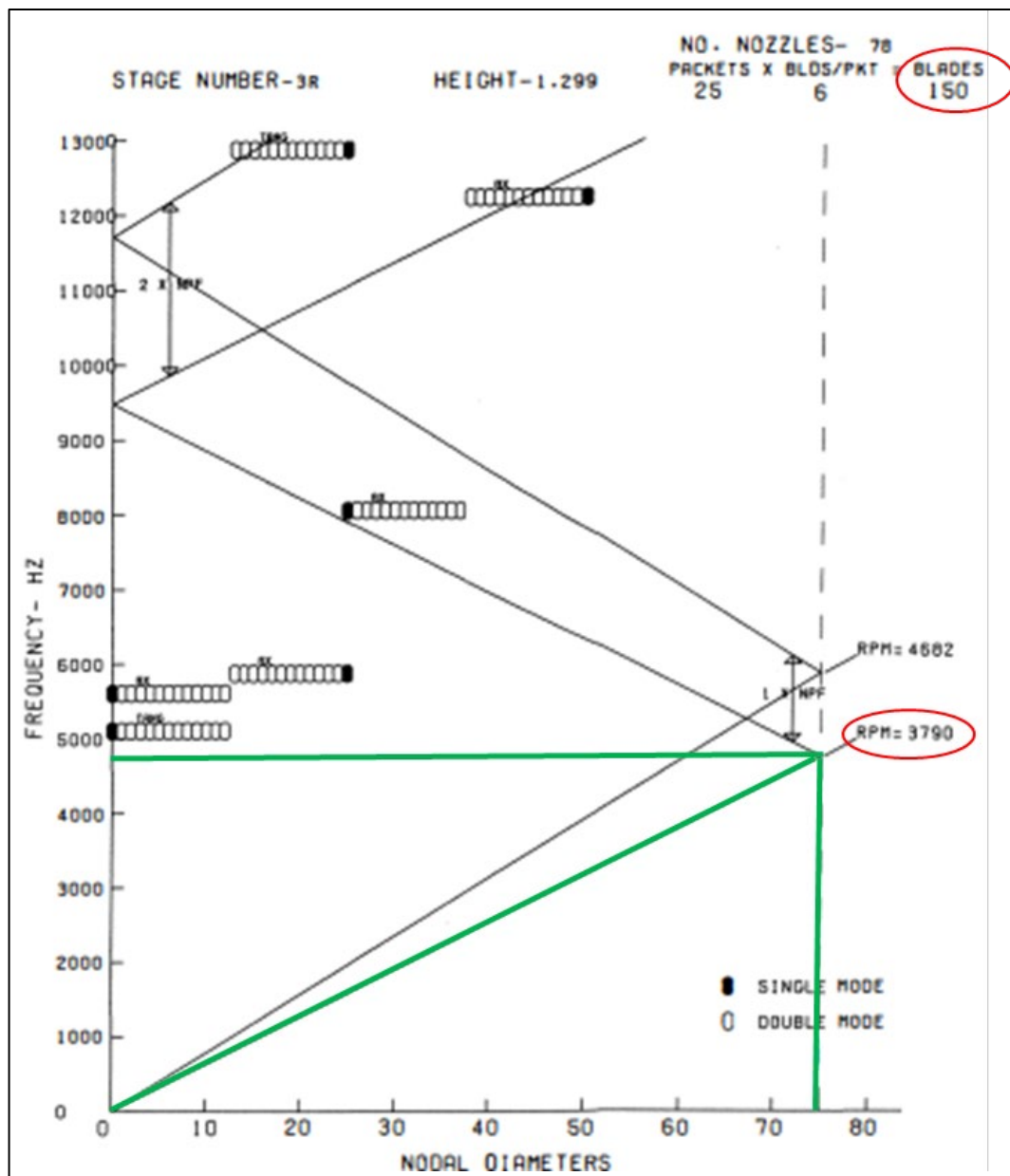


Figure 2.22: SAFE Diagram Example.

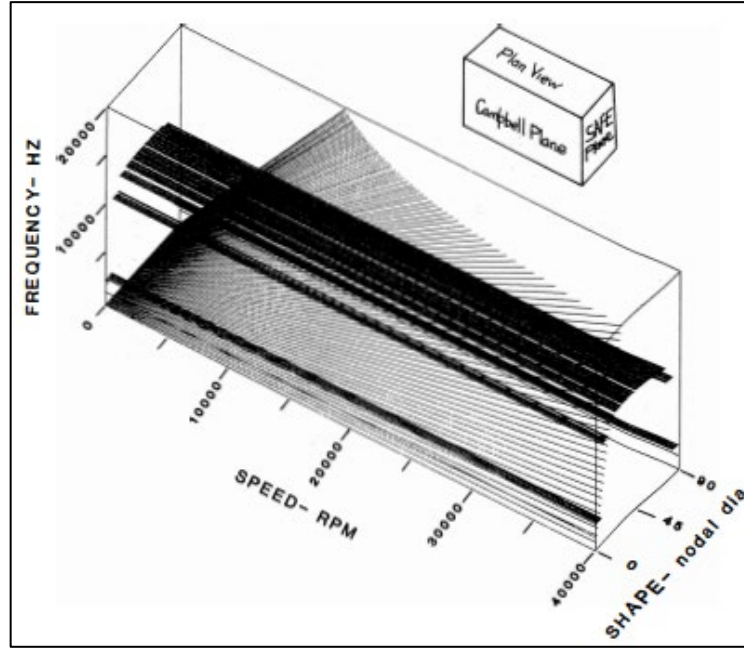


Figure 2.23: Relation of SAFE & Campbell Diagrams.

2.5. Vibration Analyses

Vibration or modal analyses are performed in order to obtain natural frequencies and mode shapes, then to evaluate the vibrational condition of the geometry. There are three types of vibration which are free vibration without damping, free vibration with damping and forced vibration with damping. Blisk geometries have extremely low structural damping because of not having dry friction damping which is caused by the connection between blade and disk. Finite element analysis tools are used to perform modal analyses because it is easier to accurately solve the complex engineering problems using FE methods than using analytical methods. In this thesis, ANSYS Static Structural and Modal analysis modules are used to obtain vibrational behaviour of the blade. In ANSYS Modal analysis module, the free vibration without damping vibration type is used and the equation of motion and natural frequency for this vibration type as follows:

Equation of motion for multiple degree of freedom (DOF) systems:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (2.10)$$

where; M is the mass matrix, C is the damping matrix, K is the stiffness matrix, x is the displacement vector, $\dot{x}$ is the speed vector, $\ddot{x}$ is the acceleration vector, $F(t)$ is the excitation force vector and t is time.

Equation of motion for free vibration without damping:

$$[M]\{\ddot{x}\} + [K]\{x\} = 0 \quad (2.11)$$

To solve above differential equation, x is the displacement for a linear system and harmonic form of the x :

$$\{x\} = \{X_i\} \sin(w_i t) \quad (2.12)$$

where, i ranges from 1 to number of DOF and X_i is the amplitude of the mode shape of the natural frequency w_i , so:

$$([K] - w_i^2 [M])\{X_i\} = 0 \quad (2.13)$$

The roots of this equation are the eigenvalues or natural frequencies, w_i , and $\{X_i\}$ values are the eigenvectors or mode shapes of the system when eigenvalues are substituted back into the equation, where. Block Lanczos method is used to extract eigenvalues and eigenvectors of large FE models in ANSYS.

2.5.1. Finite Element Analysis Model

In this thesis, blisk geometry which has cyclic symmetry boundary condition and blade-only geometry are modelled with using FEA program, ANSYS Workbench (WB) and computer aided design (CAD) program, Siemens NX.

2.5.1.1. Geometry Preparation

Firstly, geometries are prepared for discretization using Siemens NX computer aided design program. For blisk geometry, it is needed to create a sector cut model to be able to apply cyclic boundary condition and reduce the model in order to decrease

computational time. Sector model usually includes one blade geometry, so full blisk geometry is sliced into 35 pieces because the number of the blades in the stage is 35 and sector angle is $360/35$ degree. Also, blisk sector geometry is sliced on the back where the attenuation length is sufficient and to calculate attenuation length below formula is used:

$$L = 1.25\sqrt{R \times t} \quad (2.14)$$

where L is attenuation length, R is inner radius, which is equal to 29 mm, and t is thickness of the part, which is equal to 2.72 mm, at the slice location. By back substituting R and t , L is equal to 11.1 mm which is smaller than 12 mm, so the slice position on the back is sufficient in terms of not to effect deflection or stresses on the geometry [39]. Full blisk, blisk sector cut and blade-only models are shown in Figure 2.24.

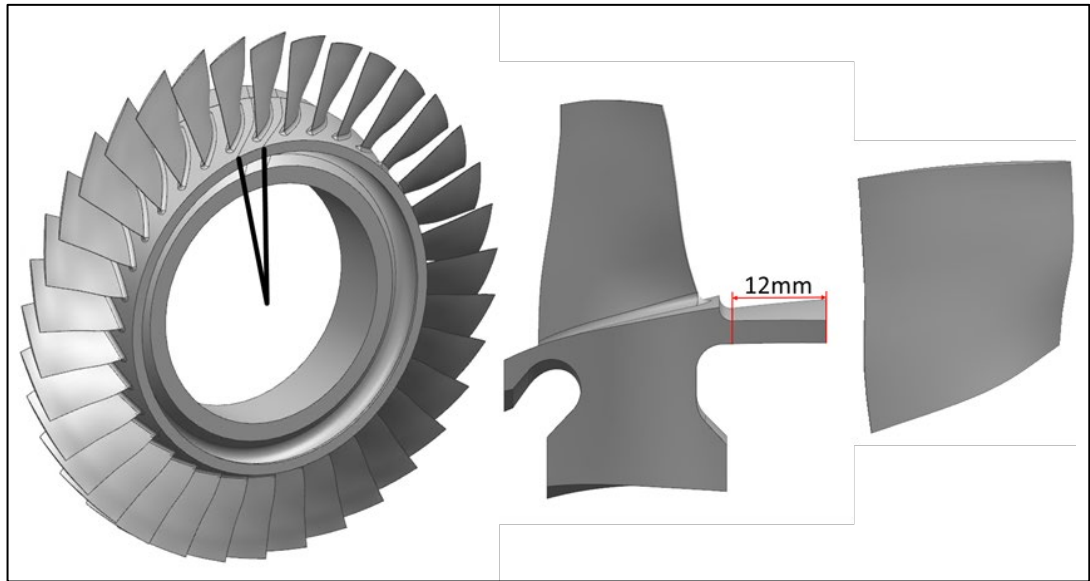


Figure 2.24: Full(left), Sector Cut(middle) and Blade-only(right) Models.

2.5.1.2. Meshing

By discretizing the geometry into elements and nodes, which is also called as meshing, finite element models are created. SOLID186 quadratic element type, which is higher order 3D 20-node solid element with mid-side nodes, is used for modelling

and geometries are meshed mostly with hexahedral option of SOLID186 which also has tetrahedral, pyramid and prism options shown in Figure 2.25.

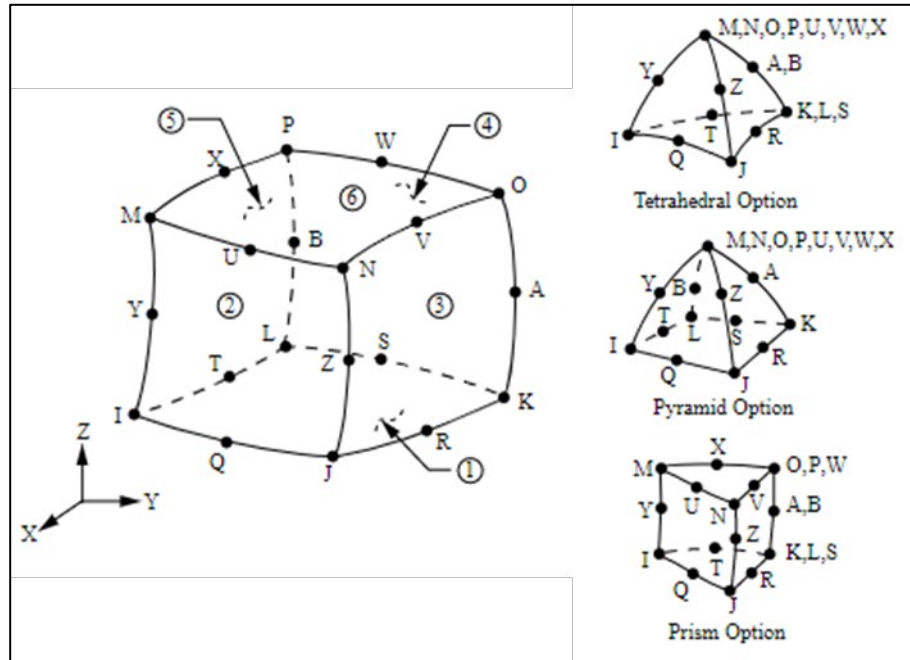


Figure 2.25: SOLID186 Element Type.

For the mesh convergence study, the convergence criterion is selected as %4. The difference between 1.25 mm and 2.5 mm mesh sizing is lower than %4 for the equivalent stress (Seqv), so, the mesh is converged for 1.25 mm mesh sizing. 43771 elements and 78301 nodes are used for blisk sector model, 424 elements and 3176 nodes are used for blade-only model and meshes of both two models are shown in Figure 2.27.

Table 2.1: Mesh Convergence.

Sizing[mm]	10 mm	5 mm	2.5 mm	1.25 mm
#Nodes	96	268	857	3176
Seqv [Mpa]	373.51	442.23	495.89	512.46
	Difference	15.54%	10.82%	3.23%
1F [Hz]	1699.20	1658.90	1646	1643.80
	Difference	2.43%	0.78%	0.13%
1T [Hz]	6129.10	6070.00	6053.30	6050.10
	Difference	0.97%	0.28%	0.05%
2F [Hz]	7897	7780	7703.5	7675.1
	Difference	1.50%	0.99%	0.37%

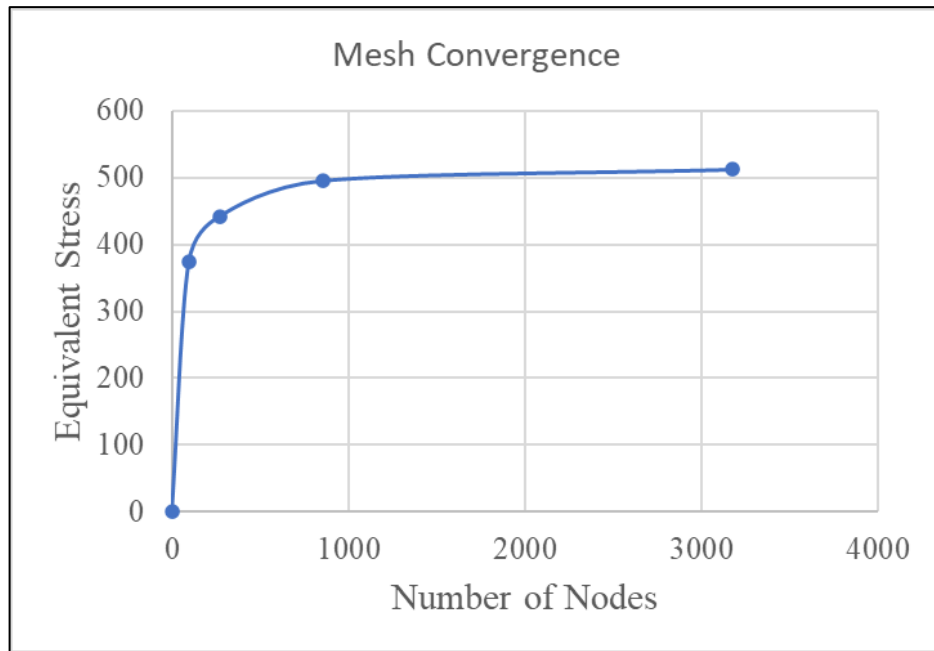


Figure 2.26: Mesh Convergence Graphic for Seqv vs. Node Number.

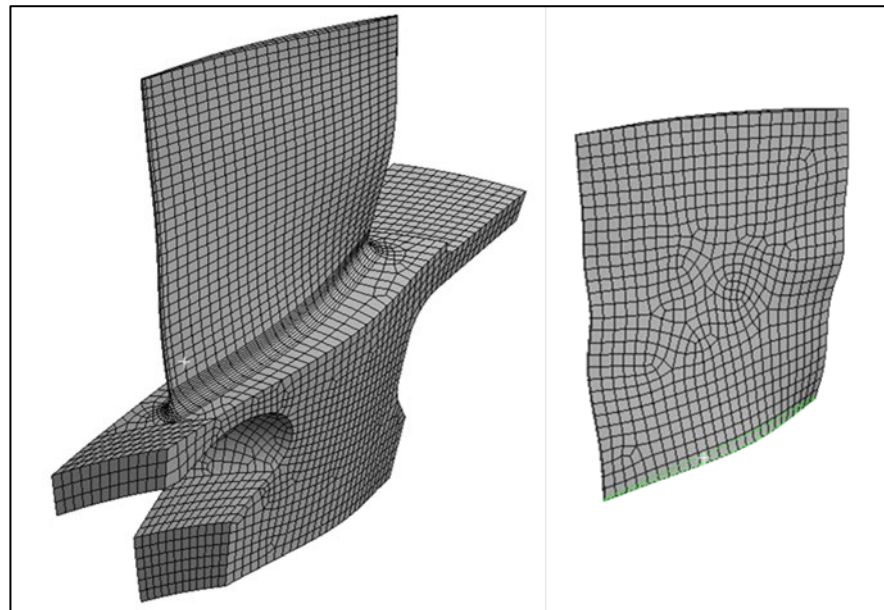


Figure 2.27: Meshes of Sector and Blade-only Models.

2.5.2. Boundary Conditions

After discretization, boundary conditions such as displacements, and loads such as temperatures, pressures, speed can be applied on the nodes for static structural analysis, displacement boundary condition is applied for stress-free modal analysis and prestressed modal analysis imports boundary conditions from the results of static

structural analysis. Firstly, material properties such as density, elastic modulus, Poisson's ratio, and thermal expansion coefficient are needed to be defined in order to be able to solve analyses. So, both sector and blade-only geometries are modelled defining Ti64 material which shows isotropic behaviour. Material properties of the Ti64 is shown at the Table 2.2 [40].

Table 2.2: Material Properties of Ti64 at room temperature.

Material Properties at room temperature	Ti-6Al-4V
Modulus of Elasticity [GPa]	117
Poisson's Ratio	0.31
Density[kg/m ³]	4430
Thermal Expansion Coefficient [1/°C]	8.82x10 ⁻⁶
Yield Strength [MPa]	880
Ultimate Tensile Strength [MPa]	950

For blisk sector model, cyclic symmetry boundary condition is applied on sector faces and the FEA models are fixed in all directions at back sliced region on disk as shown in Figure 2.28. The rotational velocity at the design point (DP) of the blade is applied at static structural analysis. DP velocity is 48000 RPM, idle/min. cruise is %87.5 of the DP which is equal to 42000 RPM and redline/max transient is %112.5 of the DP which is equal to 54000 RPM, shown in Figure 2.28.

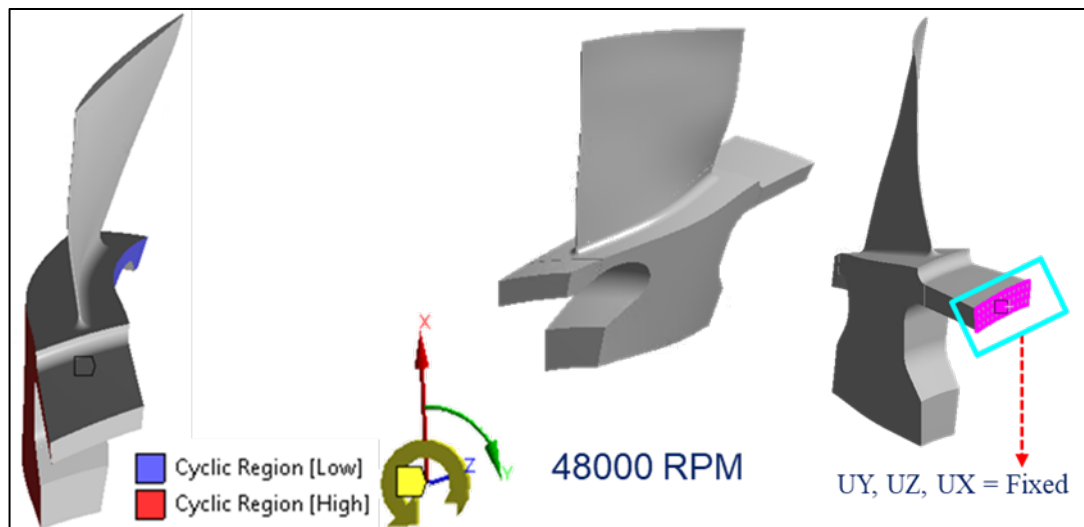


Figure 2.28: Cyclic Symmetry BC, Rotational Speed and Displacement BC.

For blade-only model, FEA models is fixed in all direction at hub the rotational velocity same as blisk sector model is applied as shown in Figure 2.29.

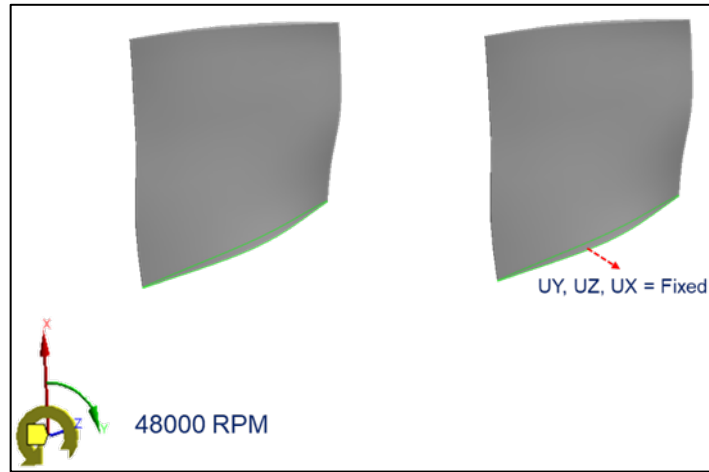


Figure 2.29: Rotational Speed and Displacement BC.

Temperature is applied and temperature distributions are shown in Figure 2.30, pressure is applied to both two models for blade and flow path on disk as shown in Figure 2.31.

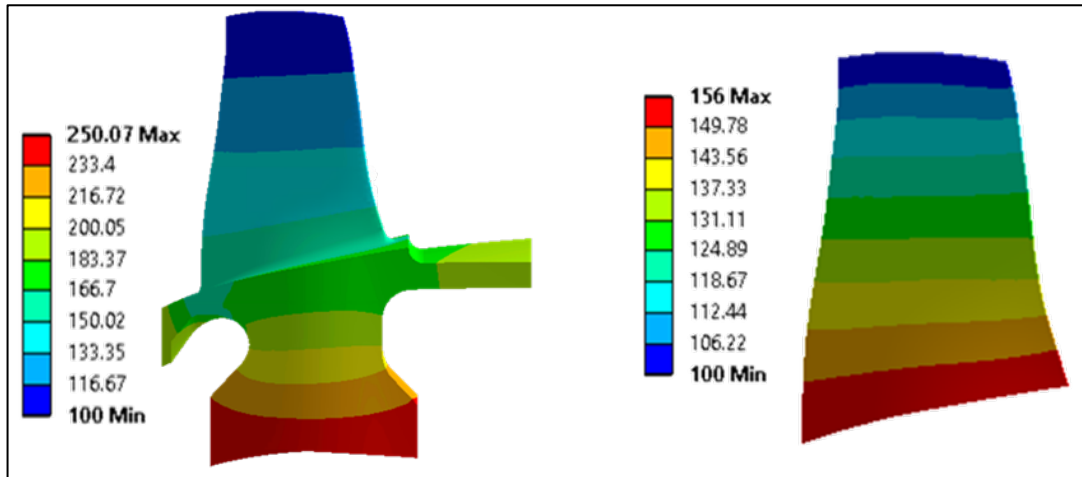


Figure 2.30: Temperature Distributions in Celsius [°C].

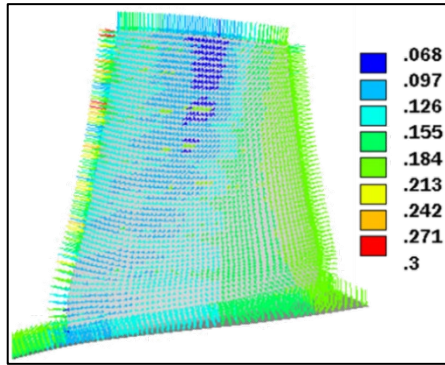


Figure 2.31: Pressure Distributions [MPa], arrows show applied direction.

Also, the large deflection option, which includes behaviours such as large rotation and stress stiffening to account for changes in stiffness due to changes in shape of the geometry, is recommended to be used during analysis.

2.5.3. Modal Analyses Results and Comparison of Results

While performing modal analyses, the geometry is assumed to show linear dynamic behaviour and in steady-state condition. So, the modal analysis performed for the geometry which is deformed under working conditions is called as prestressed modal analysis and static structural analysis is performed to find the deformed geometry under working conditions. The modal analysis performed for the undeformed geometry is called as stress-free modal analysis. The project scheme of the study conducted in this thesis is shown in Figure 2.32.

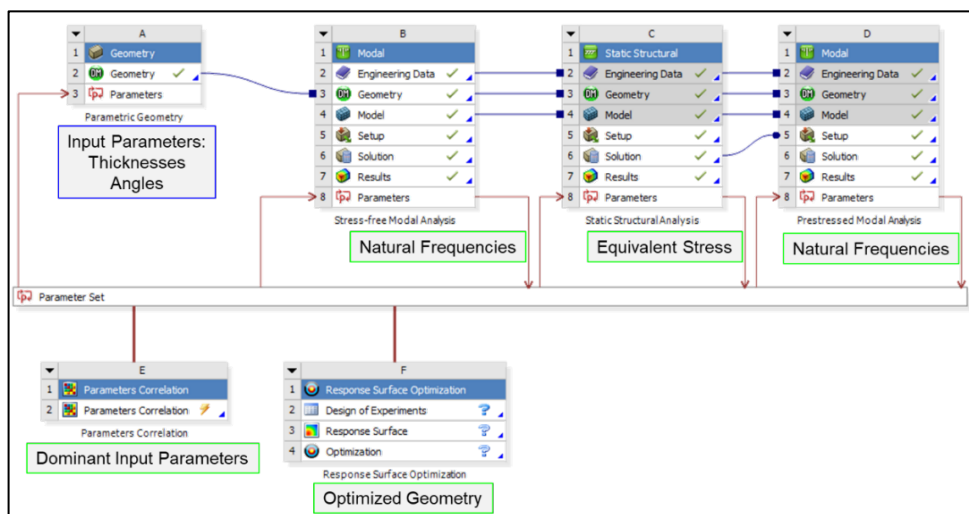


Figure 2.32: Project Scheme.

Stress-free and prestressed modal analyses results of blisk (BD) sector and blade-only (BO) models are compared at Table 2.3 and Table 2.4, mode shapes are matched up with each other using mode numbers and shown at matching columns with their differences.

Natural frequencies of same mode shapes are matched up and modes without any matching are disk dominant modes which mean the blade is not able to affect the mode frequency. Mode shapes of first three fundamental modes and their natural frequencies are shown respectively in Figures 2.33-35.

Table 2.3: Stress-free Modal Analyses Result Comparison.

Stress-free Modal Analyses			Blisk (BD)		Blade-only (BO)	
Mode Shape Matching			Modes		Modes	
BD	BO	Difference	#	[Hz]	#	[Hz]
1	1	0.74%	1	1656.1	1	1643.8
2	2	0.13%	2	6057.8	2	6050.1
3	3	0.82%	3	7612.5	3	7675
6	4	4.04%	6	13650	4	13099
7	5	1.24%	7	14970	5	14784
9	6	0.08%	9	17781	6	17795
10	7	7.31%	10	18418	7	19764
11	8	0.65%	11	22718	8	22570
12	9	4.13%	12	24815	9	25840
13	10	1.29%	13	27946	10	27585
14	11	0.69%	14	29253	11	29052
16	12	1.12%	16	34352	12	33968
18	13	0.90%	18	36883	13	36550
19	14	1.88%	19	39712	14	38965
20	15	2.66%	20	40341	15	39268

Table 2.4: Prestressed Modal Analyses Result Comparison.

Prestressed Modal Analyses			Blisk (BD)		Blade-only (BO)	
Mode Shape Matching			Modes		Modes	
BD	BO	Difference	#	[Hz]	#	[Hz]
1	1	0.58%	1	2361.1	1	2347.4
2	2	0.08%	2	6108.7	2	6103.7
3	3	2.20%	3	8252	3	8433.4
6	4	3.30%	6	13891	4	13432
7	5	1.45%	7	15059	5	14840
9	6	2.96%	9	17888	6	18417
10	7	5.00%	10	18975	7	19924
11	8	0.71%	11	22948	8	22786
12	9	3.37%	12	25262	9	26114
13	10	0.72%	13	28175	10	27971
14	11	0.89%	14	29664	11	29399
16	12	1.19%	16	34719	12	34305
18	13	0.83%	18	37022	13	36714
19	14	1.90%	19	40039	14	39277
20	15	2.62%	20	40965	15	39893

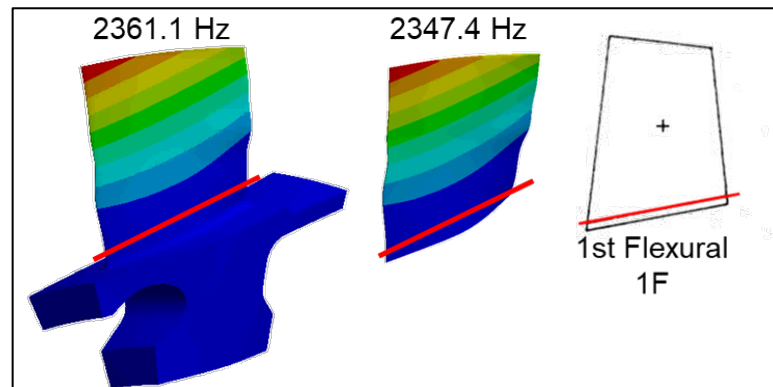


Figure 2.33: Blisk & Blade-only 1F Mode Shape.

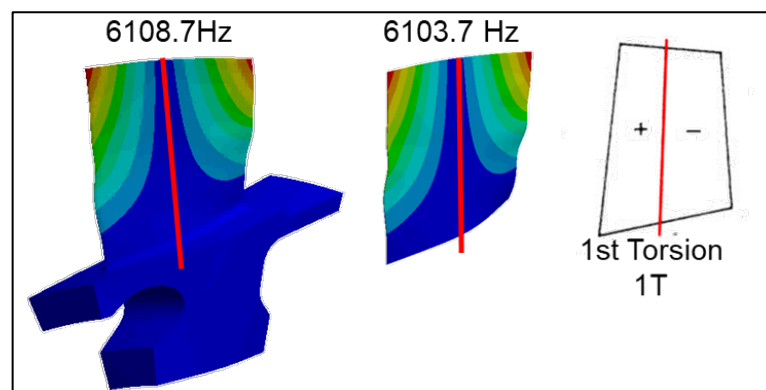


Figure 2.34: Blisk & Blade-only 1T Mode Shape.

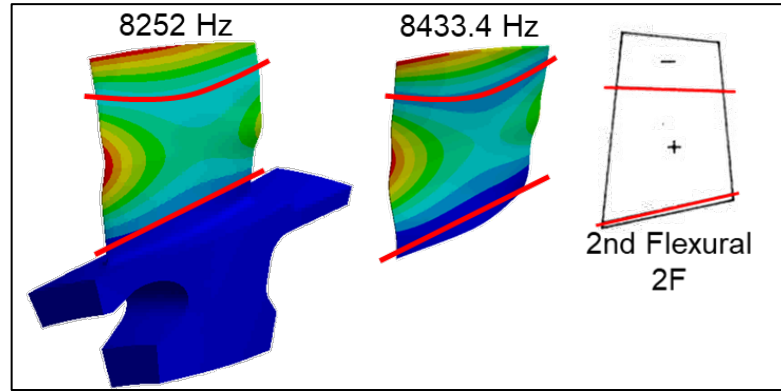


Figure 2.35: Blisk & Blade-only 2F Mode Shape.

For the first three fundamental blade modes, 1F, 1T and 2F, the differences in both results are lower than %5, so blade-only results are sufficient enough to use for parameter correlation and optimization in order to reduce computational time.

2.5.4. SAFE and Campbell Diagrams

In order to draw SAFE diagram, it is needed to use sector model result because the model includes natural frequency information for each nodal diameter. SAFE and Campbell diagrams are used to find out full resonance conditions which is needed to be optimized. Nodal diameter number is calculated using equation 2.6, where the number of the blades is 35 and consequently nodal diameter number is equal to 17. SAFE and Campbell diagrams, which are plotted using sector model results, are shown in Figure 2.36 and Figure 2.37.

Black lines represent excitation orders which are first four EO caused by inlet flow distortion, 6 struts, 34 is the difference between forward and backward excitation sources and 40 is the number of backward stator vane. As seen from the diagrams above, 1st mode – 1F has coincidences with 3rd excitation order at Campbell Diagram and with 3rd nodal diameter at SAFE Diagram, which means there is a full resonance condition in operating range. So, to have more competitive design, this full resonance condition is needed to be avoided.

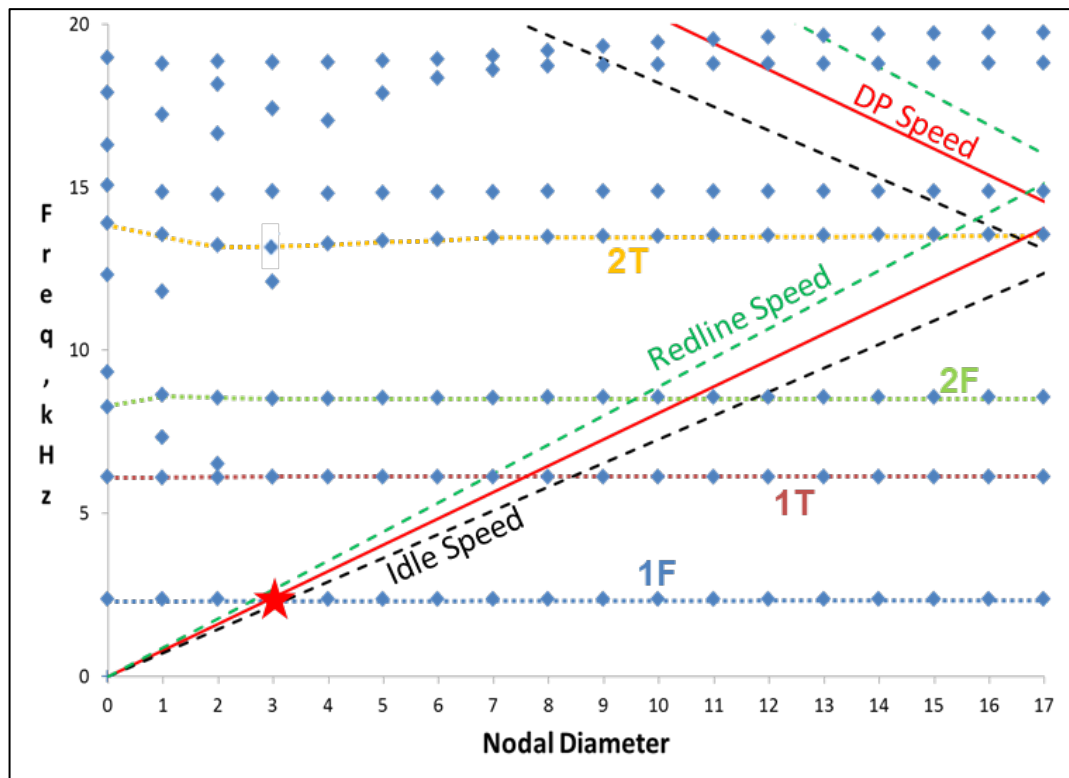


Figure 2.36: SAFE Diagram.

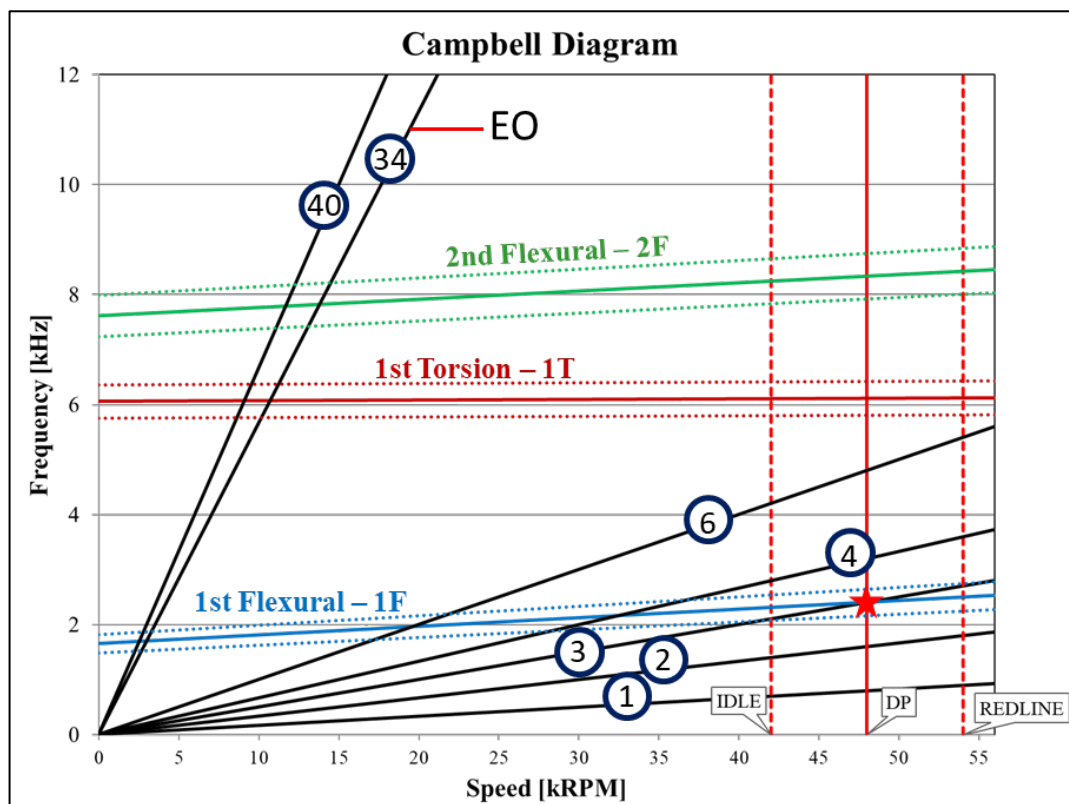


Figure 2.37: Campbell Diagram.

2.6. Parameter Correlation & Energy Density Regions

2.6.1. Parameter Correlation or Sensitivity Analysis

Parameter correlation or sensitivity analysis is a statistical method used to define the relation between the input (independent) and the output (dependent) parameters, to evaluate the degree of the relationship, and the dominant or more relevant input parameters. By finding dominant parameters, the number of input parameters is reduced for optimization and response surfaces can be created more accurately with more related input parameters. ANSYS has two correlation types or sensitivity analysis method options which are Pearson and Spearman methods. Pearson linear correlation is used when the parameters are continuous, and the data of the variables show normal distribution. Pearson method directly evaluates the relationship according to the values of the parameters. Spearman rank correlation is used to evaluate the relationship degrees of parameters by ranking the parameters by their magnitudes if input and output parameters do not show a linear distribution. Spearman is suggested to be used to find more dominant parameters and more accurate than Pearson. ANSYS calculates Spearman rank correlation coefficient for every parameter by using following formula [37]:

$$r = \frac{\sum_i^n (R_i - \bar{R})(S_i - \bar{S})}{\sqrt{\sum_i^n (R_i - \bar{R})^2} \sqrt{\sum_i^n (S_i - \bar{S})^2}} \quad (2.15)$$

where r is rank correlation coefficient, n is number of parameters, R_i is rank of input parameter, S_i is rank of output parameter, $\bar{R}$ and $\bar{S}$ average ranks of R_i and S_i respectively. Range of r is between -1 and 1 as $-1 \leq r \leq 1$. If absolute of r is close to 1, it shows the relation degree is strong and if absolute of r is close to 0, it shows the relation degree is weak. If r is close to -1, it means the relation is inversely proportional and r is close to 1, it means the relation is directly proportional. Correlation Matrix is used to define relations between parameters in ANSYS WB.

Also, coefficient of determination, r^2 , which indicates what percentage of the variation in the dependent or output parameters can be explained by the variation in

input parameters. Determination Histogram is used to define relevance between parameters in ANSYS WB.

2.6.2. Strain Energy Density Regions

Strain energy density (SED) is strain energy divided by volume of the elements and strain energy is energy storage capacity until failure. Strain energy density plots can be used to indicate the critical regions due to damage or insufficient stiffness for each mode shape of the structure. The relation between element stiffness, strain energy and natural frequency can be expressed simply as follows:

$$U = \frac{1}{2} k u^2 \quad (2.16)$$

where U is strain energy, k is stiffness, u is displacement and stiffness matrix of a four-node-element under bending is expressed as follows and variables are shown in Figure 2.38.

$$[k]_e = \frac{Et^3}{48(1-\nu^2)ab} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \quad (2.17)$$

where $[k]_e$ is stiffness matrix, N is shape function, t is plate thickness, a and b are plate width and length, E is modulus of elasticity, ν is Poisson's ratio, ρ is density of the material and $[m]_e$ is material matrix:

$$[m]_e = \rho t \int [N]^T [N] dA \quad (2.18)$$

Natural frequency formula for free vibration without damping condition is:

$$w = \sqrt{\frac{[k]_e}{[m]_e}} \propto \frac{t^3}{t} \quad (2.19)$$

So, stiffness changes with third power of thickness and mass changes directly with thickness. In order to increase a natural frequency for a mode shape, adding thickness into the high strain energy density regions affects more by increasing energy storage capacity of the weak regions of structure.

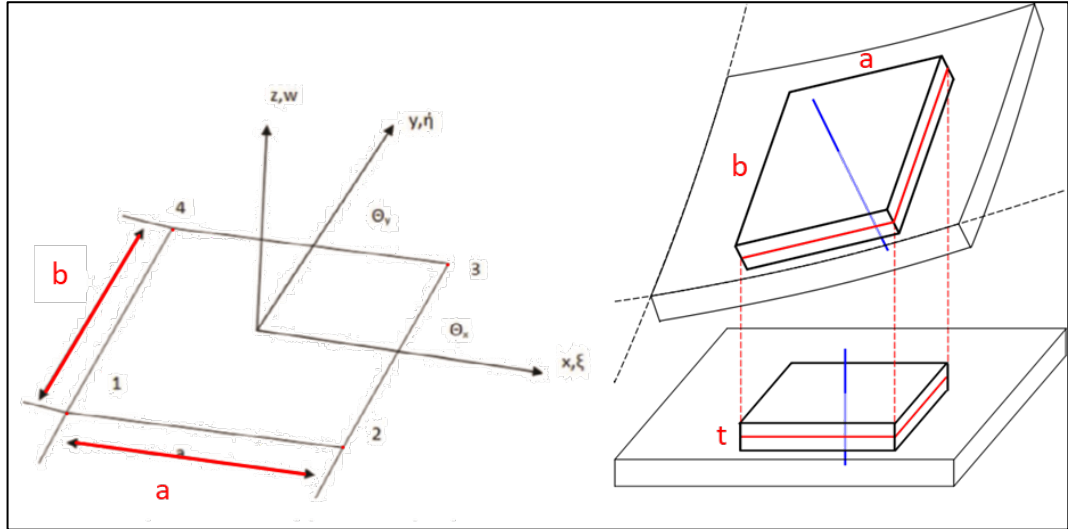


Figure 2.38: 4-node-element Variables.

2.6.3. Sensitivity Analysis Results & Strain Energy Density Regions

Dominant parameters and relations between parameters are obtained from parameter correlation results using determination histograms, sensitivity histograms and correlation matrices. Sensitivity analysis is performed on 25 thickness parameters and 9 angle parameters. ANSYS WB allows 29 parameters for sensitivity analysis, so, firstly the analysis is performed within thickness parameters and these parameters are reduced to 6 dominant parameters. Then, angle parameters are included in dominant thickness parameters to find all dominant parameters by performing another sensitivity analysis. In this way, input parameters are reduced to 6 thickness and 4 angle dominant parameters. Finally, results of sensitivity analyses are compared with strain energy density regions as shown in Figure 2.39, Figure 2.40 and Figure 2.41. Correlation matrix of dominant parameters is shown in Table 2.5. Blue and red backgrounds mean inversely and directly proportion between input and output parameters.

Table 2.5: Correlation Matrix of Dominant Parameters.

Parameter Correlation			Output Parameters			
			Natural Frequency			Equivalent Stress
			1st Flexural	1st Torsion	2nd Flexural	
Input Parameters	Thickness	L1T4	22.30%	13.10%	25.70%	-20.60%
		L1T5	17.80%	12.00%	26.10%	2.30%
		L3T5	65.50%	49.80%	22.40%	-4.90%
		L3T6	64.00%	38.10%	18.50%	-1.10%
		L5T4	9.00%	25.90%	43.30%	9.90%
		L5T5	14.40%	39.10%	40.50%	5.90%
	Angle	L5A1	0.30%	27.90%	12.80%	47.90%
		L5A3	-3.80%	-43.20%	62.20%	82.30%
		L9A1	0.70%	-13.50%	-1.50%	8.70%
		L9A3	-1.00%	-8.70%	-2.70%	3.80%

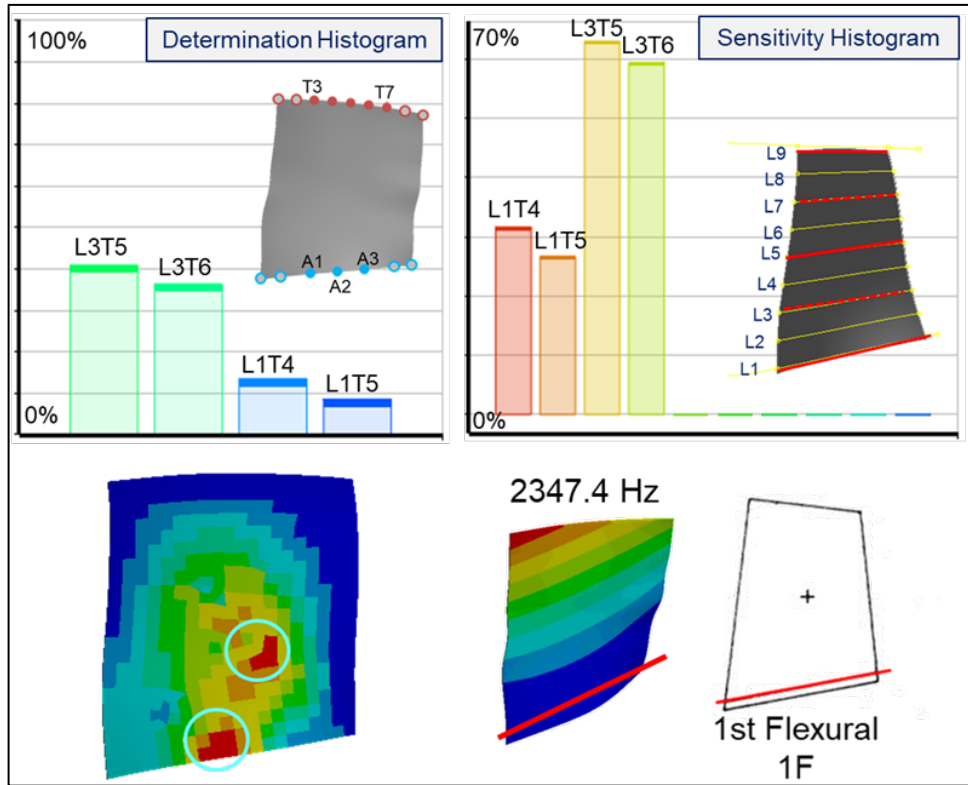


Figure 2.39: Determination(Left-Top) and Sensitivity(Right-Top) Histogram, SED Plot(Left-Bottom) and Mode Shape(Right-Bottom) for 1st Flexural Mode.

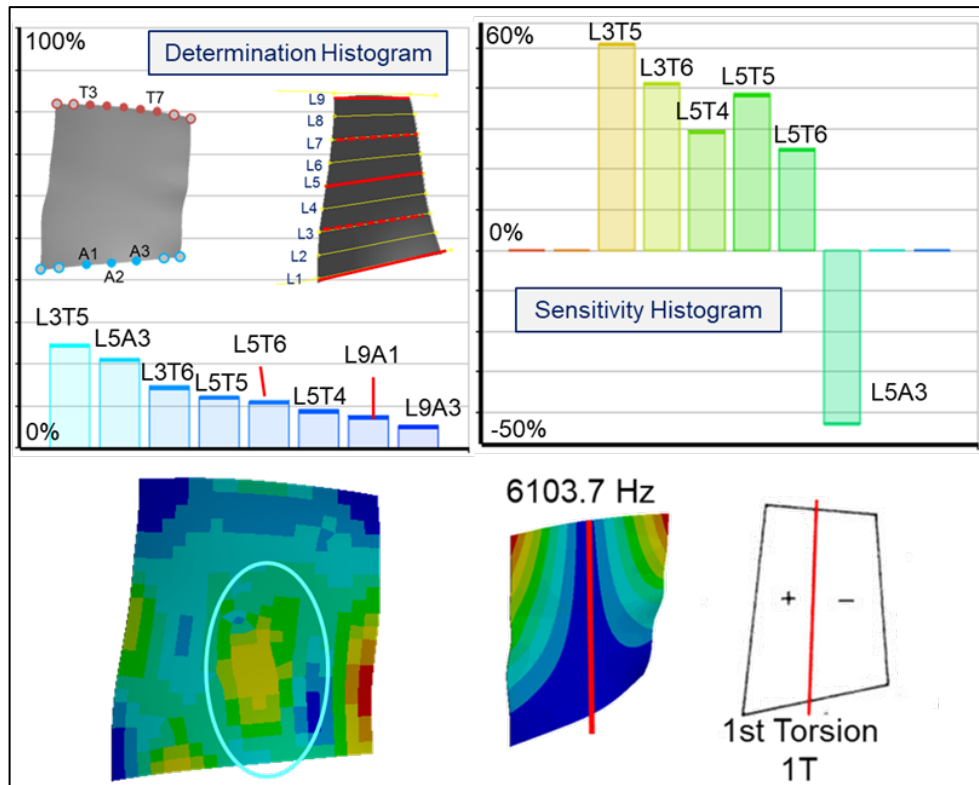


Figure 2.40: Determination(Left-Top) and Sensitivity(Right-Top) Histogram, SED Plot(Left-Bottom) and Mode Shape(Right-Bottom) for 1st Torsion Mode.

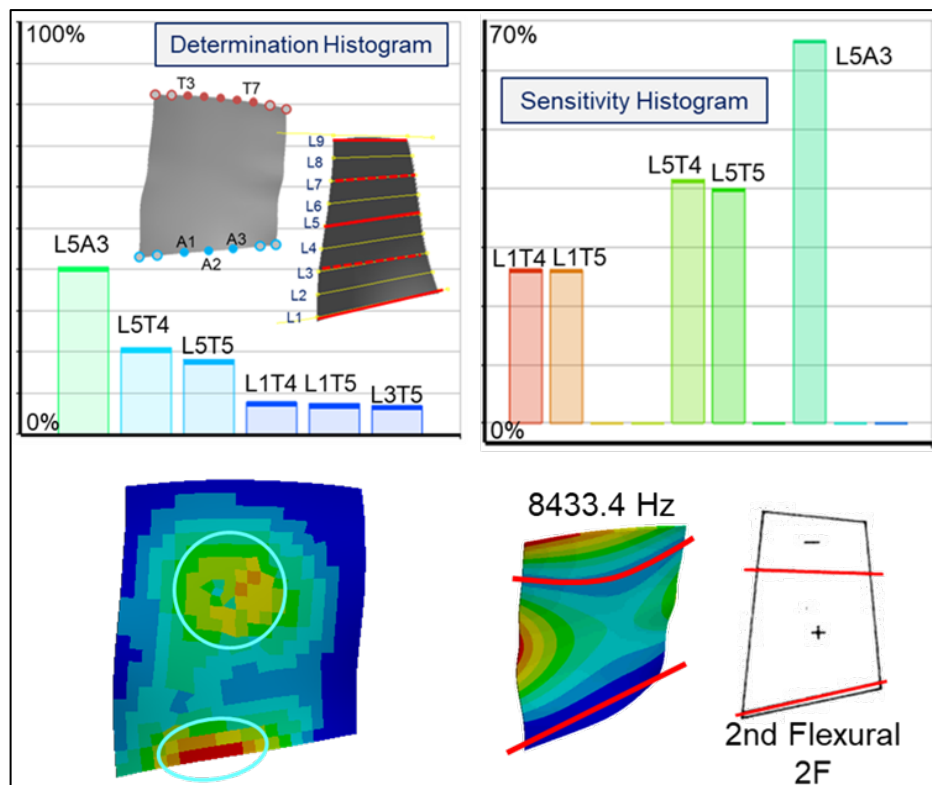


Figure 2.41: Determination(Left-Top) and Sensitivity(Right-Top) Histogram, SED Plot(Left-Bottom) and Mode Shape(Right-Bottom) for 2nd Flexural Mode.

Result comparisons show parallelism which means high SED regions and dominant parameters obtained from histograms are at the same location. Relevance levels of parameters located at hub layer stay low because fixed boundary condition makes hub region as if it has infinite stiffness, but this region does not have infinite stiffness in reality. So, parameters at the high SED locations on the blade can be taken as dominant thickness parameters. There are no dominant parameters at some of the high SED locations because parameters are fixed at that locations such as trailing edge and leading edge. Thus, dynamically weak regions can be found using only one modal analyses and strain energy regions.

2.7. Optimization Methodology

Optimization is the whole process of developing and improving a design under desired conditions iteratively creating design space. ANSYS WB provides goal driven optimization (GDO) which is a multi-objective technique that obtains the best possible designs using design points and goals. There are two GDO systems which are Response Surface Optimization (RSO) that is economically beneficial and Direct Optimization (DO) in ANSYS WB DesignXplorer module.

DO is gradient-based that finds local optima and requires more computational effort because it uses real analysis results rather than metamodels and RSO is gradient-free, much more time efficient and also provides accurate results creating metamodels or surrogate models of problems. Metamodels are used to represent statistical relations between parameters in order to subrogate expensive computational times. In this study, Response Surface Optimization method is used due to its computational effort benefits.

First step of the RSO is to create design space using DOE method. Then, using the design space, response surfaces are generated. Finally, optimization is carried out.

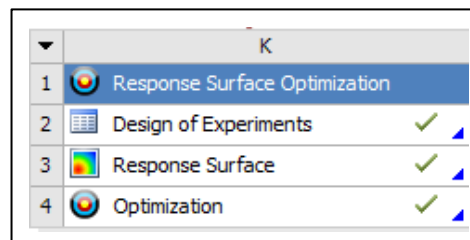


Figure 2.42: RSO Scheme.

2.7.1. Design of Experiment (DOE)

RSO utilizes DOE methodology that effectively samples design points to create the design space in order to obtain statistical models which help predicting optimum design points accurately.

There several DOE design space filling techniques that try to obtain responses of the design with some sampling points. In this thesis, Central Composite Design (CCD) technique, which provides screening set in order to evaluate the trends of statistical model or metamodel, is used. Enhanced face-centred design type is chosen to have high accurate response prediction and better fit for response surfaces. CCD combines one point at the centre, points on the direction of input parameters and points evaluated by using fractional factorial design. Sampling space configuration for 3D CDD is shown in Figure 2.43 [36].

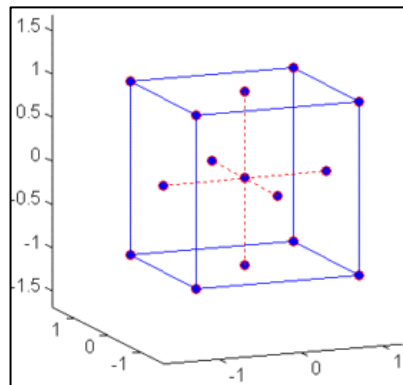


Figure 2.43: Sampling space configuration for 3D Face-centred CCD.

Design of Experiments	
Design of Experiments Type	Central Composite Design
Design Type	Face-Centered
Template Type	Enhanced

Figure 2.44: Selected DOE Options.

CCD allows maximum of 20 input parameters while defining statistical relation between input and output parameters. So, the maximum required design point or analyses with respect to number of input parameters is calculated by using following formula:

$$N=I+ 2^{(n-f)}+2n \quad (2.20)$$

where n is the number of input parameters, f is factorial number and N is the number of required design points and 149 design points is required for 10 input parameters. The list of the number of required design point with respect to the number of input parameter is shown below table:

Table 2.6: The number of Required Design Points.

No. of Input Parameters, n	Factorial Number, f	No. of Design Points, N
1	0	5
2	0	9
3	0	15
4	0	25
5	1	27
6	1	45
7	1	79
8	2	81
9	2	147
10	3	149
11	4	151
12	4	281
13	5	283
14	6	285
15	7	287
16	8	289
17	9	291
18	9	549
19	10	551
20	11	553

The lower and upper bounds for the input parameters are chosen as 50 percent of initial conditions which are shown in Figure 2.45.

		Enabled		
Design of Experiments				
Input Parameters				
Geometry (A1)				
	P11 - L1T3	☐	L1T4 = 1.5 mm	
	P12 - L1T4	☒	Lower Bound	0.75
	P13 - L1T5	☒	Upper Bound	2.25
	P14 - L1T6	☐	L1T5 = 1.44 mm	
	P15 - L1T7	☐	Lower Bound	0.72
	P16 - L3T3	☐	Upper Bound	2.16
	P17 - L3T4	☐	L3T5 = 1.45 mm	
	P18 - L3T5	☒	Lower Bound	0.725
	P19 - L3T6	☒	Upper Bound	2.175
	P20 - L3T7	☐	L3T6 = 1.4625 mm	
	P21 - L5T3	☐	Lower Bound	0.73127
	P22 - L5T4	☒	Upper Bound	2.1938
	P23 - L5T5	☒	L5T4 = 1.25 mm	
	P24 - L5T6	☐	Lower Bound	0.625
	P25 - L5T7	☐	Upper Bound	1.875
	P26 - L7T3	☐	L5T5 = 1.18 mm	
	P27 - L7T4	☐	Lower Bound	0.59
	P28 - L7T5	☐	Upper Bound	1.77
	P29 - L7T6	☐	L5A1 = 1.95°	
	P30 - L7T7	☐	Lower Bound	0.975
	P31 - L9T3	☐	Upper Bound	2.925
	P32 - L9T4	☐	L5A3 = -3.15°	
	P33 - L9T5	☐	Lower Bound	-4.725
	P34 - L9T6	☐	Upper Bound	-1.575
	P35 - L9T7	☐	L9A1 = 2.502°	
	P67 - L1A1	☐	Lower Bound	1.251
	P68 - L1A2	☐	Upper Bound	3.753
	P69 - L1A3	☐	L9A3 = -2.4°	
	P70 - L5A1	☒	Lower Bound	-3.6
	P71 - L5A2	☐	Upper Bound	-1.2
	P72 - L5A3	☒		
	P73 - L9A1	☒		
	P74 - L9A2	☐		
	P75 - L9A3	☒		

Figure 2.45: Enabled Input Parameters for RSO, Lower and Upper Bounds.

2.7.2. Response Surfaces

After creating design space, response surfaces are generated using design points, which are sampled in DOE, in order to obtain response functions of the output parameters with respect to input parameters using metamodels. Efficiently selected sample points in DOE increase the accuracy of the response surfaces while reducing the required number of design points. ANSYS DesignXplorer module provides algorithms to create response surfaces such as Full Second Order Polynomial, Non-parametric Regression, Kriging, Neural Network, Sparse Grid and Genetic Aggregation. Genetic Aggregation method creates metamodel populations with different response surface settings to define which algorithm is the best choice for a particular output parameter in order to have more effective response surfaces. Thus, the Genetic Aggregation method is used in this current study to automatize selection of response surface algorithm and generation of response surface. Thickness parameters show linear correlation with natural frequencies in response surfaces. Prestressed 1F natural frequency response surfaces with angle parameters are shown following figures.

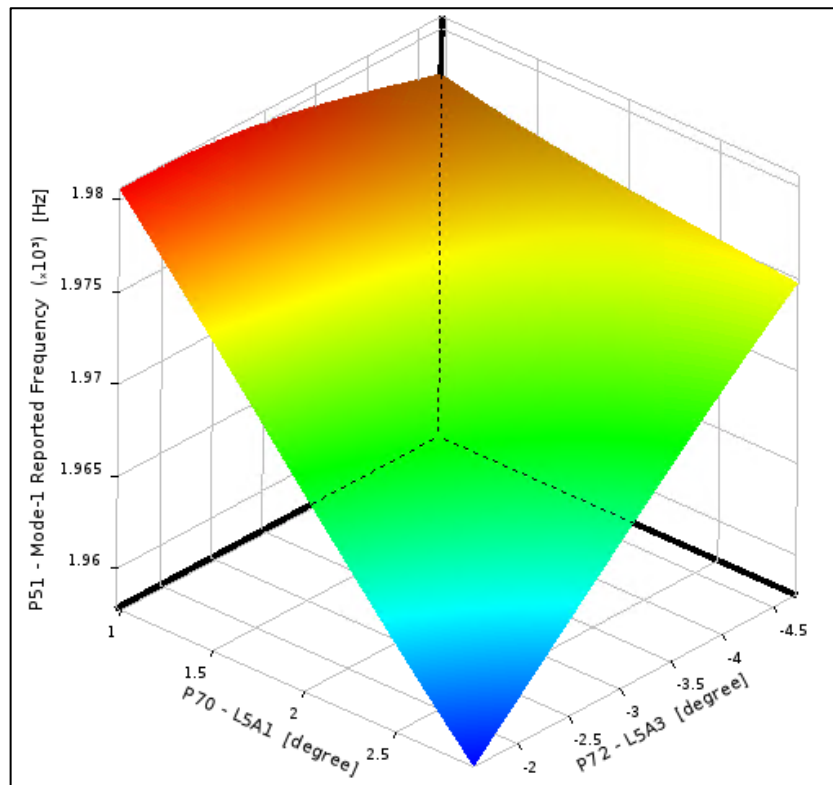


Figure 2.46: Response surface of 1F mode versus angle parameters at 5th layer.

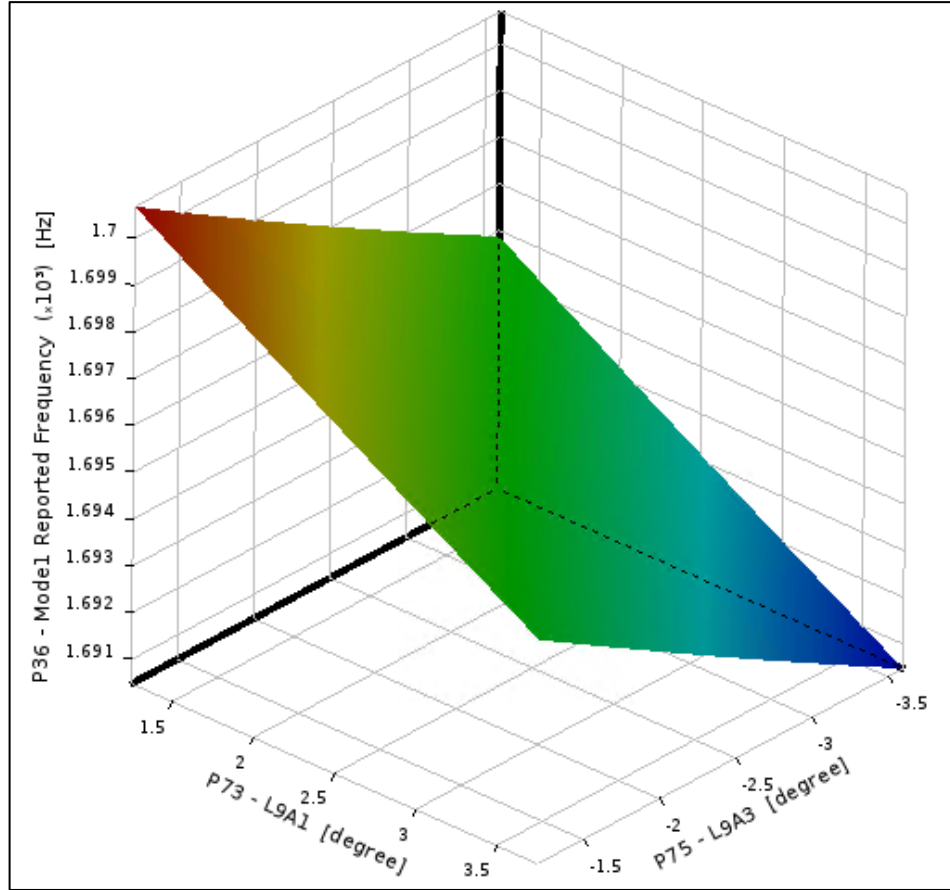


Figure 2.47: Response surface of 1F mode versus angle parameters at 9th layer.

2.7.3. Optimization with Multi-Objective Genetic Algorithm

After generating the response surfaces, exploration of design and correlations of input-output parameters have been discovered, so the next step is to perform optimization by determining constraints and objectives. ANSYS provides several methods for response surface optimization such as Screening, MOGA, Nonlinear Programming by Quadratic Lagrangian (NLPQL) and Mixed-Integer Sequential Quadratic Programming (MISQP). MOGA uses genetic algorithm in order to optimize the design iteratively and it is a better approach to find global optima, while other approaches are ideally suited for finding local optimum design. In this thesis, MOGA approach, which firstly generates the initial samples using genetic algorithm and then uses cross over and mutation for the next populations to iteratively search for the feasible design points, also called as candidate design points using Pareto fronts, is used. Feasible solutions, that satisfy all objectives, are rare. In this situations, Pareto fronts or Pareto optimal solutions are used which are set of solutions when it is not

possible to find a single solution for all objectives and ANSYS uses Pareto ranking method that is a non-dominated sorting method which does not need any penalty functions and ensures that feasible solutions are always ranked higher than infeasible solutions. In summary, MOGA provides a more refined approach than others by going through several iterations and retaining elite design points, which are strong parents at each generation, to help identify global minima or maxima with providing several candidates accurately and allows the samples genetically evolve until the best Pareto set is obtained. MOGA workflow is shown in below figure [36]:

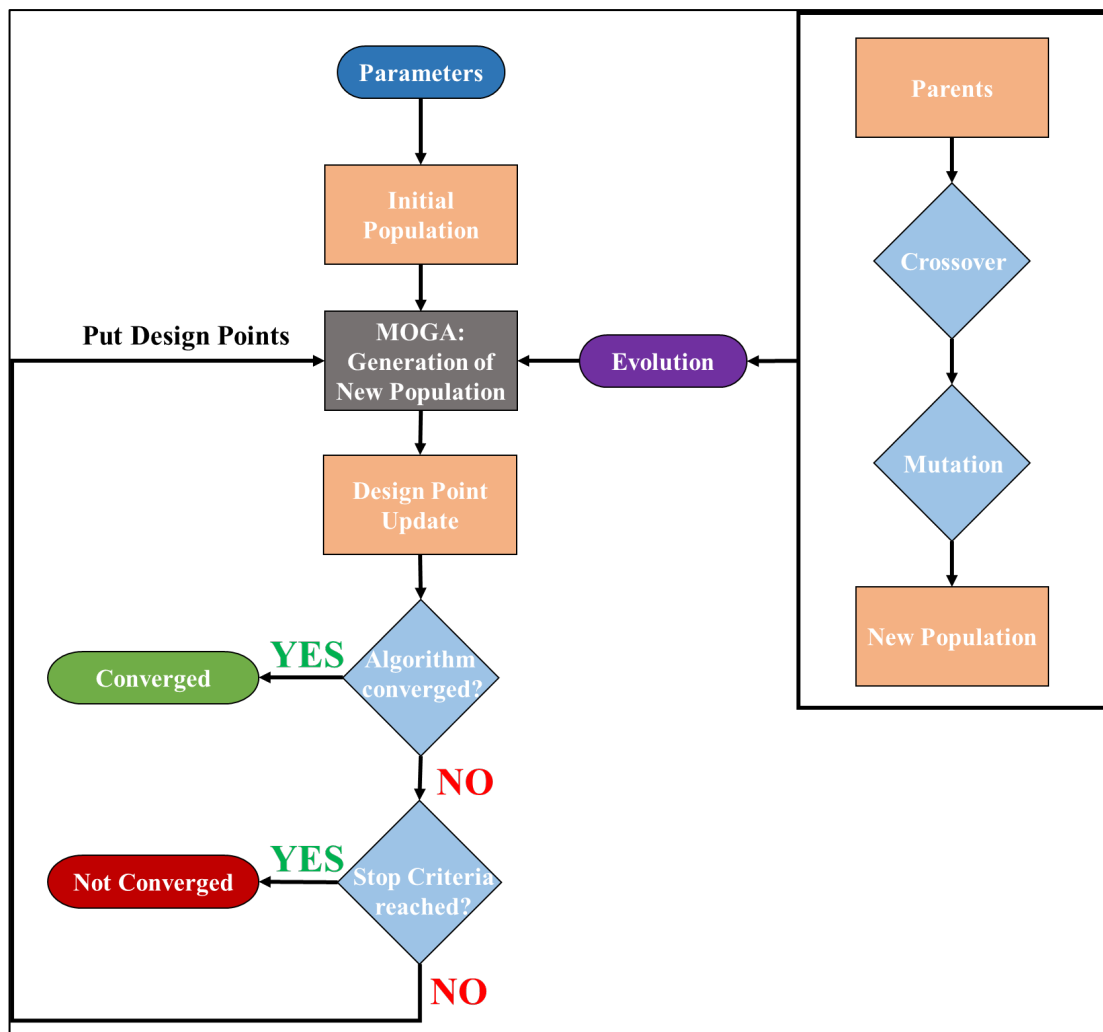


Figure 2.48: MOGA Workflow.

The crossover process linearly combines information two parents (design points) to produce a two new design points using a user-definable crossover probability. The idea behind crossover is that the new design points may be better than both parents if

it takes the best characteristics from each of the parents. Crossover is performed by using below equations, where a is crossover probability constant:

$$NewDesignPoint1 = a \times Parent1 + (1-a) \times Parent2 \quad (2.21)$$

$$NewDesignPoint2 = (1-a) \times Parent1 + a \times Parent2 \quad (2.22)$$

For example, assume two parents with four input parameters selected to perform crossover:

$$Parent1 = (0.3)(1.4)(0.2)(7.4) \text{ and } Parent2 = (0.5)(4.5)(0.1)(5.6) \quad (2.23)$$

If crossover probability constant, a is equal to 0.7, following two new design points are produced:

$$NewDesignPoint1 = (0.36)(2.33)(0.17)(6.86) \quad (2.24)$$

$$NewDesignPoint2 = (0.402)(2.981)(0.149)(6.842) \quad (2.25)$$

The mutation process alters one or more values of input parameters in a design point to result in entirely new design point. With these new design points, the genetic algorithm may be able to get better solutions using user-defined mutation probability constant. Mutation prevents the population from stagnating at any local optima. Mutation is performed by using below equation, where δ is mutation probability constant:

$$NewDesignPoint = (UpperBound-LowerBound)\delta \quad (2.26)$$

2.7.4. Optimization Results

The minimum number of cycles is recommended to be 10 times the number of the input parameters, so the number of input parameters is 10 and the number of initial samples is 100. The maximum number of cycles is 10 to make the process faster and the number of samples per iteration, which are iterated and updated within each

iteration, is recommended to be 100 for RSO. Maximum allowable Pareto percentage is the convergence criteria of the optimization which shows the ratio of the number of desired Pareto points to the number of samples per iteration and better to be between %55 and %75. If it is higher then %80 or lower then %30, convergences will slow down or will result prematurely, so the maximum allowable Pareto percentage is %70. Convergence stability percentage option is the stability convergence criteria of the population based on mean and standard deviation of population. Lastly, maximum number of iterations is the stop criteria which is set to 10. These optimization options and optimization status, which shows result status of optimization, shown above in Figure 2.49.

Optimization			
Method Selection	Manual		
Method Name	MOGA		
Estimated Number of Evaluations	1000		
Verify Candidate Points			
Number of Initial Samples	100		
Number of Samples Per Iteration	100		
Maximum Allowable Pareto Percentage	70		
Convergence Stability Percentage	2		
Maximum Number of Iterations	10		
Mutation Probability	0.01		
Crossover Probability	0.98		
Maximum Number of Candidates	3		

Optimization Status	
Converged	Yes
Pareto Percentage	10
Stability Percentage	1.8223
Number of Iterations	10
Number of Evaluations	986
Number of Failures	0
Size of Generated Sample Set	100
Number of Candidates	3

Figure 2.49: Optimization Options(Left) and Status(Right).

In this study it is main objective and critical to maintain integrity in operating range, so, in order to have more alternating stress margin, equivalent stress on the blade is minimized and to replace the coincidence between 3rd EO and 1F mode out of the operating range a derived parameter is maximized to be higher than 56000 RPM. The derived coincidence RPM parameter is created by using intersection of the two lines formula, where M(equals 3 for 3rd EO) is the number of EO and the intersection is shown in Figure 2.51:

$$1F=ax+b \text{ and } EO=bx+d \quad (2.27)$$

$$ax+c=bx+d \text{ and } d=0 \quad (2.28)$$

$$x = \frac{-c}{a-b} \quad (2.29)$$

$$a = \frac{f_s - f_0}{56000} \text{ and } c = f_0 \quad (2.30)$$

$$b = \frac{M}{60} \quad (2.31)$$

P82	$f_s - f_0$	Output Parameter	870.71
P83	$(f_s - f_0)/56000$	Output Parameter	0.015548
P84	$M/60$	Output Parameter	0.05
P85	$a - b$	Output Parameter	-0.034452
P86	$(-f_0)/(a - b)$	Coincidence RPM-3.EO	49217

Figure 2.50: Derived Parameter.

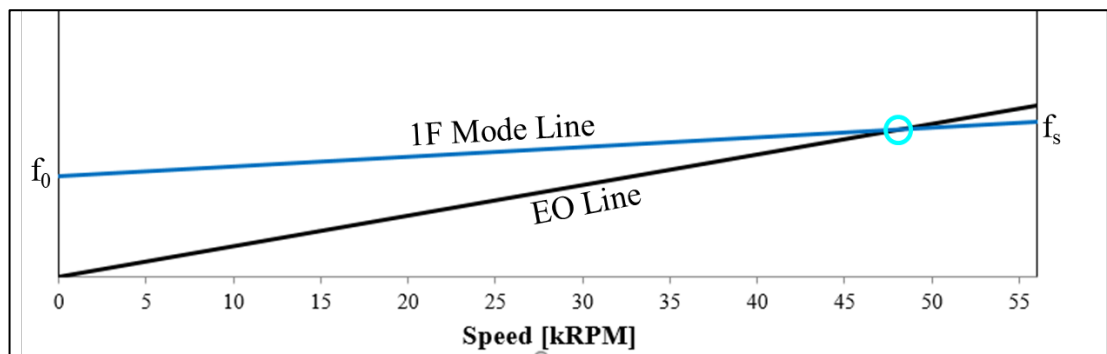


Figure 2.51: Intersection of 3.EO Line and 1F Mode Line.

Objectives, constraints, input parameters and convergence graphic of the derived parameter are shown in following figures.

		Enabled	Monitoring
✓ Optimization			
[-] Objectives and Constraints			
Maximize P86; P86 >= 56000			
Minimize P66			
[-] Domain			
[-] Geometry (A1)			
P12 - L1T4		☒	
P13 - L1T5		☒	
P18 - L3T5		☒	
P19 - L3T6		☒	
P22 - L5T4		☒	
P23 - L5T5		☒	
P70 - L5A1		☒	
P72 - L5A3		☒	
P73 - L9A1		☒	
P75 - L9A3		☒	

Name	Parameter	Objective		Constraint		
		Type	Target	Type	Lower Bound	Upper Bound
Maximize P86; P86 >= 56000	P86 - Coincidence RPM-3.EO	Maximize		Values >= Lower Bound	56000	
Minimize P66	P66 - Equivalent Stress Maximum	Minimize		No Constraint		

Figure 2.52: Objectives, Constraints and Input Parameters.

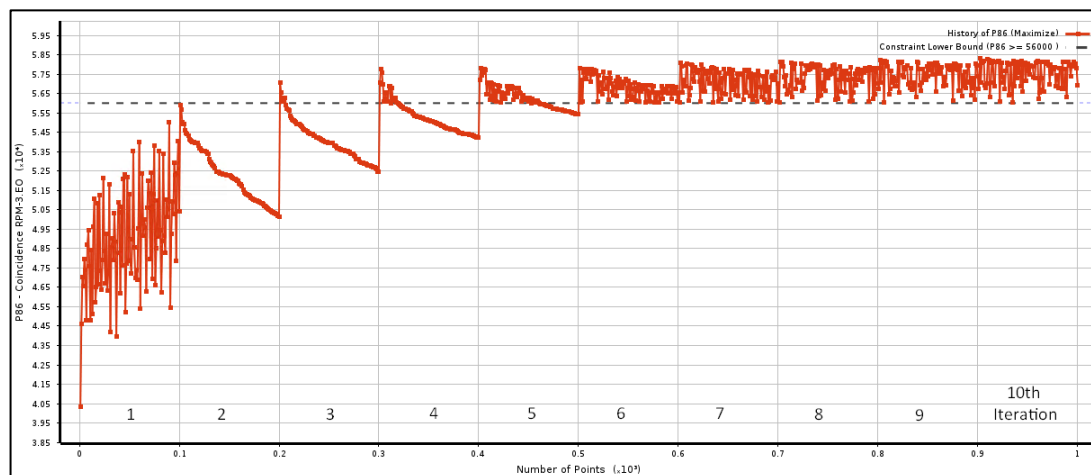


Figure 2.53: The Derived Coincidence RPM Parameter Convergence Graphic.

After the optimization is run in 10 iterations, optimizer provides predicted optimum candidate design points as shown in Figure 2.54 and Figure 2.55.

Candidate Points			
	Candidate Point 1	Candidate Point 2	Candidate Point 3
P12 - L1T4 (mm)	2.2381	2.25	2.2379
P13 - L1T5 (mm)	2.123	2.16	2.1031
P18 - L3T5 (mm)	2.1734	2.175	2.1734
P19 - L3T6 (mm)	2.186	2.1938	2.1858
P22 - L5T4 (mm)	1.8513	1.875	1.8402
P23 - L5T5 (mm)	1.7305	1.77	1.7301
P70 - L5A1 (degree)	1.2727	1.95	1.2727
P72 - L5A3 (degree)	-1.8168	-3.78	-1.8141
P73 - L9A1 (degree)	3.4845	3.002	3.4845
P75 - L9A3 (degree)	-1.3822	-2.88	-2.2589
P66 - Equivalent Stress Maximum (MPa)	★ 528.01	★★ 452.75	★ 525.85
P86 - Coincidence RPM-3.EO	★★★ 58507	★★★ 58416	★★★ 58398

Figure 2.54: Candidate Design Points.

Candidate Points				
	Candidate Point 1	Candidate Point 2	Candidate Point 2 (verified)	Candidate Point 3
P12 - L1T4 (mm)	2.2381	2.25		2.2379
P13 - L1T5 (mm)	2.123	2.16		2.1031
P18 - L3T5 (mm)	2.1734	2.175		2.1734
P19 - L3T6 (mm)	2.186	2.1938		2.1858
P22 - L5T4 (mm)	1.8513	1.875		1.8402
P23 - L5T5 (mm)	1.7305	1.77		1.7301
P70 - L5A1 (degree)	1.2727	1.95		1.2727
P72 - L5A3 (degree)	-1.8168	-3.78		-1.8141
P73 - L9A1 (degree)	3.4845	3.002		3.4845
P75 - L9A3 (degree)	-1.3822	-2.88		-2.2589
P66 - Equivalent Stress Maximum (MPa)	★ 528.01	★★ 452.75	★★ 459.15	★ 525.85
P86 - Coincidence RPM-3.EO	★★★ 58507	★★★ 58416	★★★ 58454	★★★ 58398

Figure 2.55: Candidate Design Points and Verification of DP2.

As seen from above figures, 2nd candidate satisfies the objectives and constraint, so, the design is optimized in terms of integrity concerns. The design comparison of both initial and optimum design is shown below:

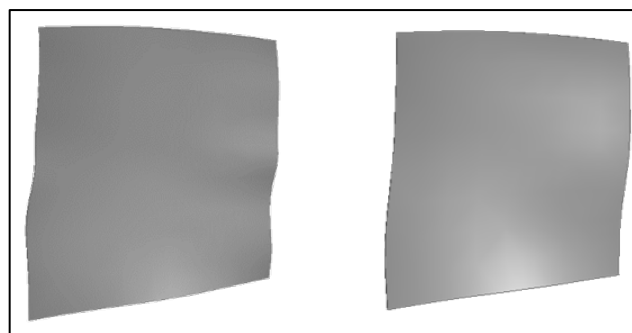


Figure 2.56: Initial Design (Left) and Optimum Design (Right).

Table 2.7: Input Parameter Comparison of Designs.

Optimization			Design Points	
			Initial (2.295 gram)	Optimum (2.539 gram)
Input Parameters	Thickness [mm]	L1T4	1.5	2.25
		L1T5	1.44	2.16
		L3T5	1.45	2.175
		L3T6	1.463	2.194
		L5T4	1.25	1.875
		L5T5	1.18	1.77
	Angles [°]	L5A1	1.95	1.56
		L5A3	-3.15	-3.78
		L9A1	2.502	3.002
		L9A3	-2.4	-2.88
Output Parameters	Equivalent Stress[MPa]		512.46	459.12
	1F (Stress-Free)[Hz]		1643.8	2156.9
	1F (Prestressed)[Hz]		2347.4	2721.7
	1T (Stress-Free)[Hz]		6050.1	6862.1
	1T (Prestressed)[Hz]		6103.7	6968.8
	2F (Stress-Free)[Hz]		7675	8922.9
	2F (Prestressed)[Hz]		8433.4	9520.6
	Resonance		Yes	No

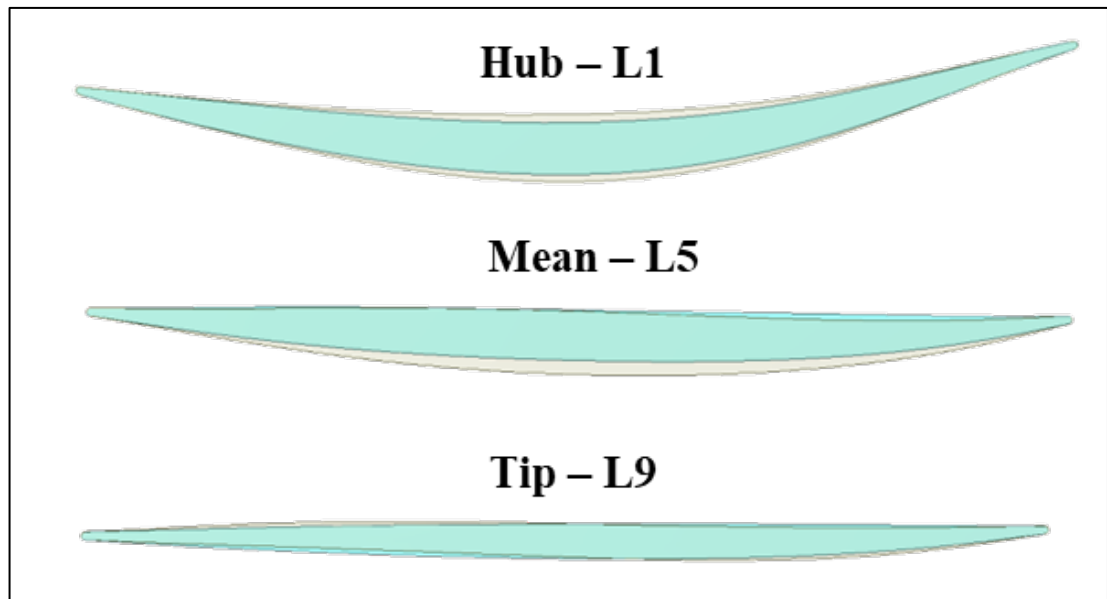


Figure 2.57: Initial Design(Green) and Optimum Design(Grey) Section Comparison.

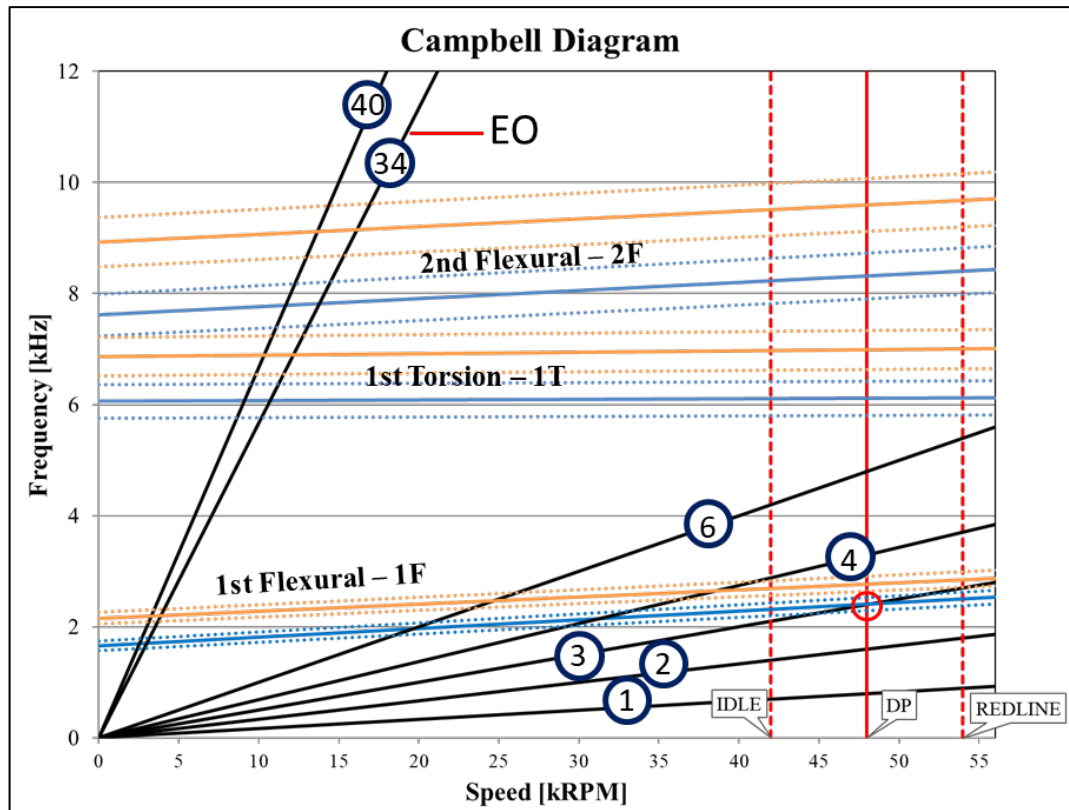


Figure 2.58: Campbell Diagram Comparison: Initial (Blue) and Optimum (Orange).

As seen from Campbell diagram comparison, the coincidence location is moved from the operating range and there is not any other resonance condition for the optimum design and equivalent stress is reduced from 512.46 MPa to 459.12 MPa shown in figure below, also the overall stresses are reduced which can be seen from stress contours. But, the mass of the blade increased from 2.295 gram to 2.539 gram.

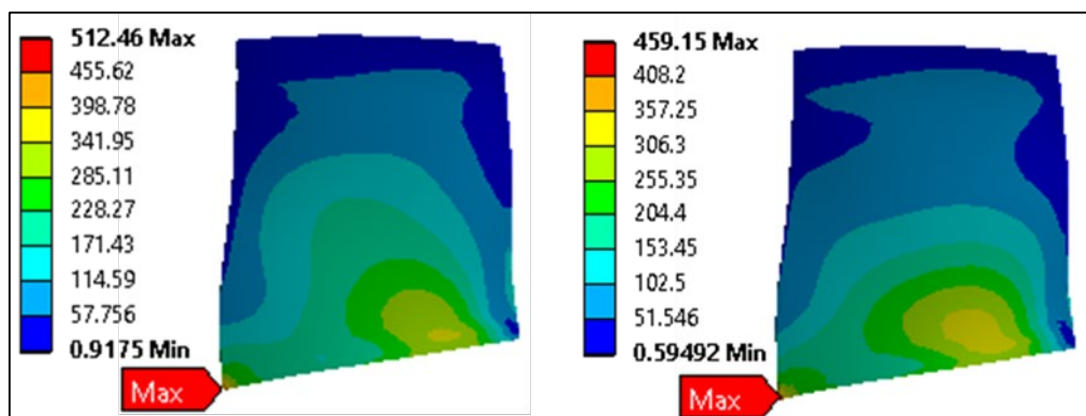


Figure 2.59: Equivalent Stresses[MPa] of Initial(Left) and Optimum(Right) Designs.

2.8. Experimental Modal Analysis

After optimization process is performed, initial and optimum designs are manufactured as blisk using Ti-6Al-4V material whose material properties are shown at the Table 2.2. A bigger and heavier block under the blade is used in order to avoid affecting natural frequency of the blade and to have ability to fix the part to the bench. The designs are shown in below Figure 2.60 and Figure 2.61.

The method, which is used to measure the responses caused by vibrations on the structures and to determine the dynamic characteristic of the structure using the measurement data, is called as experimental modal analysis. In experimental modal analysis, the structures can either be excited with a known force, or measurements can be made by considering the vibration records. To avoid measurement loading effects for small structures, non-contact sensors, such as microphones and laser displacement sensors, are used. Among non-contact sensors, microphones are the least expensive and easy to use and install sensors. The microphones measure the sound waves and are suited for high frequency measurements due to their sensitivity to pick up noise. Impact hammers are the most used excitation sources within a force sensor to measure the force applied to the structure. The tip hardness of the impact hammer controls the frequency ranges of the excitation so the tip by selecting to have enough frequency range to excite the modes. The mass of the hammer controls the amplitude of the force.

In this thesis, a measuring system; which contains a small impact hammer which is used tap onto the blade to excite, microphone which records the excited sound to minimize mass loading effect for the blade and the signal analyser which records the measured amplitude-frequency characteristics of the blades, is used to find natural frequencies of the blade in a quiet room.

The signal from the microphone is analysed using a Fast Fourier Transform (FFT) analyser to obtain natural frequencies. FFT decomposes mixed signals and indicates what frequency and intensity of vibration occurs in order to convert signals from time domain into frequency domain. FFT does not take non-repeating signals into account and identifies the periodic complex signals and divides them into harmonic components.

Using impact hammer and microphone, the input excitations and response outputs are combined acoustically and mechanically, thus, the force (N) is used as

input and the acceleration (g) is used as output in order to calculate Frequency Response Function (FRF).

FRF is used to quantify the response of the blade to the excitation generated by the hammer, normalized by the magnitude of this excitation, in the frequency domain. Using FRF measurement, peaks indicating the natural frequencies of the blade under the test can be observed. FRFs are shown using Bode plots of amplitude and phase. Amplitude has peaks at the natural frequencies/resonances of the blade and phase shifts at resonant frequency. Also, FRF is a complex function and using imaginary and real terms, the amplitude($\sqrt{\text{Imag}^2 + \text{Real}^2}$) and the phase($\tan^{-1}(\frac{\text{Imag}}{\text{Real}})$) are able to be transformed. A general scheme of the experimental setup is shown in Figure 2.62.

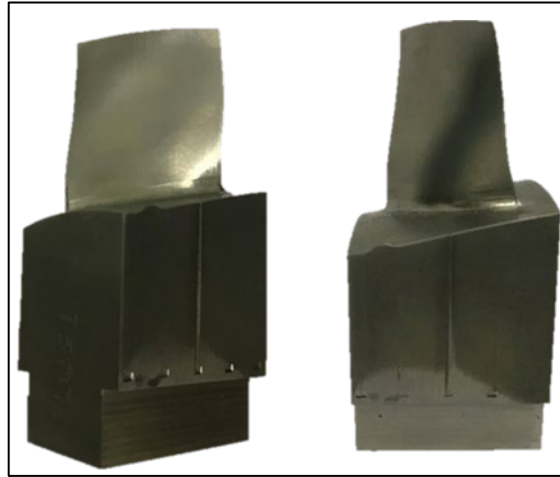


Figure 2.60: Initial Design.

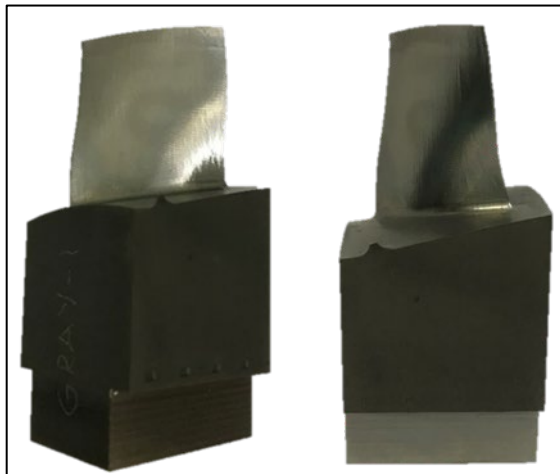


Figure 2.61: Optimum Design.

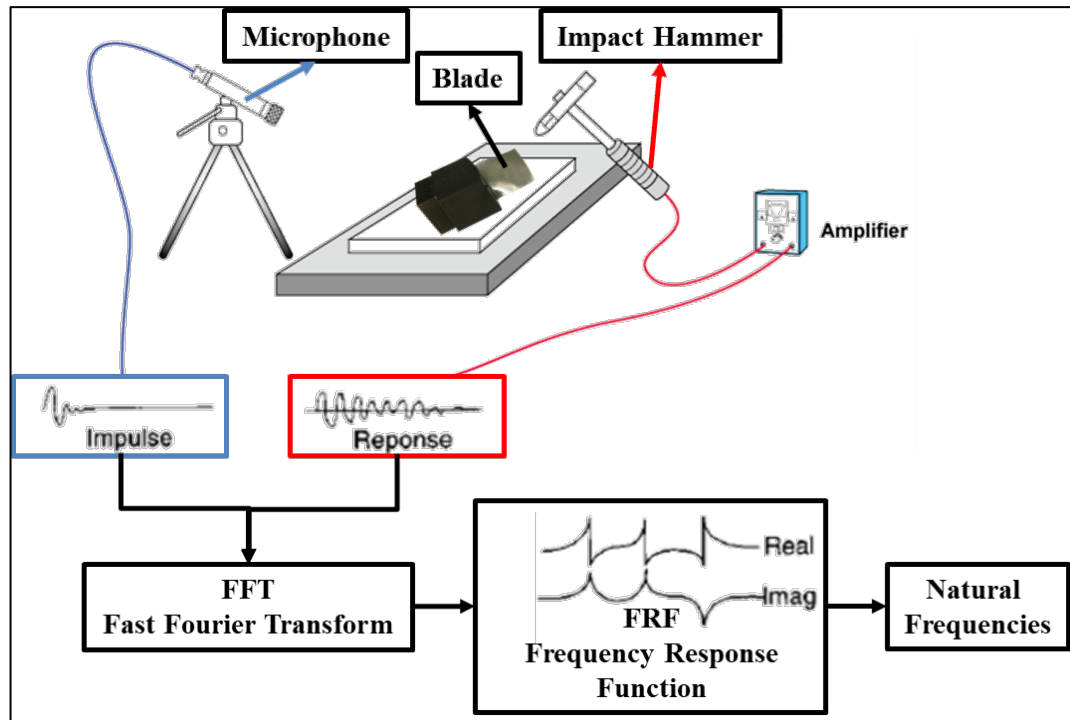


Figure 2.62: General Scheme of the Experimental Setup.

In the test setup, PCB Impact Hammer(086E80) with a hard(metal) tip to be able to evaluate high frequency response of the blade, PCB Microphone(426B03) and Siemens LMS Scadas Mobile as FFT analyser. Test is performed tapping properly onto the fixed blade tip three times to average the response and FRFs are obtained. The experimental modal analysis test setup is shown in Figure 2.63.



Figure 2.63: Test Setup with Microphone, Hammer, Clamp and Blade.

Using FRF bode plots of amplitude and phase which are shown in as Figure 2.64 and Figure 2.65, natural frequencies are obtained for the first three modes.

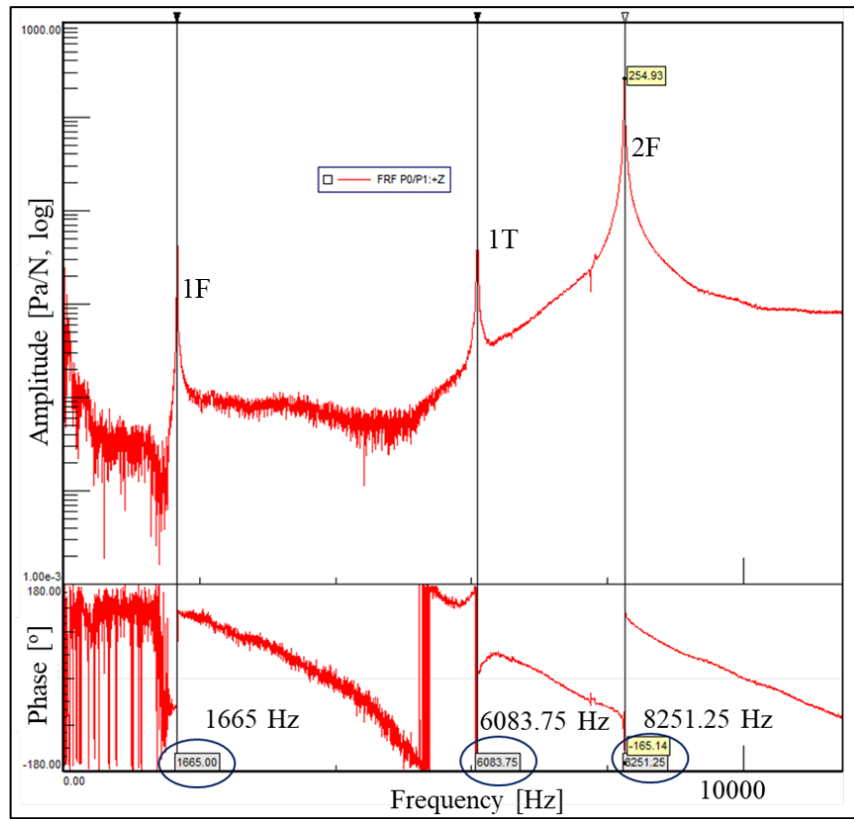


Figure 2.64: FRF Bode Plot of the Initial Design.

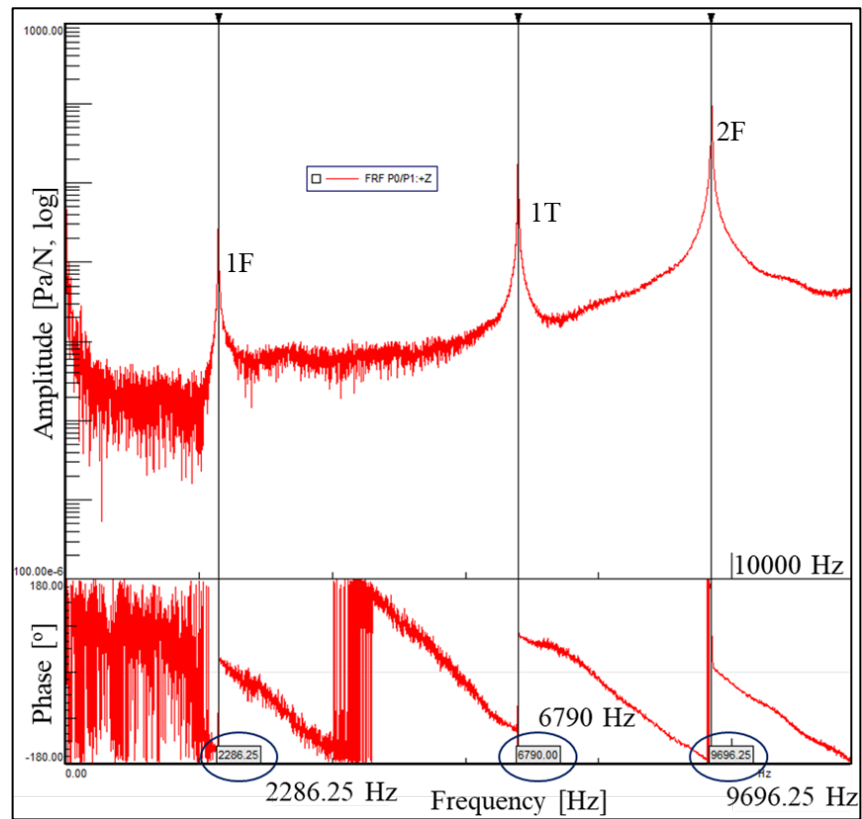


Figure 2.65: FRF Bode Plot of the Optimum Design.

After evaluating the FRF plots, FEA and experimental modal analysis test results for initial and optimum designs are compared at the Table 2.8 and Table 2.9.

Table 2.8: FEA and Test Results Comparison of Initial Design.

Result Comparison of Initial Design	FEA Results		Test Results	
	Modes		Modes	
Difference	#	[Hz]	#	[Hz]
1.27%	1	1643.8	1	1665
0.55%	2	6050.1	2	6083.75
6.98%	3	7675	3	8251.25

Table 2.9: FEA and Test Results Comparison of Optimum Design.

Result Comparison of Optimum Design	FEA Results		Test Results	
	Modes		Modes	
Difference	#	[Hz]	#	[Hz]
5.66%	1	2156.9	1	2286.25
1.06%	2	6862.1	2	6790
7.98%	3	8922.9	3	9696.25

Result differences for initial and optimum designs are within the %10 frequency margin that defined at the Campbell Diagram section, so results show compatibility with each other.

3. CONCLUSION

In this study, a detailed methodology to avoid from a full resonance condition of an axial transonic compressor blade is described. Avoiding from such critical coincidence between 1F and 3rd EO in operating speed zone is important to have a competitive design.

Thus, the blisk geometry, which is modelled using cyclic symmetry boundary condition, and blade-only geometry modal analysis results are compared by matching up natural frequencies of same mode shapes of both blisk and blade-only geometry and the modes without any matches are disk dominant modes. This comparison shows that the blade dominant modes does not depend on the disk in blisk geometries and being independent of disk geometry provides ability to perform parameter correlation and optimization process using the blade-only model, which expedites these time-consuming processes. The modal analysis results of the bladed disk are used to determine the critical coincidence or full resonance conditions between 1F and 3rd EO in critical operating zone is evaluated plotting Campbell and SAFE diagrams. After, parameter correlation or sensitivity analyses are performed to obtain dominant input parameters and relations between input and output parameters using Spearman Correlation method. Then, parameter correlation results are compared to high SED regions, which represents dynamically weak regions, to show parallelism between energy regions and dominant parameters.

In order to avoid from the full resonance condition and to validate the main aim of this thesis which is the integrity of the blade design, equivalent stress on the blade is minimized and the derived coincidence RPM parameter is maximized to replace out of the operating range, using RSO method and experimental modal analysis.

RSO method is used due to its computational effort benefits and first step of the RSO is to create the design space using DOE method. CCD technique, which provides better metamodels is used as DOE method. After DOE creation, next step is to obtain response surfaces and Genetic Aggregation method to automatize selection of response surface algorithm, is used. Then, optimization is performed using MOGA, which provides more refined approach to identify global minimum and maximum, and a candidate design point, that satisfies the objectives and the constraint, is obtained. Then, the initial and optimum designs are compared, the Campbell Diagram

comparison show that the coincidence RPM is replaced from the operating range and the equivalent stress is reduced from 512.46 to 459.12 MPa, but the mass of the blade is increased 0.244 gram.

Finally, the initial and optimum designs are manufactured with stiff block using Ti-6Al-4V material, the material properties are shown at the Table 2.2, in order to perform experimental modal analyses. Then, FEA and experimental modal analysis, which is performed using a microphone and hammer with a metal tip, results are compared, and results shows compatibility for initial and optimum designs where the difference between the results are within the %10 frequency probability margin that defined at the Campbell Diagram section.

For future works, the HCF Life of the blade can be calculated, input parameter effects on the aerodynamic performance can be evaluated using numerical or experimental methods and multidisciplinary optimization can be performed.

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BIOGRAPHY

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