

**T.R.
GEBZE TECHNICAL UNIVERSITY
GRADUATE SCHOOL**

**INVESTIGATION OF STRESS AND VELOCITY RELATION IN
COMPLEX GEOMETRIC STRUCTURES:
CALCULATION OF THE CONSTANT COEFFICIENT**

SAMET GÖNER

**A THESIS OF MASTER OF SCIENCE
DEPARTMENT OF MECHANICAL ENGINEERING**

ADVISOR: ASSIST. PROF. DR. FATİH USTA

FEBRUARY 2025

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LİSANSÜSTÜ EĞİTİM ENSTİTÜSÜ

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GERİLME VE HIZ İLİŞKİSİNİN İNCELENMESİ:
SABİT KATSAYININ HESAPLANMASI

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*To My
Family*

ABSTRACT

In this study, the effects of time-dependent loads were examined through the responses of the geometry, focusing particularly on stress and velocity parameters. The dynamic stress-velocity coefficients known in the literature were calculated using the finite element method by applying unit loads separately in tension, bending, and torsion to a simple cylindrical geometry, and the results were compared with those in the literature. The analyses revealed a maximum discrepancy of 4% between the finite element results and the literature data.

For round shaft with shoulder fillet and hole, dynamic stress-velocity coefficients were determined using the finite element method and compared with the stress concentration factor. The relationship between these parameters was analyzed in terms of stress and velocity variations. The analyses indicated that in filleted shafts, a parallel relationship exists between the dynamic stress-velocity coefficient and the stress concentration factor, allowing for the determination of a constant coefficient. However, no such parallelism was observed in perforated structures. It was concluded that while stress and the dynamic stress-velocity coefficient are directly proportional, no similar relationship exists between velocity and the dynamic stress-velocity coefficient.

Additionally, the stiffness and frequency-dependent natural modes of complex geometries, such as round shaft with shoulder fillet and hole, were calculated using static finite element analysis and modal analysis. Based on these values, analytical calculations were performed, and this approach was named the "Combined Method." A maximum discrepancy of 5% was found between the results obtained from the Combined Method and those from the finite element analysis.

Finally, generating dynamic stress-velocity coefficient graphs for round shaft with shoulder fillet and hole in a manner similar to the stress concentration factor graphs in the literature aims to contribute to future reference materials on stress concentration factors. This would enable the analytical calculation of dynamic stresses alongside stress analyses performed under static loading conditions.

Keywords: Dynamic Stress-Velocity Coefficient, Stress concentration factor, Finite Element Analysis, Round shaft with shoulder fillet, Round shaft with hole, Modal Finite Element Analysis

ÖZET

Bu çalışmada, zamana bağlı olarak uygulanan yüklerin etkisi, geometrinin tepkileri üzerinden incelenmiş; özellikle gerilme ve hız parametrelerine odaklanılmıştır. Literatürde bilinen dinamik gerilme-hız katsayıları, basit silindirik geometriye birim yüklerin çekme, eğilme ve burulma tiplerinde ayrı ayrı uygulanmasıyla sonlu elemanlar yöntemi kullanılarak hesaplanmış ve literatürdeki verilerle karşılaştırılmıştır. Analizler sonucunda, elde edilen sonuçlar ile literatür arasındaki maksimum farkın %4 olduğu belirlenmiştir.

Radyüslü ve delikli miller için dinamik gerilme-hız katsayıları, sonlu elemanlar yöntemiyle hesaplanarak gerilme yığılma faktörüyle kıyaslanmış ve bu parametreler arasındaki ilişki, gerilme ve hız değişimleri açısından incelenmiştir. Analizler, radyüslü millerde dinamik gerilme-hız katsayısı ile gerilme yığılma faktörü arasında bir paralellik olduğunu ve sabit bir katsayı belirlenebileceğini göstermiştir. Ancak delikli yapılarda benzer bir paralellik gözlemlenmemiştir. Çalışmada, gerilme ile dinamik gerilme-hız katsayısının doğru orantılı olduğu, ancak hız ile dinamik gerilme-hız katsayısı arasında benzer bir ilişkinin bulunmadığı sonucuna varılmıştır.

Ayrıca, radyüslü ve delikli miller gibi karmaşık geometrilerin rijitlik ve frekansa bağlı doğal modları, statik sonlu elemanlar analizi ve modal analiz yöntemiyle hesaplanmış; elde edilen değerler doğrultusunda analitik hesaplamalar yapılmış ve bu yöntemle "Combined Method" adı verilmiştir. Combined Method ile sonlu elemanlar analizi sonuçları arasında maksimum %5 fark tespit edilmiştir.

Son olarak, radyüslü ve delikli miller için elde edilen dinamik gerilme-hız katsayısı grafiklerinin, literatürdeki gerilme yığılma faktörü grafiklerine benzer şekilde oluşturulması, gelecekte gerilme yığılma faktörlerine yönelik kaynaklara katkı sağlamayı amaçlamaktadır. Böylece, statik yüklemelerle yapılan gerilme analizlerinin yanı sıra dinamik gerilmeler de analitik olarak hesaplanabilecektir.

Anahtar Kelimeler: Dinamik gerilme-hız katsayısı, Gerilme yığılma faktörü, Sonlu elemanlar yöntemi, Radyüslü miller, Delikli miller, Modal Sonlu Elemanlar Analizi

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LIST OF SYMBOLS AND ABBREVIATIONS

σ	: Stress [Pa]
ϵ	: Strain [m/m]
REMTM	: Ratio of effective mass of total mass
FEA	: Finite Element Analysis
FEM	: Finite Element Method
F	: Applied Force [N]
T	: Torque [Nm]
l	: Length [m]
E	: Young's modulus of the material [Pa]
G	: Shear Modulus [Pa]
A	: Cross-sectional area [m ²]
θ	: Angle of twist [rad]
I	: Cross-sectional area moment of inertia about neutral axis [m ⁴]
J	: Polar second moment of area [m ⁴]
r	: Radius of geometry [m]
ζ	: Damping ratio
X	: Amplitude of Motion [m]
t	: Time [s]
ϕ	: Phase [radians]
x	: Distance along the rod, measured from a reference point. [m]
y	: Deflection of neutral surface [m]
k	: Wave number
u	: Displacement of a plane located at position x. [m]
ρ	: Density [kg/m ³]
η	: Radius of gyration
h	: Maximum cross-sectional distance from neutral axis [m]
Tet10	: 10 nodes tetrahedral element from Ansys Mesh Database
Hex20	: 20 nodes hexahedral element from Ansys Mesh Database
Wed15	: 15 nodes wedge element from Ansys Mesh Database
m	: Meter
mm	: Millimeter
Pa	: Pascal
N	: Newton
Nm	: Newton-meter
Hz	: Hertz
s	: Second
rad	: Radian
DOF	: Degree of freedom
E	: Exponent
TX	: Translational at X direction
TY	: Translational at Y direction
TZ	: Translational at Z direction
RX	: Rotation at X axial
RY	: Rotation at Y axial
RZ	: Rotation at Z axial
°	: Degree
r	: Radius [m]

D	: Diameter [m]
d	: Diameter [m]
L1	: Length [m]
L2	: Length [m]
RMS	: Root mean square



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1. INTRODUCTION

Dynamic loads refer to forces transmitted over a unit of time at a specific speed, typically encountered in systems operating under moving or variable conditions. These loads exhibit a time-dependent magnitude, distinguishing them fundamentally from static loads. The influence of dynamic loads is critical in numerous engineering challenges and plays a pivotal role in the design and analysis of mechanical systems.

For instance, in the context of an internal combustion engine, the combustion process within the cylinder generates pressure loads on components such as the piston and crankshaft. These loads not only induce forces but also significantly affect the system's speed, vibration characteristics, and overall durability. Similarly, in a tennis racket scenario, the impact of a ball striking the racket at a given velocity results in deformation of the racket structure. Moreover, this impact force propagates to the wrist holding the racket, inducing rotational moments and vibrational effects.

The responses elicited by such dynamic loads—particularly stress, deformation, and material durability—are crucial factors in engineering design. Accurate analysis of these responses is imperative for developing safe, efficient, and reliable systems.

This study seeks to investigate the effects of dynamic loads on complex geometries by focusing on two primary responses: stress and velocity, as well as the stress-velocity relationship. The complex geometries in this study include geometries with stress concentration regions. The analysis will be conducted using a combination of experimental data, numerical simulations, and theoretical modeling to elucidate the complex interactions induced by dynamic loads. A key objective is to optimize the stress-velocity relationship and underscore its relevance to system design. Additionally, the study will examine how dynamic load effects vary with different material properties and design geometries, providing a comprehensive understanding of these interactions.

In conclusion, this study aims to provide a detailed explanation of the stress-velocity relationship resulting from dynamic loads in engineering systems. Furthermore, it

seeks to emphasize the significance of this relationship in enabling the development of more efficient and robust designs.

1.1. Stress and Velocity Relation in Literature

The relationship between velocity and stress in impact situations was dynamically studied, formulated, and applied by Hopkinson [1]. Hunt developed a formula to calculate the stress on a rod caused by a tension vibration applied in the direction of the rod using the velocity and material wave speed properties [2]. Hunt aims to calculate the maximum velocity (vibration velocity) that a material can achieve while staying within its elastic limits in this formulation [2]. Hunt has developed formulations for calculating the stress generated by the vertical vibration velocity in bar, plate, and tapered rod and bar (such as cone, wedge, etc.) [2]. In general, designs avoid resonance regions, but when constraints are present, there is a need to calculate vibration-induced stresses to design structures that can operate within resonance regions [3]. In his work, Ungar has formulated the relationship between stress and velocity for circular and rectangular plates, considering both fully clamped and simply supported boundary conditions [3]. Additionally, he defined coefficients to express the effects of boundary conditions and used these values in the formulation [3]. Crandall calculated that in regions with stress concentration, the stress-to-velocity ratio would be higher. This elevated value exceeds the coefficient calculations presented in Hunt's study [2], demonstrating that the range identified by Hunt encompasses a broader set of values [4]. Robert stated that shock loads impact the component in waveforms across different frequency bands. In his study, he demonstrated through shock spectrum graphs that using velocity instead of acceleration provides a more accurate assessment of the stress-strain effects on the component [5]. Geberson and Chalmers demonstrated that modal velocity measurement provides more accurate results in stress calculations compared to acceleration data, as illustrated through graphs of velocity, acceleration, and displacement measurements. Additionally, they explained, using examples, that the relationship between velocity and stress varies depending on the yield strength of materials and the geometry of their cross-sections [6].

In the calculations performed using Ungar's statistical energy analysis method, it has been stated that deriving regional stress concentrations from average velocity values

does not yield precise results. To investigate this issue, a study was conducted using the bending wave field on two plates with different cross-sectional ratios. As an example, velocity and stress measurements were carried out in the transition regions of plates with cross-sectional ratios of 4:1 and 2:1, and these measurements were compared with the results obtained through wave theory [7].

For the plate with a cross-sectional ratio of 4:1, a strong agreement was observed between experimental results and theoretical calculations. However, for the plate with a cross-sectional ratio of 2:1, significant discrepancies were noted between experimental and theoretical results. This discrepancy is attributed to the lower stress concentration in the transition regions of plates with similar cross-sectional areas, rendering the theory insufficient for calculating minor variations [7].

Lyon explained the formulation involving the relationship between velocity and stress using the statistical energy analysis method. They also stated that the design target should ensure that the sum of the average response level and 2 or 3 standard deviations remains below the specification limits, depending on the required confidence level [8]. This approach is consistent with the method applied by Ungar.

Norton and Fahr presented some experimental results on the correlation between dynamic stress and strain with pipe wall vibrations for small-diameter cylinders typical of industrial pipeline systems. The specimens used in the experiment were prepared by considering both unconstrained cylindrical sections and constrained cylindrical sections with regions of high stress concentration. Special attention was given to the statistics of the distribution of the stress/strain ratio with pipe wall velocity, and comparisons were made between the predicted and actual rms stress/strain levels [9].

Karczub and Norton aimed to expand the existing literature in this field by presenting a consistent approach that integrates experimental measurements with a solid theoretical foundation for narrow-band and broad-band vibrations, as well as resonant and non-resonant vibrations. The relationship derived between stress and velocity in their study is applicable across low, mid, and high-frequency ranges, and is independent of the type and amplitude of excitation [10].

This approach enables the estimation of spatial maximum dynamic stress specific to each frequency band by correlating the spatial maxima of dynamic stress and velocity. Subsequently, the total maximum dynamic stress is conservatively predicted by

summing these values. Furthermore, the correlation between the spatial maxima of dynamic stress and velocity is designed to directly account for dynamic stress concentration effects [10].

Finnveden and Pinnington conducted a comprehensive investigation of a velocity-based method for estimating dynamic strain and stress in pipe structures. This method relies on the use of predicted or measured spatial average vibration velocity in conjunction with theoretically derived strain factors to estimate the maximum strain at the ends of the pipes. Their tests confirmed that this method can be represented by a descriptive non-dimensional quantity. Within the scope of the study, numerical experiments were carried out using the spectral finite element method by varying pipe parameters and considering 16 different homogeneous boundary conditions. While these experiments revealed potential limitations of the method when equipment is mounted on the pipes, they also validated the accuracy of the theoretical results. Furthermore, the researchers indicated that this velocity-based method could serve as a valuable tool in engineering applications, particularly for fatigue life assessments [11].

In their study, Kersch, Schmidt, and Woschke developed an approach to calculate fatigue damage from three-dimensional vibration measurements and defined a reliable measurement quantity to predict local stresses. Using the relationship between modal velocity and stress from existing theoretical concepts, they performed hysteresis counting based on velocity measurements and damage accumulation using S–N curves. Furthermore, instead of focusing solely on uniaxial test setups as in existing methods, they performed fatigue calculations under three-dimensional vibration setups. They also derived a fatigue calculation method for three axes using the equivalent stress concept and validated the derived calculation with experimental data [12].

Bateman has stated that, among the parameters used to define the structural response in the shock spectrum—displacement, velocity, and acceleration—velocity is of the greatest importance for damage analysis. The reason for this is that stresses that occur in a structure subjected to dynamic loading are typically caused by the responses of the structure's normal modes, i.e., the responses at natural frequencies. Additionally, he has formulated that natural frequency is proportional to velocity [13].

1.2. Purpose of Study

In the literature, it has been demonstrated in the previous section that the stress in components subjected to time-dependent dynamic loads is influenced by the loading rate. However, most of these studies have primarily focused on simple and uniform geometries. The complexity of geometry such as variations in cross-section, the presence of radiused surfaces, and holes can significantly affect stress distribution.

Irvine [14] conducted calculations on how variations in cross-sectional geometry and shifts in the center of gravity influence the stress coefficient. Additionally, Crandall performed various analyses on geometries with stress concentration regions. In this thesis study, the effects of such geometric variations on the stress-speed relationship will be examined, and the connection of this relationship with the stress concentration factor will be investigated.

This study focuses on different geometries that have not been extensively addressed in the literature to date. Specifically, the stress concentration factor and the dynamic stress-speed relationship coefficient in shafts with radiused and varying diameters, as well as in shafts with holes of different shapes, have been calculated using the finite element method and analytical calculations.

Another aim of this study is to determine how finite element analysis (FEA) and analytical calculations can be combined efficiently. The stiffness and frequency-dependent natural modes of complex geometries, such as round shaft with shoulder fillet and hole, were calculated using static finite element analysis and modal analysis. Based on these values, analytical calculations were performed, and this approach was named the “Combined Method”. The results of static and harmonic analyses conducted in Section 2 were compared with analytical calculations. To develop a more efficient method, an approach was devised in which some inputs were obtained from finite element analysis, while analytical calculations were utilized more intensively. This method is shown as schematic in the **Figure 1.1**. In calculating $\sigma_{\max}$ (Green Color); K_d (Orange Color), δ_{st_max} (Light Blue Color), REMTM (Light Blue Color) and w_n (Light Blue Color) values are taken as inputs from finite element analysis, Equations and Other Inputs (Blue Color) are taken as inputs from literature and the calculations are performed accordingly. Each case study in Section 2 will consist of three parts. In

the first part, a static calculation will be conducted to determine the static displacement value. In the second part, harmonic response analysis will be performed to calculate K_d and REMTM values, followed by applying the analytical calculations using the method described below. Finally, the maximum stress and velocity values will be compared with FEA results.

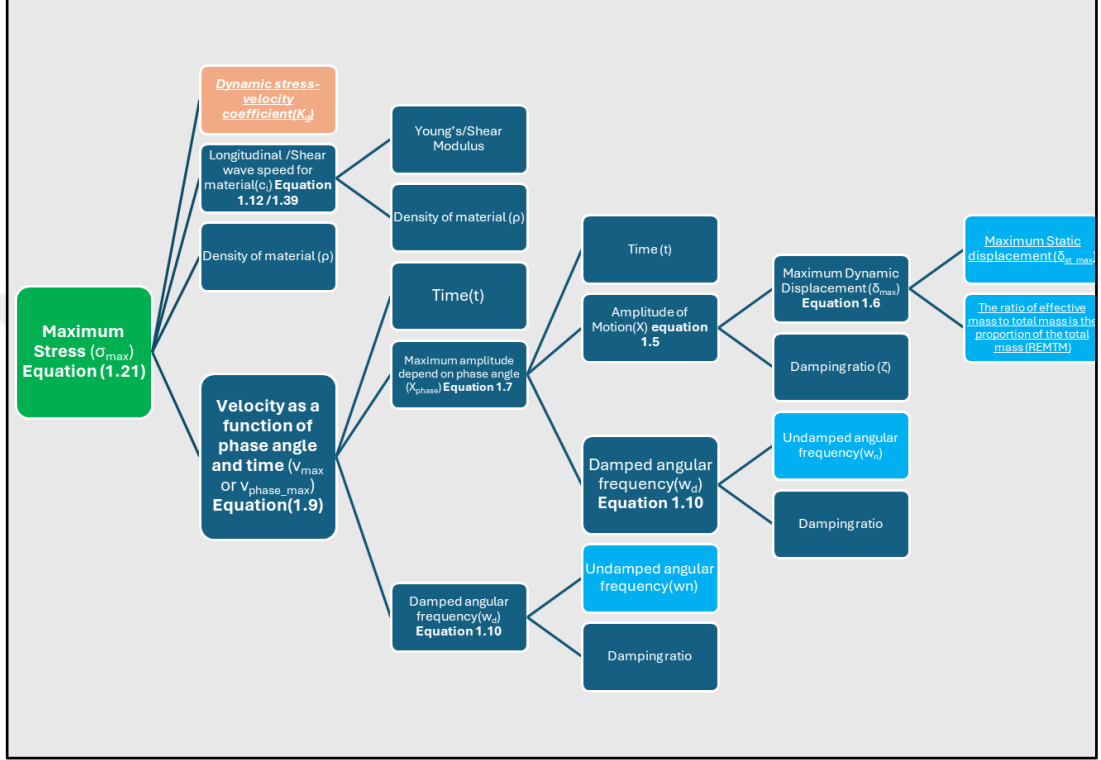


Figure 1.1: Representation of analytical and FEM inputs in the flowchart

1.3. Static and Forced Frequency Response Displacement Theory

1.3.1. Static Displacement Theory for Tension Load

Displacement for tension load (δ_t) is calculated using Equation (1.1) provided below [15]. This equation represents the linear elastic deformation under axial loading.

$$\delta_t = \frac{Fl}{EA} \quad (1.1)$$

where δ_t is displacement of tension load (m), F is applied force (N), l is length of the geometry (m), E is Young's modulus of the material (Pa), A is cross-sectional area of the geometry (m²).

1.3.2. Static Displacement Theory for Bending Load

Displacement for bending load (δ_b) is calculated using Equation (1.2) provided below [15]. This equation describes the maximum deflection of a simply supported beam under a centrally applied load.

$$\delta_b = \frac{Fl^3}{3EI} \quad (1.2)$$

where δ_b is displacement of bending load (m), I is moment of inertia of the geometry's cross-section (m⁴).

1.3.3. Static Displacement Theory for Torsional Load

Displacement for torsional load (δ_{tr}) is calculated using equation (1.3) provided below [16]. This equation describes the maximum deflection of a simply supported beam under a centrally applied load.

$$\delta_t = \theta * r \quad (1.3)$$

where δ_t is displacement of torsional load (m), θ : Angle of twist (rad), r is Radius of geometry (m).

The *angle of twist*, in radians, for a solid round bar is [15]

$$\theta = \frac{Tl}{GJ} \quad (1.4)$$

where T is Torque (Nm), l is Length of the geometry (m), G is Shear Modulus (Pa), J is Polar second moment of area (m⁴).

1.3.4. Displacement and Velocity Theory for Forced Frequency Response

X (Amplitude of Motion) is calculated by using equation (1.5) is given by [17]

$$X = \frac{\delta_{max}}{2*\zeta*\sqrt{1-\zeta^2}} \quad (1.5)$$

where δ_{max} is maximum dynamic displacement (m). This represents the largest amount of displacement or deformation for tension or bending or torsional load in the system. (m). ζ is Damping ratio, with a value defined as 0.05. δ_{max} is calculated by using equation (1.6) is given by

$$\delta_{max} = \delta_{st_max} * REMTM \quad (1.6)$$

where δ_{st_max} is deformation of the geometry caused by the application of unit loads in bending, torsion, or tension directions. (m), $REMTM$ is the ratio of effective mass to total mass is the proportion of the total mass that contributes to the system's dynamic behavior, particularly in vibrational or oscillatory motion. Depending on phase angle X_{phase} formula is calculated by using equation (1.7) is given by

$$X_{phase} = X * \cos(w_n * t - \Phi) \quad (1.7)$$

where X_{phase} is the amplitude is a function of phase angle and time, w_n is the undamped angular frequency (rad/s), which accounts for the effects of damping on the system's oscillation. t is time (s), representing the moment at which the displacement is being calculated. Φ is the phase angle (radians), which represents the initial phase or the phase shift of the oscillation, indicating where the oscillation starts relative to a reference point. Time (t) is calculated by using equation (1.8) is given by

$$t = \frac{1}{w_d} \quad (1.8)$$

This equation describes the oscillatory motion of a damp system, incorporating both the amplitude and phase shift. Depending on phase angle V_{phase} formula is calculated by using equation (1.9) is given by

$$V_{\text{phase}} = X_{\text{phase}} * w_n * \sin(w_n * t - \Phi) \quad (1.9)$$

where V_{phase} is the velocity as a function of phase angle and time and amplitude (m/s). Undamped angular frequency (w_d) is calculated by using equation(1.10) is given by

$$w_d = \sqrt{1 - 2 * \zeta^2} * w_n \quad (1.10)$$

where w_d is damped natural frequency (rad/s). This equation shows how the w_d is related to w_n and ζ , with damping reducing the frequency of oscillation.

1.4. Stress and Velocity Relation Theory

Mechanical loads are composed of normal stress and shear stresses. For every stress component a particular relationship has to be formulated. For the derivation of generalized relationships, rods and Euler-Bernoulli beams are sufficient. Thus, the effects of transverse shear strain are neglected.

1.4.1. Longitudinal vibrations in rod

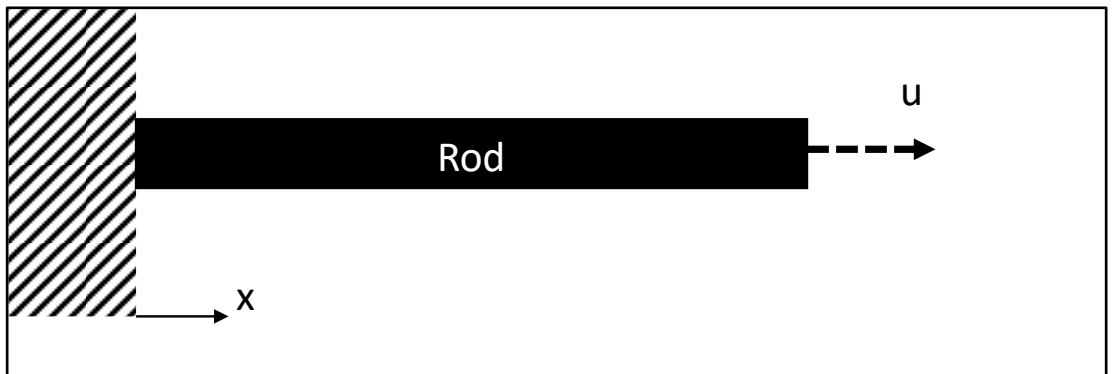


Figure 1.2: Longitudinal vibrations in Rod

The following consideration assumes that during longitudinal vibration of a prismatic bar the cross sections of the bar remain plane and the particles in these cross sections

perform only motion in an axial direction of the bar. In **Figure 1.2**, the longitudinal extensions and compressions which take place during such a vibration of the bar will certainly be accompanied by some lateral deformation, but in the following only those cases will be considered where the length of the longitudinal waves is large in comparison with the cross sectional dimensions of the bar [18]. Assume now that the bar motion performs a natural mode of vibration, which can be expressed as [6]:

$$u_l = \left(C \cos\left(\frac{wx}{c_l}\right) + D \sin\left(\frac{wx}{c_l}\right) \right) (A \cos(wt) + B \sin(wt)) \quad (1.11)$$

where x is distance along the rod, measured from a reference point, u is displacement of a plane located at position x, w is angular velocity, t is time; A, B, C, D are constants, c_l is longitudinal wave speed (m/s), E is Young's Modulus, ρ is density. c_l is calculated using following equation:

$$c_l = \sqrt{E/\rho} \quad (1.12)$$

At the initial moment (t=0), the equation of motion (1.11) simplifies to:

$$u = A \cos(wt) \cos\left(\frac{wx}{c_l}\right) \quad (1.13)$$

According to Hooke's Law, the stress (σ) is directly proportional to the strain (ϵ) and can be expressed as:

$$\sigma = E\epsilon = E \frac{\partial u}{\partial x} = -EA \frac{w}{c_l} \sin\left(\frac{wx}{c_l}\right) \cos(wt) \quad (1.14)$$

The maximum value of stress is given by:

$$\sigma_{max} = \frac{EAw}{c_l} \quad (1.15)$$

The displacement is a function of the driving frequency. From the simplified motion equation (1.13), the maximum displacement (u_{max}), velocity (v_{max}), and acceleration (a_{max}) can be derived as follows:

$$u_{max} = A \quad (1.16)$$

$$v_{max} = A\omega \quad (1.17)$$

$$a_{max} = A\omega^2 \quad (1.18)$$

where a_{max} is acceleration (m^2/s).

In the equation (1.15), if the term v_{max} is added instead of the $A\omega$, the maximum stress can be reformulated as:

$$\sigma_{max} = \frac{E v_{max}}{c_l} \quad (1.19)$$

Young's modulus is expressed as:

$$E = (c_l)^2 \rho \quad (1.20)$$

In the equation (1.19), if the term $(c_l)^2 \rho$ is added instead of the E , the maximum stress can be reformulated as:

$$\sigma_{max} = K_d c_l \rho v_{max} \quad (1.21)$$

where K_d is dynamic Stress-Velocity Coefficient of tension load, c_l : Longitudinal wave speed for material, ρ is density for material (kg/m^3), v_{max} is maximum velocity as a function of phase angle and time is same meaning with V_{phase_max} . It has been proved and illustrated that the maximum stress due to long-wave longitudinal vibrations in

rods is completely specified by the material properties and the maximum modal velocity [6].

1.4.2. Transverse Vibrations in Beam Bending

Transverse uniform beam vibrations can also be classified according to maximum modal velocities. According to Timoshenko's bending theory, the free vibrations of the neutral surface of such beams are expressed by the following solution:

$$y = (C_1 \cos(kx) + C_2 \cos(kx) + C_3 \sinh(kx) + C_4 \cosh(kx))x(B \cos(wt) + D \sin(wt)) \quad (1.22)$$

where k , the wave number, and w , angular velocity, are related by [18]

$$k^4 = \frac{w^2 \rho}{\eta^2 E} \quad (1.23)$$

In these equations the following definitions are used: y is deflection of neutral surface, x is distance along the rod, measured from a reference point; B , C_1 , C_2 , C_3 , C_4 , D are arbitrary constants, ρ is density, E is Young's Modulus, η is radius of gyration ($\frac{I}{A}$), I is cross-sectional area moment of inertia about neutral axis, A is cross-sectional area.

Equation (1.22) to consider a semi-infinite beam, which starts at $x = 0$ and continues out to infinity. The semi-infinite beam can accept sinusoidal vibration at all frequencies with amplitudes that do not exceed the elastic limit. The simplest case is that with zero shear and slope at its end, as shown in **Figure 1.3**.

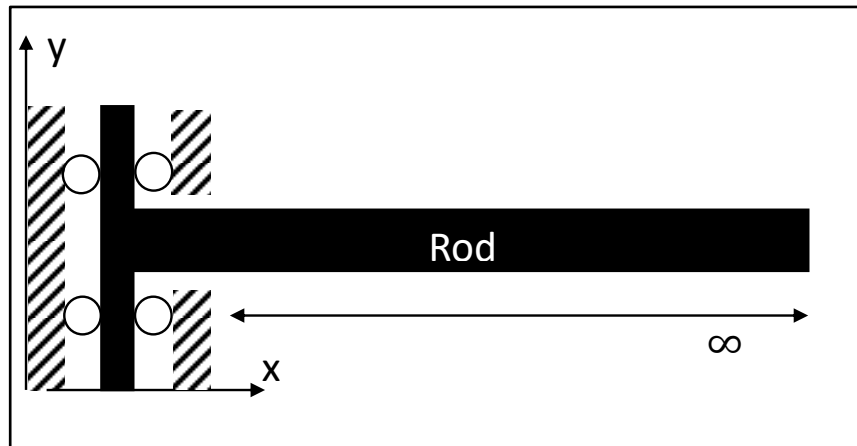


Figure 1.3: Semi-infinite beam with zero slope and shear at end [6]

The boundary conditions to be imposed on Equation (1.21) are:

1. The solution remains finite for very large x ,
2. Zero slope at $x = 0$; $y' = 0$ at $x = 0$,
3. Zero shear at $x = 0$; $y'' = 0$ at $x = 0$,

The only way in which condition 1 can be satisfied is

$$C_3 = -C_4$$

Thus the hyperbolic sine and cosine terms from Equation (1.22) may be written

$$(C_3 \sinh(kx) + C_4 \cosh(kx)) = (C_3 (\sinh(kx) - \cosh(kx))) \quad (1.24)$$

The substitution of the definitions of the hyperbolic functions yields

$$C_3(\sinh(kx) - \cosh(kx)) = C_3 e^{-kx} \quad (1.25)$$

Thus, the shape portion of equation (1.24) may be written as

$$y = (C_1 \sin(kx) + C_2 \cos(kx) + C_3 e^{-kx}) \quad (1.26)$$

By applying boundary condition 2, we obtain

$$0 = (C_1 - C_3')$$

or

$$C_1 = C_3'$$

hence becomes

$$y = (C_1 \sin(kx) + C_2 \cos(kx) + C_1 e^{-kx}) \quad (1.27)$$

Finally, application of boundary condition 3 proves that C_1 must be zero and so for this semi-infinite beam, the shape function is simply

$$y = C \cos(kx) \quad (1.28)$$

Again, without loss of generality, time can start when the deflection is a maximum making B zero, in equation (1.22). Thus, the deflection of this beam may be expressed simply as

$$y = C \cos(wt) \cos(kx) \quad (1.29)$$

The maximum stress in any initially straight beam bent to a curvature given approximately by $\frac{\partial^2 y}{\partial x^2}$ is shown in beam theory to be given by

$$\sigma = Eh \frac{\partial^2 y}{\partial x^2} \quad (1.30)$$

where E is Young's Modulus, h is Maximum cross-sectional distance from neutral axis. Substituting the maximum value of the second derivative of equation (1.29) into equation (1.30) gives the maximum stress to be

$$\sigma_{max} = C E h k^2 \quad (1.31)$$

Substitution of the value of k^2 from equation (1.23) gives the expression for maximum stress as a function of the maximum deflection, C, to be

$$\sigma_{max} = C h \frac{w}{\eta} \sqrt{E\rho} \quad (1.32)$$

Note that in this case the stress is a function of both the deflection, C, and the angular velocity, w. Now the maximum value of the first-time derivative of the equation (1.29) shows the maximum velocity, v_{max} , to be

$$v_{max} = Aw = Cw \quad (1.33)$$

The substitution of this value into equation (1.32) yields our result:

$$\sigma_{max} = v_{max} \frac{h}{\eta} \sqrt{E\rho} \quad (1.34)$$

Again, the maximum stress for any possible free vibration shape when specified by the maximum velocity does not depend upon frequency, but only on material properties and a beam cross-sectional shape factor, $\frac{h}{\eta} \cdot \frac{h}{\eta}$ is symbolized as K_{db} , where K_{db} is Stress-velocity relationship coefficient of dynamic bending or Shape factor [6]. If K_{db} and c_l are added instead of $\frac{h}{\eta}$ and E , equation (1.32) is reformulation as;

$$\sigma_{max} = K_{db} c_l \rho v_{max} \quad (1.35)$$

Table 1.1 values of the beam shape factor (K_{db}) of equation (1.35) for simple cross sections.

Table 1.1: Kdb values of dynamic bending depend on cross sections

Cross sections	K_{db}
Solid rectangle	$\sqrt{3}$
Solid round bar	2
Solid triangle	$2\sqrt{2}$
Thin hollow tube	$\sqrt{2}$
Thin hollow square	$\sqrt{6}/2$

1.4.3. Transverse Vibrations in Beam Shear

The L length beam can accept sinusoidal vibration at all frequencies with amplitudes that do not exceed the elastic limit. The simplest case is that with shear and slope at its end, as shown in **Figure 1.4**.

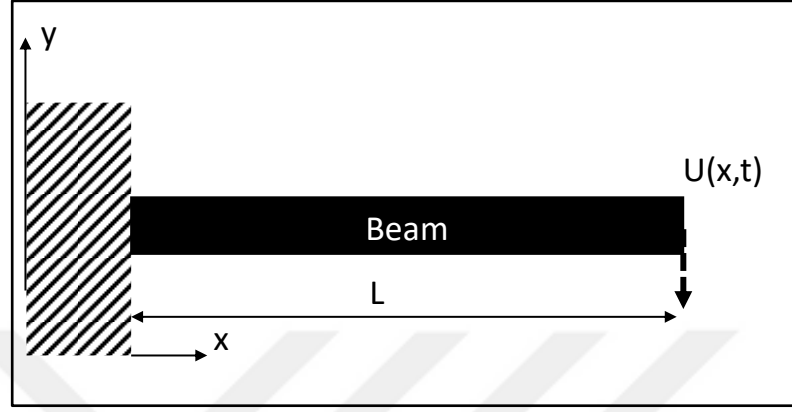


Figure 1.4: Beam Shear

Timoshenko theory considers both the rotary inertia and the shear deformation of the beam [19]. Assuming a uniform cross-section and mass density, the transverse shear displacement $u(x,t)$ is described by the following equation:

$$\rho A \frac{\partial^2 u}{\partial t^2} = k_s G A \frac{\partial^2 u}{\partial x^2} \quad (1.36)$$

In this equation ρ is the mass density of the beam, A is the cross-sectional area, k_s is the shear shape factor, G is the shear modulus, and $u(x,t)$ represents the transverse shear displacement. By substituting the equation, perform the partial differentiation.

$$\left[\frac{k_s G}{\rho} \right] U''(x) T(t) = U(x) \ddot{T}(t) \quad (1.37)$$

where $U(x)$ is the spatial displacement function, $T(t)$ is the time-dependent function, $U''(x)$ represents the second derivative of the displacement with respect to x , $\ddot{T}(t)$ is the second derivative of $T(t)$ with respect to time. This formulation separates the spatial and time-dependent variables, allowing for easier analysis of the beam's dynamic

response under shear forces. The detailed differential calculation is explained in reference [14]. The relevant equation is shared below:

$$\tau_{max} = 1.5 \frac{v_{max} K_{dt} c_s \rho}{k_s} \quad (1.38)$$

where τ_{max} is maximum Shear Stress (Pa), K_{dt} is Stress-velocity relation coefficient of dynamic torsional or shear loads, k_s is shape factor, c_s is material shear wave speed (m/s), ρ is material density (kg/m³).

$$c_s = \sqrt{(k_s G) / \rho} \quad (1.39)$$

where G is Shear Modulus (Pa).

The shear factors for two cross-sections are given in the following **Table 1.2** [14].

Table 1.2: Shape factor (k_s)

Cross-Sections	Values
Rectangular	2/3
Circular	3/4

2. CASE STUDY WITH DIFFERENT GEOMETRIES AND LOADS

To calculate the stress-velocity relationship coefficient, finite element analysis and analytical calculations were initially performed on a simple cylindrical geometry. The analyses were conducted under tension, bending, and torsional loading conditions, respectively. In each static analysis study, mesh sensitivity (section 0) was applied to identify the appropriate element size that resulted in the maximum stress ($\sigma_{\max}$) at the critical location. Using this mesh configuration, harmonic analysis was performed.

By utilizing the maximum stress ($\sigma_{\max}$) and maximum velocity ($v_{\max}$) values obtained from the harmonic analyses, the K_d coefficient was calculated using the equations provided in Section 1.4, and the K_d coefficient calculated for the cylinder was compared with values in the **Table 1.1**. After validating the formulas from the literature, these coefficients were calculated for radiused shafts and shafts with holes, and their relationship with the stress concentration factor was investigated.

2.1. Mesh Rules and Mesh Sensitivity

Mesh model is shown in the **Figure 2.1**. Mesh model detail fillet is shown in the **Figure 2.2**.

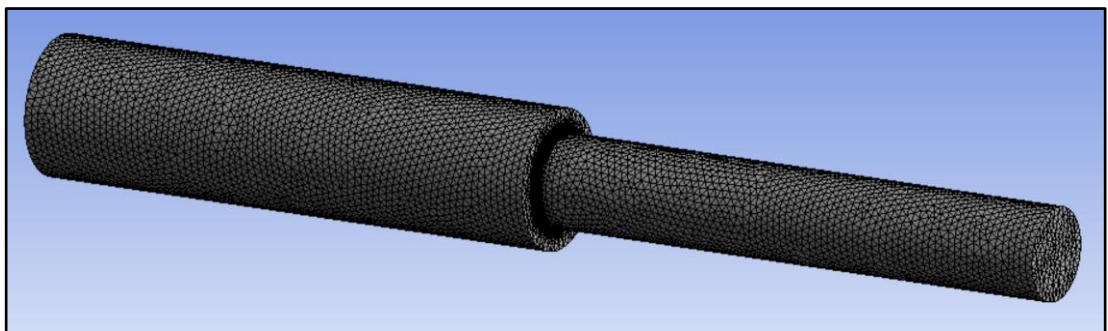


Figure 2.1: Mesh model r/d: 0.2 from **Table 2.14**

In the **Figure 2.2**, Fillet area radius contains at least 3 elements. Element type is Tet10, which is a high-order element from Ansys Mesh database [20]. The average mesh

element quality is greater than or equal to 0.80, as specified in the Ansys Mesh Documentation [20]. However, a mesh sensitivity analysis is conducted to ensure accurate calculation of stress, deformation, and velocity results in critical areas (such as radii or holes) before performing modal and harmonic analysis.

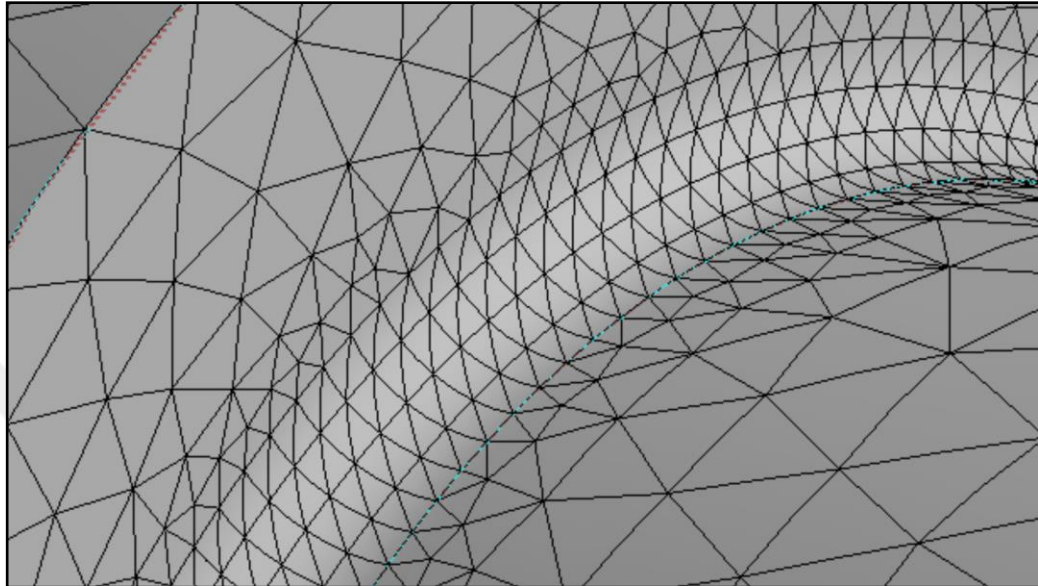


Figure 2.2: Fillet mesh detail

For the mesh sensitivity analysis, a static analysis is performed to ensure the most accurate stress results by using three different element sizes. For Mesh sensitivity analysis, static analysis is made and ensure the best accurate stress is tried with different element sizes. Case study example is shared in the **Figure 2.3**. Geometry is fillet radius shaft and Geometry length, diameter 1 and 2 shared in the **Figure 2.42**. Radius is 10.2 mm.

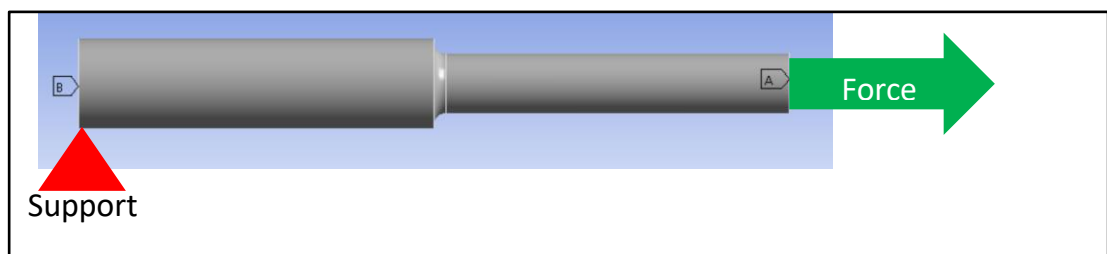


Figure 2.3: Mesh Sensitivity Analysis Case Study

The base mesh model has an element size of 6 mm, while the fillet region has an element size of 6 mm. To ensure accurate stress calculations, static analyses were

performed using five different element sizes (10 mm, 6 mm, 4 mm, 2 mm, 1 mm, and 0.5 mm) in sequence.

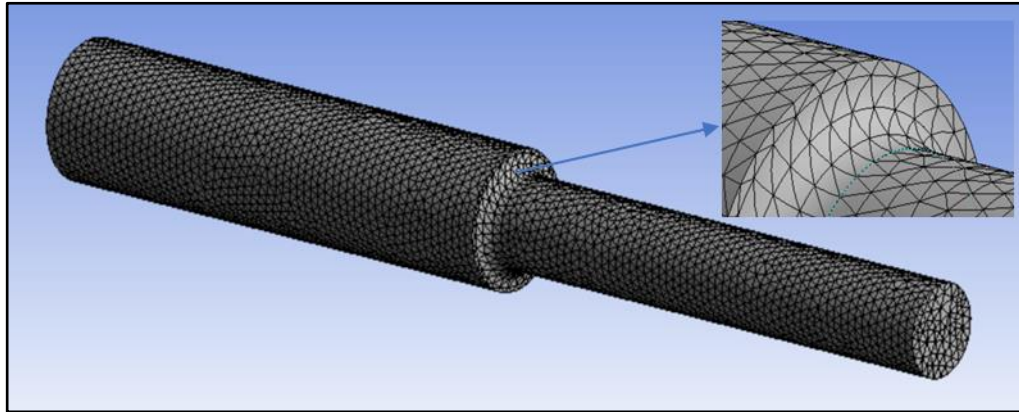


Figure 2.4: Mesh sensitivity mesh model

Averaged Equivalent (von Mises) stress results are shown in the **Figure 2.5**.

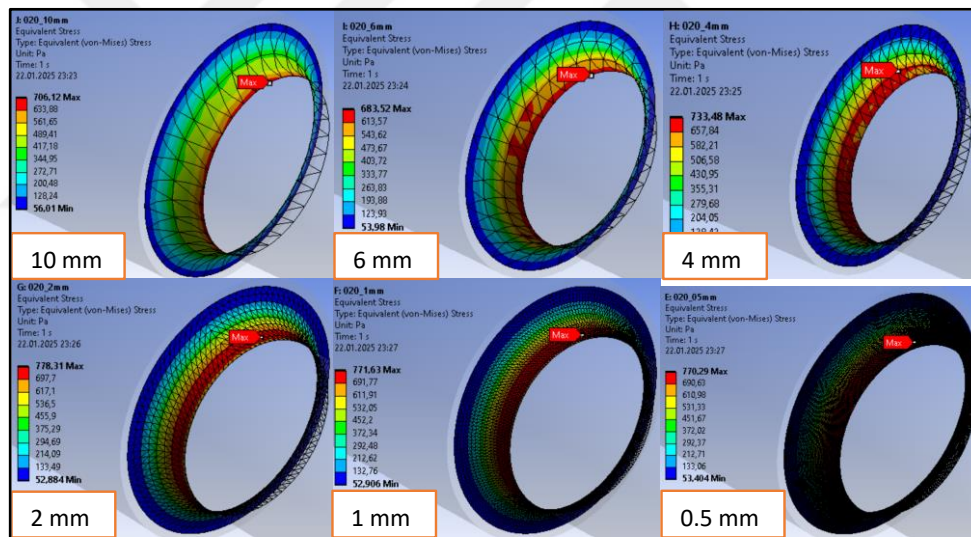


Figure 2.5: Averaged Equivalent (von Mises) Stress Results

The relationship between element size and both Unaveraged and Averaged Maximum stress results is illustrated in detail in **Figure 2.6**. It is observed that the Maximum stress values for both Averaged and Unaveraged results converge when the element size reaches 0.5. This indicates that a finer mesh (element size ≤ 0.5) does not significantly alter the Maximum stress outcomes, confirming the convergence and reliability of the results at this mesh resolution. The primary purpose of comparing Averaged and Unaveraged results is to verify that when both results are in agreement,

the element quality is ideal, and the calculated stress values accurately represent the true stress. This approach and the mesh size convergence criteria were consistently applied across all case studies to ensure uniformity and accuracy in the analysis.

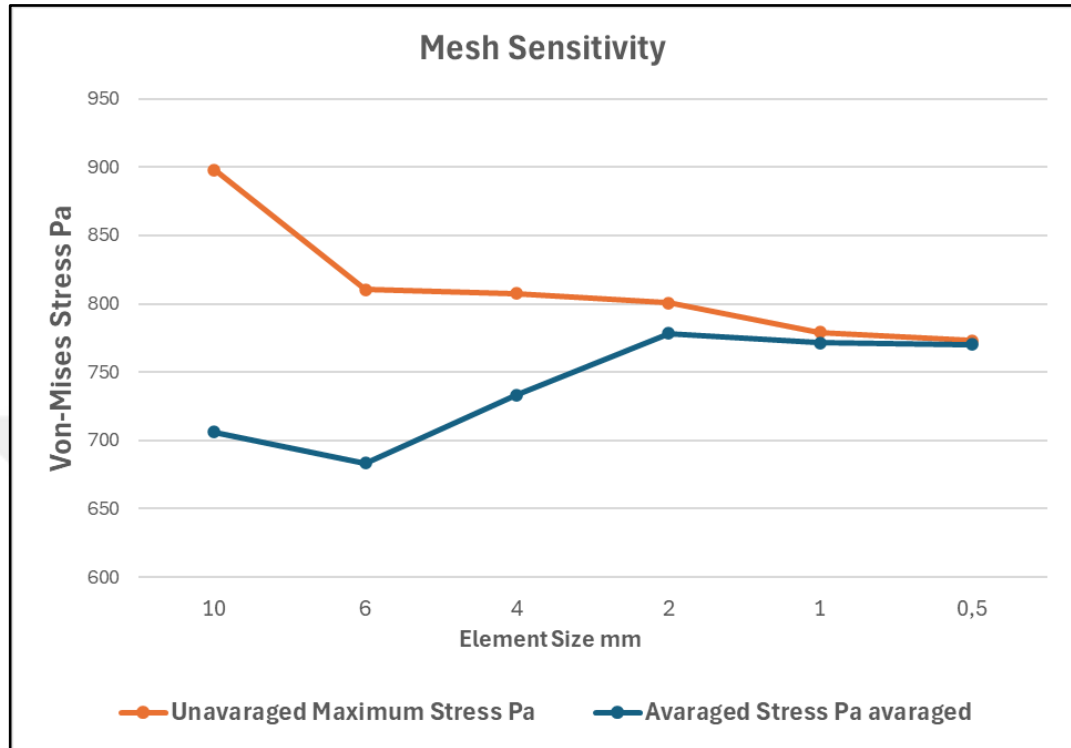


Figure 2.6: Convergence of Stress Averaged and unaveraged Result

2.2. Case Study - 1 - Simple Cylindrical Model

2.2.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Geometry is as shown at **Figure 2.7**. Geometry properties, diameter and length are shared at **Table 2.1**.

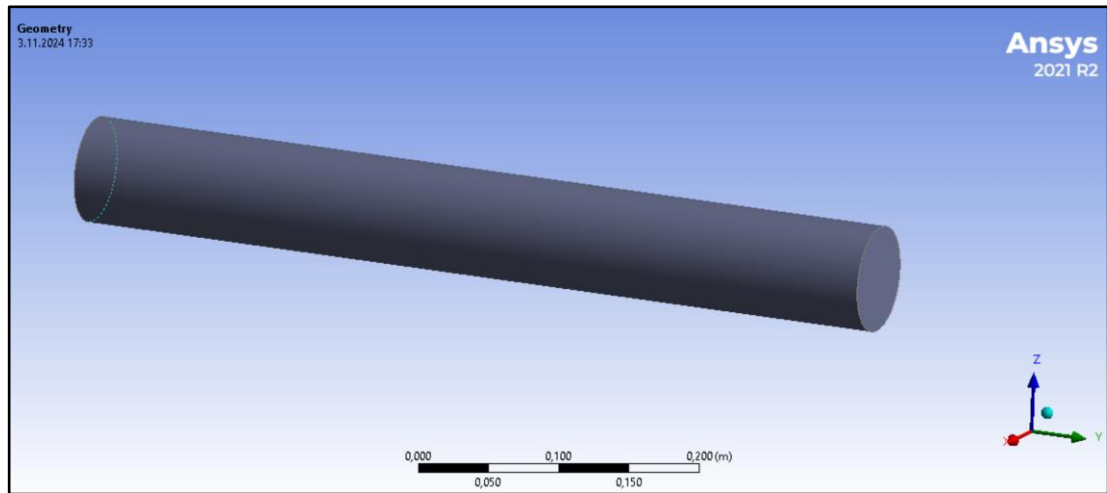


Figure 2.7: Simple model geometry

Table 2.1: Geometry properties

Parameter	Value
Length [m]	0,6096
Radius [m]	0,0381
Area [m ²]	4,56E-3
Polar second moment of area [m ⁴]	3,31E-6
Second moment of area [m ⁴]	1,65E-06

Material properties are shared at **Table 2.2**. Geometries which used for all analysis in the thesis have same material properties.

Table 2.2: Material Properties

Parameters	Value
Name	Aluminum
Density [kg/m ³]	2700
Young Modulus [Pa]	7E10
Poisson Ratio	0,3
Shear Modulus [Pa]	2,69E10

Mesh model is as shown in the **Figure 2.8**.

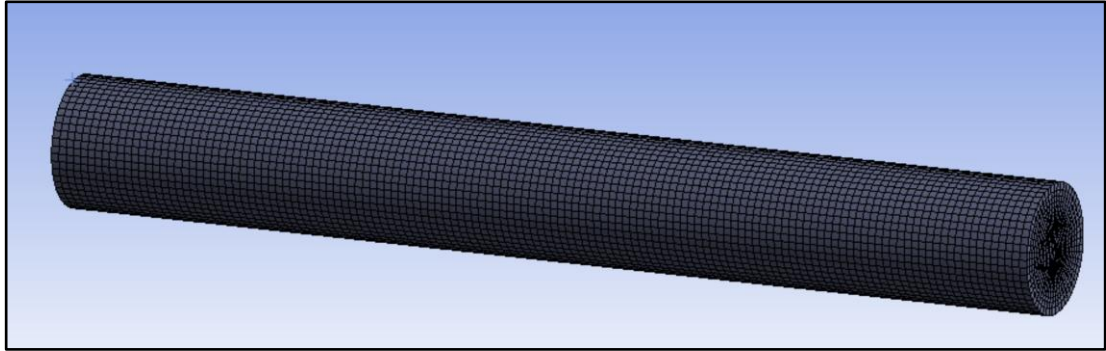


Figure 2.8: Mesh Model

Mesh model element property is shared at **Table 2.3**.

Table 2.3: Element Property

Element Library Name	Ansys Structural
Element Type	Hex 20 and Wed 15
Element Size	4,5E-3
Element Quality	0,75
Element Number	81135
Node Number	335975

Boundary condition is selected 6 DOF included 3 Rotation and 3 Translation are define as “0”. Fixed support is shown at **Figure 2.9**.

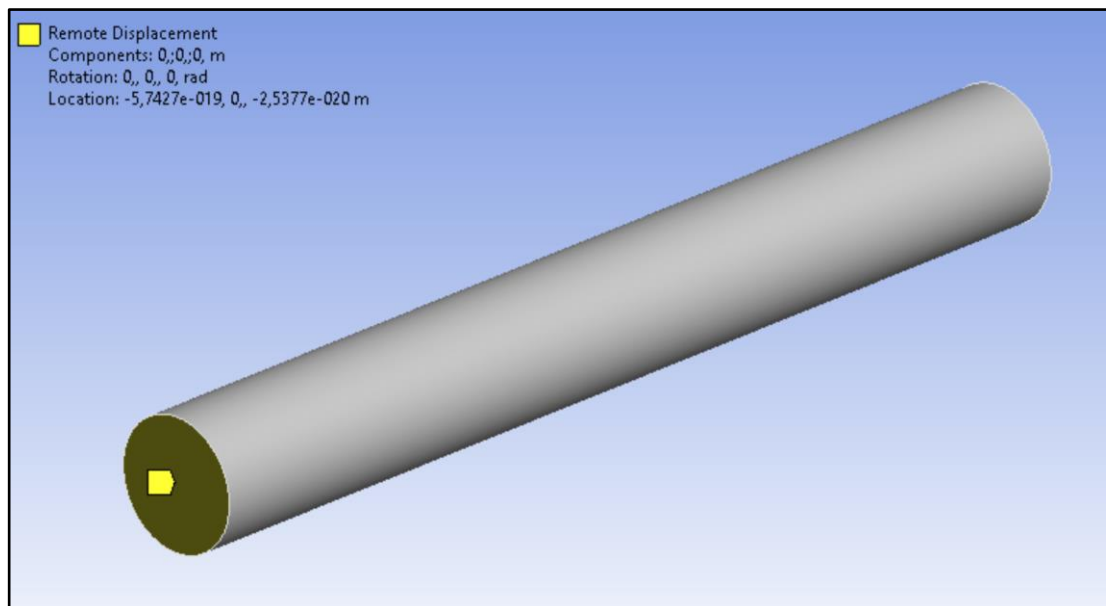


Figure 2.9: Fixed Support is name as Remote Displacement

Loads are defined as Tension, Bending and Torsional. They are applied to different cases. Tension, Bending and Torsional Loads are shown as **Figure 2.10**.

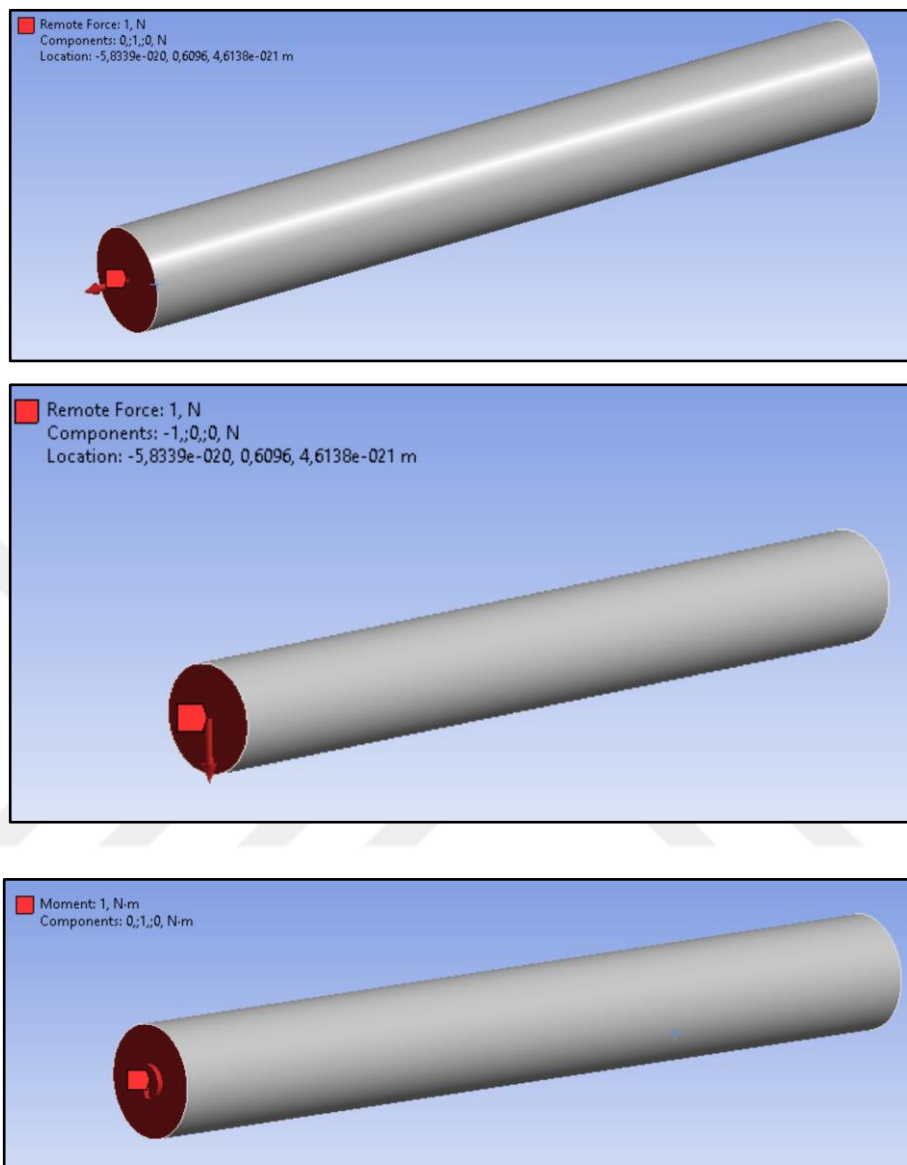


Figure 2.10: Tension, Bending and Torsional

2.2.2. Static Analysis Result

Static analysis result included deformation at Load direction Y directions is shown **Figure 2.11**. Maximum deformation is 1,89E-9 m.

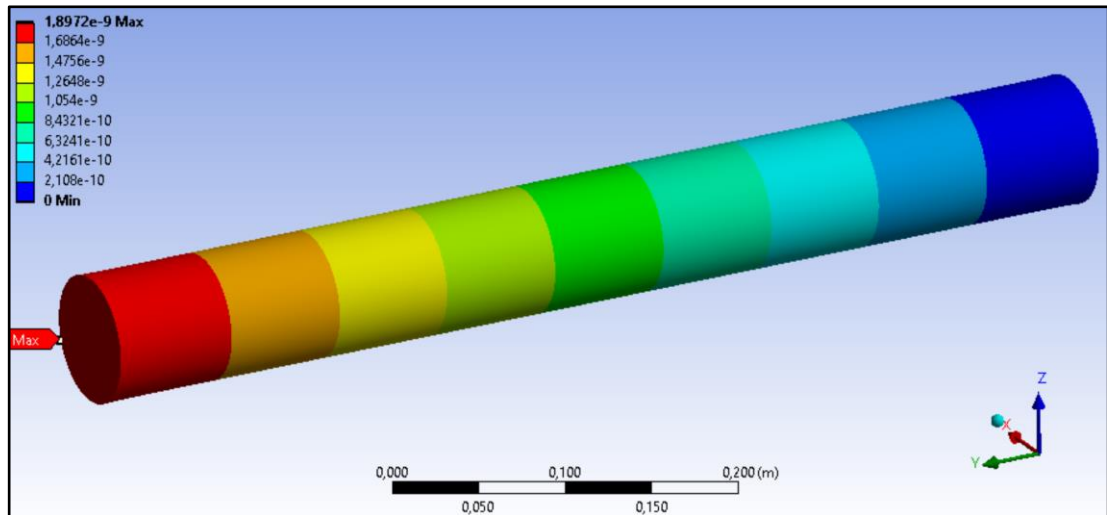


Figure 2.11: Tension Deformation at Y direction

Validation of the FEM result used equation (1.1) [15] . When we put the value to the formula; $F=1$ N, $l=0,6096$ m, $R=0,0381$ m, $A= 0,045372$ m². The result is $\delta= 1,91\text{E-}9$ m. Difference between Analytical and FEM result is shared in the **Table 2.4**.

Table 2.4: Analytical and FEM Result Comparison for Tension Deformation

Parameter	Analytical	FEM	Difference
Tension deformation	1,91E-9	1,89E-9	%1

For bending loads, at X direction deformation is $6,5248\text{E-}7$ m as shown in **Figure 2.12**.

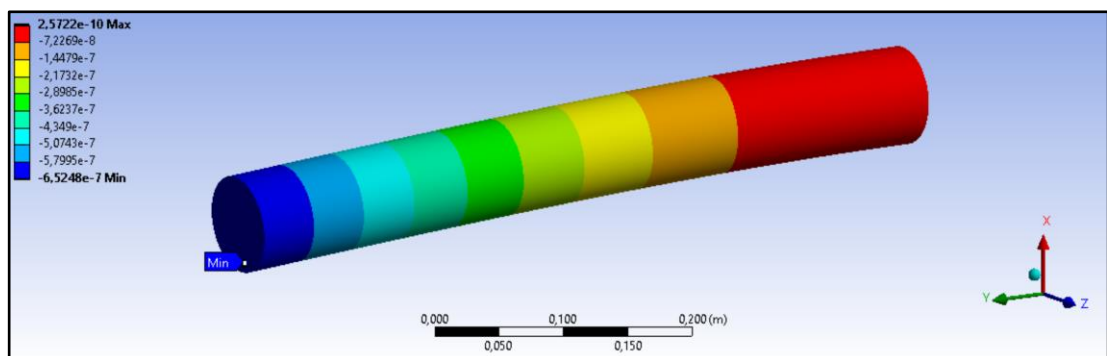


Figure 2.12: Bending deformation result at X direction

Validation of the FEM result used equation (1.2) [15]. When we put the value to the formula; $F=1$ N, $l=0,6096$ m, $E= 7E10$ Pa, $I=6,62E-6$ m⁴. The result is $\delta= 6,5E-7$ m. Difference between Analytical and FEM result is shared in the **Table 2.5**.

Table 2.5: Analytical and FEM Result Comparison for bending loading

Parameter	Analytical	FEM	Difference
Bending deformation	6,52E-07	6,53E-07	%0,1

For torsional loads, cylinder load surface is defined as a cylindrical coordinate system as shown in the **Figure 2.13**.

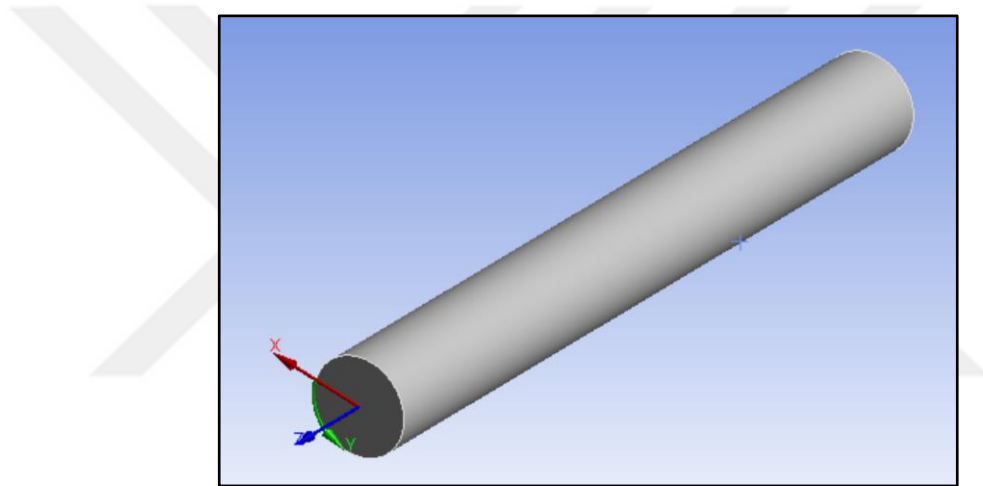


Figure 2.13: Cylindrical Coordinate System

Maximum deformation at Y direction is $2,61E-7$ m in the **Figure 2.14**.

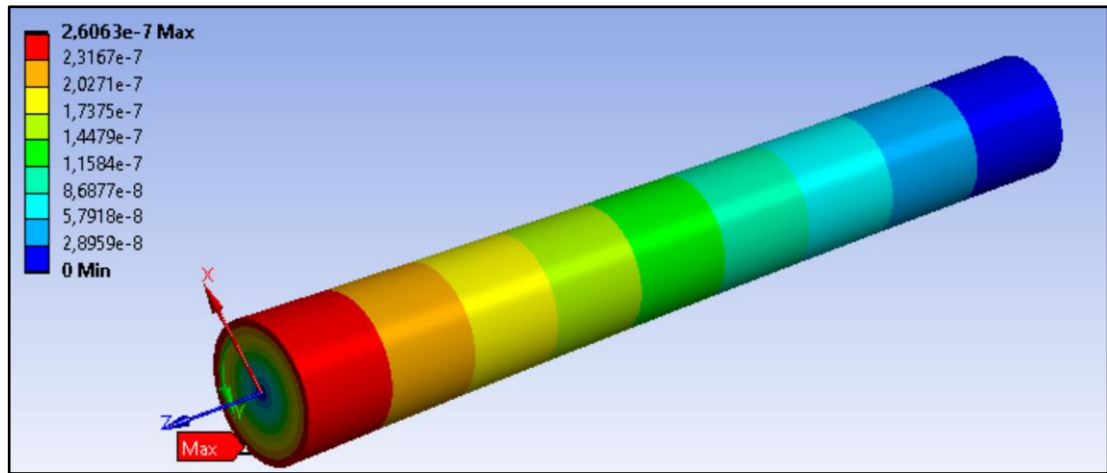


Figure 2.14: Maximum Deformation for Torsional Load

Validation of the FEM result used equation (1.4) [15]. When we put the value to the formula; $T=1$ Nm, $l=0,6096$ m, $G= 2,69E10$ Pa, $J=6,62E-6$ m⁴. The result is $6,8E-6$ radian. Then the value is converted to the deformation used equation (1.3) [16] .When we put the values to the formula ; $\theta = 6,8E-6$ radian, $r = 0,0381$. The result is $2,61E-07$ m. Difference between Analytical and FEM result is shared in the **Table 2.6**.

Table 2.6: Analytical and FEM Result Comparison for Torsional Loading

Parameter	Analytical	FEM	Difference
Torsional deformation	2,61E-07	2,61E-7	%0

2.2.3. Harmonic Analysis Result

For harmonic response analysis, load (Tension, Bending and Torsional) is applied depend on 0-3000 Hz. Analytical and FEM results (velocity, natural frequency). Then K_d is calculated for the different loads. Load effect is evaluated on the K_d coefficient.

Geometry, Material and Boundary Conditions is same model at 2.2.1 Section. Modal analysis is made 0-3000 Hz. The Ratio of Effective Mass to Total Mass results are shared in the Table 2.7. For Analytical calculation, we need to use the ratio of effective mass to total mass value. For Tension, the maximum value is 0,81 at Mode 6 2087,8 Hz of the TY column and Tension load direction is Y. For Bending, the maximum values are 0,608 and 0,363 at Mode 1 and 2 144,81 and 144,82 of the RZ column.

Mode 1 and 2 frequency values are almost the same, so Total of the values are used for calculation. Total value is $0,608+0,363=0,971$. For Torsional, maximum value is 0,811 at Mode 5 1295,1 Hz of the RY column.

Table 2.7: The Ratio of Effective Mass to Total Mass

Mode	Frequency [Hz]	TX	TY	TZ	RX	RY	RZ
1	144,81	3,84E-01	1,16E-12	2,29E-01	3,63E-01	7,72E-11	6,08E-01
2	144,82	2,29E-01	5,01E-12	3,84E-01	6,08E-01	7,92E-15	3,63E-01
3	864,65	3,89E-02	1,26E-09	1,54E-01	1,96E-02	6,41E-10	4,94E-03
4	864,74	1,54E-01	9,15E-10	3,89E-02	4,94E-03	5,33E-09	1,96E-02
5	1295,10	1,06E-09	5,48E-10	1,54E-09	1,08E-10	8,11E-01	7,48E-11
6	2087,80	1,37E-08	8,10E-01	1,01E-08	1,06E-12	3,26E-09	7,11E-12
7	2264,10	5,31E-04	2,20E-09	6,73E-02	3,01E-03	2,37E-10	2,33E-05
8	2264,30	6,73E-02	1,38E-10	5,18E-04	2,33E-05	3,25E-08	3,01E-03
Sum		8,88E-01	8,10E-01	8,96E-01	9,99E-01	9,01E-01	9,99E-01

Tension Mode 6 result is shown in the **Figure 2.15**.

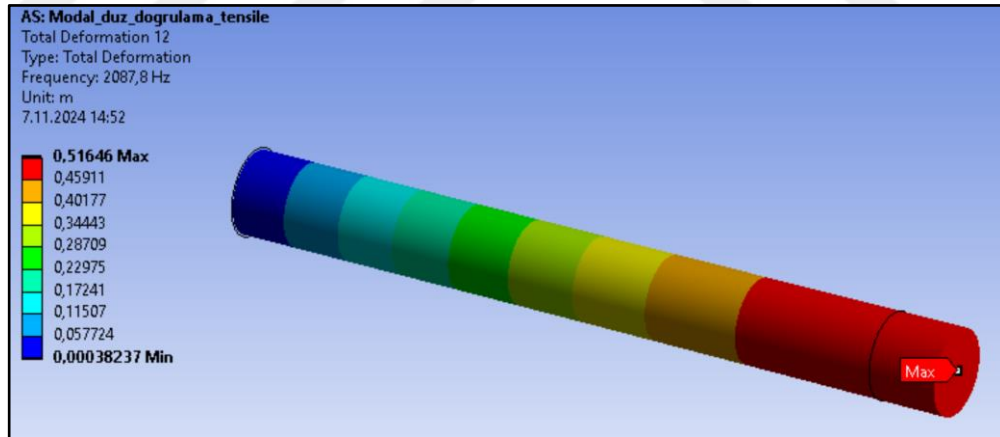


Figure 2.15: Mode 6 – Tension

Bending Mode 1 result is shown in the **Figure 2.16**.

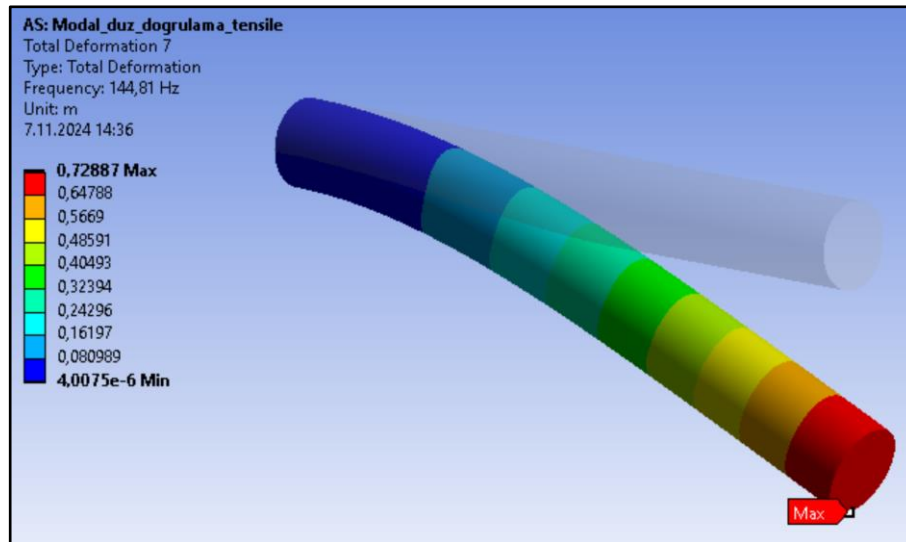


Figure 2.16: Mode 1 - Bending

Torsional Mode 5 result is shown in the **Figure 2.17**.

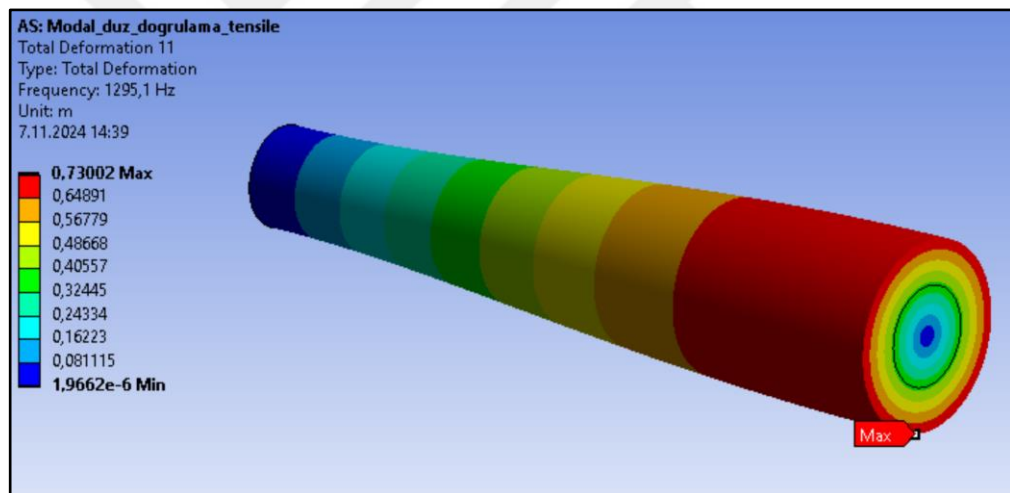


Figure 2.17: Mode 5 – Torsional

For Harmonic Response Analysis, damping coefficient (ζ) is defined as 0,05.

Tension load direction is shown in the **Figure 2.18** and load is applied as “1 N”.

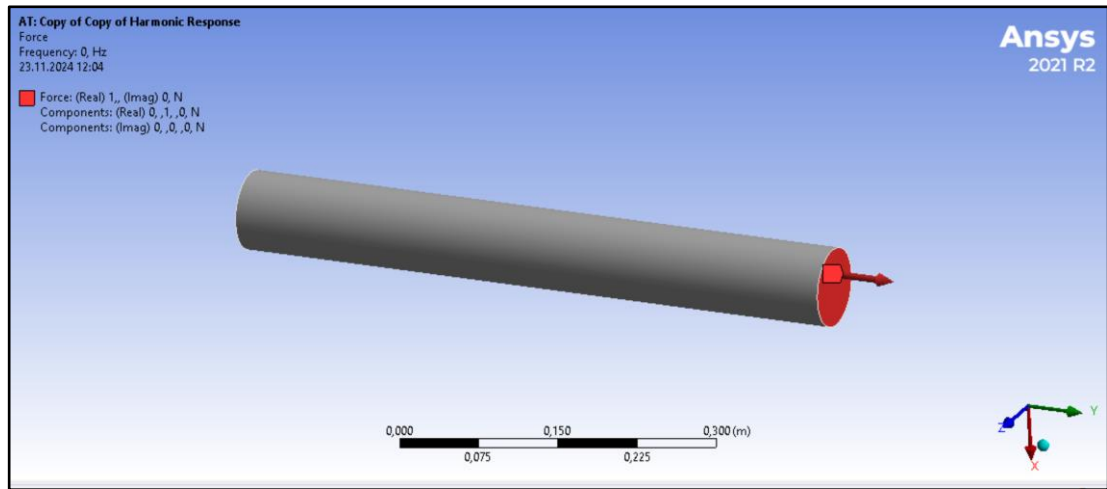


Figure 2.18: Tension Load direction

Maximum velocity is top of the cylinder shown in the **Figure 2.19**.

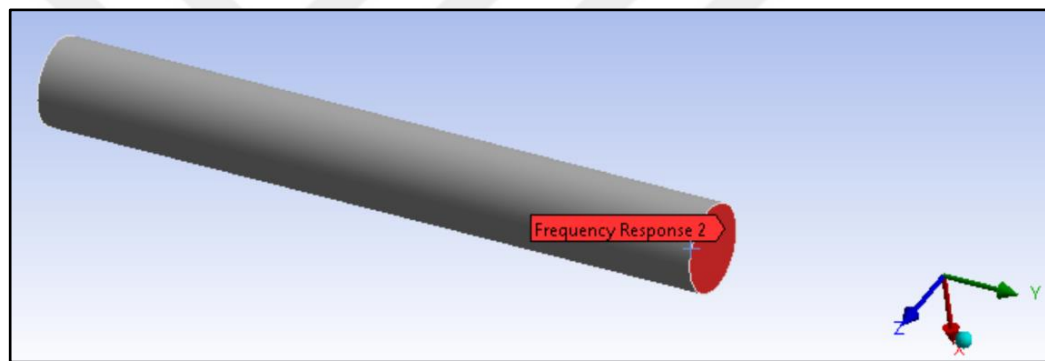


Figure 2.19: Top of the cylinder

The Maximum Velocity value at Y direction is 2,0307E-4 m/s at 2087,8 Hz in the **Figure 2.20**.

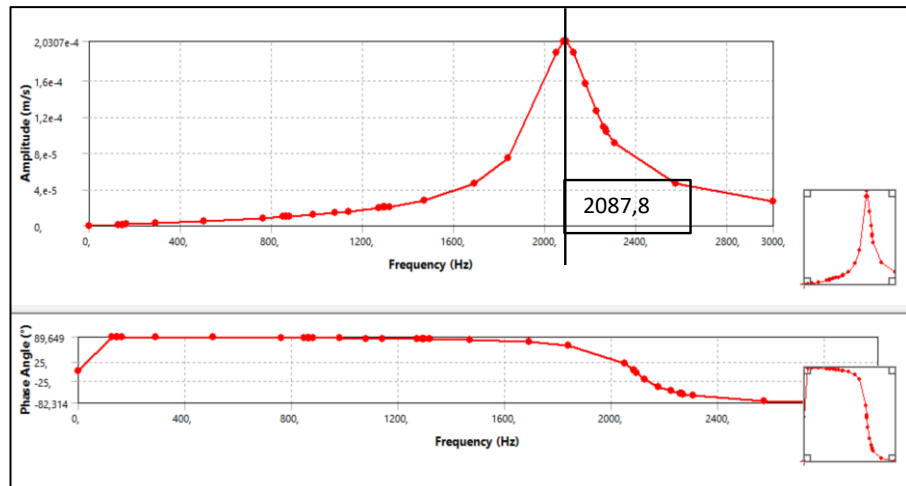


Figure 2.20: Maximum velocity at Y direction

When I calculate maximum velocity at Maximum Phase and 2087,8 Hz, The Maximum Velocity is 2,0318E-4 m/s at 2087,8 Hz and 0° phase angle shown in the **Figure 2.21**.

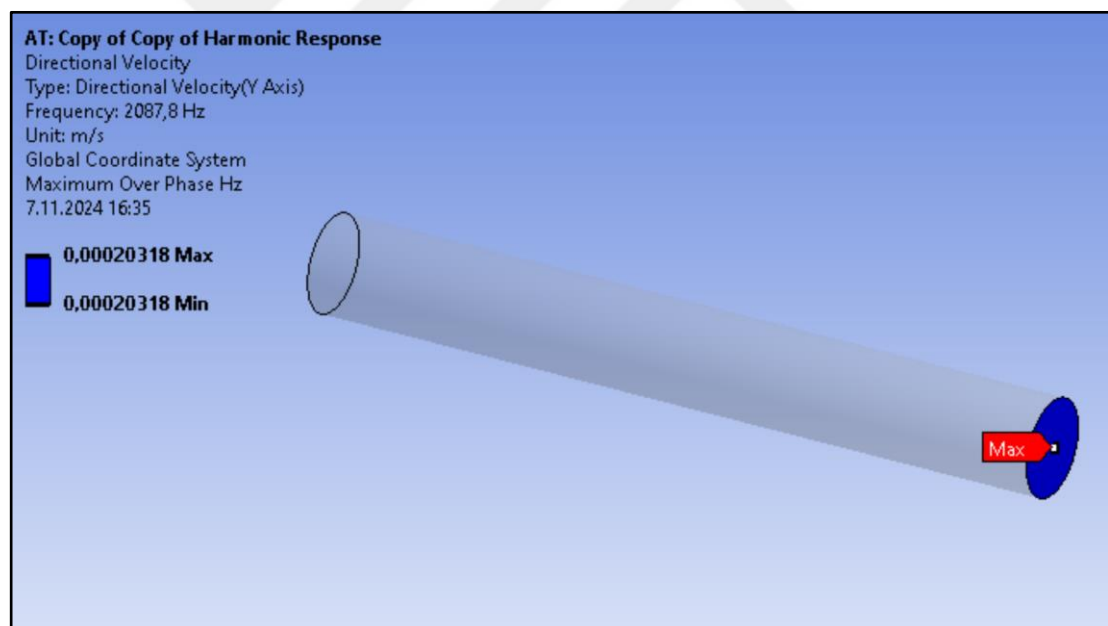


Figure 2.21: Maximum Velocity at 2087,8 Hz and 0° phase angle

Maximum Normal Stress is taken red face shown in the **Figure 2.22**.

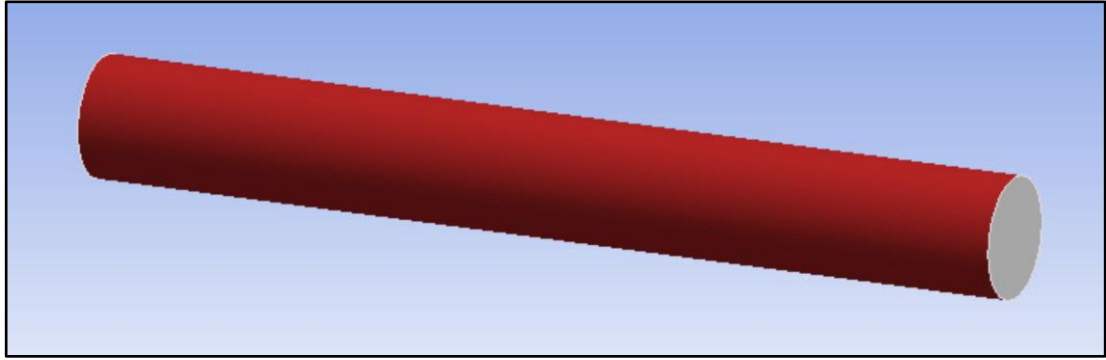


Figure 2.22: Maximum Normal Stress is taken on the red face

The Maximum Normal Stress value is 1776,2 Pa at 2082,6 Hz in the **Figure 2.23**.

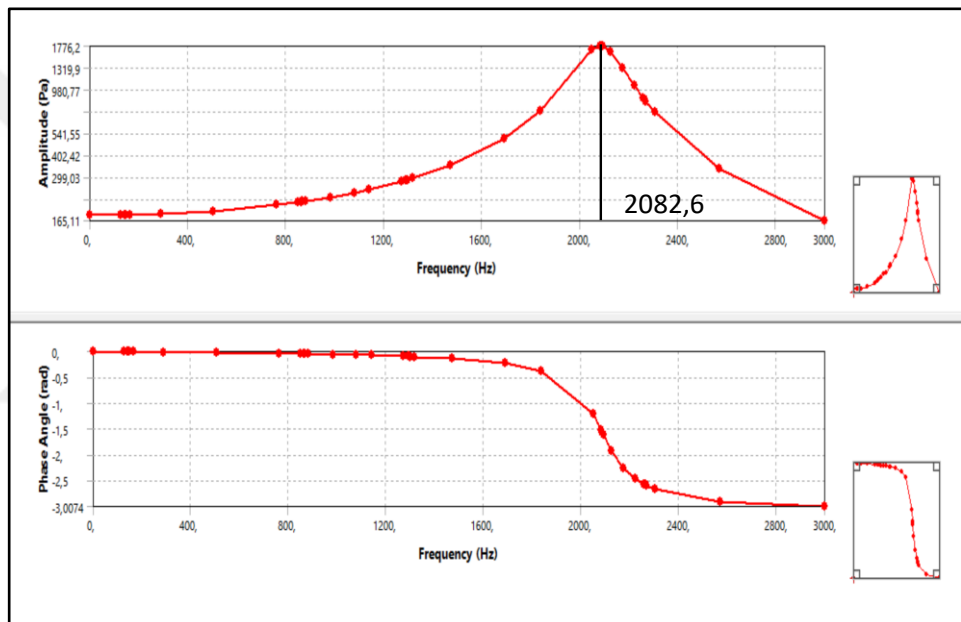


Figure 2.23: Stress Results depend on Frequency for tension

When I calculate maximum Von Mises Stress at Maximum Phase and 2082,6 Hz, The Maximum Stress is 2888,4 Pa at 2082,6 Hz and -90° phase angle shown in the **Figure 2.24**.

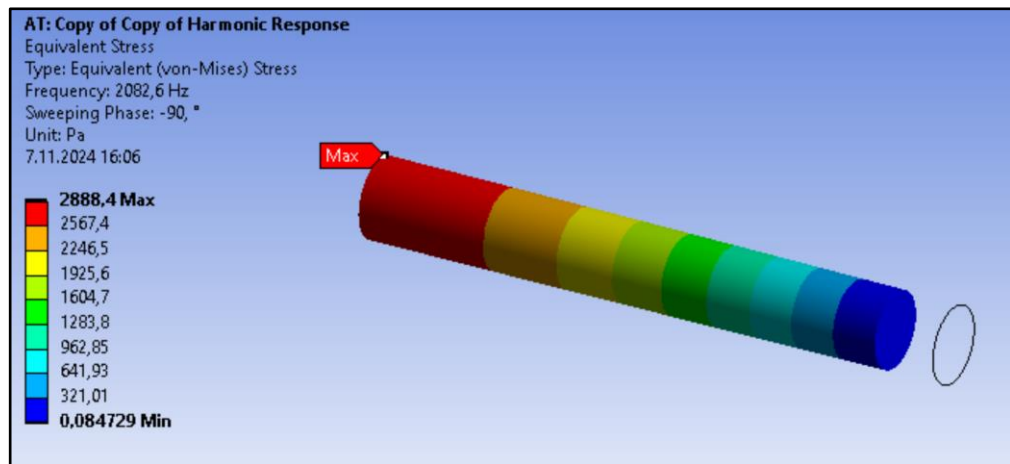


Figure 2.24: Maximum Equivalent Stress for tension

Bending load direction is shown in the **Figure 2.25**. Bending load is applied as “1 N”.

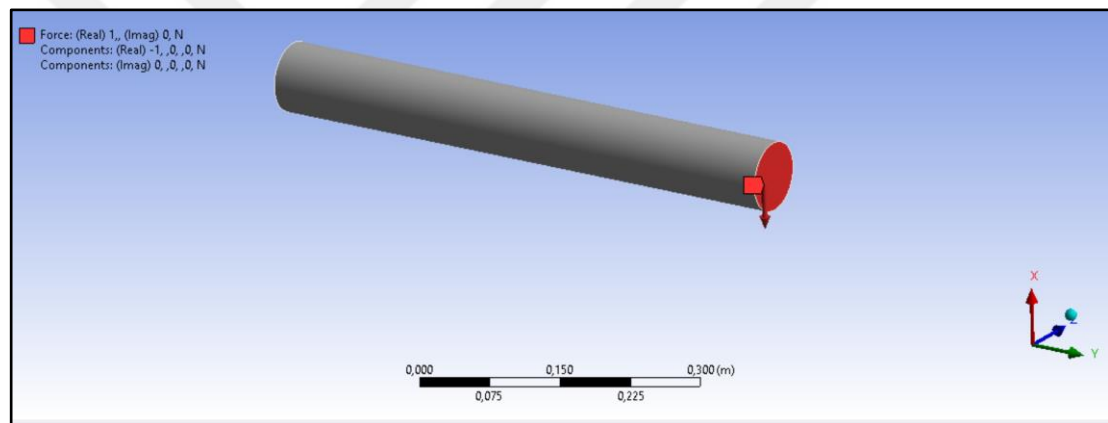


Figure 2.25: Bending Load direction

Maximum velocity is top of the cylinder shown in the **Figure 2.26**.

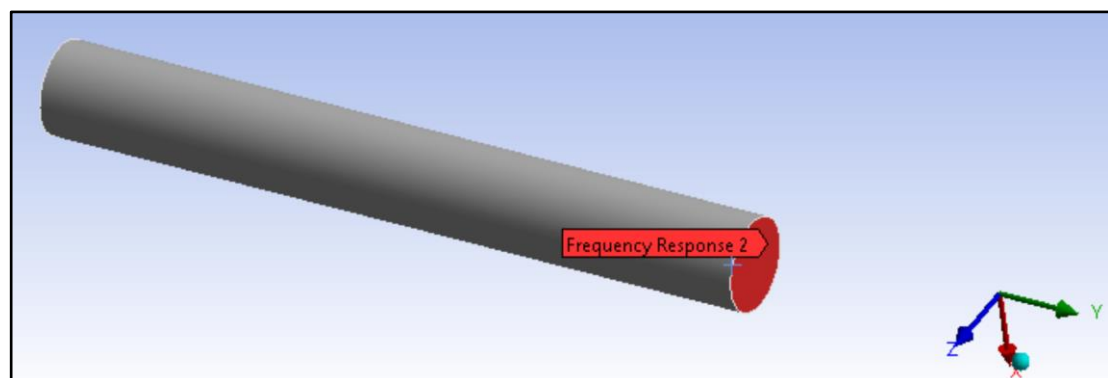


Figure 2.26: Top of the cylinder

Maximum velocity is 144,82 Hz in the **Figure 2.27**.

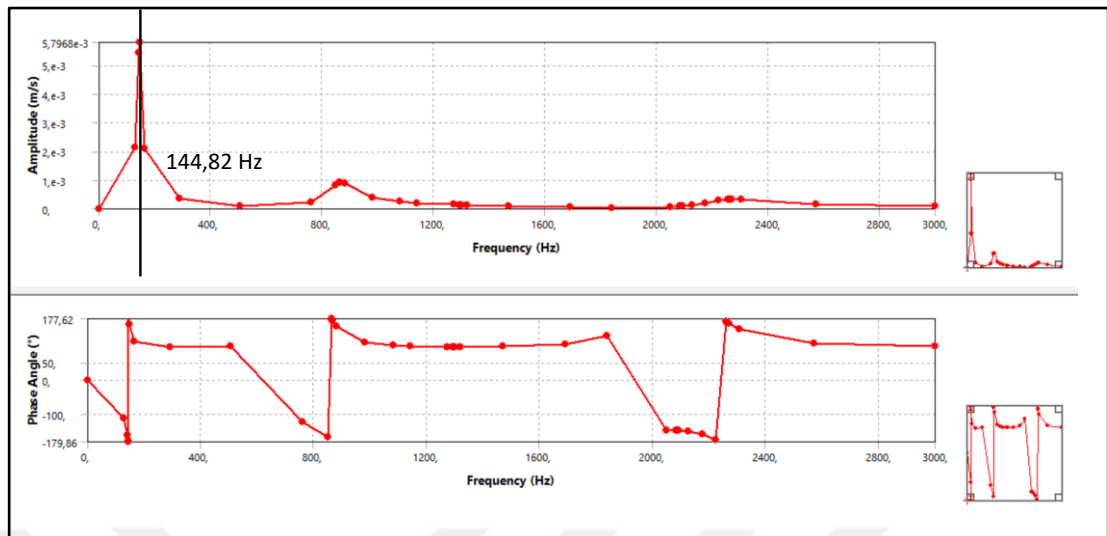


Figure 2.27: Velocity depends on Frequency and Phase Angle

When I calculate maximum velocity at Maximum Phase and 144,82 Hz, The Maximum Velocity is 0,0057967 m/s at 144,82 and -179,86 ° phase angle shown in the **Figure 2.28**.

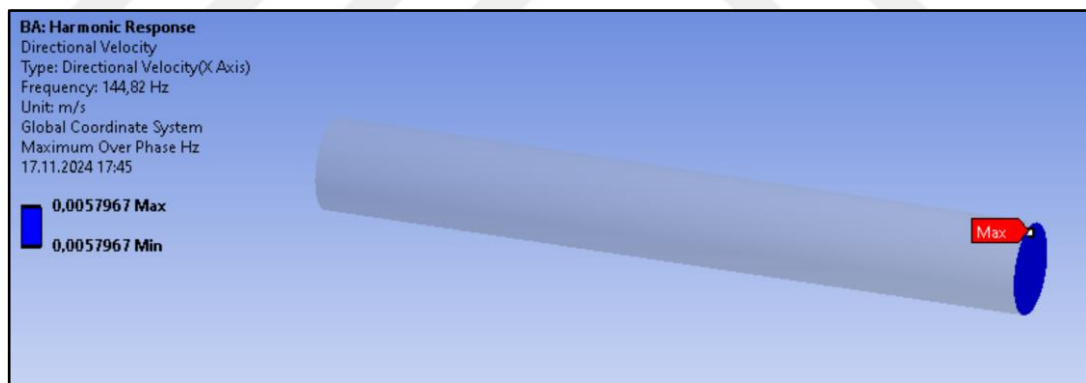


Figure 2.28: Maximum Velocity at 144.82 Hz and -179,86 °

Maximum Normal Stress is taken red face shown in the **Figure 2.29**.

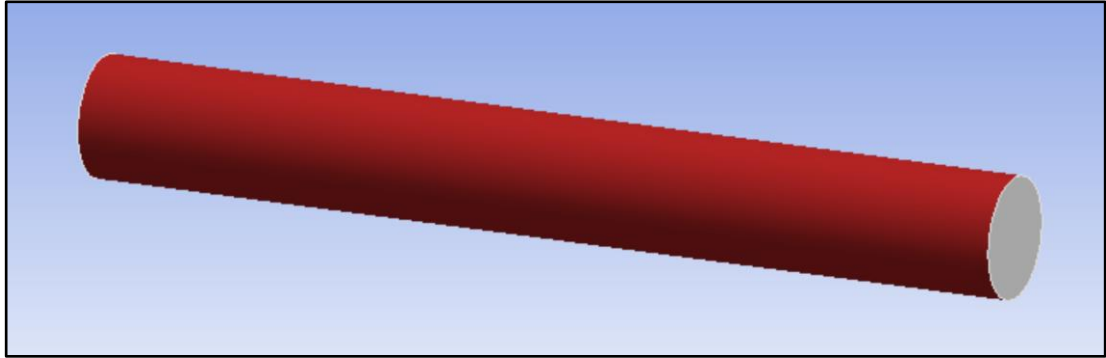


Figure 2.29: Maximum Normal Stress is taken on the red face

The Maximum Normal Stress value is calculated at 862,49 Hz, however there is singularity for the region in the **Figure 2.30**. So Maximum stress is at 142,19 Hz in the **Figure 2.30**.

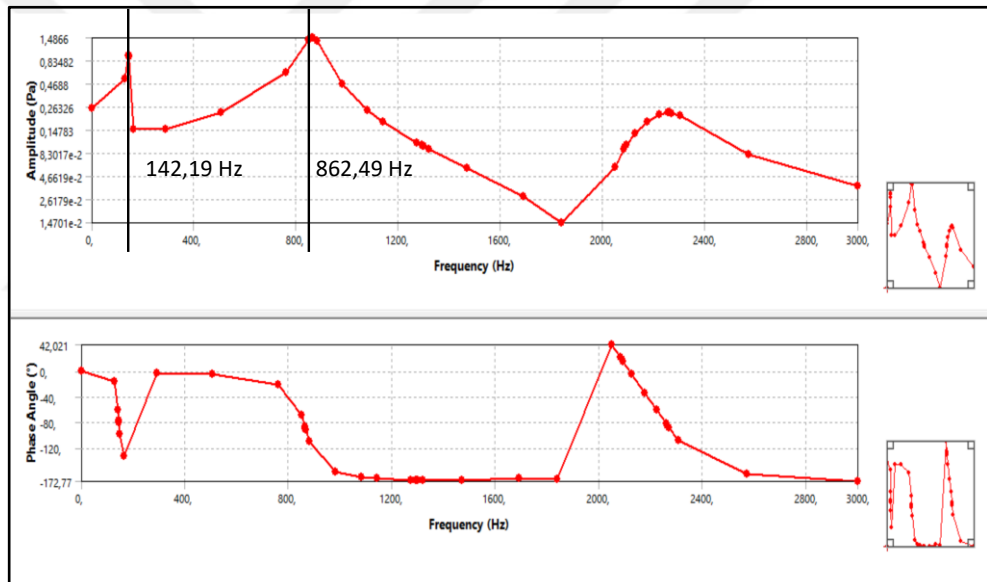


Figure 2.30: Stress Results depend on Frequency for bending load

Also, at 144,82 Hz, Stress and velocity is maximum. When I calculate maximum Von Mises Stress at Maximum Phase and 144,82 Hz, The Maximum Stress is 1,5597E5 Pa at 144,82 Hz and -179,86° phase angle shown in the **Figure 2.31**.

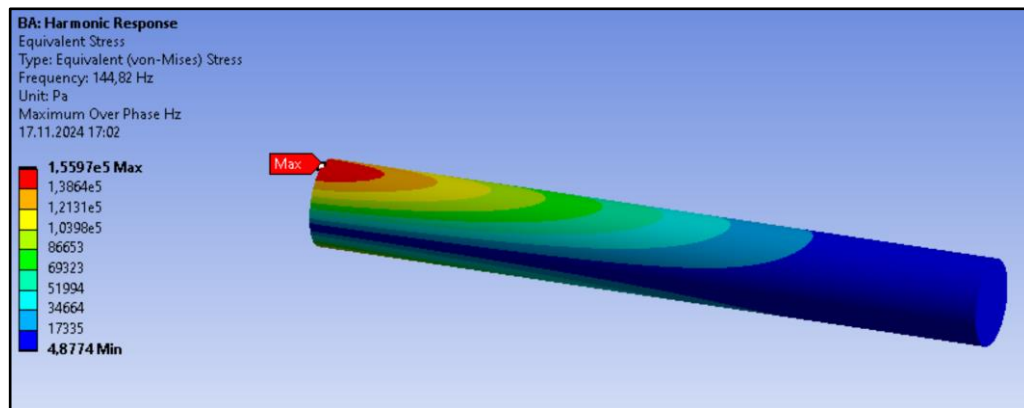


Figure 2.31: Maximum Equivalent Stress

Torsional load direction is shown in the **Figure 2.32**. Torsional load is applied as “1 N”.

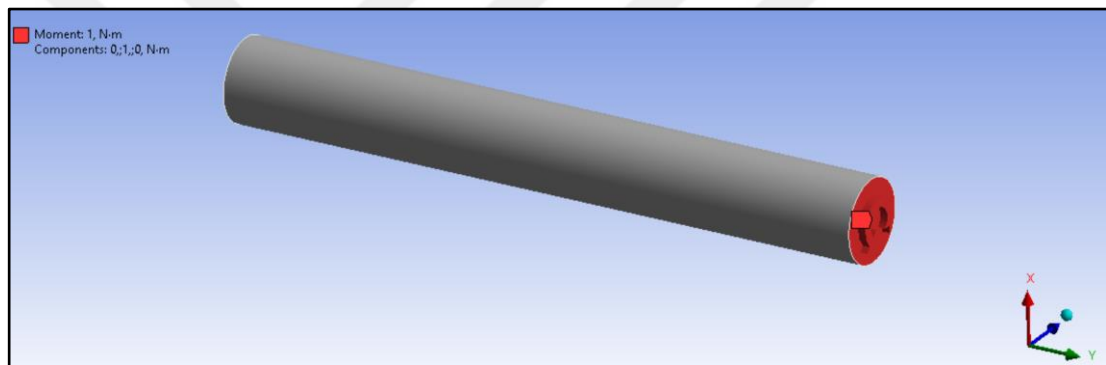


Figure 2.32: Torsional Load direction

Maximum velocity is 1295,1 Hz in the **Figure 2.33**.

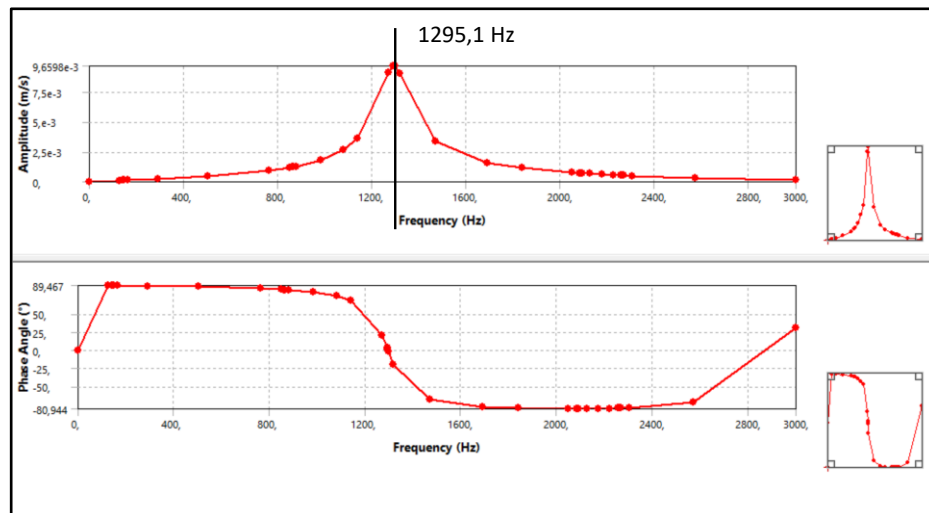


Figure 2.33: Velocity depends on Frequency and Phase Angle

The Maximum Stress is 2,5654E5 Pa at 1295,1 Hz and -91,906° phase angle shown in the **Figure 2.34**.

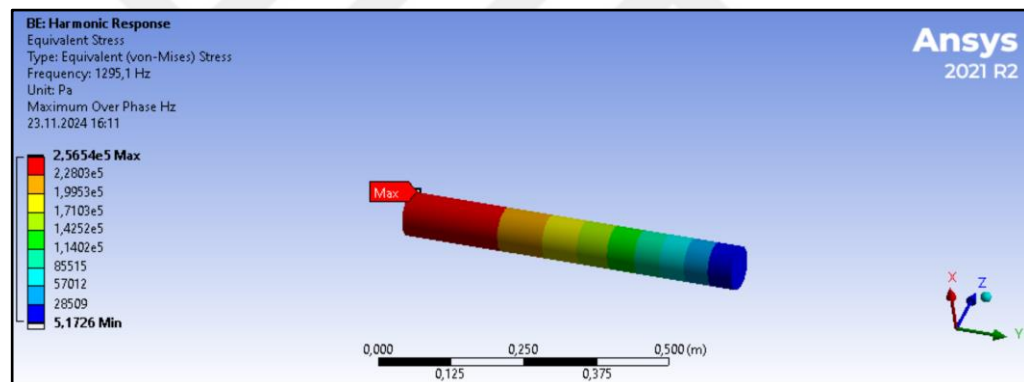


Figure 2.34: Maximum Equivalent Stress for Torsional Load

Maximum velocity is top of the cylinder shown in the **Figure 2.35**.

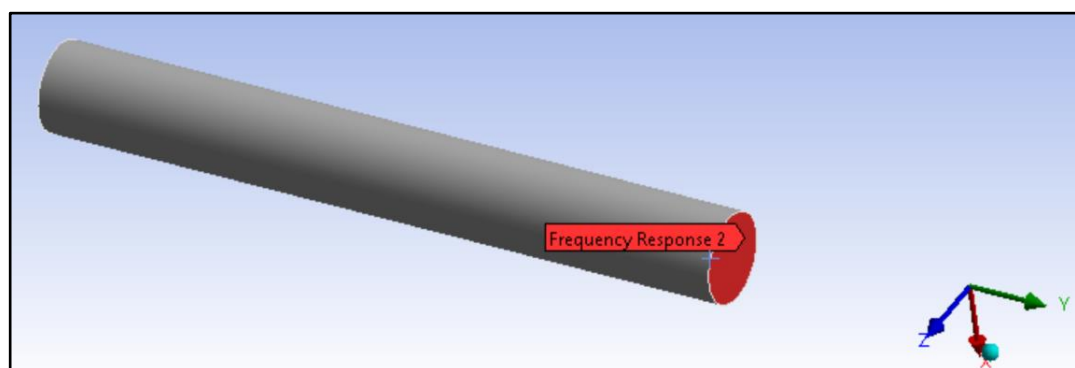


Figure 2.35: Top of the cylinder for Torsional Load

When I calculate maximum velocity at Maximum Phase and 1295,1 Hz, The Maximum Velocity is 0.017197 m/s at 1295,1 Hz and -91,906° phase angle shown in the **Figure 2.36**.

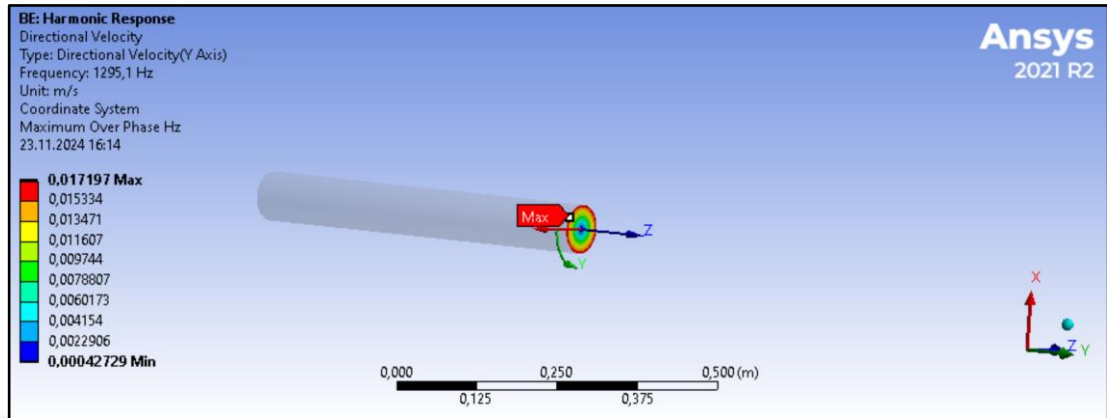


Figure 2.36: Maximum Velocity at 1295,1 Hz and -91,906° phase angle

Maximum Normal Stress is taken red face shown in the **Figure 2.37**.

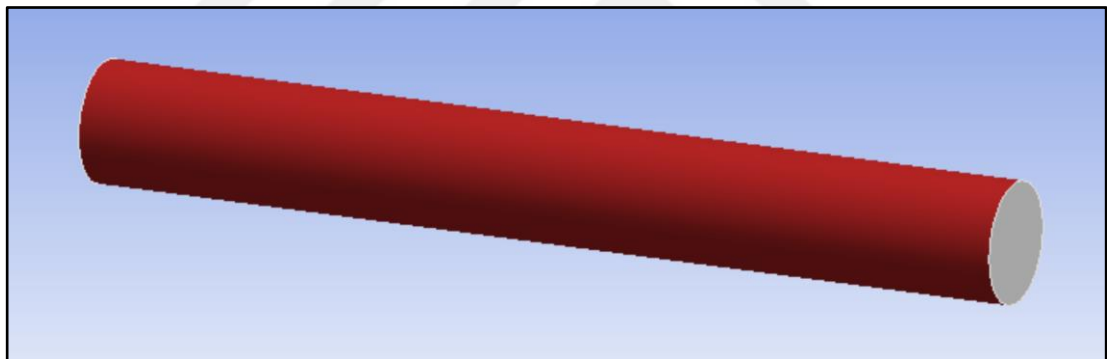


Figure 2.37: Maximum Normal Stress is taken on the red face

The Maximum Stress value is calculated at 1295,1 Hz, however there is singularity for the region in the **Figure 2.37**. So Maximum stress is 1295,1 Hz in the **Figure 2.38**.

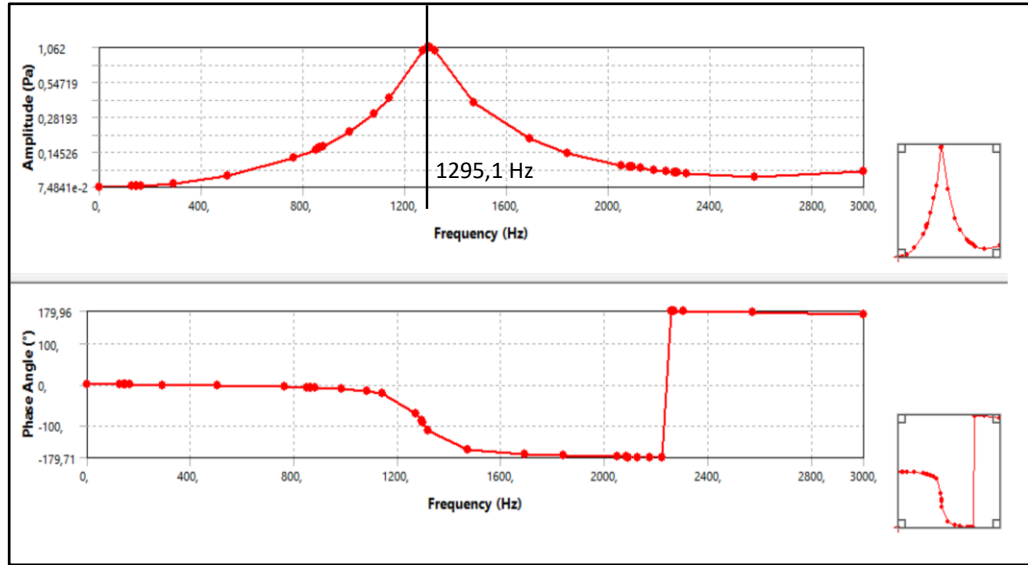


Figure 2.38: Stress Results depend on Frequency for Torsional Load

2.2.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.2.4.1. Maximum Velocity and Stress for Tension Load

Amplitude of Motion(X) was calculated. This amplitude was determined using equation (1.5) [17]. Parameters of the equation(1.5) are Maximum Dynamic Displacement ($\delta_{\max}$) and Damping ratio (ζ). $\delta_{\max}$ value is calculated by using equation (1.6). Parameters of the equation (1.6) are the ratio of effective mass to total mass is the proportion of the total mass (REMTM) and Maximum Static displacement of unit load (δ_{st_max}) at tension load direction. REMTM is the ratio of effective mass to total mass is the proportion of the total mass at tension load direction(TY). The value is on the unit tension load direction is 0,81 from Table 2.7. δ_{st_max} is taken from FEM analysis result (Table 2.4). δ_{st_max} value is 1,91E-9 m from the Table 2.4. $\delta_{\max}$ value is 1.54E-9 m. ζ is taken as 0.05. X value is 1.54E-8 m. Amplitude as a function of phase angle and time(X_{phase}) is calculated by using equation (1.7) [17]. Parameters of the equation(1.7) are X, time(t), phase angle(Φ) and w_d . t is calculated by using equation (1.8) and t value is 4.8E-4 s. Damped angular frequency(w_d) is calculated by using equation(1.10) [17]. Parameters of the equation(1.10) are ζ and undamped angular

frequency(w_n). W_n value corresponding to REMTM (0.81) and tension load direction (TY) is selected from Table 2.7 and converted from Hz to rad/s. w_n and w_d values are 13118.5 rad/s and 13085.7 rad/s. X_{phase} results depend on (0-360° phase angle) are shown in the Figure 2.39. Maximum amplitude depend on Φ ($X_{\text{phase_max}}$) is 1.54E-8 m. V_{phase} (Velocity as a function of phase angle and time) is calculated by using (1.9) and $V_{\text{phase_max}}$ (Velocity as a function of phase angle and time) is 2.01E-4m/s. The value is shown in the **Figure 2.39**.

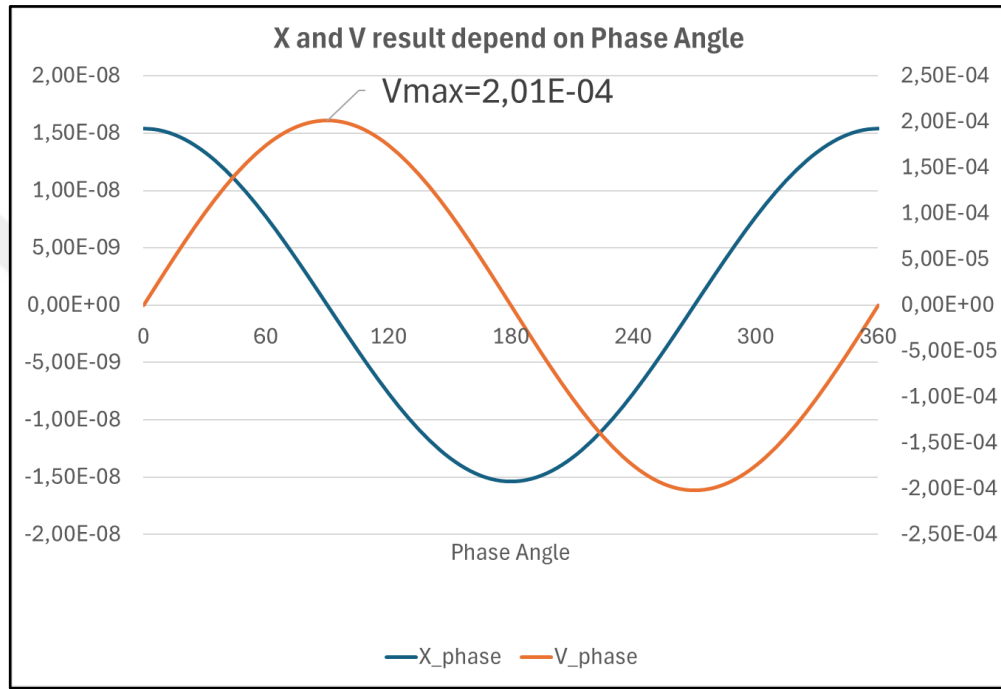


Figure 2.39: Velocity value in Analytical Calculation for Tension loading

If Results of Analytical with FEM Parameters and only FEM result (**Figure 2.21**) are compared, the results are shared in the **Table 2.8**.

Table 2.8: Comparison of Analytical and FEM Velocity Results for Tension Load

Parameter	Combined	FEM	Difference
Max. Velocity	2.01E-4	2.03E-4	%0.9

Maximum Stress (σ_{max}) is calculated by using equation (1.21) [2]. Parameters of equation (1.21) are dynamic stress-velocity coefficient(K_d), longitudinal wave speed for material(c_l), density of material (ρ) and velocity as a function of phase angle and time (v_{max} or $v_{\text{phase_max}}$). c_l is calculated by using equation (1.12). Parameters of the

equation (1.12) are Young's Modulus (E) and ρ . E and ρ values are 7E10 Pa and 2700 kg/m³. c_1 is calculated 5091.75 m/s. v_{max} value is 2.01E-4 m/s from Table 2.8. σ_{max} is calculated 2.77E3 Pa. If Results of Analytical with FEM Parameters and only FEM result (Figure 2.24) are compared, the results are shared in the **Table 2.9**.

Table 2.9: Comparison of Analytical and FEM Stress Results for Tension Load

Parameter	Combined	FEM	Difference
Max. Stress	2.77E3	2.89E3	%4

2.2.4.2. Maximum Velocity and Stress for Bending Load

Amplitude of Motion(X) was calculated. This amplitude was determined using equation (1.5) [17]. Parameters of the equation(1.5) are Maximum Dynamic Displacement (δ_{max}) and Damping ratio (ζ). δ_{max} value is calculated by using equation (1.6). Parameters of the equation (1.6) are the ratio of effective mass to total mass is the proportion of the total mass (REMTM) and Maximum Static displacement (δ_{st_max}) at tension load direction. REMTM is the ratio of effective mass to total mass is the proportion of the total mass at Bending load direction (RZ). The value is on the unit tension load direction is 0,97 from Table 2.7. δ_{st_max} is taken from FEM analysis result (Table 2.5). δ_{st_max} value is 6.57E-7 m from the Table 2.5. δ_{max} value is 6.34E-7. ζ is taken as 0.05. X value is 6.34E-6 m. Amplitude as a function of phase angle and time(X_{phase}) is calculated by using equation (1.7) [17]. Parameters of the equation(1.7) are X, time(t), phase angle(Φ) and w_d . t is calculated by using equation (1.8) and t value is 6.91E-3 s. Damped angular frequency(w_d) is calculated by using equation(1.10) [17]. Parameters of the equation(1.10) are ζ and undamped angular frequency(w_n). w_n value corresponding to REMTM (0.97) and bending load direction (RZ) is selected from Table 2.7 and converted from Hz to rad/s. W_n and W_d values are 909,6,9 rad/s and 907,6 rad/s. X_{phase} results depend on (0-360° phase angle) are shown in the **Figure 2.40**. Maximum amplitude depend on Φ (X_{phase_max}) is 6.35E-6 m. V_{phase} (Velocity as a function of phase angle and time) is calculated by using (1.9) and V_{phase_max} (Velocity as a function of phase angle and time) is 5.76E-3 m/s. The value is shown in the **Figure 2.40**.

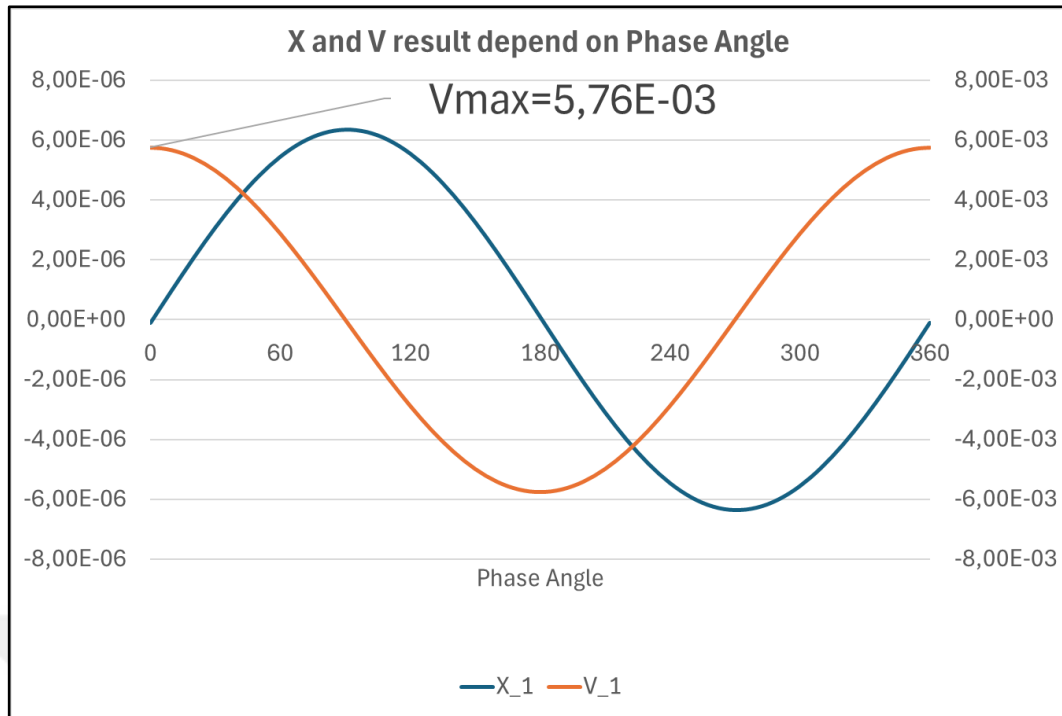


Figure 2.40: Velocity value in analytical calculation for bending loading

If Results of Analytical with FEM Parameters and only FEM result (**Figure 2.28**) are compared, the results are shared in the **Table 2.10**.

Table 2.10: Comparison of Analytical and FEM Velocity Results

Parameter	Combined	FEM	Difference
Max. Velocity	5.77E-03	5.80E-03	%0.7

Maximum Stress (σ_{max}) is calculated by using equation (1.35) [2]. Parameters of equation(1.35) are dynamic stress-velocity coefficient($K_{db}=2$), longitudinal wave speed for material(c_l), density of material (ρ) and velocity as a function of phase angle and time (v_{max} or v_{phase_max}). c_l is calculated by using equation (1.12). Parameters of the equation (1.12) are Young's Modulus (E) and ρ . E and ρ values are $7E10$ Pa and 2700 kg/m³. c_l is calculated 5091.75 m/s. v_{max} value $5.77E-3$ m/s from Table 2.8. σ_{max} is calculated $1.58E5$ Pa. If Results of Analytical with FEM Parameters and only FEM result (**Figure 2.30**) are compared, the results are shared in the **Table 2.11**.

Table 2.11: Comparison of Analytical and FEM values for Bending Load

Parameter	Combined	FEM	Difference
Max. Stress	1.58E+05	1,56E+05	%1.5

2.2.4.3. Maximum Velocity and Stress for Torsional Load

Amplitude of Motion(X) was calculated. This amplitude was determined by using equation (1.5) [17]. Parameters of the equation(1.5) are Maximum Dynamic Displacement ($\delta_{\max}$) and Damping ratio (ζ). $\delta_{\max}$ value is calculated by using equation (1.6). Parameters of the equation (1.6) are the ratio of effective mass to total mass is the proportion of the total mass (REMTM) and Maximum Static displacement ($\delta_{\text{st_max}}$) at tension load direction. REMTM is the ratio of effective mass to total mass is the proportion of the total mass at Torsional load direction (RY). The value is on the unit tension load direction is 0,81 from Table 2.7. $\delta_{\text{st_max}}$ is taken from FEM analysis result (Table 2.6). $\delta_{\text{st_max}}$ value is 2.61E-7 m from the Table 2.6. $\delta_{\max}$ value is 2.1E-7. ζ is taken as 0.05. X value is 6.34E-6 m. Amplitude as a function of phase angle and time(X_{phase}) is calculated by using equation (1.7) [17]. Parameters of the equation(1.7) are X, time(t), phase angle(Φ) and w_d . t is calculated by using equation (1.8) and t value is 7.72E-4 s. Damped angular frequency(w_d) is calculated by using equation(1.10) [17]. Parameters of the equation(1.10) are ζ and undamped angular frequency(w_n). w_n value corresponding to REMTM (0.81) and torsional load direction (RY) is selected from Table 2.7 and converted from Hz to rad/s. w_n and w_d values are 8137.4 rad/s and 8117 rad/s. X_{phase} results depend on (0-360° phase angle) are shown in the Figure 2.41. Maximum amplitude depend on Φ ($X_{\text{phase_max}}$) is 2.12E-6 m. V_{phase} (Velocity as a function of phase angle and time) is calculated by using (1.9) and $V_{\text{phase_max}}$ (Velocity as a function of phase angle and time) is 1.72E-2 m/s. The value is shown in the **Figure 2.41**.

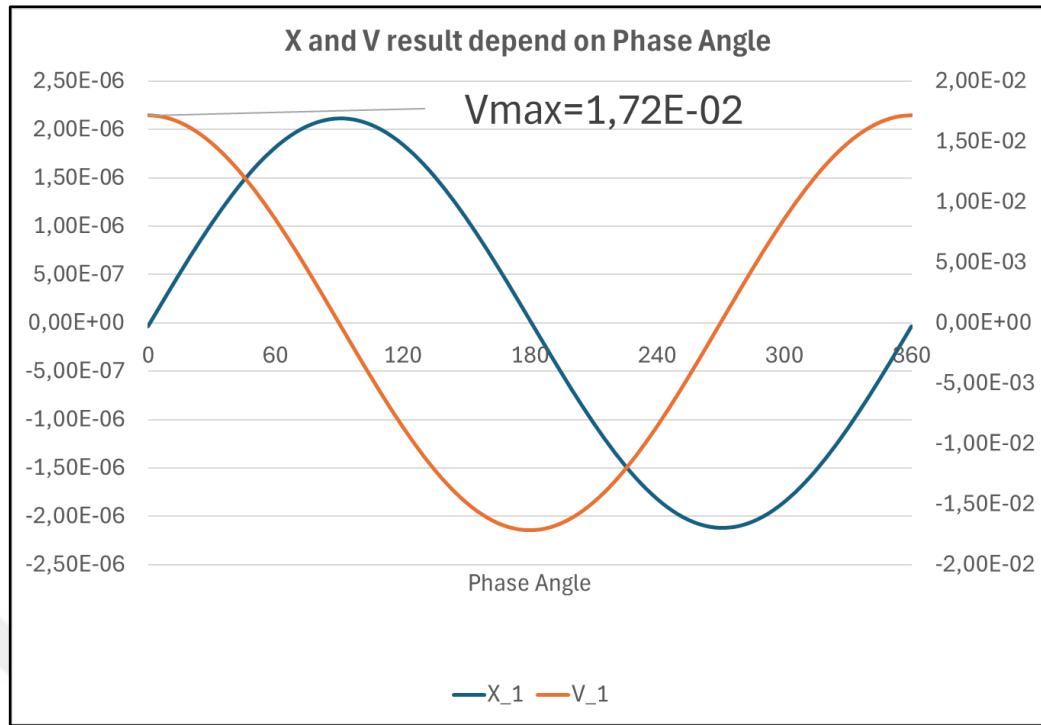


Figure 2.41: Velocity value in analytical calculation for torsional loading

If Results of Analytical with FEM Parameters and only FEM result (**Figure 2.33**) are compared, the results are shared in the **Table 2.12**.

Table 2.12: Comparison of Analytical and FEM Velocity values

Parameter	Combined	FEM	Difference
Max. Velocity	1.717E-02	1.720E-02	%0,2

Maximum Stress (τ_{max}) is calculated by using equation (1.38) [2]. Parameters of equation(1.35) are dynamic stress-velocity coefficient ($K_{dt}=1$), shear wave speed for material(c_s), density of material (ρ) and velocity as a function of phase angle and time (v_{max} or v_{phase_max}). c_s is calculated by using equation (1.39). Parameters of the equation (1.39) are Shear Modulus (G) and ρ . E and ρ values are $2.69E10$ Pa and 2700 kg/m³. c_s is calculated 2734.71 m/s. v_{max} value $1.717E-2$ m/s from Table 2.12. σ_{max} is calculated $2.54E5$ Pa. If Results of Analytical with FEM Parameters and only FEM result (Figure 2.38) are compared, the results are shared in the **Table 2.13**.

Table 2.13: Comparison of Analytical and FEM Stress values for Torsional Load

Parameter	Combined	FEM	Difference
Max. Stress	2.54E+05	2.56E+05	%1

2.3. Case study – 2 - Round shaft with shoulder fillet

2.3.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Round shaft with shoulder fillet is selected as sample. Geometry is shown in the **Figure 2.42**. Geometry dimensions are shared in the **Table 2.14**.

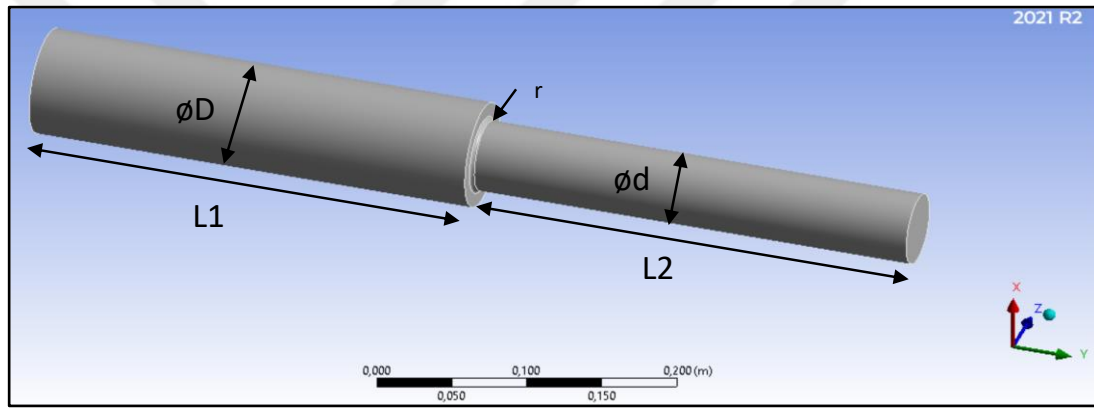


Figure 2.42: Round shaft with shoulder fillet

Table 2.14: Geometry dimensions and properties

Parameter	Value					
L1 [m]	0.3048					
L2 [m]	0.3048					
D [m]	0.0762					
d [m]	0.0508					
r /d	0.05	0.1	0.15	0.2	0.25	
r [m]	2.54E-3	5.08E-3	7.62E-3	1.02E-2	1.27E-2	
K _t for tension	2.4	1.9	1.65	1.55	1.45	
K _t for bending	2.05	1.66	1.5	1.41	1.39	

The mesh is shown in the **Figure 2.43**.

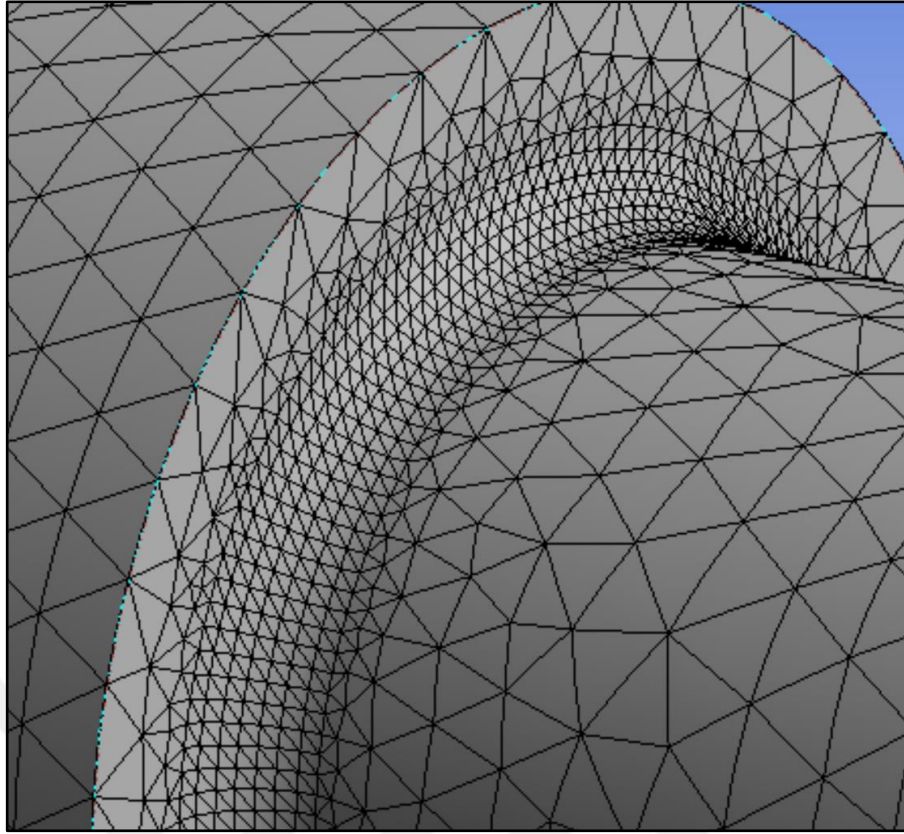


Figure 2.43: The mesh with updated radius ($r/d=0,1$)

The mesh is shown in the **Figure 2.44**.

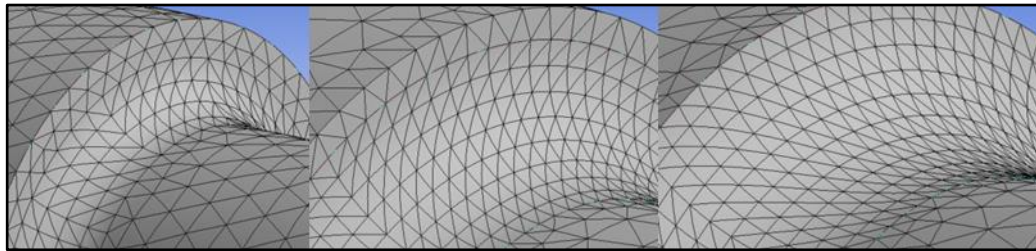


Figure 2.44: The mesh with updated radius ($r/d=0.15-0.20-0.25$)

Static, Modal and Harmonic Response Analysis is made. Boundary Condition is fixed 6 DOF and shown for modal analysis in the **Figure 2.45**.

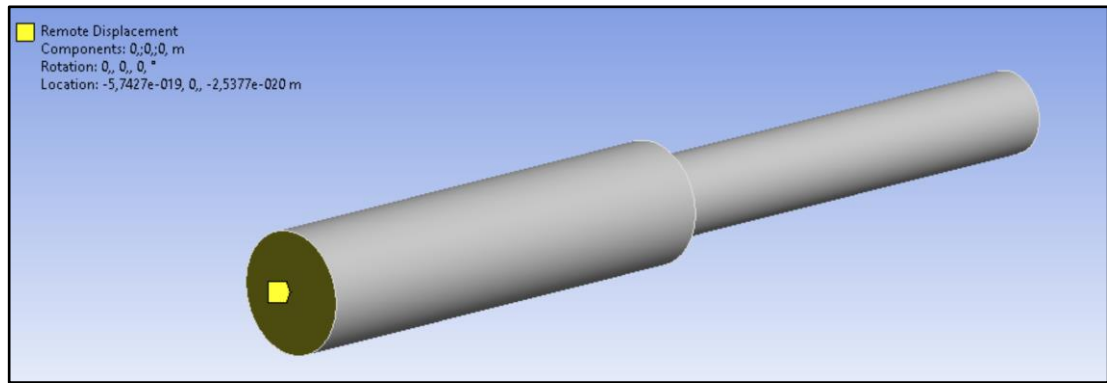


Figure 2.45: Boundary Conditions on the yellow face

Applied tension and bending loads are shown in the **Figure 2.46-Figure 2.47**.

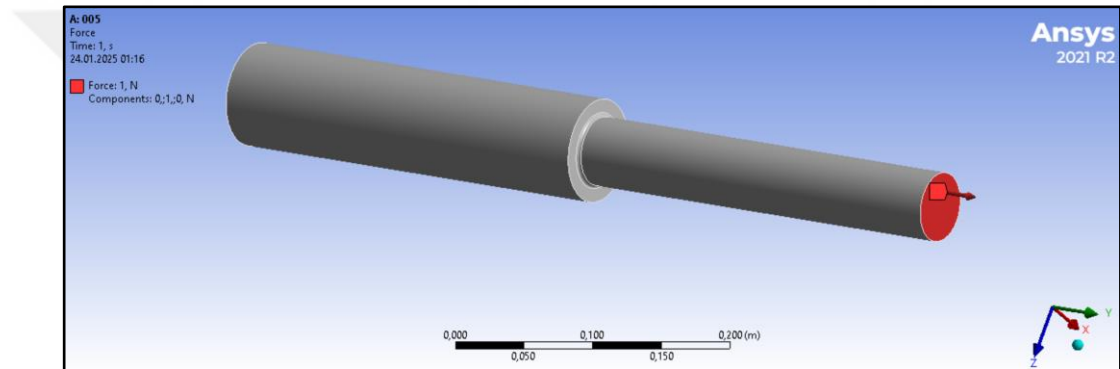


Figure 2.46: Tension Load Direction

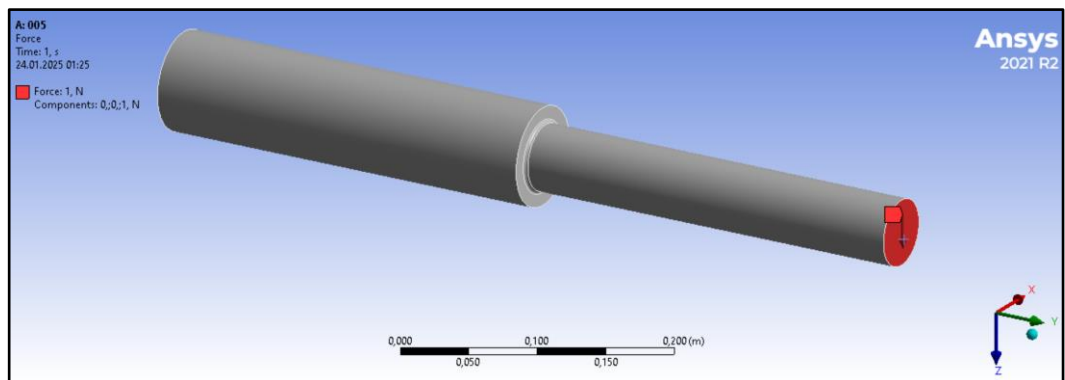


Figure 2.47: Bending Load Direction

According to different r/d ratio, there are 5 geometries with 5 fillet radius dimensions. In the **Figure 2.48**, Stress Concentration Factor graphics are shown for Tension Load. In this study, there is calculation K_d for 3 different D/d ratios and 5 different r/d ratios.

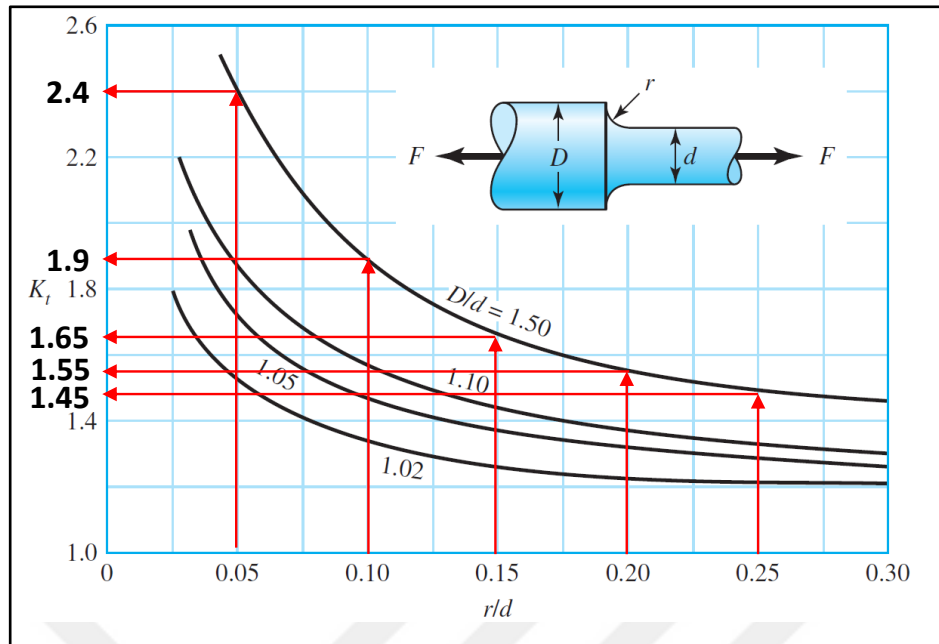


Figure 2.48: K_t depend on r/d and D/d for tension [15]

For bending load, Stress concentration factor is shown in the **Figure 2.49**.

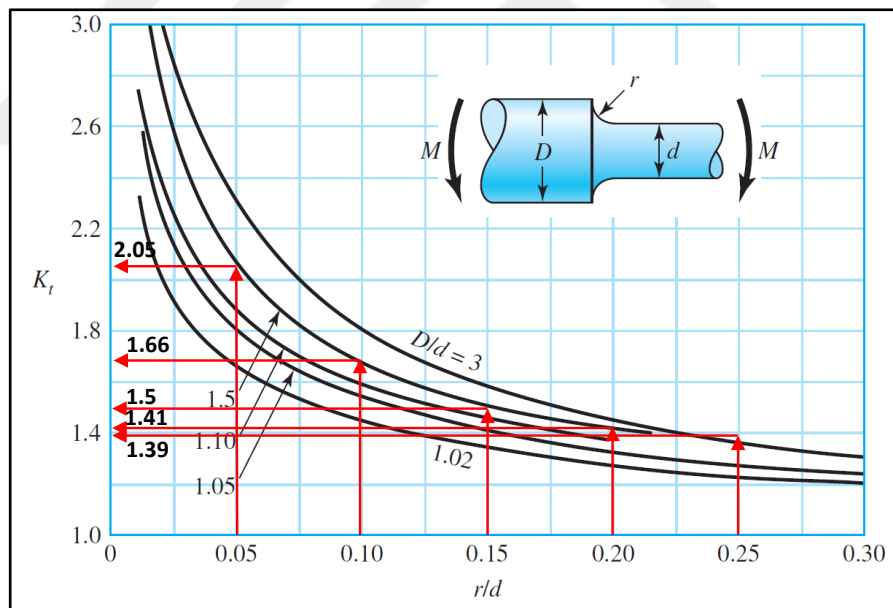


Figure 2.49: K_t depend on r/d and D/d for bending [15]

2.3.2. Static Analysis Result

Static analysis result for $r/d:0.05$ round shaft with shoulder fillet included deformation at Load direction Y directions is shown **Figure 2.50**. Maximum deformation is $3.144\text{E-}9$ m.

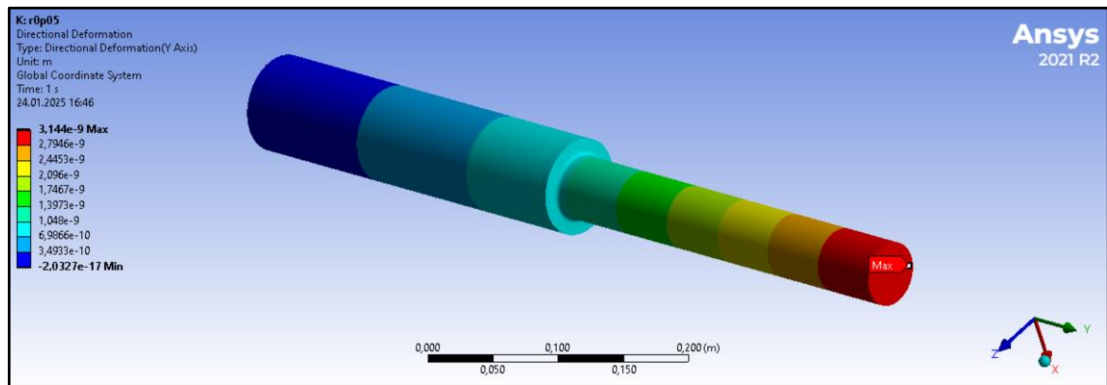


Figure 2.50: Tension Deformation at Y direction for r/d:0.05

Static analysis result for r/d:0.25 round shaft with shoulder fillet included deformation at Load direction Y directions is shown **Figure 2.51**. Maximum deformation is 3.11E-9 m.

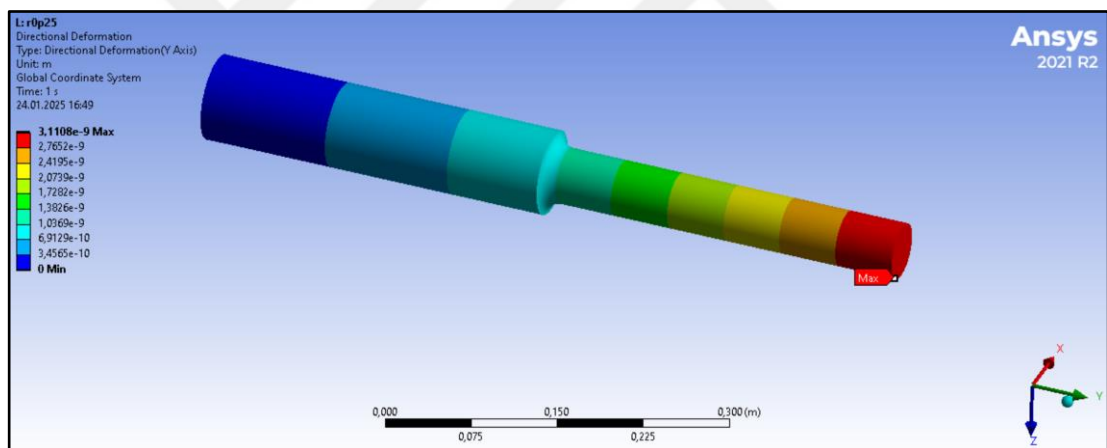


Figure 2.51: Tension Deformation at Y direction for r/d:0.25

Static analysis result for r/d:0.05 round shaft with shoulder fillet included deformation at Load direction X directions is shown **Figure 2.52** . Maximum deformation is 1E-6 m.

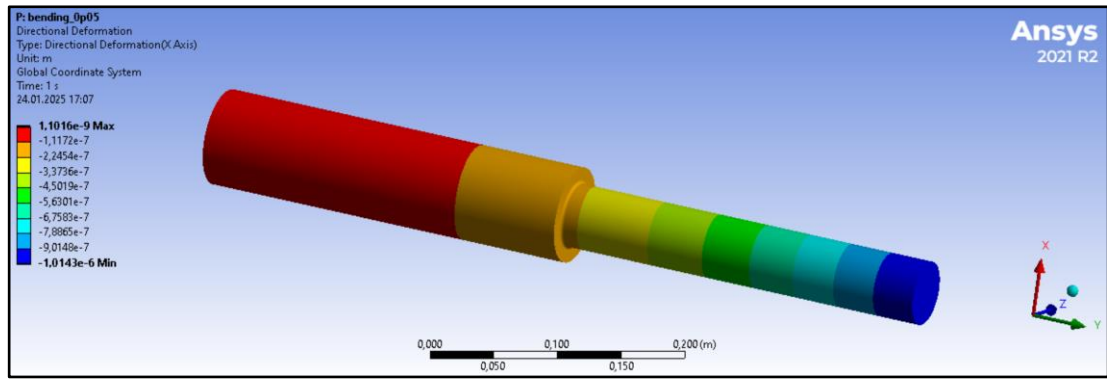


Figure 2.52: Bending Deformation at X direction for r/d:0.05

Static analysis result for r/d:0.25 round shaft with shoulder fillet included deformation at Load direction X directions is shown **Figure 2.53**. Maximum deformation is 9.92E-7E-6 m.

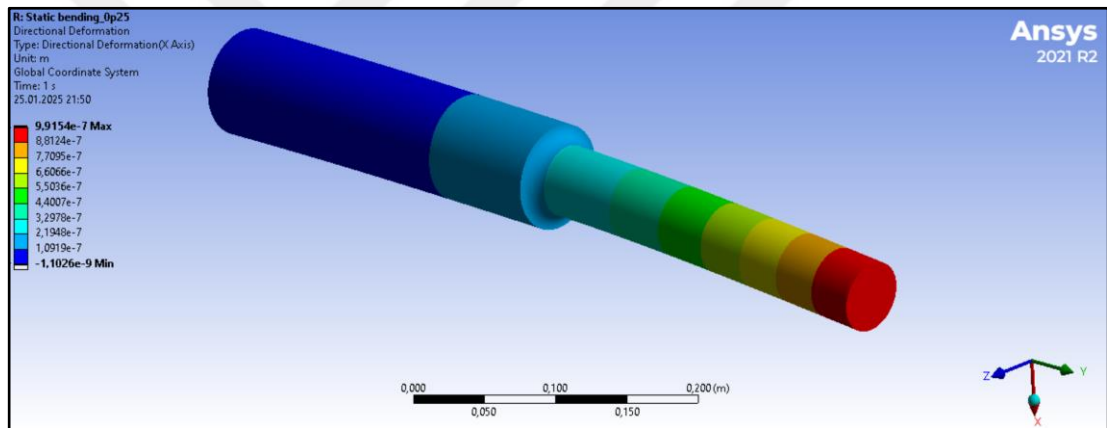


Figure 2.53: Bending Deformation at X direction for r/d:0.05

2.3.3. Harmonic Load Analysis Result

10 modes are calculated in the modal analysis. 3 critical mode shapes are selected for tension, bending and torsional modes. They are 2594,2 Hz for Y direction (Tension), 191,33 Hz for X direction (Bending) and 1890,7 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.15**.

Table 2.15: The Ratio of Effective Mass to Total Mass for Round Shaft

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	191,33	4,52E-01	1,27E-14	2,06E-03	4,15E-03	3,50E-11	9,12E-01
2	191,33	2,06E-03	6,87E-14	4,52E-01	9,12E-01	1,00E-11	4,15E-03
3	698,47	2,47E-01	3,30E-13	7,12E-06	2,06E-06	4,24E-10	7,34E-02
4	698,47	6,90E-06	3,96E-12	2,47E-01	7,34E-02	8,45E-11	2,06E-06
5	1890,70	4,54E-12	1,55E-15	2,48E-12	3,14E-14	6,22E-01	4,92E-16
6	1946,60	9,45E-02	9,39E-12	2,03E-03	1,48E-04	1,74E-09	6,89E-03
7	1946,60	2,04E-03	6,92E-11	9,45E-02	6,89E-03	7,91E-11	1,48E-04
8	2594,20	3,96E-09	7,11E-01	5,85E-09	1,14E-17	2,44E-09	1,56E-15
Sum		7,98E-01	7,11E-01	7,98E-01	9,96E-01	6,22E-01	9,96E-01

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, Tension force is added in the **Figure 2.54**. Damping coefficient is defined as 0.05.

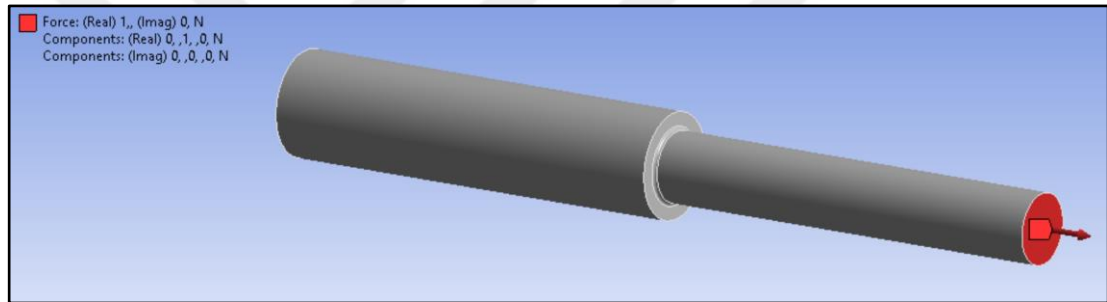


Figure 2.54: Tension Force in the Harmonic response analysis.

Normal Stress depends on frequency result is shown in the **Figure 2.55**.

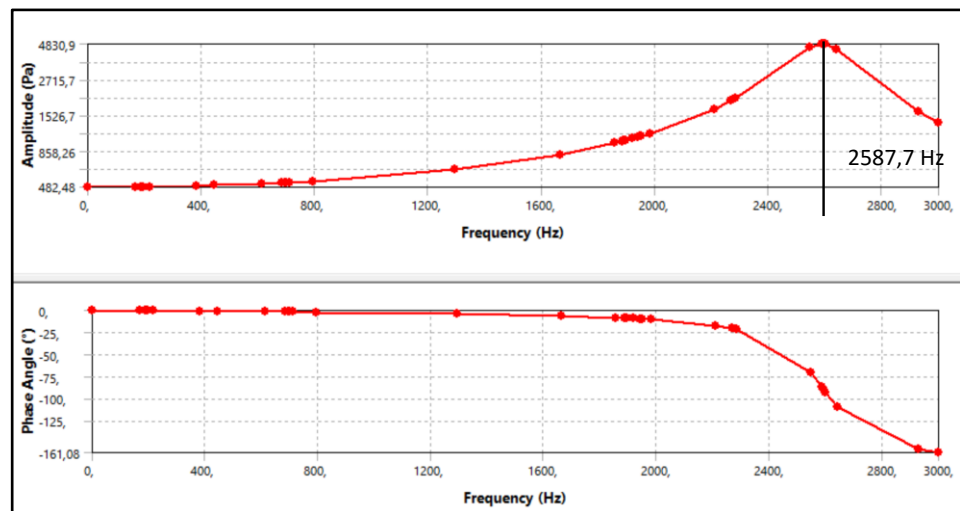


Figure 2.55: Normal Stress result depends on frequency

Maximum von Mises stress at 2587,7 Hz is shown in the **Figure 2.56**.

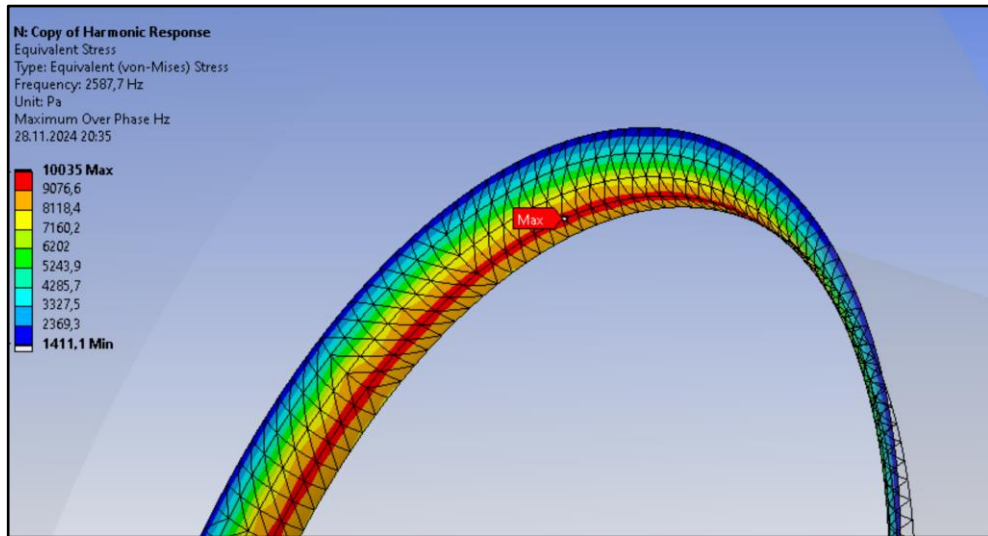


Figure 2.56: Maximum Von Mises at 2587,7 Hz for Tension Loading

Velocity depends on frequency result is shown in the **Figure 2.57**.

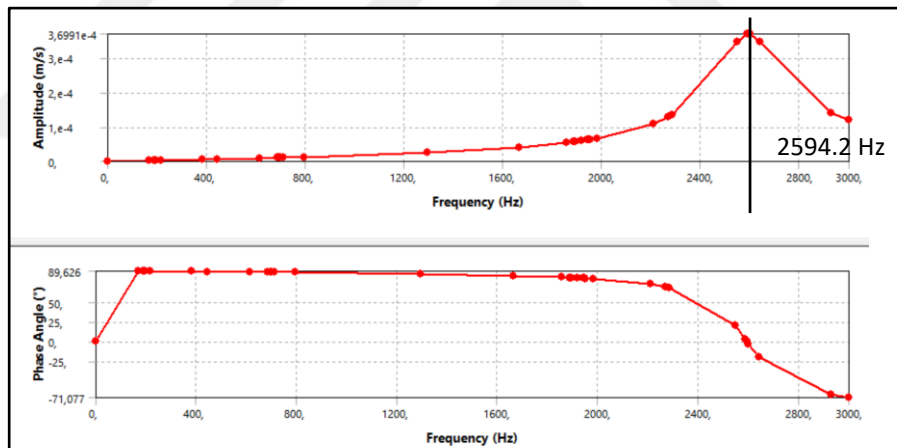


Figure 2.57: Velocity depends on frequency result for tension loading

Maximum velocity result at 2594.2 Hz is shown in the **Figure 2.58**.

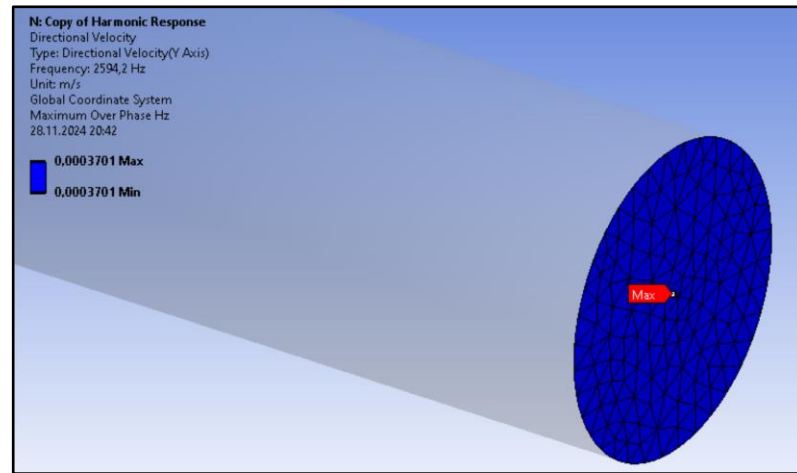


Figure 2.58: Maximum Velocity Result for tension loading

Only fillet radius dimensions are updated to 5,08E-3 m from 2 ,54E-3 m in the Table 2.14. Maximum stress and velocity result is shown in the **Figure 2.59**.

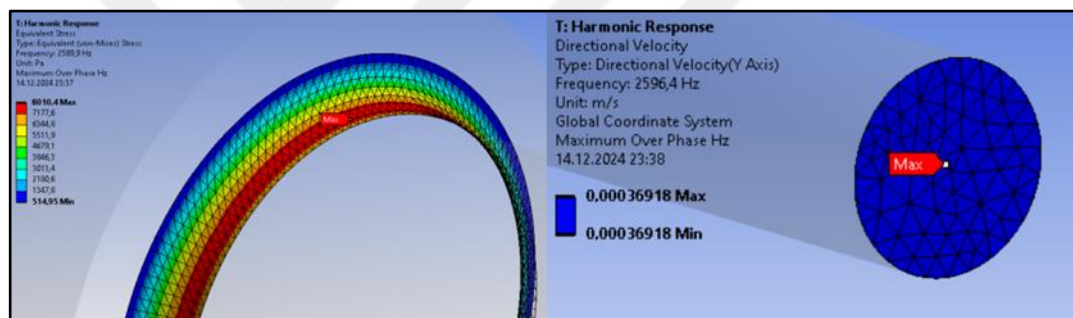


Figure 2.59: Maximum stress and velocity result ($r/d=0,1$)

The remaining fillet radius dimensions are 7.62E-3, 1.02E-2 and 1.27E-2 m in the Figure 2.44. For $r/d = 0.15-0.2-0.25$, the maximum stress and velocity results are shown in **Figure 2.60**.

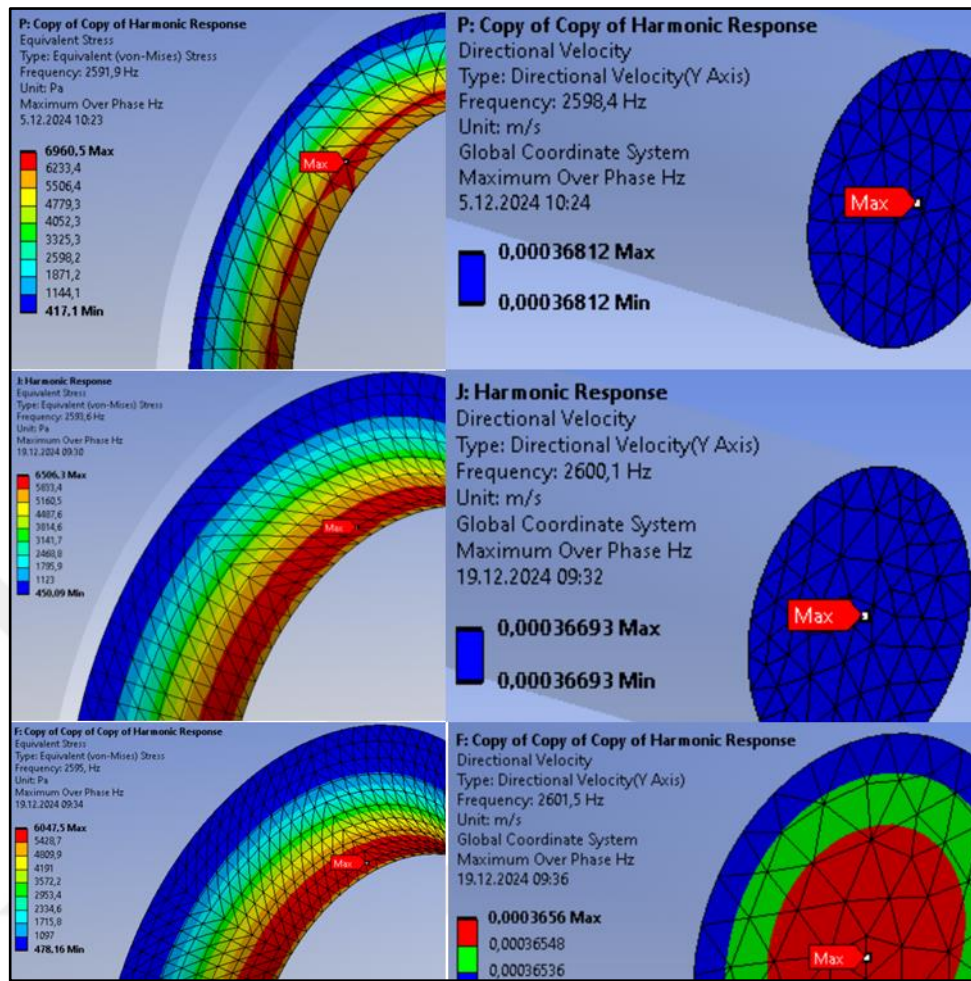


Figure 2.60: Maximum Stress and Velocity Result for $r/d=0.15-0.2-0.25$

K_d coefficient is calculated by using equation (1.21). Maximum stress, maximum velocity, K_d , K_t values are shared in the **Table 2.16**.

Table 2.16: Stress, Velocity and Coefficient Result for tension loading

r/d	0,05	0,1	0,15	0,2	0,25
Maksimum Stress Pa	10035	8010,4	6960,5	6506,3	5929,6
Maksimum Velocity m/sn	3,70E-04	3,69E-04	3,68E-04	3,67E-04	3,65E-04
K_d	1,97	1,58	1,38	1,29	1,18
K_t	2,4	1,9	1,65	1,55	1,45

In **Table 19** and **Figure 2.61**, When the ratio of K_d/K_t is evaluated for different r/d ratios, the following results are obtained: an average value of 0.83, an RMS value of 0.83 and an RMS error of 0.01. The observation of a consistent ratio across different

r/d values indicates a relationship between the K_d and K_t coefficients. This constant value (K_d/K_t : Constant) is accepted as 0.83.

Table 2.17: K_t and K_d comparison for tension loading

r/d	0,05	0,1	0,15	0,2	0,25
K_d/K_t	0.82	0.83	0.83	0.83	0.81
Average	0.83				
RMS	0.83				
RMS error	0.01				

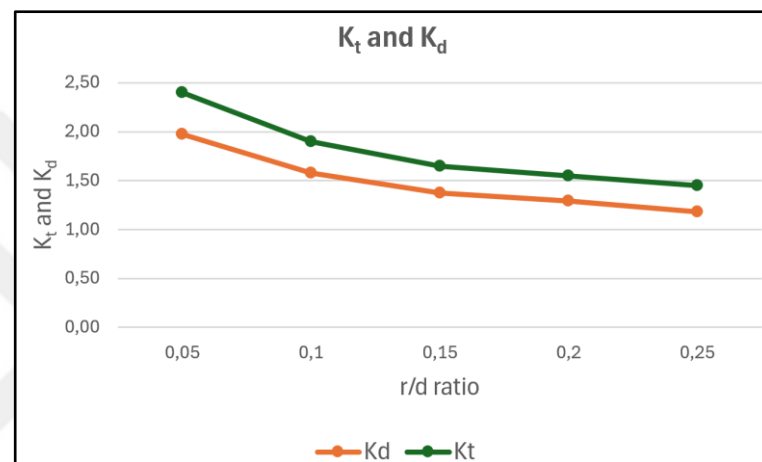


Figure 2.61: K_t and K_d values depend on different r/d ratio for tension loading

Bending moment is added in the **Figure 2.62**.

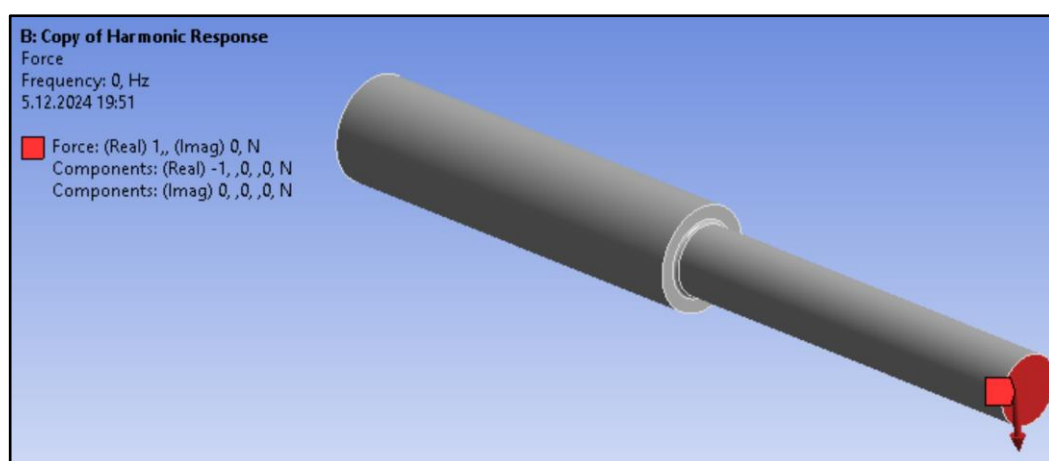


Figure 2.62: Bending moment in the Harmonic response analysis.

The variation of normal stress with frequency for a geometry with an r/d ratio of 0.05 is presented in **Figure 2.63**, where the X-axis represents frequency, and the Y-axis represents normal stress.

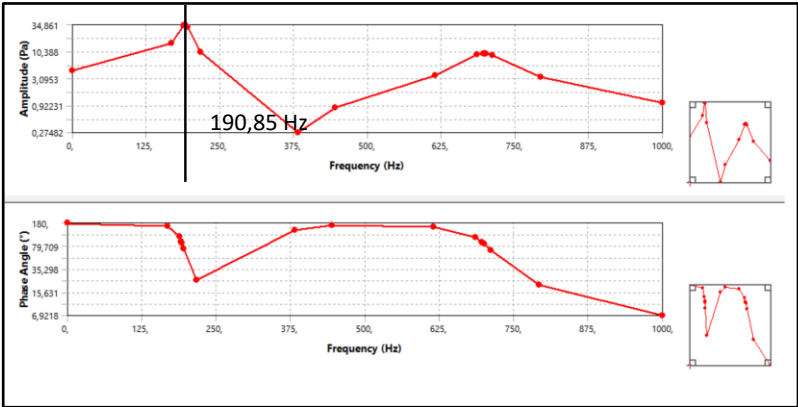


Figure 2.63: Normal Stress result depends on frequency

Maximum Von Mises stress at 190.85 Hz is shown in the **Figure 2.64**.

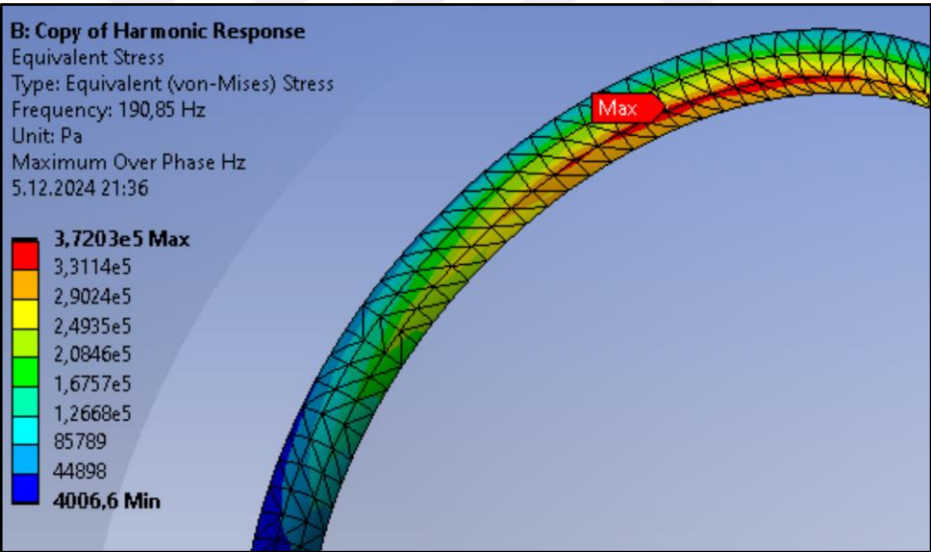


Figure 2.64: Maximum Von Mises at 190,85 Hz for bending loading

Velocity depends on frequency result is shown in the **Figure 2.65**.

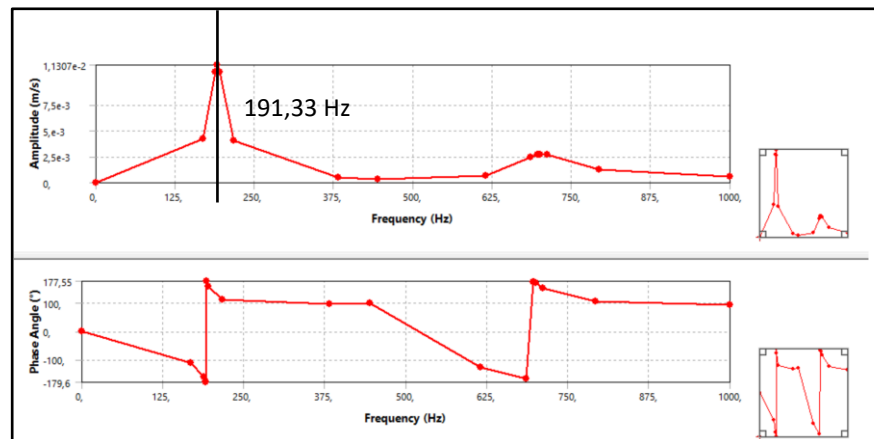


Figure 2.65: Velocity depends on frequency result for bending loading

Maximum velocity result at 191,33 Hz is shown in the **Figure 2.66**.

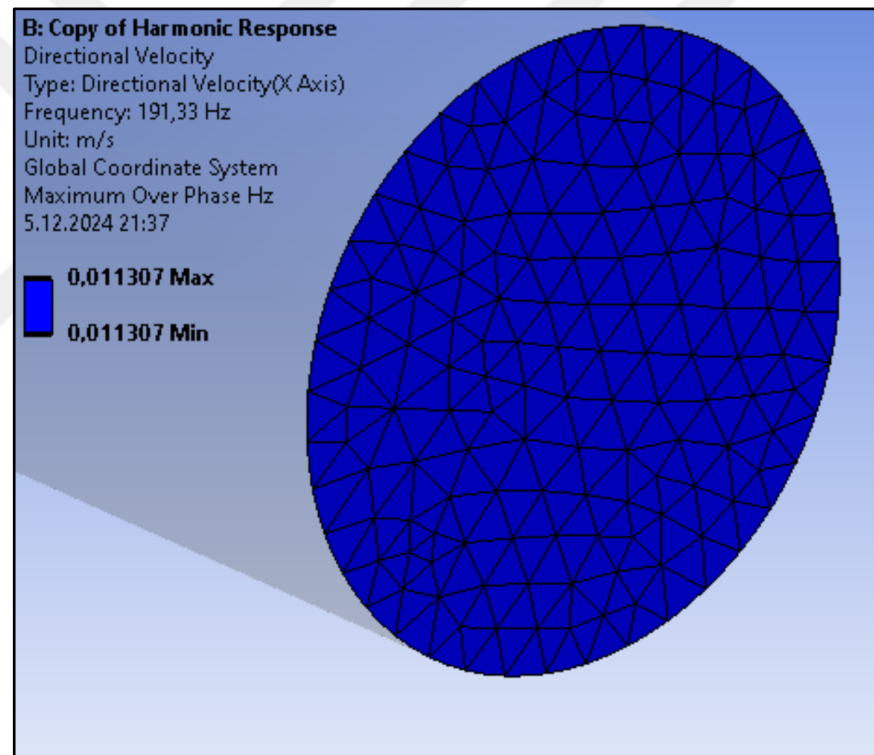


Figure 2.66: Maximum Velocity Result for bending loading

For $r/d = 0.1-0.15-0.2-0.25$, the maximum stress and velocity results are shown in the **Figure 2.67**.

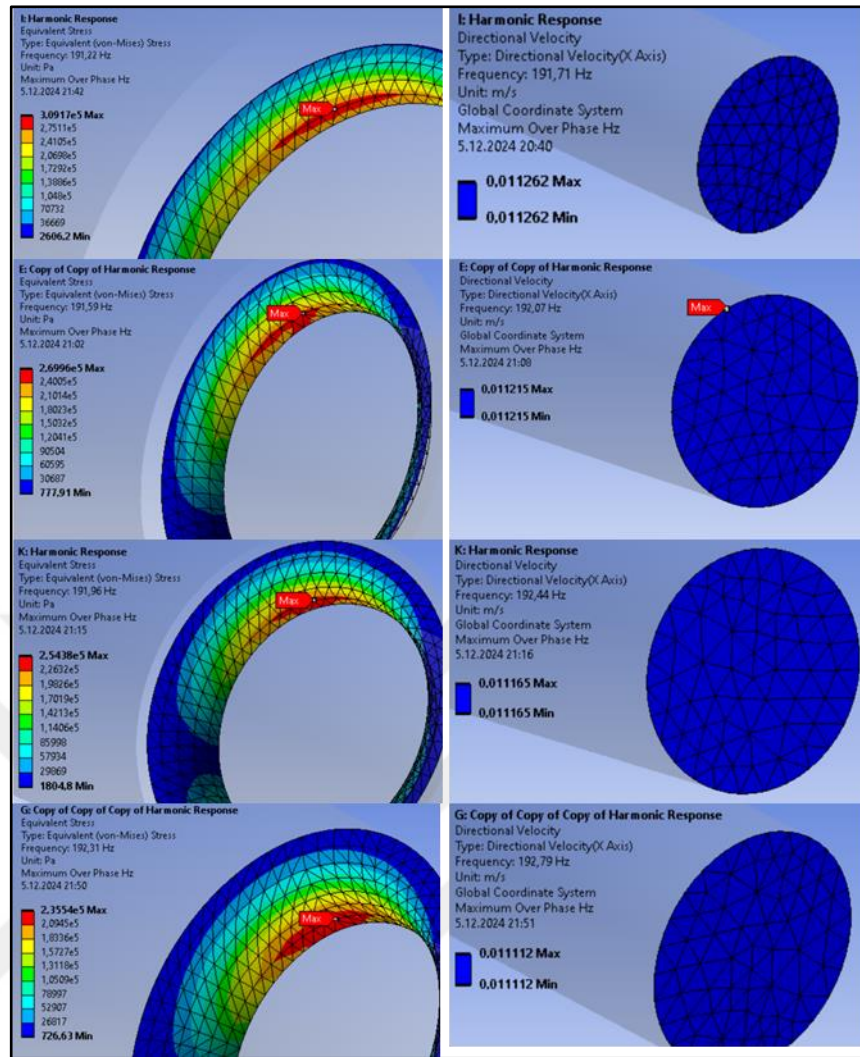


Figure 2.67: Maximum Stress and Velocity Result for $r/d=0.05-0.25$

K_{db} coefficient formula is calculated by using equation (1.35). Maximum stress, maximum velocity, K_{db} , K_t values are shared in the **Table 2.18**.

Table 2.18: Stress, Velocity and Coefficient Result for bending loading

r/d	0,05	0,1	0,15	0,2	0,25
Maksimum Stress Pa	4,00E+05	3,16E+05	2,79E+05	2,55E+05	2,41E+05
Maksimum Velocity m/sn	1,13E-02	1,13E-02	1,12E-02	1,12E-02	1,11E-02
K_{db}	2,58	2,04	1,81	1,66	1,58
K_t	2,05	1,66	1,5	1,41	1,39

When K_t is compared with K_{db} , the average difference is 1.2, the RMS difference is 1.2 and the RMS error is 0.04 K_t and K_{db} values have a constant relationship with respect to the change in r/d in the Table 2.19. K_{db} and K_t values are shown as graph in

the Figure 2.68. K_{db} is 1.2 times K_t . However, the difference between $K_{t_tension}$ (Stress concentration factor of tension load) and K_{db} decreases as the r/d ratio increases from 0.05 to 0.25.

Table 2.19: K_t and K_{db} comparison for bending loading

r/d	0,05	0,1	0,15	0,2	0,25
(K_{db}/K_t)	1,26	1,23	1,21	1,18	1,14
Avarage	1.2				
RMS=K	1.2				
RMS error	0.04				

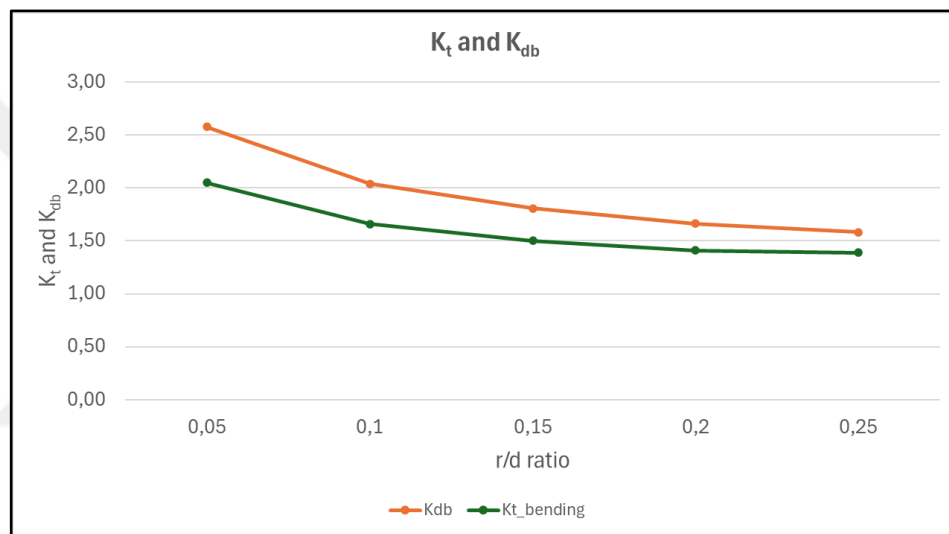


Figure 2.68: K_t and K_{db} values depend on r/d ratio for bending loading

To better understand the cause of this behavior, the graphs shown in the **Figure 2.69**, of $K_{t_tension}$, $K_{t_bending}$ (Stress concentration factor of bending load), K_d , and K_{db} were analyzed.

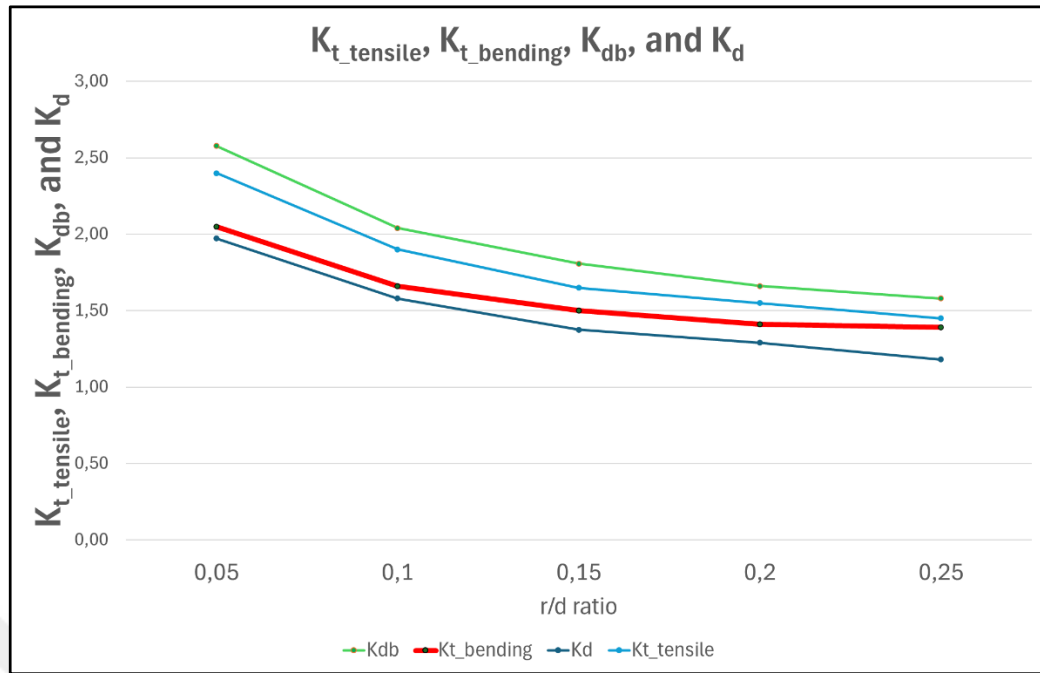


Figure 2.69: K_{t_tension}, K_{t_bending}, K_d, and K_{db} values

The analysis revealed that the K_{t_bending} value exhibits a noticeable deviation compared to the other curves. The parallel trend between K_d and K_{db} indicates that, although these coefficients are distinct, there might be a proportional relationship between them. Conversely, no parallelism was observed between the K_t values, suggesting that no consistent ratio can be established between these coefficients. The stress and velocity values used to calculate K_d and K_{db} are shown in **Figure 2.70-Figure 2.71**. When these values are examined, a similar parallelism to that observed between K_d and K_{db} can be identified.

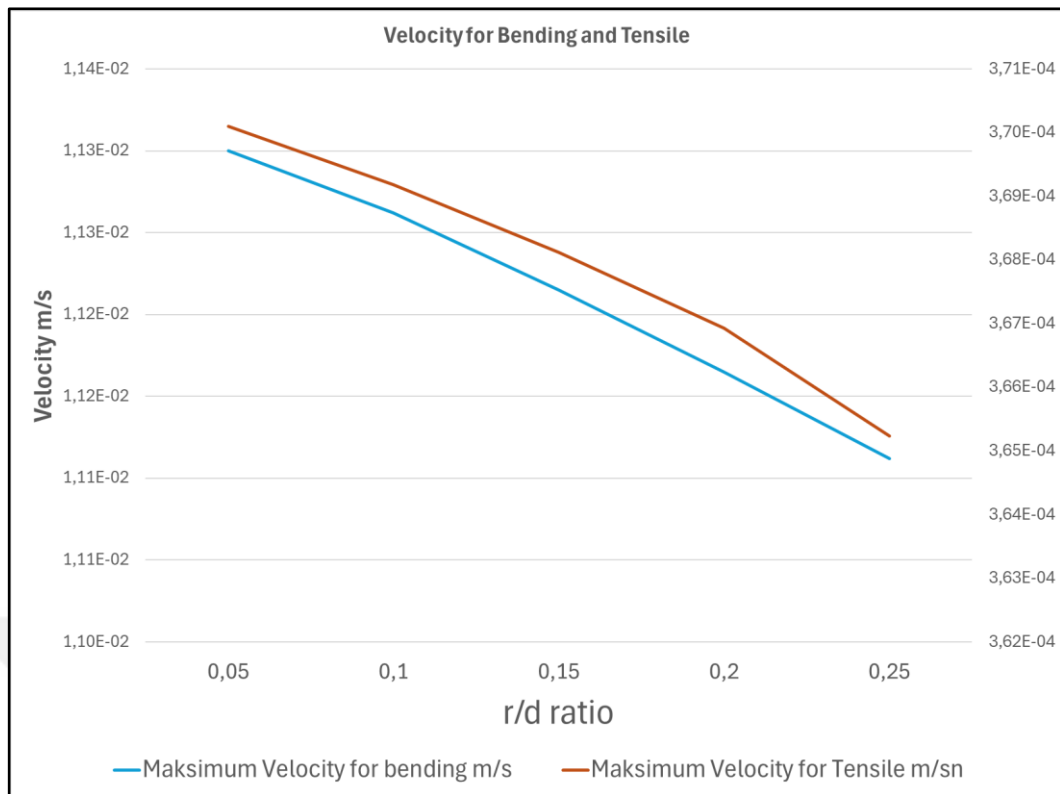


Figure 2.70: Velocity values of K_d and K_{db} coefficient

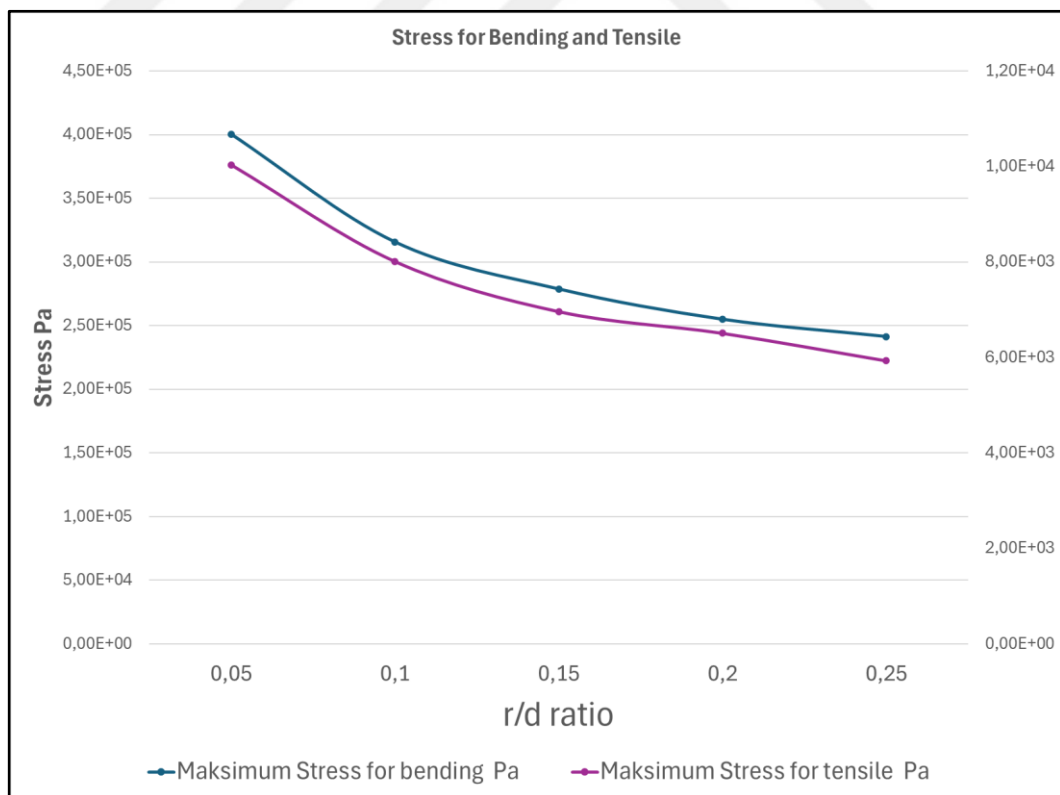


Figure 2.71: Stress values of K_d and K_{db} coefficient

2.3.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.3.4.1. Maximum Velocity and Stress for Tension Load

These calculation values are shown on the schematic from **Figure 1.1**. If Analytical calculation is made for r/d: 0.05 shaft with fillet radius, The Results are shown in the **Figure 2.72**.

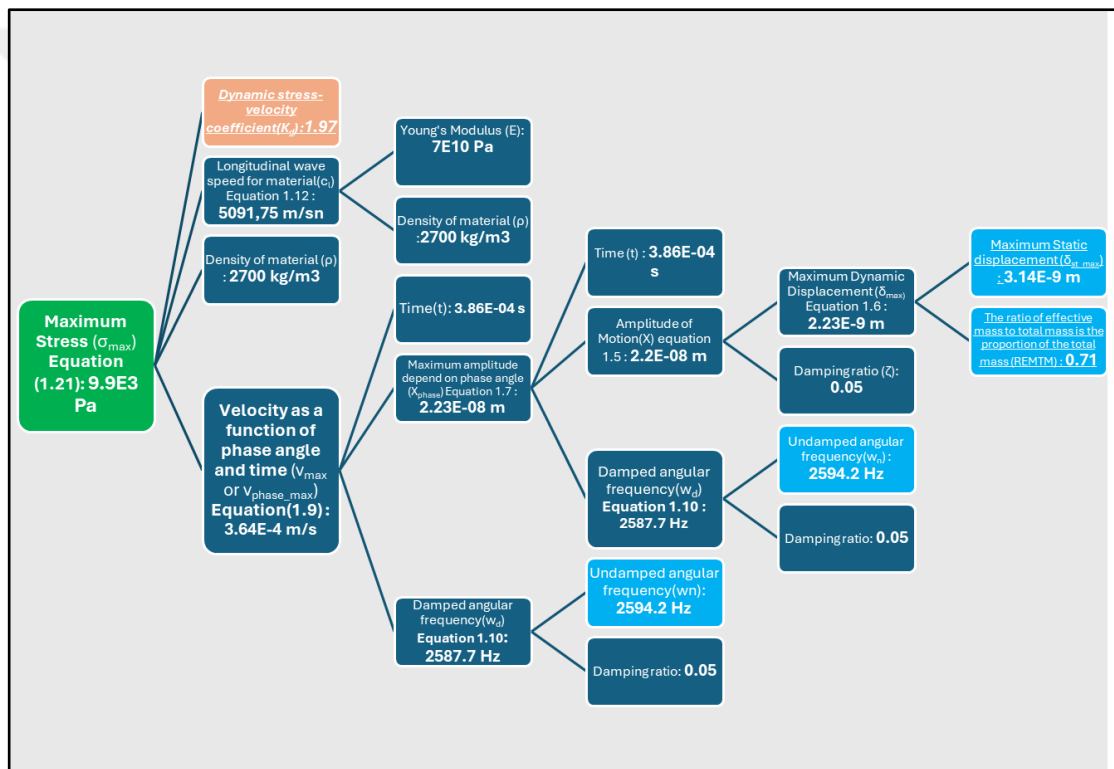


Figure 2.72: Analytical Calculation for r/d:0.05 on tension load

If Analytical calculation is made for r/d: 0.25 shaft with fillet radius, The Results are shown in the **Figure 2.73**.

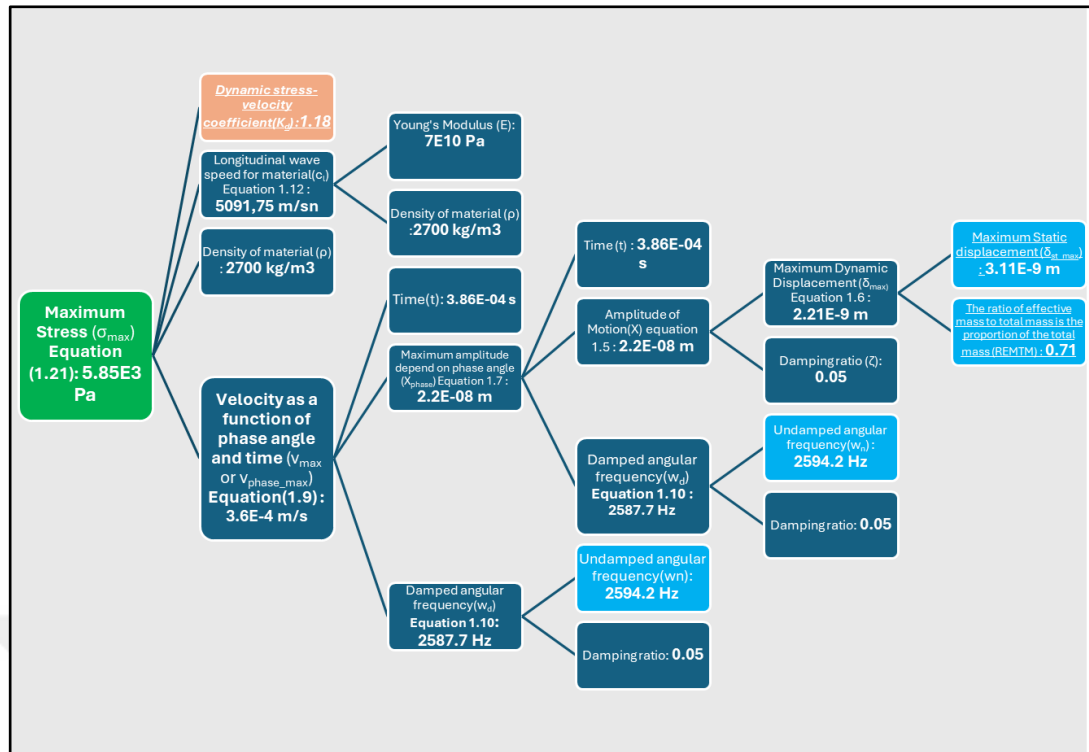


Figure 2.73: Analytical Calculation for r/d:0.25 on tension load

Comparison of FEM and Analytical results are shared in the **Table 2.20**. The difference shows that some analytical parameter can be used instead of FEM parameter.

Table 2.20: Comparison of FEM and Analytical Results

r/d	FEM Stress	Combined method Stress	Difference
0.05	1.0E+04	9.9E+03	2%
0.25	5.9E+03	5.9E+03	1%
	Velocity	Velocity	
0.05	3.7E-4	3.64E-04	2%
0.25	3.65E-04	3.6E-4	1%

2.3.4.2. Maximum Velocity and Stress for Bending Load

These calculation values are shown on the schematic from Figure 1.1. If Analytical calculation is made for r/d: 0.05 shaft with fillet radius, The Results are shown in the **Figure 2.74**.

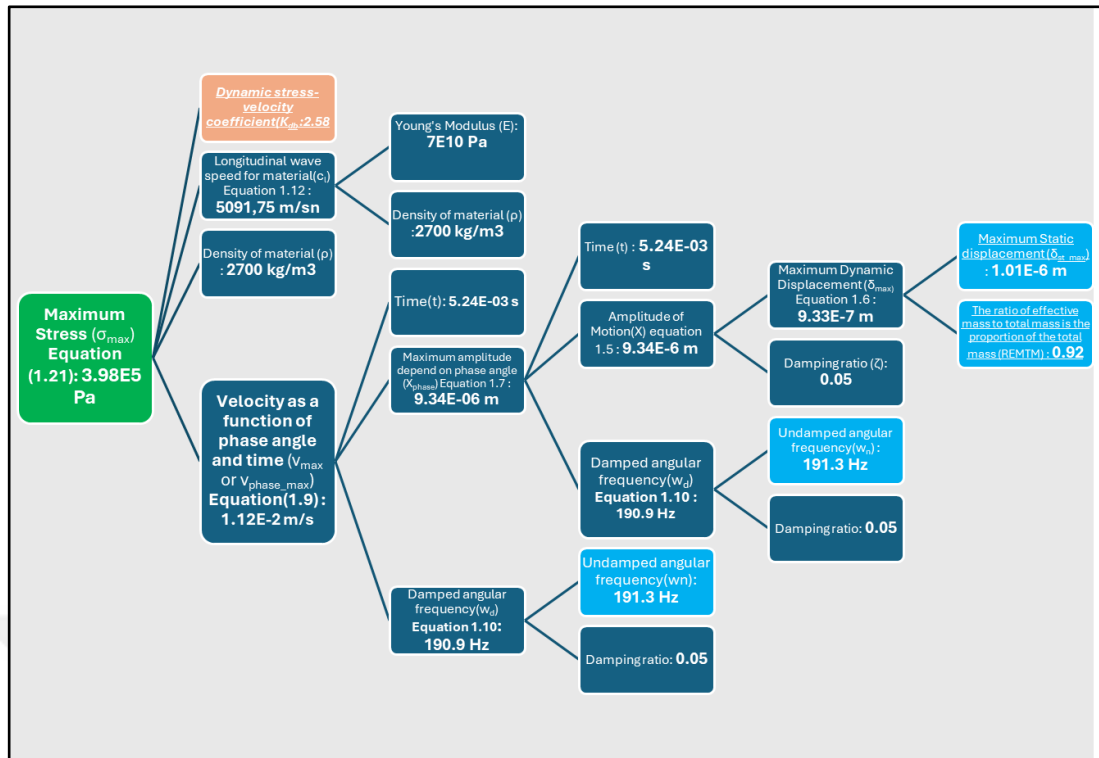


Figure 2.74: Analytical Calculation for r/d:0.05 on bending load

If Analytical calculation is made for r/d: 0.25 shaft with fillet radius, The Results are shown in the **Figure 2.75**.

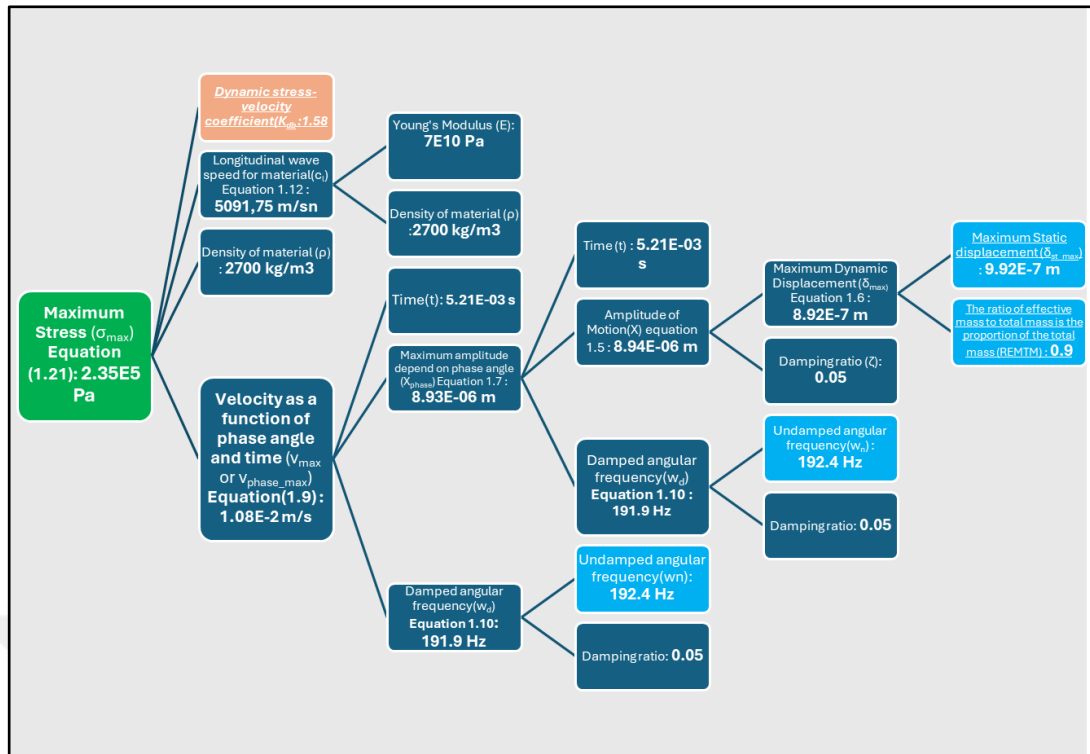


Figure 2.75: Analytical Calculation for r/d:0.25 on bending load

Comparison of FEM and Analytical results are shared in the **Table 2.21**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.21: Comparison of FEM and Analytical Results

r/d	FEM _Stress	Combined Method Stress	Difference
0.05	4E5	3.8E5	1%
0.25	2.41E5	2.35E5	2.9%
	Velocity	Velocity	
0.05	1.13E-02	1.12E-02	0.6%
0.25	1.11E-02	1.08E-02	2.9%

2.4. Case Study - 3 – Round shaft with hole

2.4.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Round shaft with hole is selected as sample. Geometry is shown in the **Figure 2.76**.

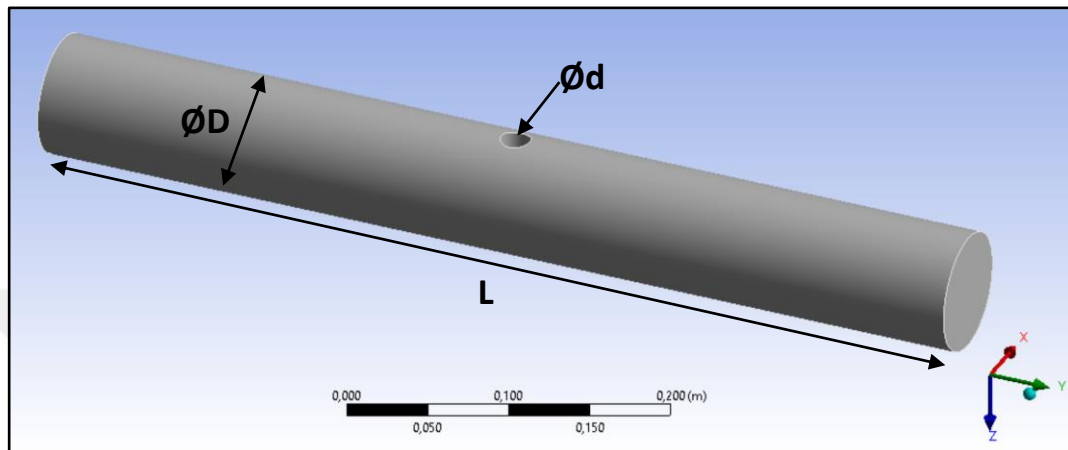


Figure 2.76: Round shaft with hole

Geometry parameters are shared in the **Table 2.22**. K_{ts} , A and d values are chosen to depend on d/D ratio values in the **Figure 2.82**.

Table 2.22: Round shaft with transverse hole.

Parameter	Value		
L [m]	0.6096		
D [m]	0.0762		
d/D	0.05	0.15	0.25
d [m]	3.810E-03	1.143E-02	1.91E-02
K_t for bending	2.45	2.1	1.95
K_{ts} , A for torsional	3.55	3.15	2.95

where K_{ts} is stress concentration factor of torsional load.

The mesh is shown in the **Figure 2.77**.

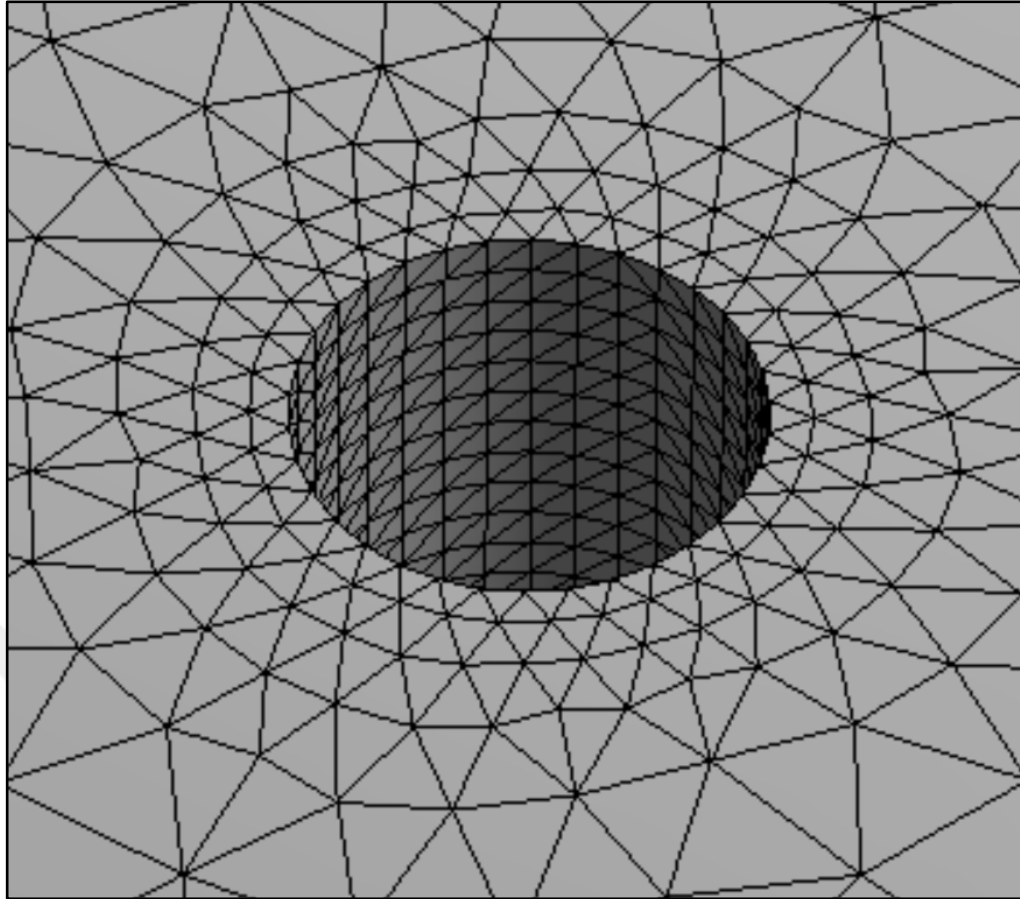


Figure 2.77: The mesh with updated radius ($r/d=0.05$)

Static, Modal and Harmonic Response Analysis is made. Boundary Condition is fixed 6 DOF and shown for modal analysis in the **Figure 2.78**.

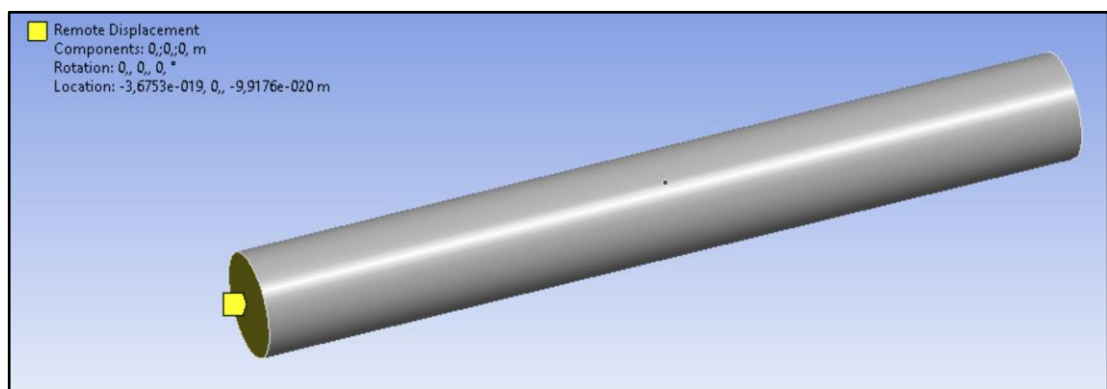


Figure 2.78: Boundary Conditions on the yellow face

Applied bending and torsional load is shown in the **Figure 2.79-Figure 2.80**. Bending and torsional unit load is applied on the variable geometries. 3 different geometries are used for bending and torsional loading analysis.

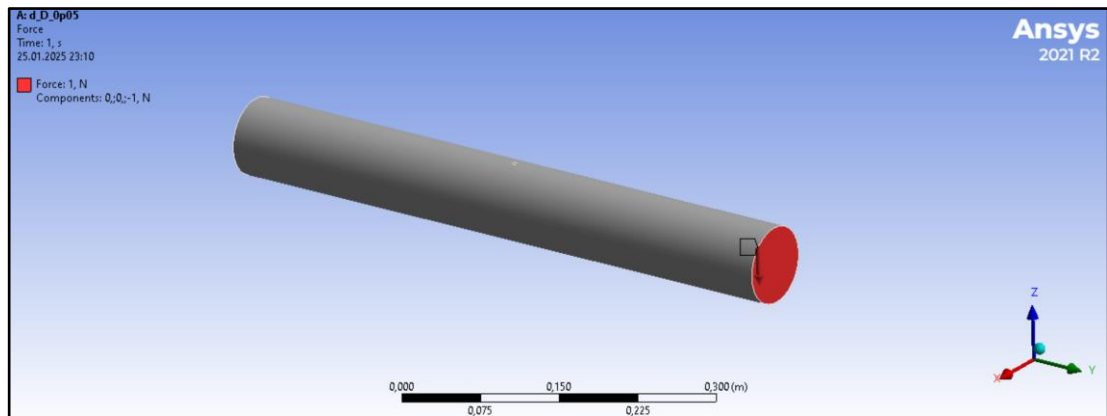


Figure 2.79: Bending Load Direction

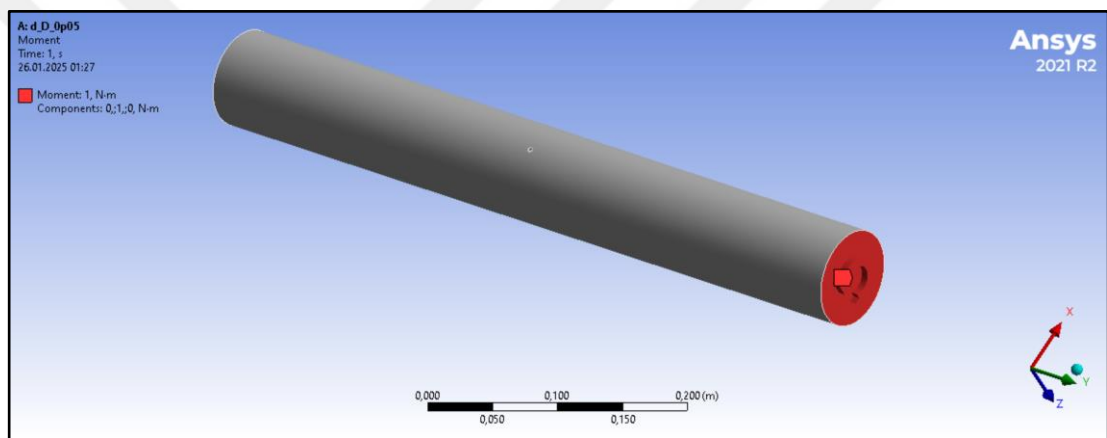


Figure 2.80: Torsional Load Direction

According to different d/D ratio, there are 3 geometries with 3 hole diameter dimensions. In the **Figure 2.81- Figure 2.82**, Stress Concentration Factor graphics are shown for bending and torsional loads. In this study, there is calculation K_t and K_{ts} for 3 different d/D ratios.

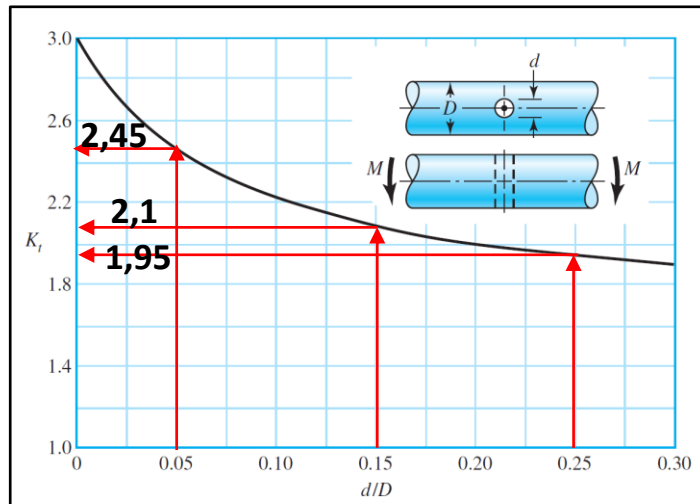


Figure 2.81: K_t depend on d/D for bending [15]

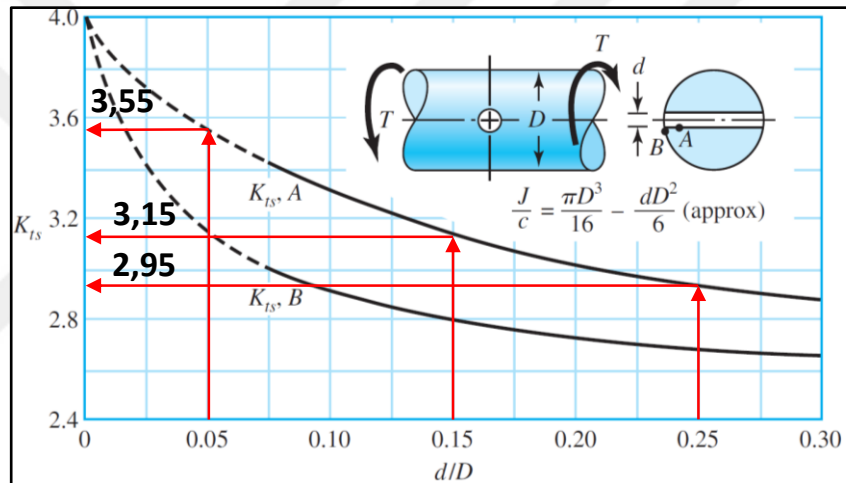


Figure 2.82: K_{ts} depend on d/D for torsional [15]

2.4.2. Static Analysis Result

Static analysis result for $d/D:0.05$ round shaft with hole included deformation at Load direction Z directions is shown **Figure 2.83**. Maximum deformation is $6.61E-7$ m.

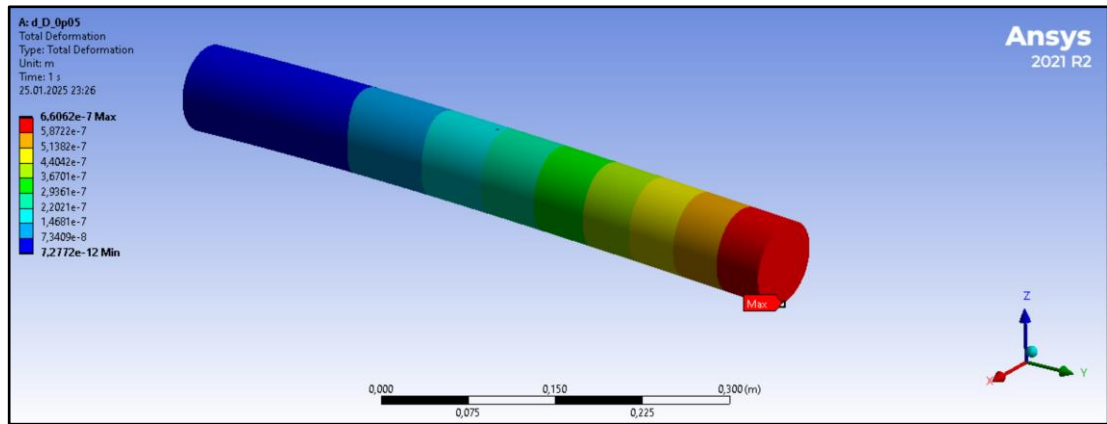


Figure 2.83: Bending Deformation at Z direction for d/D:0.05

Static analysis result for d/D:0.25 round shaft with hole included deformation at Load direction Z directions is shown **Figure 2.84**. Maximum deformation is 6.77E-7 m.

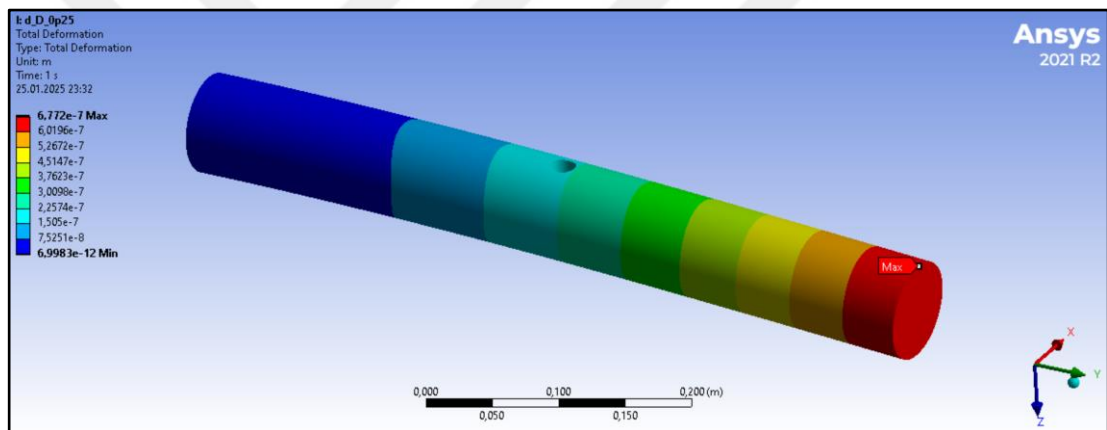


Figure 2.84: Bending Deformation at Z direction for d/D:0.25

Static analysis result for d/D:0.05 round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.85**. Maximum deformation is 2.61E-7 m.

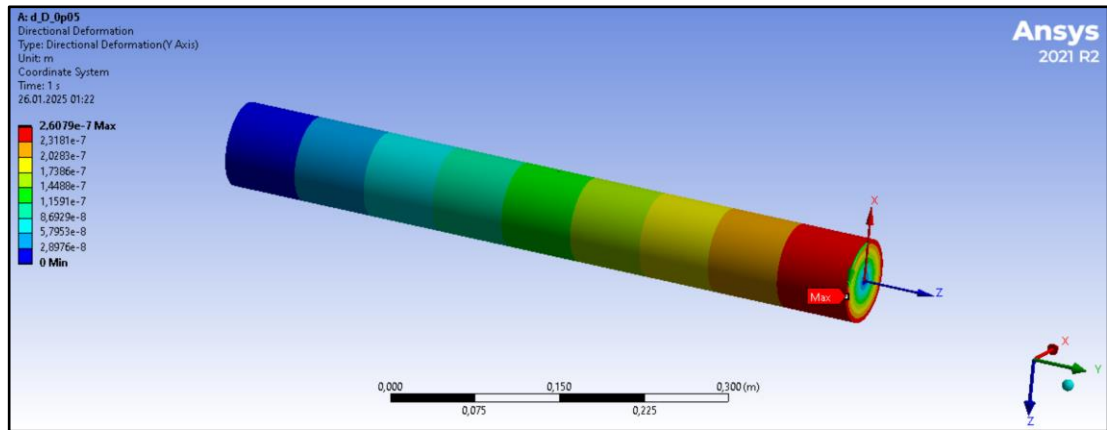


Figure 2.85: Torsional Deformation at RY direction for $r/d:0.05$

Static analysis result for $d/D:0.25$ round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.84**. Maximum deformation is $2.65E-7$ m.

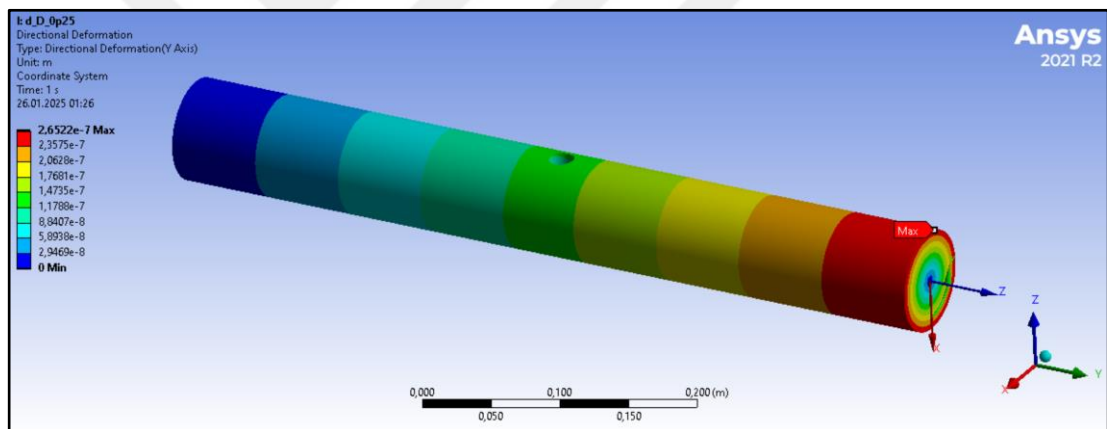


Figure 2.86: Torsional Deformation at RY direction for $r/d:0.25$

2.4.3. Harmonic Load Analysis Result

8 modes are calculated in the modal analysis. 2 critical mode shapes are selected for bending and torsional modes. They are 144.79 Hz for RX direction (Bending) and 1294.8 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.23**.

Table 2.23: The Ratio of Effective Mass to Total Mass for Round Shaft with hole

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	144,79	1,1E-09	1,8E-17	6,1E-01	9,7E-01	4,3E-12	1,4E-09
2	144,82	6,1E-01	8,5E-15	8,1E-10	1,4E-09	3,3E-12	9,7E-01
3	863,98	3,4E-09	1,3E-11	1,9E-01	2,5E-02	9,6E-11	2,7E-10
4	864,96	1,9E-01	9,7E-12	1,5E-09	2,8E-10	1,1E-10	2,5E-02
5	1294,8	3,3E-13	2,7E-17	1,8E-13	1,7E-17	8,1E-01	3,1E-17
6	2087,1	4,0E-11	8,1E-01	1,9E-09	9,8E-15	1,3E-09	1,1E-17
7	2263,7	6,8E-02	1,8E-10	3,0E-07	1,3E-08	6,2E-10	3,0E-03
8	2264	2,6E-07	2,7E-10	6,8E-02	3,0E-03	3,7E-10	1,3E-08
Sum		8,7E-01	8,1E-01	8,7E-01	1,0E+00	8,1E-01	1,0E+00

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, damping coefficient is defined as 0.05. For Harmonic analysis with bending load velocity and maximum von mises stress results are shown in **Figure 2.87-**

Figure 2.89.

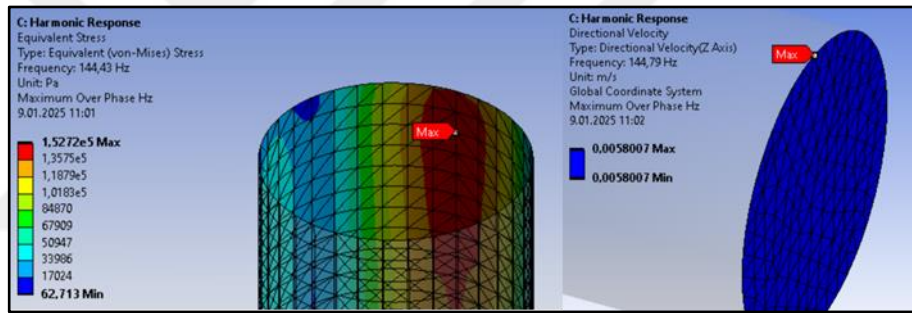


Figure 2.87: Max. Von Mises Stress and Velocity results for $d/D=0.05$

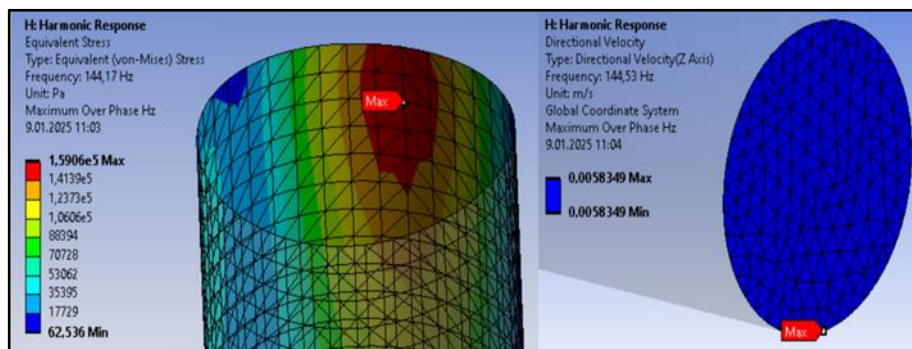


Figure 2.88: Max. Von Mises Stress and Velocity results for $d/D=0.15$

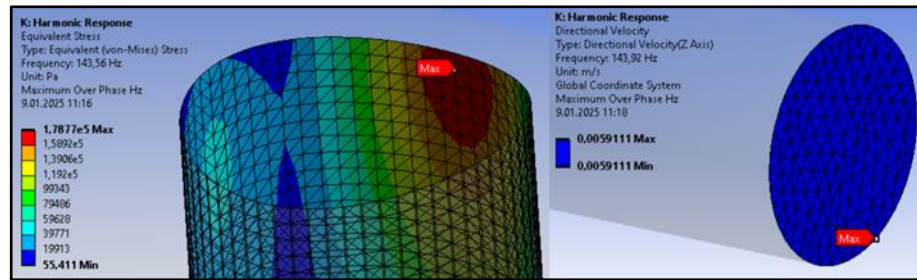


Figure 2.89: Maximum Von Mises Stress and Velocity results for $d/D=0.25$

For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.90**-**Figure 2.91**-**Figure 2.92**.

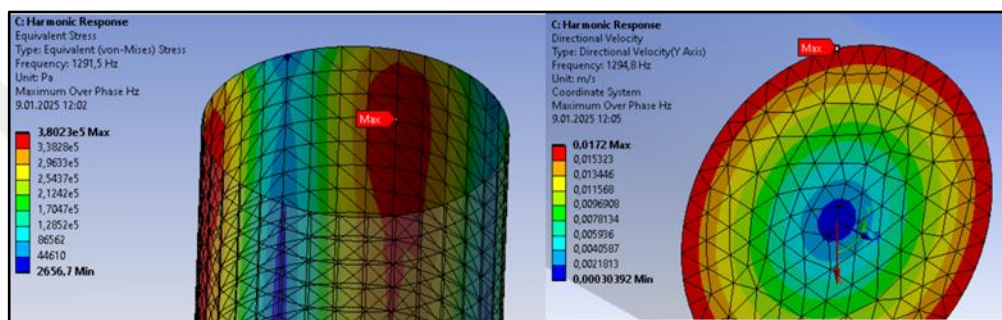


Figure 2.90: Round shaft with transverse hole for $d/D:0.05$

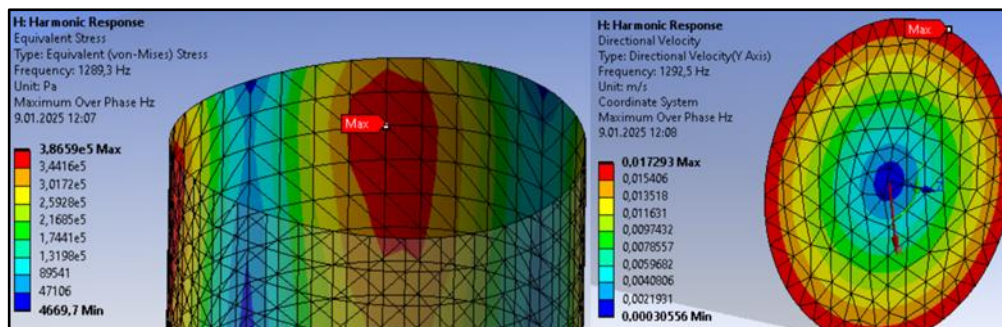


Figure 2.91: Round shaft with transverse hole for $d/D:0.15$

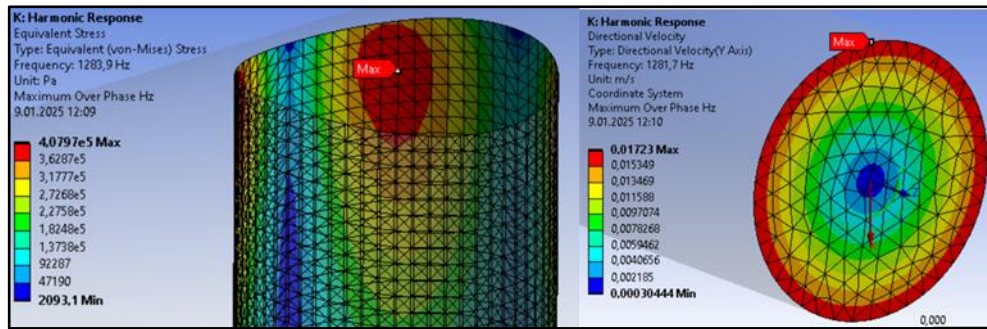


Figure 2.92: Round shaft with transverse hole for $d/D:0.25$

K_{db} coefficient is calculated by using Equation (1.35). The results are shared in the Table 2.24.

Table 2.24: Stress, velocity, K_{db} and K_t results

d/D	0.05	0.15	0.25
Maksimum Stress Pa	1.53E+05	1.59E+05	1.79E+05
Maksimum Velocity m/s	5.80E-03	5.83E-03	5.91E-03
K_{db}	1.92	1.98	2.20
K_t	2.45	2.1	1.96

K_{db} and K_t results are shown as graph in the Figure 2.93.

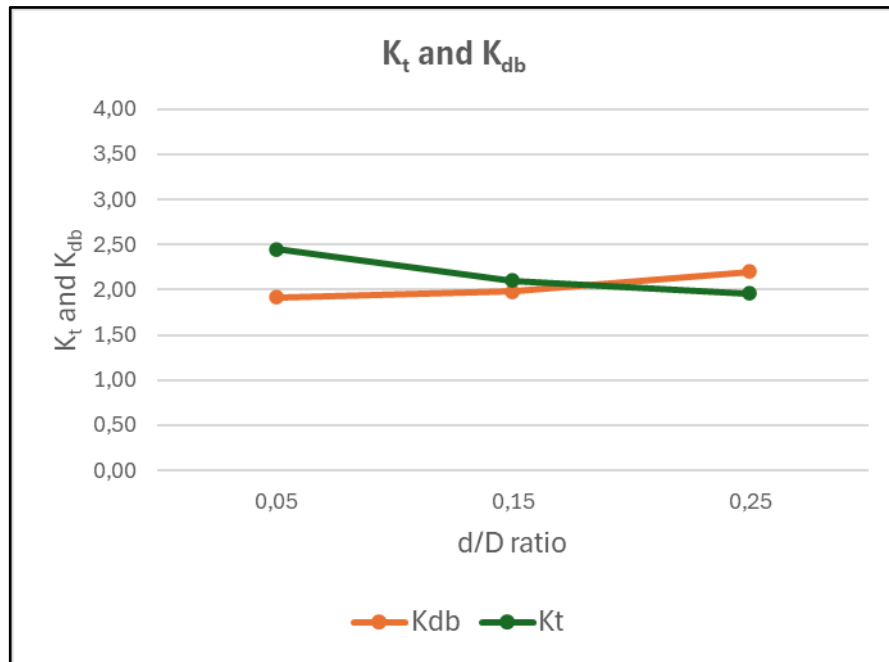


Figure 2.93: Round shaft with hole K_t and K_{db} values with bending

For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.90-Figure 2.91-Figure 2.92**. K_{dt} coefficient is converted by using Equation (1.38). The results are shared in the **Table 2.25**.

Table 2.25: Stress, velocity, Kdt and Kts results for torsional load and hole

d/D	0.05	0.15	0.25
Maksimum Stress Pa	3.80E+05	3.87E+05	4.08E+05
Maksimum Velocity m/s	1.72E-02	1.73E-02	1.72E-02
K_{dt}	1.50	1.51	1.60
K_{ts}	3.55	3.15	2.95

K_{db} and K_{ts} values are shown as graph in the **Figure 2.94**.

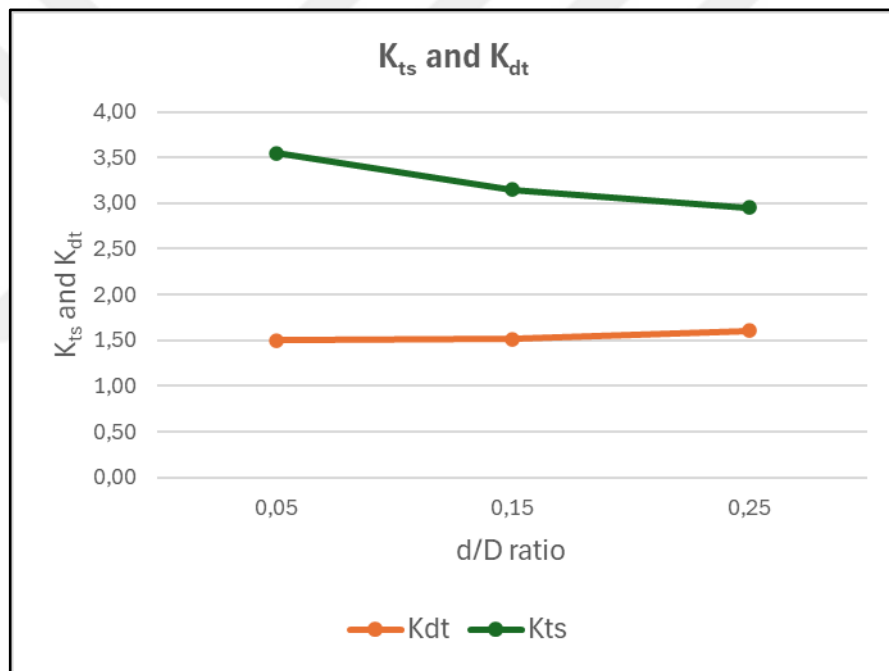


Figure 2.94: Round shaft with hole K_{ts} and K_{dt} values with torsional

In **Figure 2.93-Figure 2.94**, it is observed that the K_{db} - K_{ds} and K_t - K_{ts} values move in opposite directions as the d/D ratio increases. To better understand this behavior, we examined the stress- K_{dt} , stress- K_{ts} , velocity- K_{dt} , and velocity- K_{ts} graphs in the **Figure 2.95-Figure 2.98**. The analysis revealed that stress and K_{dt} are directly proportional, whereas stress and K_{ts} are inversely proportional. The reason for this inverse relationship with K_{ts} is that as the d/D ratio increases, the section area where the stress is maximum decreases due to the larger hole. This reduction in the cross-sectional area

causes stress to increase according to the formulation, ensuring that the correct stress value is calculated even when the coefficient decreases.

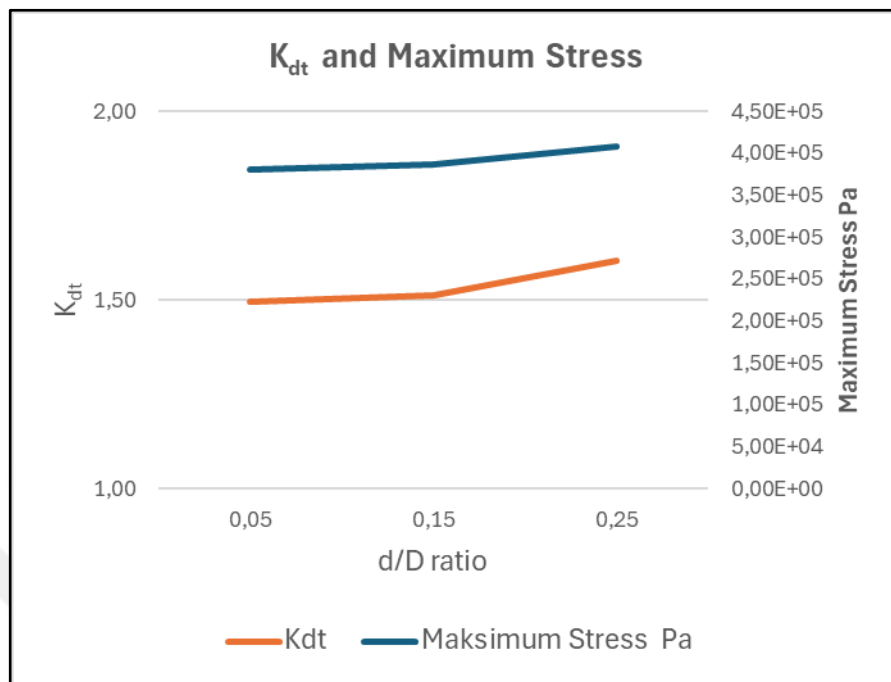


Figure 2.95: K_{dt} and Maximum Stress

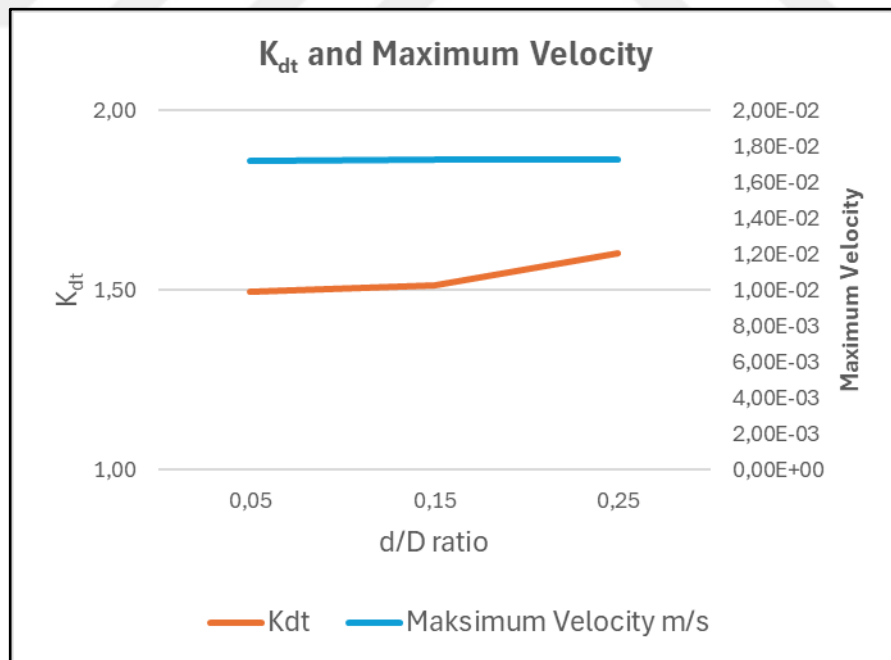


Figure 2.96: K_{dt} and Maximum Velocity

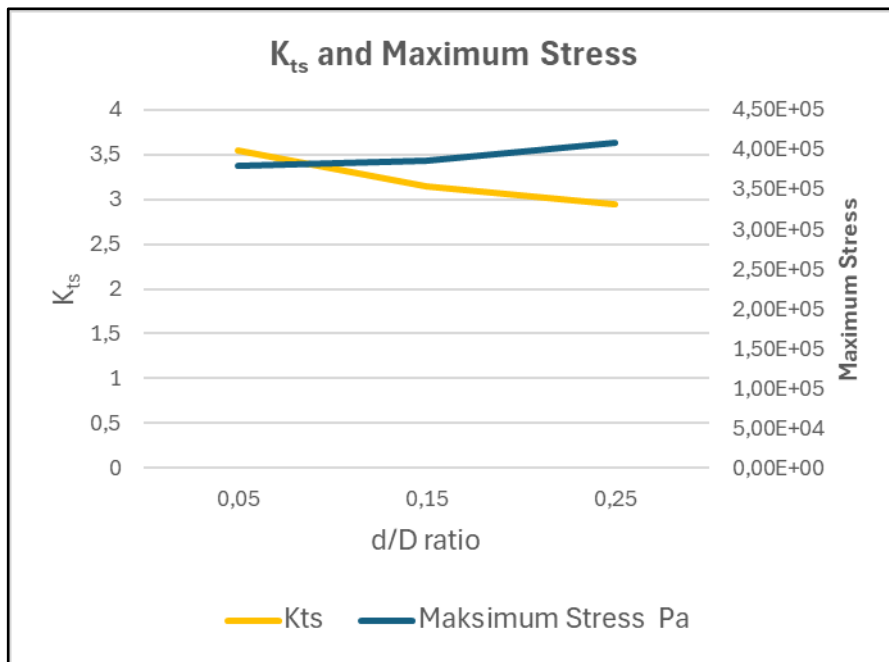


Figure 2.97: K_{ts} and Maximum Stress

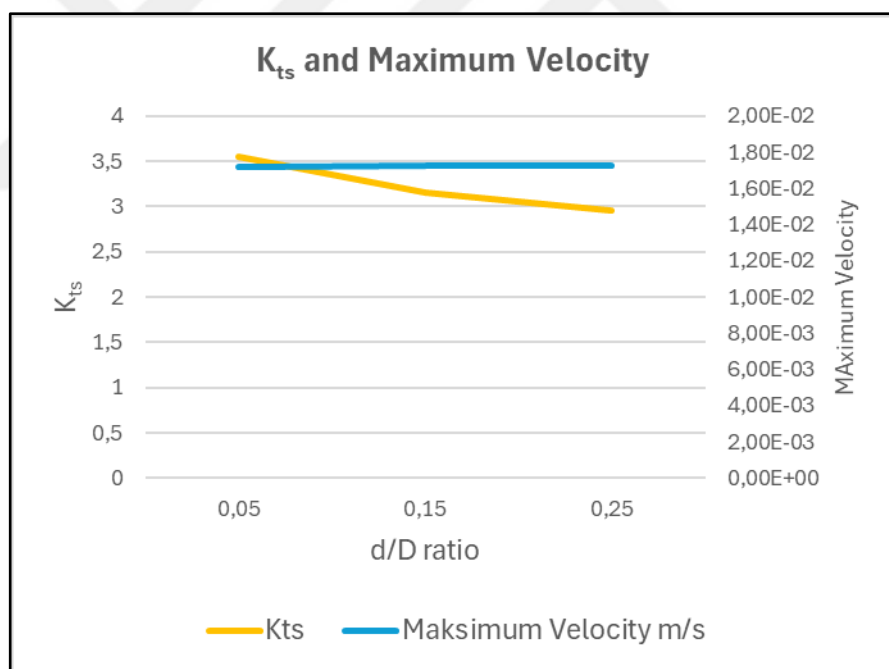


Figure 2.98: K_{ts} and Maximum Velocity

2.4.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.4.4.1. Maximum Velocity and Stress for Bending Load

These calculation values are shown on the schematic from **Figure 1.1**. If Analytical calculation is made for d/D: 0.05 shaft with hole. The Results are shown in the **Figure 2.99**.

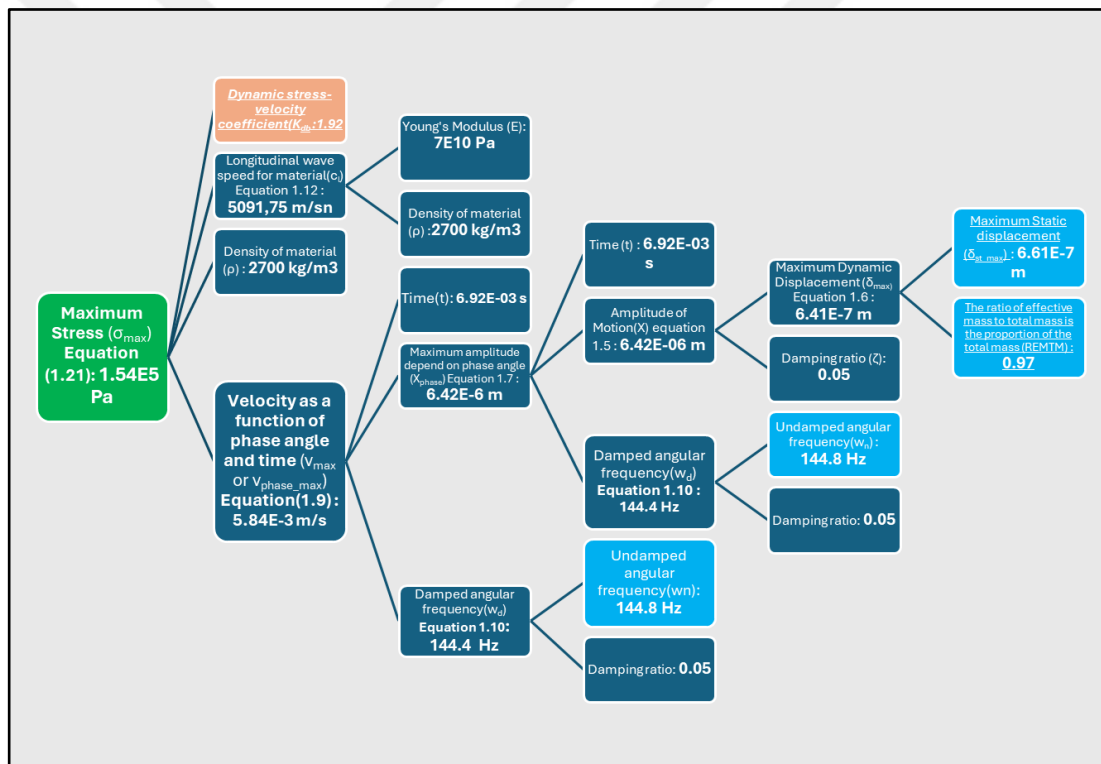


Figure 2.99: Analytical Calculation for d/D:0.05 on bending load

If Analytical calculation is made for d/D : 0.25 shaft with hole. The Results are shown in the **Figure 2.100**.

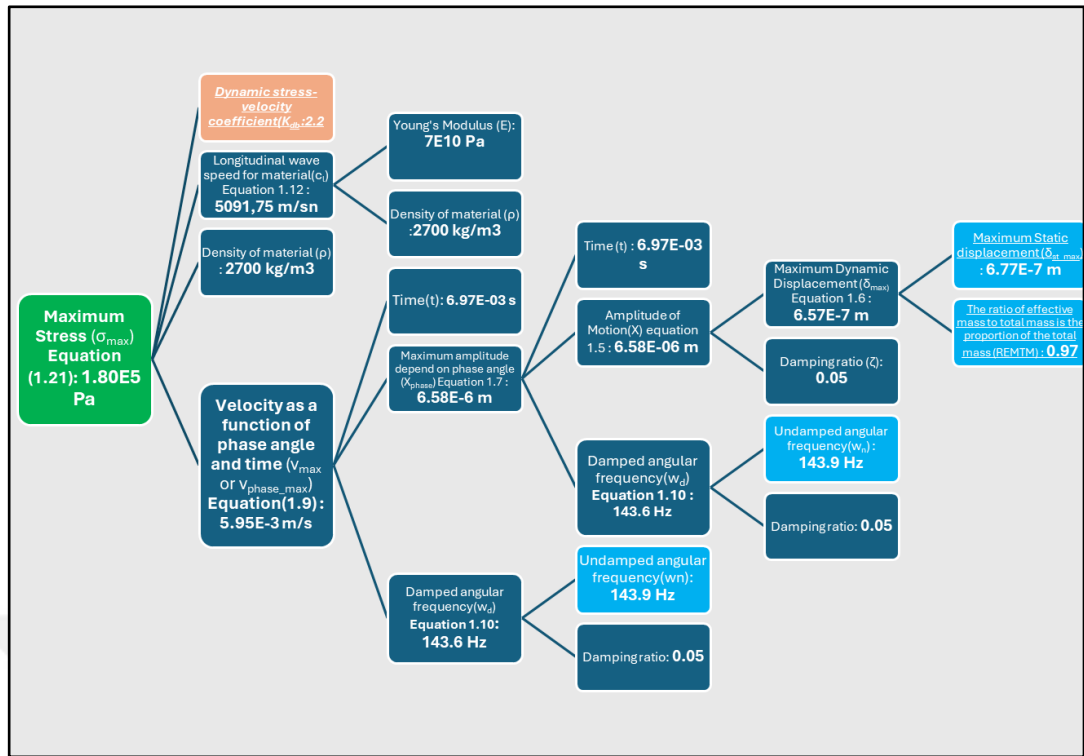


Figure 2.100: Analytical Calculation for d/D:0.25 on bending load

Comparison of FEM and Analytical results are shared in the **Table 2.26** .The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.26: Comparison of FEM and Analytical Results

r/d	FEM Stress	Combined Method Stress	Difference
0.05	1.53E+05	1.54E+05	0.6%
0.25	1.8E+05	1.8E+05	0.6%
	Velocity	Velocity	
0.05	5.80E-03	5.84E-03	0.6%
0.25	5.91E-03	5.95E-03	0.6%

2.4.4.2. Calculation of Maximum Velocity and Stress for Torsional Load

These calculation values are shown on the schematic from Figure 1.1. If Analytical calculation is made for d/D: 0.05 shaft with hole. The Results are shown in the **Figure 2.101**.

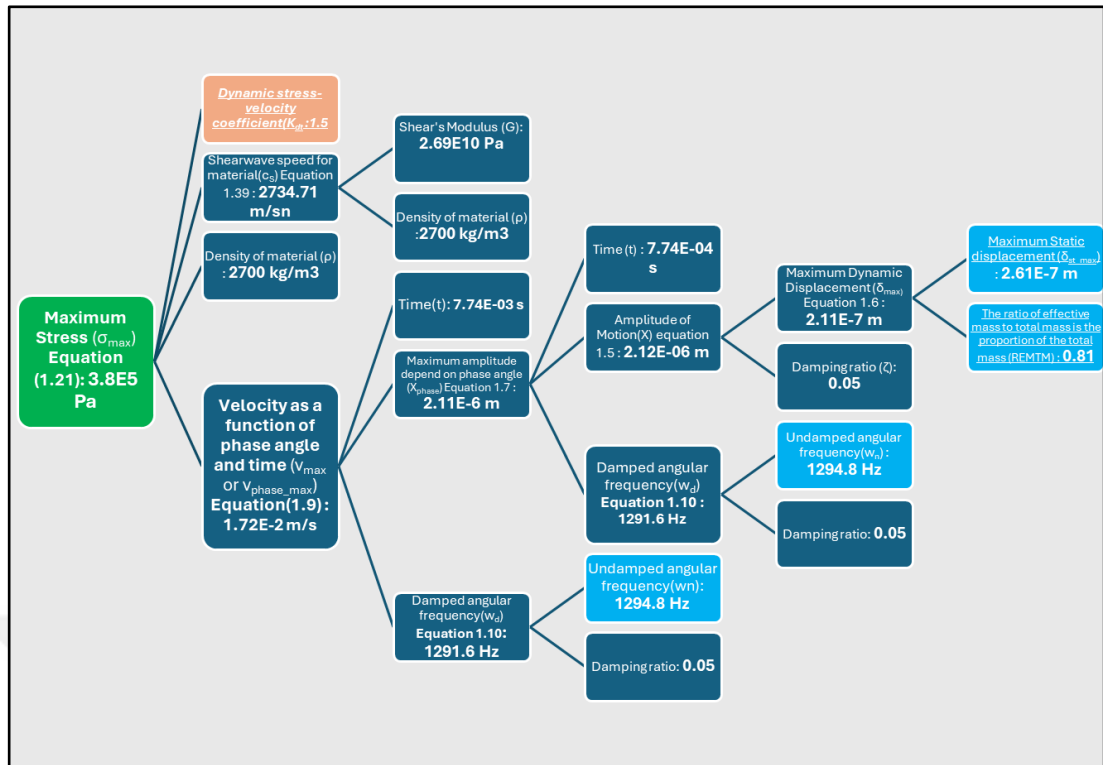


Figure 2.101: Analytical Calculation for d/D:0.05 on torsional load

If Analytical calculation is made for d/D: 0.25 shaft with hole. The Results are shown in the **Figure 2.102**.

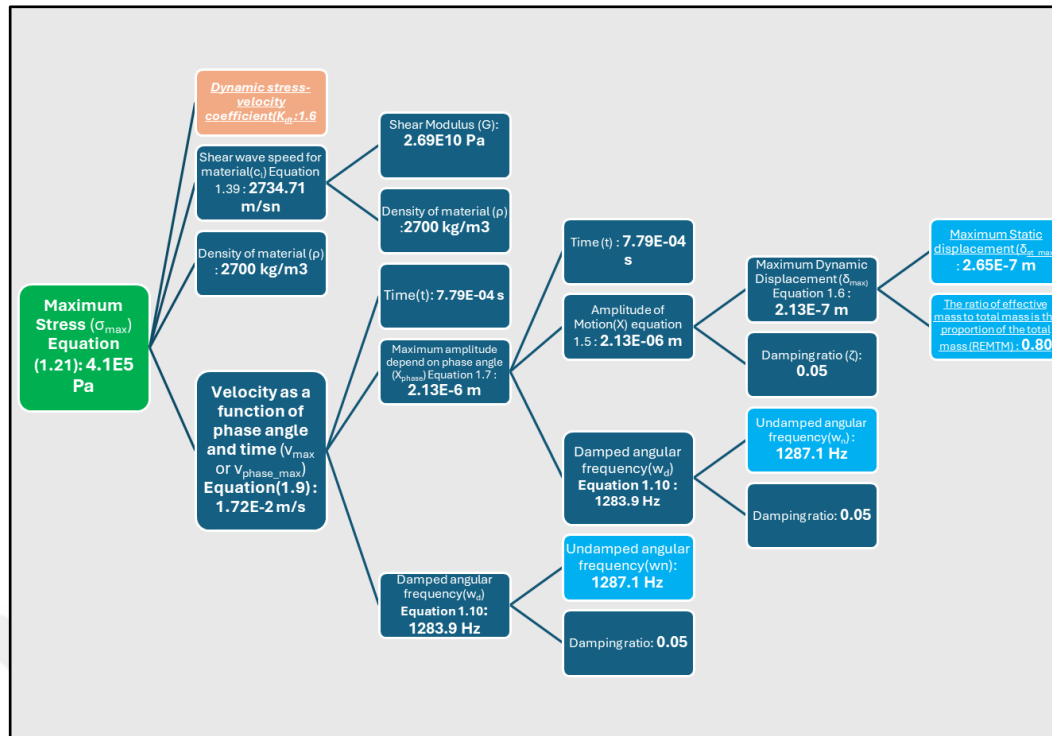


Figure 2.102: Analytical Calculation for d/D:0.25 on torsional load

Comparison of FEM and Analytical results are shared in the **Table 2.27**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.27: Comparison of FEM and Analytical Results

r/d	FEM Stress	Combined Method Stress	Difference
0.05	3.8E+05	3.8E+05	~0%
0.25	4.1E+05	4.1E+05	~0%
	Velocity	Velocity	
0.05	5.80E-03	5.84E-03	~0%
0.25	5.9E-03	5.95E-03	~0%

2.5. Case Study-4-Round shaft with square hole

2.5.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Round shaft with hole is selected as sample. The geometry is shown in the **Figure 2.103**.

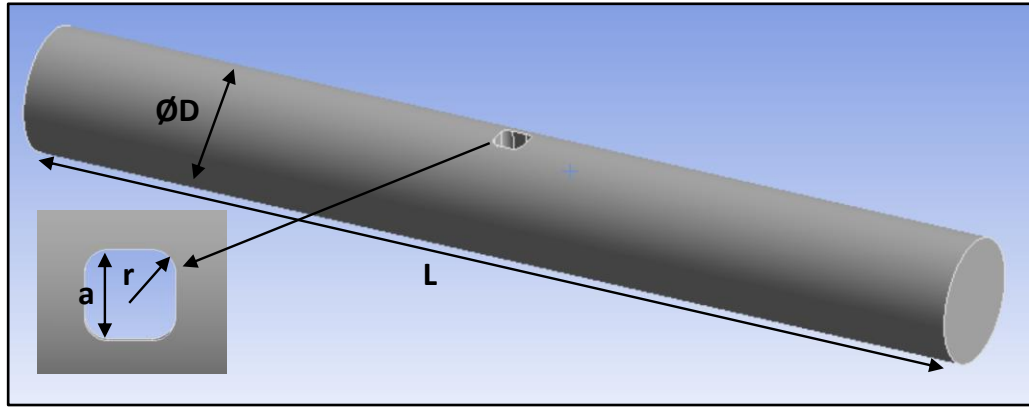


Figure 2.103: Round shaft with square hole

Geometry is shown in the **Figure 2.103**. There is only a and r difference parameter between circle and square hole. The geometry dimensions are shared in the **Table 2.28**.

Table 2.28: Square hole dimensions

Parameter	Value		
L [m]	0.6096		
D [m]	0.0762		
a/D	0.05	0.15	0.25
a [m]	3.810E-03	1.143E-02	1.91E-02
r [m]	1E-03	3E-03	5E-03

The mesh is shown in the **Figure 2.104**.

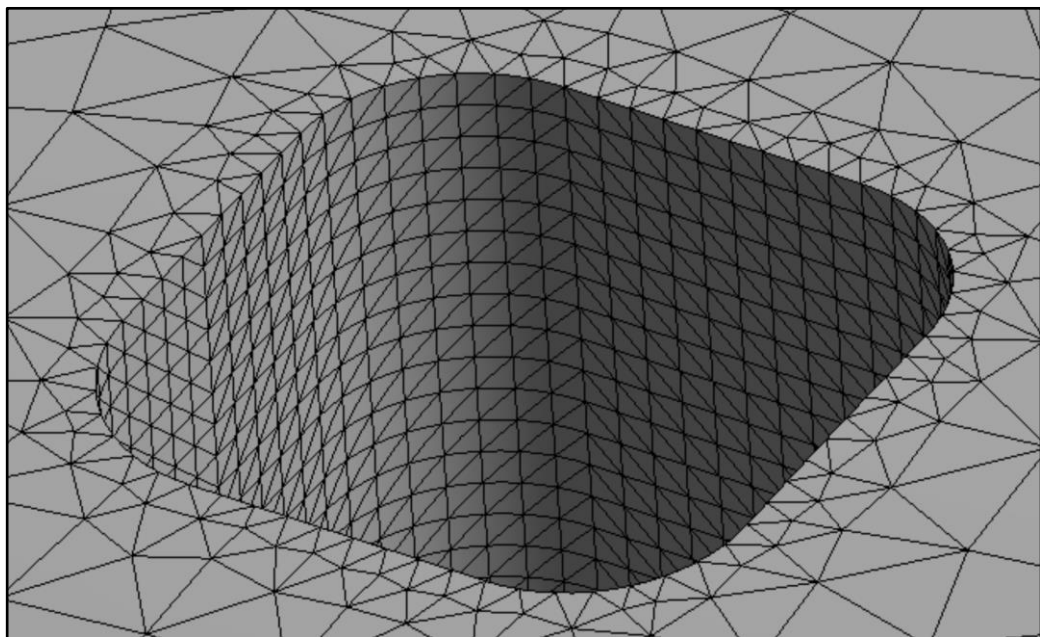


Figure 2.104: The mesh with updated radius (a/D=0.05)

Static, Modal and Harmonic Response Analysis is made. Boundary Condition is fixed 6 DOF and shown for modal analysis in the **Figure 2.105**.

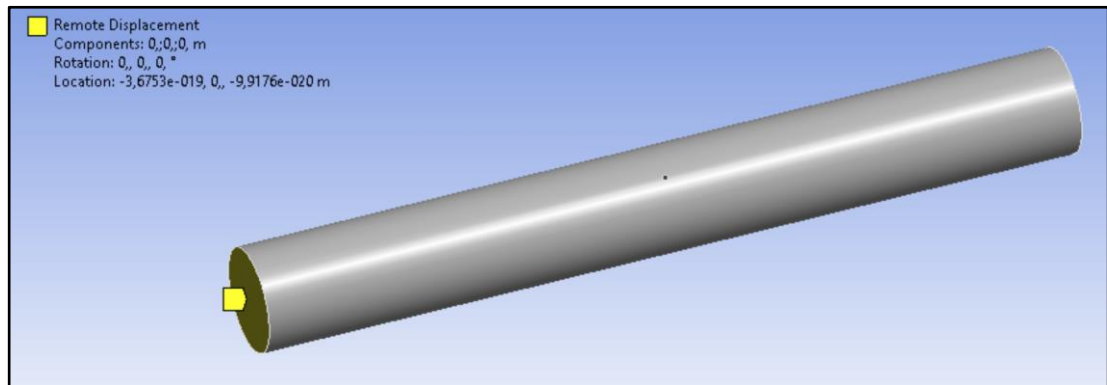


Figure 2.105: Boundary Conditions on the yellow face

Applied torsional load is shown in the **Figure 2.106**. Torsional unit load is applied on the variable geometries. 3 different geometries are used for torsional loading analysis.

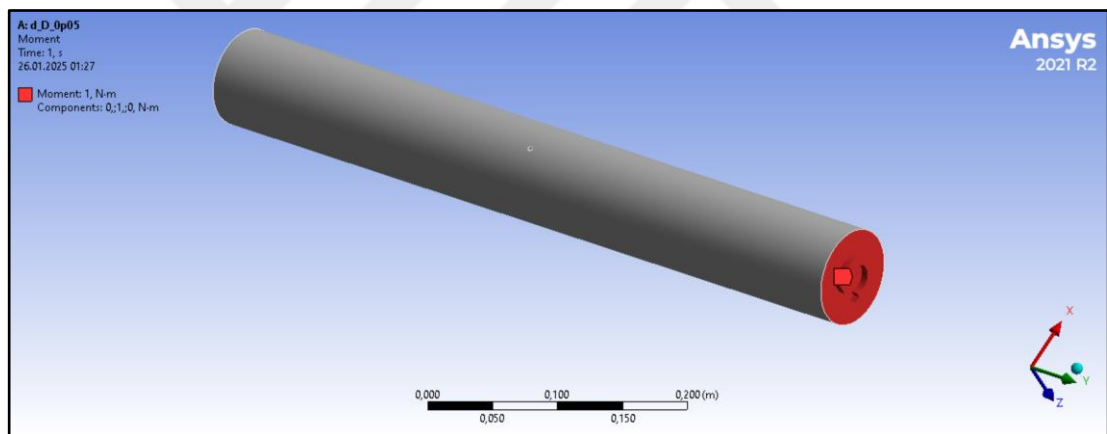


Figure 2.106: Torsional Load Direction

2.5.2. Static Analysis Result

Static analysis result for $a/D:0.05$ round shaft with transverse hole included deformation at Load direction RY directions is shown Figure 2.107. Maximum deformation is $2.61E-7$ m.

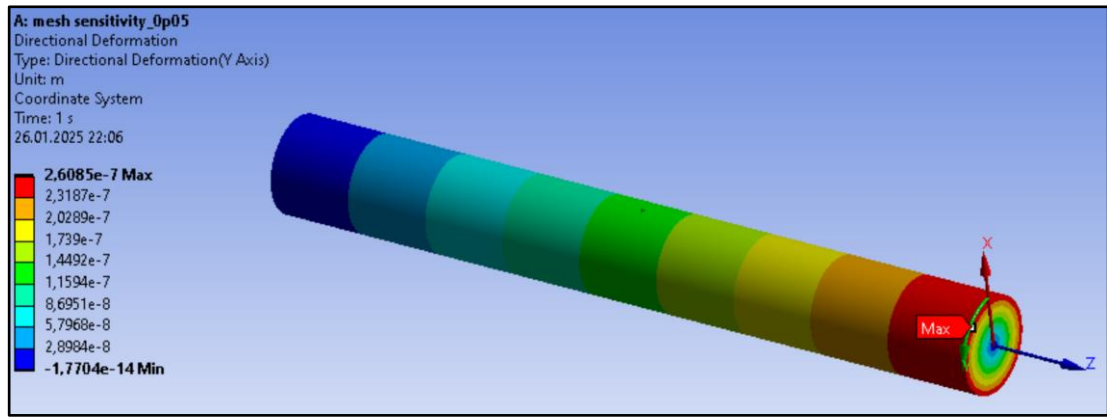


Figure 2.107: Torsional Deformation at RY direction for a/D:0.05

Static analysis result for a/D:0.25 round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.108-Figure 2.86**. Maximum deformation is 2.67E-7 m.

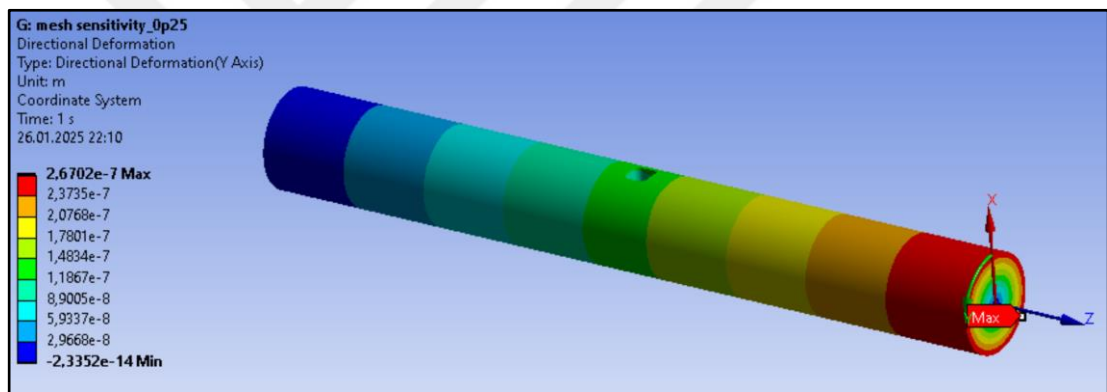


Figure 2.108: Torsional Deformation at RY direction for a/D:0.25

2.5.3. Harmonic Load Analysis Result

8 modes are calculated in the modal analysis. One critical mode shapes are selected for bending and torsional modes. This is 1283.4 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.29**.

Table 2.29: REMTM for Round Shaft with square hole

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	144,38	1,1E-10	9,7E-14	6,1E-01	9,7E-01	2,8E-13	1,7E-10
2	145,68	6,1E-01	3,9E-15	9,2E-11	1,6E-10	1,4E-13	9,7E-01
3	844,26	1,0E-09	4,2E-12	2,0E-01	2,7E-02	1,8E-11	1,5E-10
4	874,83	1,9E-01	4,8E-11	9,9E-10	1,2E-10	3,4E-11	2,5E-02
5	1283,4	7,3E-12	2,4E-11	4,8E-12	6,3E-14	8,0E-01	3,9E-13
6	2072,2	1,1E-09	8,0E-01	6,8E-11	1,5E-12	1,3E-11	3,9E-11
7	2251	6,8E-02	1,1E-08	1,0E-09	4,7E-11	6,6E-12	3,2E-03
8	2264,4	9,3E-10	7,2E-10	6,9E-02	3,2E-03	2,3E-12	4,0E-11
Sum		8,7E-01	8,0E-01	8,7E-01	1,0E+00	8,0E-01	1,0E+00

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, damping coefficient is defined as 0.05. For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.109-Figure 2.110-Figure 2.111** .

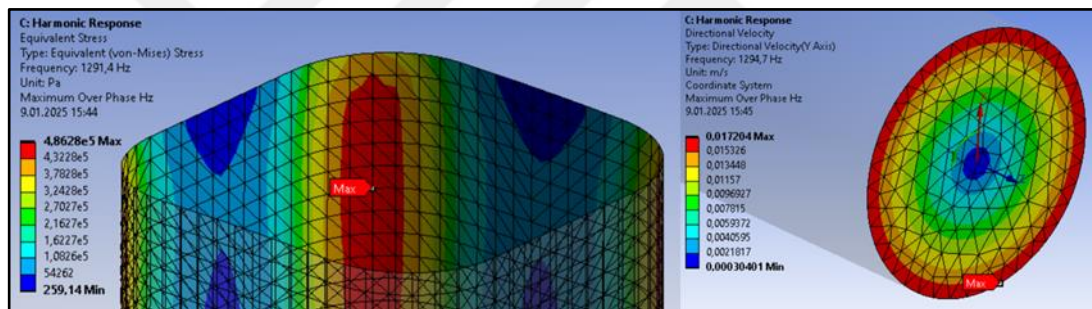


Figure 2.109: Round shaft with square hole for a/D:0.05

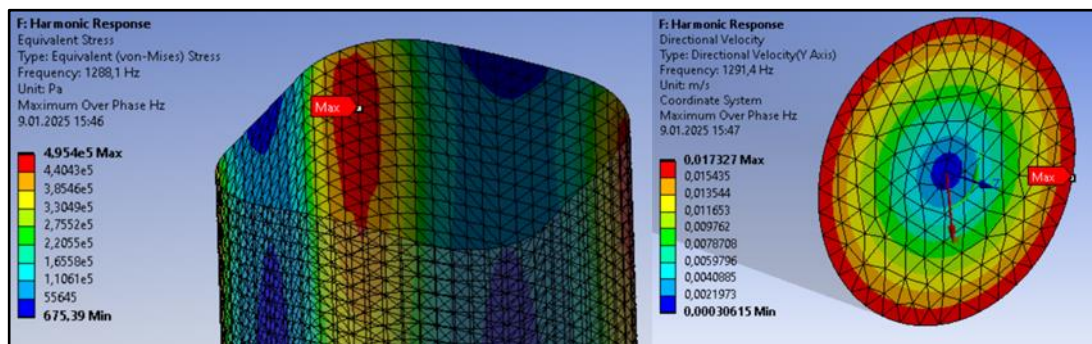


Figure 2.110: Round shaft with square hole for a/D:0.15

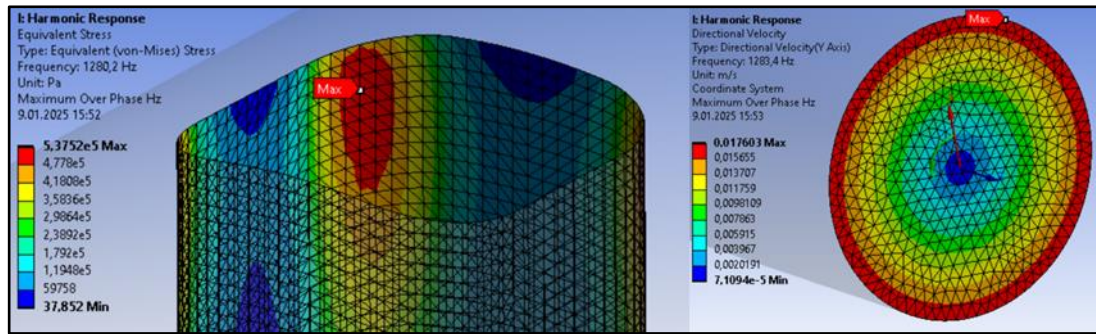


Figure 2.111: Round shaft with square hole for $a/D:0.25$

K_{dt_square} and K_{dt_circle} coefficient is calculated by using Equation (1.35) .The results are shared in the **Table 2.30**.

Table 2.30: Stress, velocity, K_{dt_square} and K_{dt_circle} results

a/D	0.05	0.15	0.25
Maksimum Stress Pa	4.86E+05	4.95E+05	5.38E+05
Maksimum Velocity m/sn	1.72E-02	1.73E-02	1.76E-02
K_{dt_square}	1.91	1.94	2.07
K_{dt_circle}	1.50	1.51	1.60
$K_{dt_square} / K_{dt_circle}$	1.28	1.28	1.29
Average	1.28		
RMS	1.28		
RMS error	0.01		

K_{dt_square} and K_{dt_circle} values are shown as graph in the **Figure 2.112**.

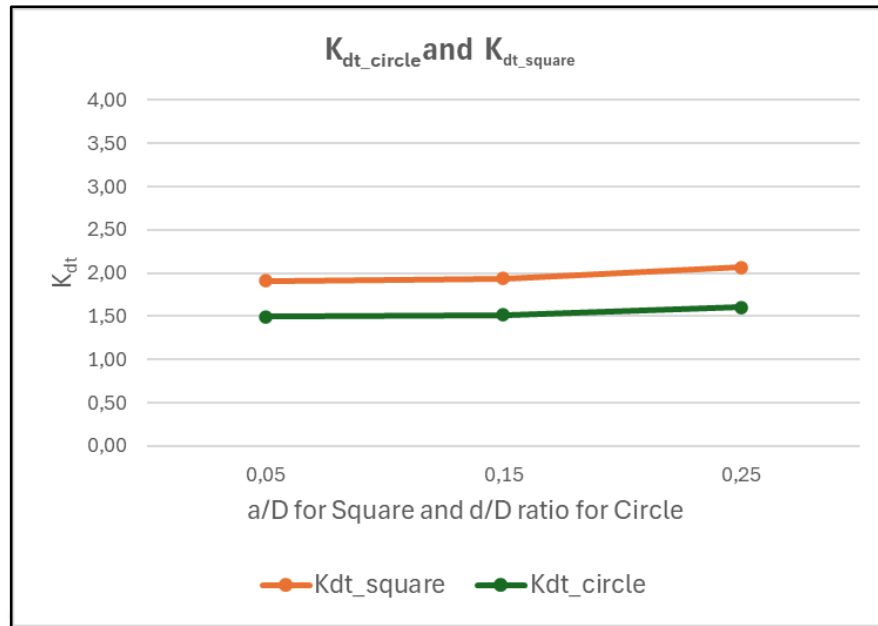


Figure 2.112: K_{dt} values for square and circle holes

In **Figure 2.112**, it is shown that as the a/D and d/D ratios of square and circular holes increase proportionally, there is a constant coefficient relationship. This coefficient is calculated as an average of 1.28, as shared in **Table 2.30**. When comparing the maximum stresses in circular and square holes in the **Figure 2.113**, they increase linearly with each other, similar to the K_{dt} coefficient. However, when we examine the velocities in the **Figure 2.114**, they do not increase proportionally. Notably, after the a/D - d/D ratio reaches 0.15, they change inversely proportional to each other.

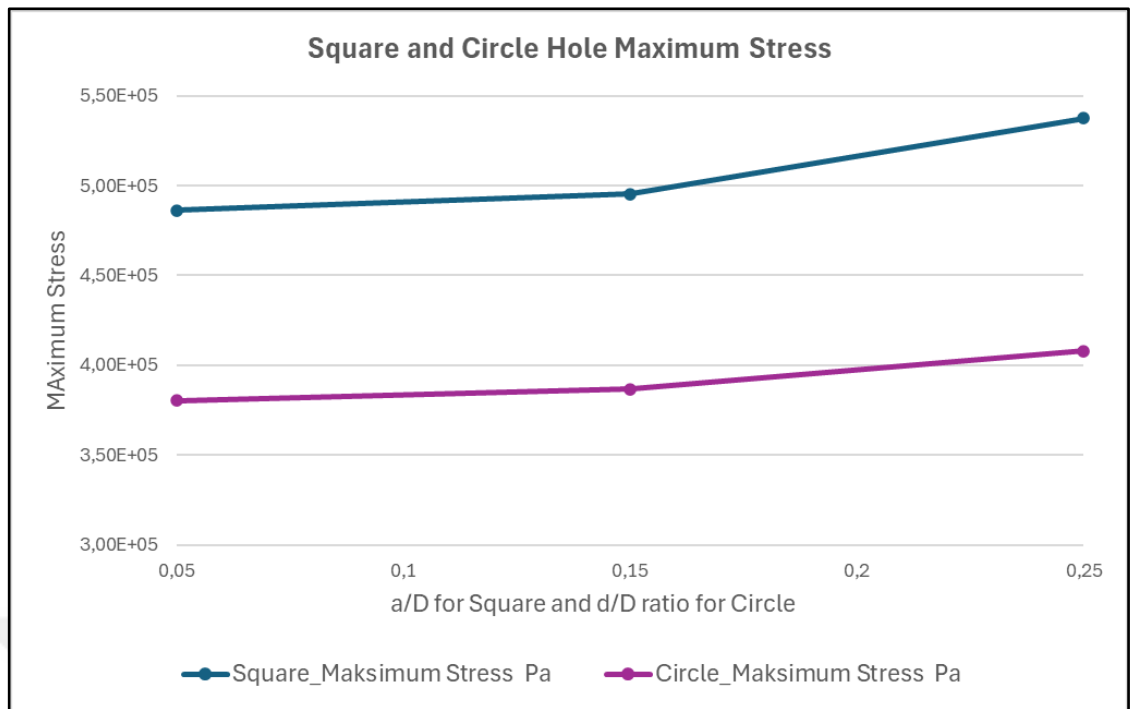


Figure 2.113: Maximum Stress of Circle and Square Holes

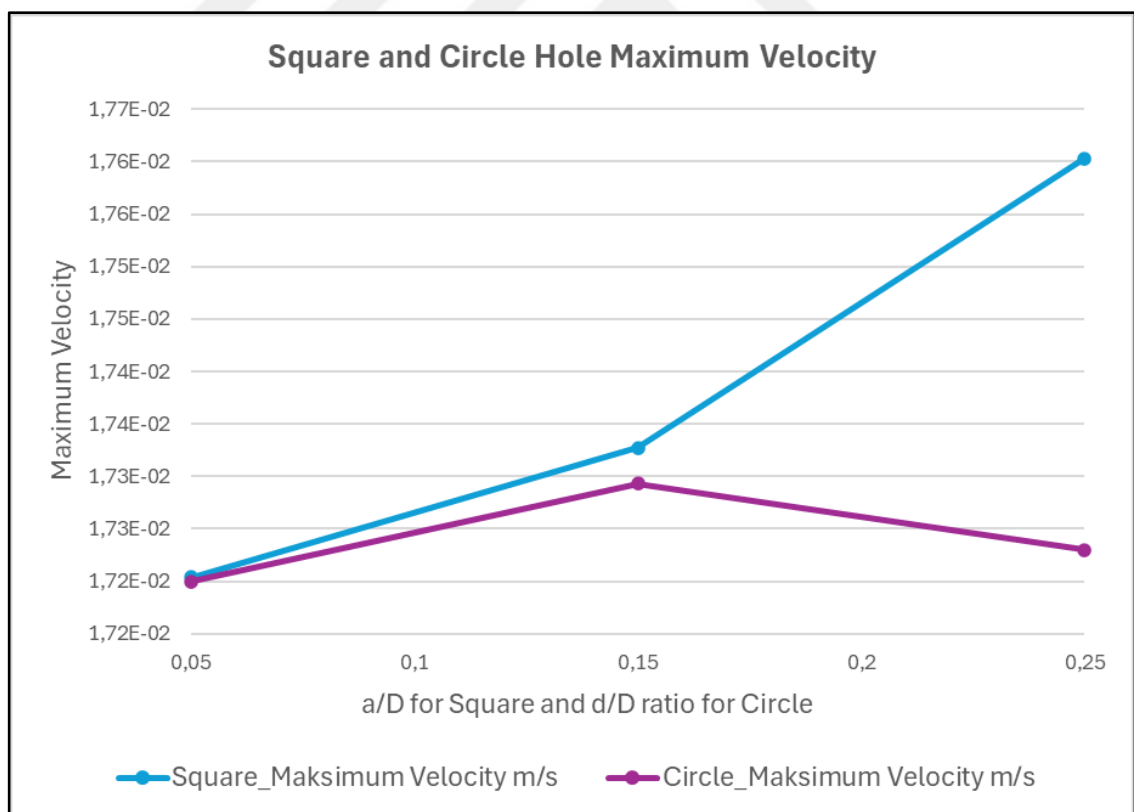


Figure 2.114: Maximum Velocity of Circle and Square Holes

2.5.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.5.4.1. Maximum Velocity and Stress for Torsional Load

These calculation values are shown on the schematic from Figure 1.1. If Analytical calculation is made for a/D : 0.05 shaft with square hole. The Results are shown in the **Figure 2.115**.

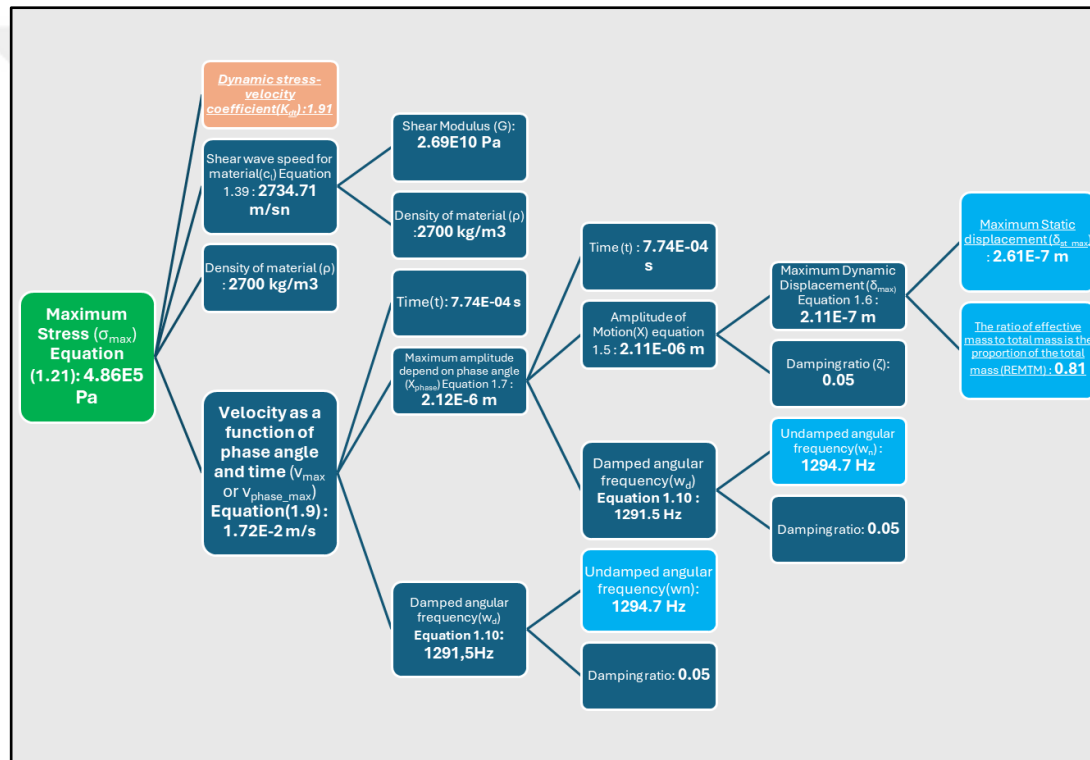


Figure 2.115: Analytical Calculation for a/D :0.05 on torsional load

If Analytical calculation is made for a/D : 0.25 shaft with square hole. The Results are shown in the **Figure 2.116**.

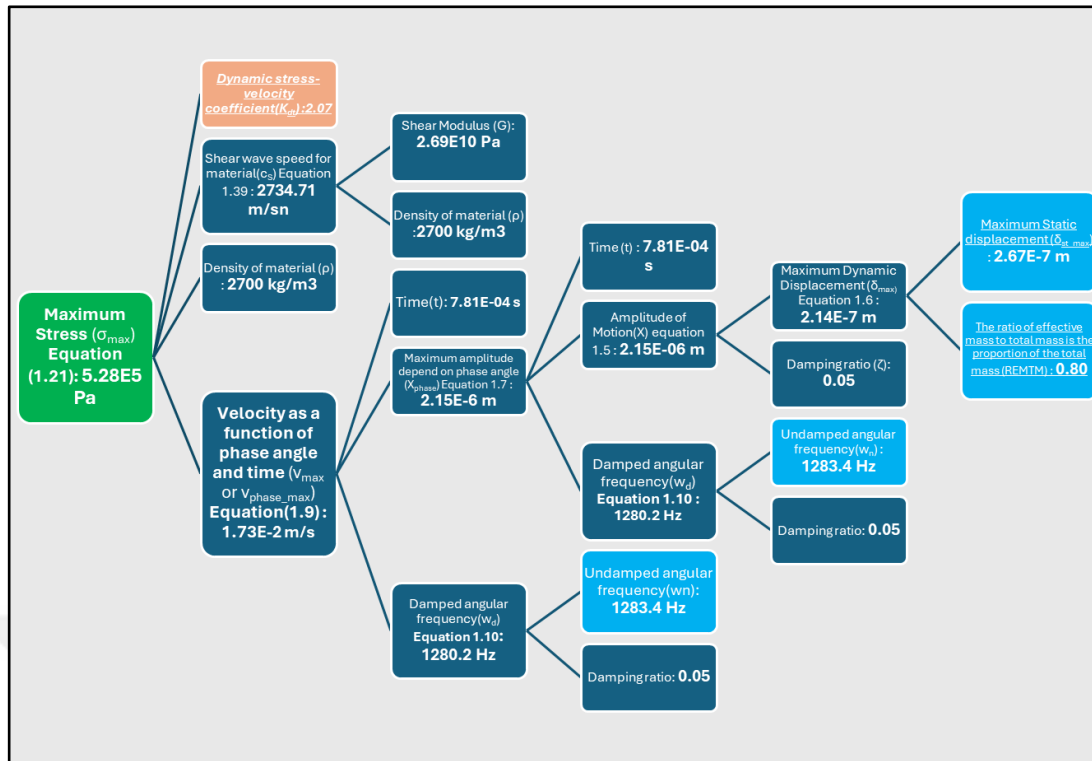


Figure 2.116: Analytical Calculation for a/D:0.25 on torsional load

Comparison of FEM and Analytical results are shared in the **Table 2.31**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.31: Comparison of FEM and Analytical Results

a/D	FEM Stress	Combined Method Stress	Comparison
0.05	4.9E+05	4.9E+05	~0%
0.25	5.4E+05	5.3E+05	~2%
	Velocity	Velocity	
0.05	1.72E-2	1.72E-2	~0%
0.25	1.76E-2	1.73E-2	~2%

2.6. Case Study-5-Round shaft with square hole rotate 45°

2.6.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Geometry is shown in the **Figure 2.117**. Square hole is rotated by 45° and like the rhombus.

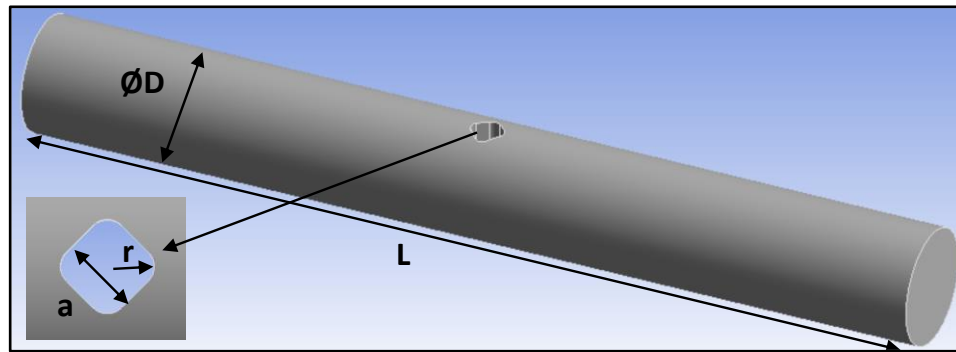


Figure 2.117: Round shaft with square hole rotate 45°

Geometry dimensions are the same with square hole. Applied torsional load is shown in the **Figure 2.118**. Torsional unit load is applied on the variable geometries. 3 different geometries are used for torsional loading analysis.

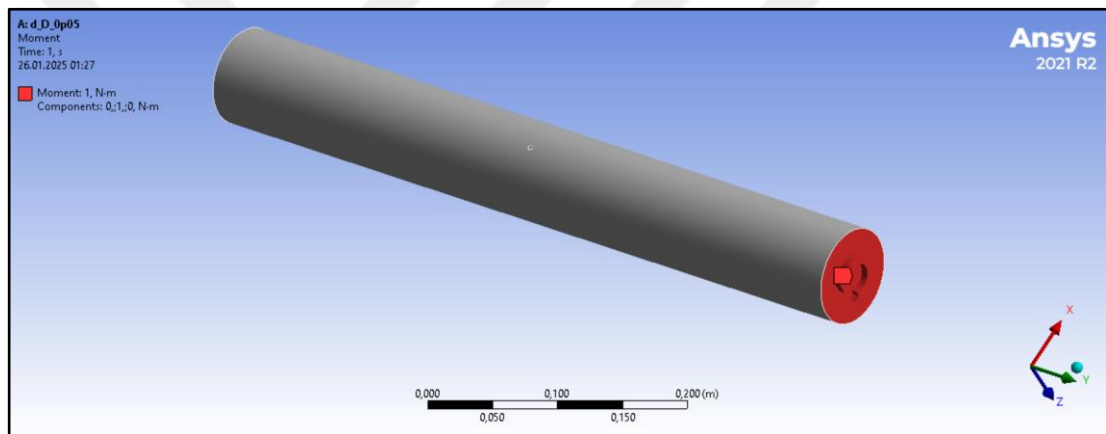


Figure 2.118: Torsional Load Direction

2.6.2. Static Analysis Result

Static analysis result for $a/D:0.05$ round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.119**. Maximum deformation is $2.61E-7$ m.

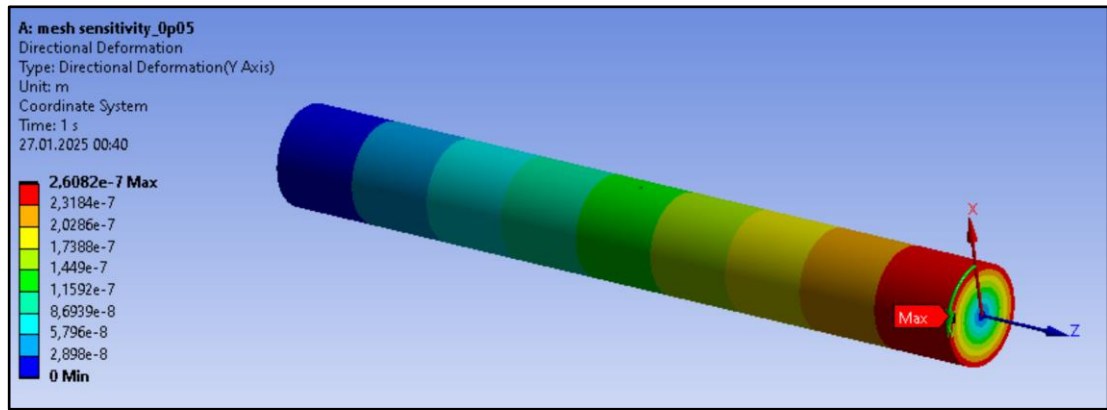


Figure 2.119: Torsional Deformation at RY direction for a/D:0.05

Static analysis result for a/D:0.25 round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.120**. Maximum deformation is 2.66E-7 m.

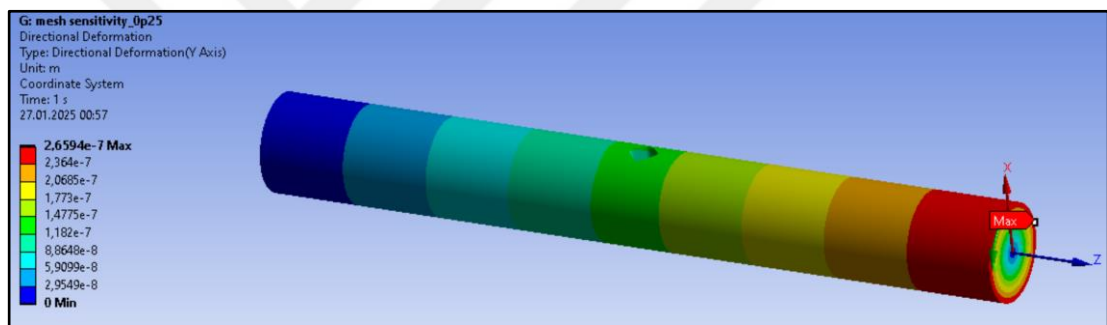


Figure 2.120: Torsional Deformation at RY direction for a/D:0.25

2.6.3. Harmonic Load Analysis Result

8 modes are calculated in the modal analysis. One critical mode shapes are selected for torsional modes. This is 1294.7 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.32**.

Table 2.32: REMTM for Round Shaft with square hole rotate 450

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	145,46	4,4E-06	3,3E-12	6,1E-01	9,7E-01	8,1E-14	7,0E-06
2	145,51	6,1E-01	3,2E-14	4,4E-06	7,0E-06	4,6E-15	9,7E-01
3	866,85	2,4E-09	5,0E-11	1,9E-01	2,5E-02	3,3E-12	3,7E-10
4	868,15	1,9E-01	4,1E-13	2,3E-09	2,7E-10	1,7E-11	2,5E-02
5	1294,7	1,1E-11	3,7E-11	3,9E-12	2,4E-17	8,1E-01	1,3E-12
6	2093,7	3,1E-10	8,1E-01	4,1E-10	5,1E-12	1,9E-11	1,3E-11
7	2270,3	6,8E-02	2,7E-09	1,1E-04	5,1E-06	1,2E-11	3,1E-03
8	2270,5	1,1E-04	4,3E-09	6,8E-02	3,1E-03	3,5E-11	5,0E-06
Sum		8,7E-01	8,1E-01	8,7E-01	1,0E+00	8,1E-01	1,0E+00

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, damping coefficient is defined as 0.05. For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.121-Figure 2.122-Figure 2.123**.

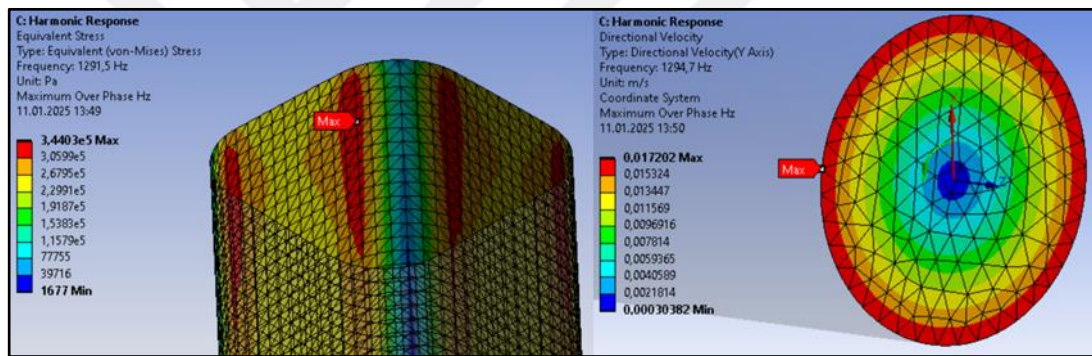


Figure 2.121: Round shaft with square hole rotate 45° for a/D:0.05

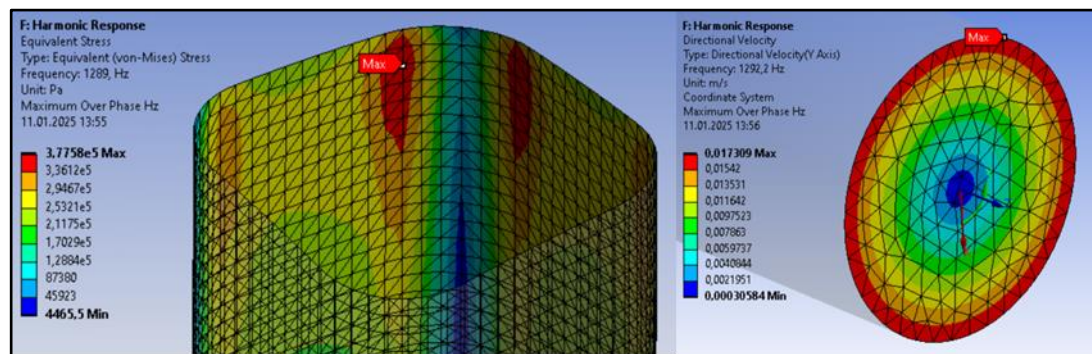


Figure 2.122: Round shaft with square hole rotate 45° for a/D:0.15

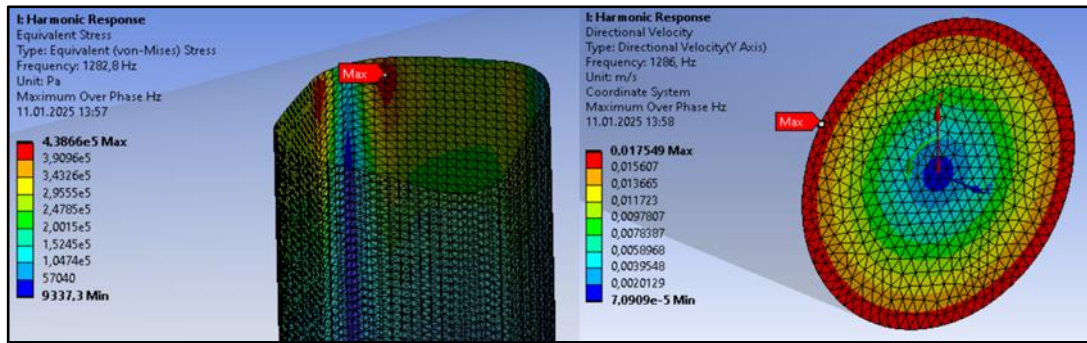


Figure 2.123: Round shaft with square hole rotate 45° for a/D:0.25

$K_{dt_square_rotate_45}$ and K_{dt_circle} coefficient is calculated by using Equation (1.35). The results are shared in the Table 2.33.

Table 2.33: Stress, velocity, $K_{dt_square_rotate_45}$ and K_{dt_circle} results

a/D	0.05	0.15	0.25
Maksimum Stress Pa	3.44E+05	3.78E+05	4.39E+05
Maksimum Velocity m/sn	1.72E-02	1.73E-02	1.75E-02
$K_{dt_square_rotate_45}$	1,35	1,48	1,69
K_{dt_circle}	1.50	1.51	1.60
$K_{dt_square_rotate_45}/K_{dt_circle}$	1,11	1,02	0,95
Average	1.03		
RMS	1.03		
RMS error	0.06		

$K_{dt_square_rotate_45}$ and K_{dt_circle} values are shown as graph in the **Figure 2.124**.

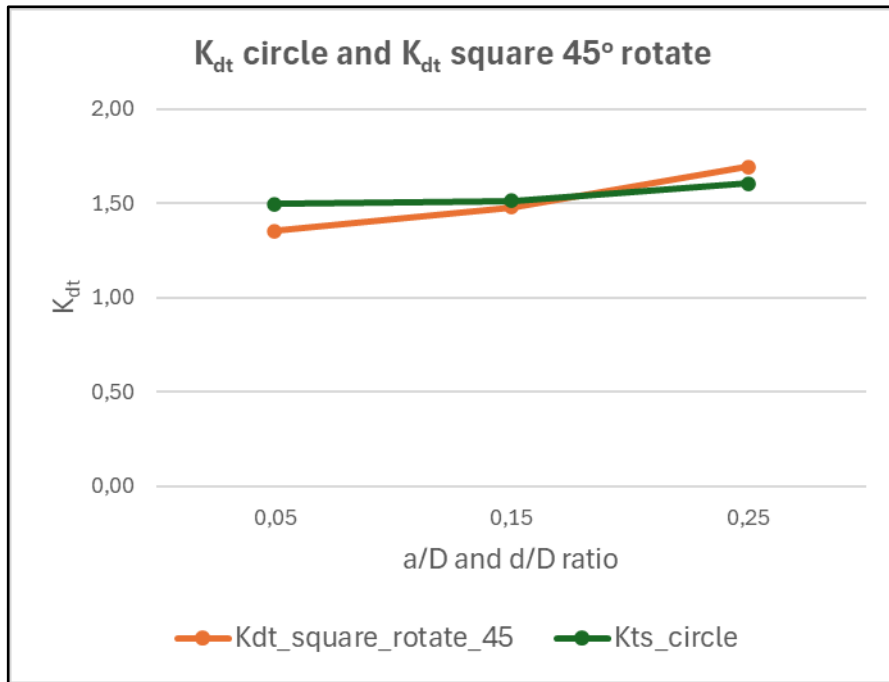


Figure 2.124: K_{dt} values for square rotate 45° and circle holes

In Figure 2.124, it is shown that as the a/D and d/D ratios of square and circular holes increase proportionally, there is no constant coefficient relationship. When the graphs in the Figure 2.125-Figure 2.126 are examined, it is observed that the stresses vary in parallel with the coefficients. However, the velocity exhibits variations at different rates.

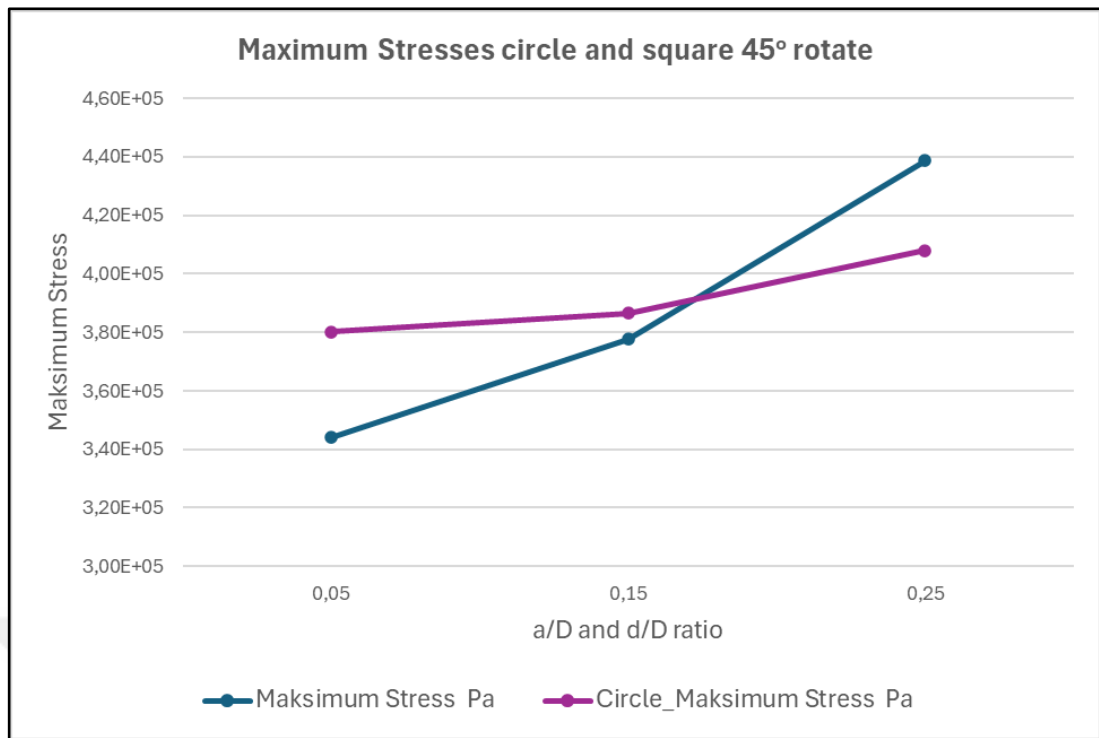


Figure 2.125: Maximum Stress of Circle and Square rotate 45° Holes

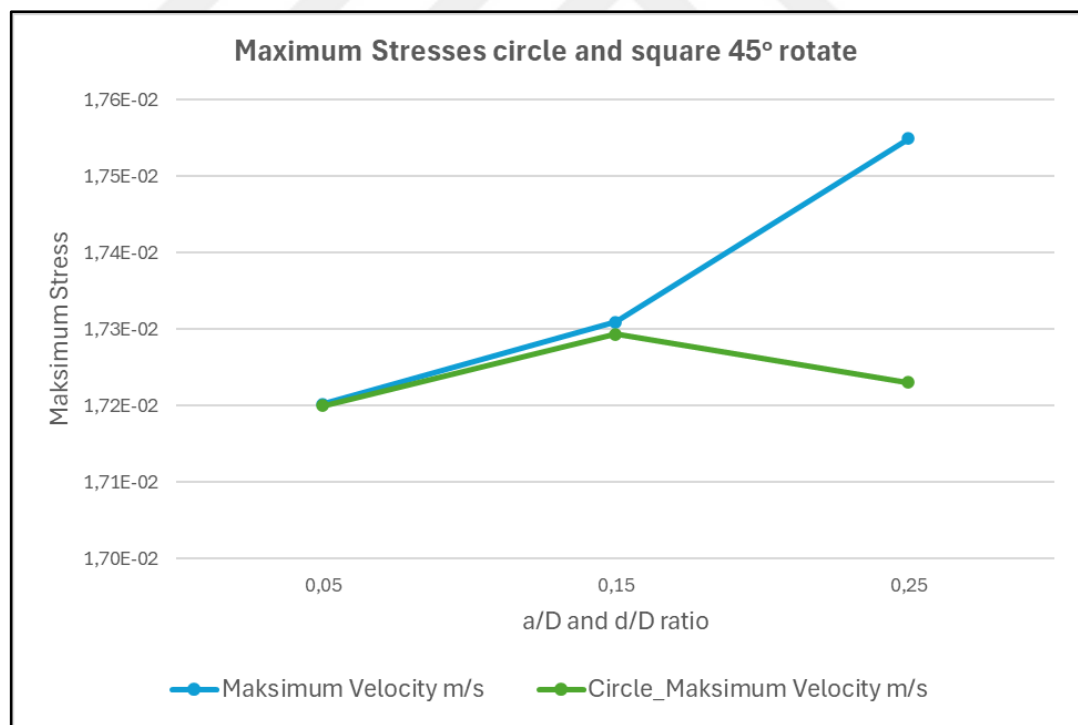


Figure 2.126: Maximum Velocity of Circle and Square rotates 45° Holes

2.6.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.6.4.1. Maximum Velocity and Stress for Torsional Load

These calculation values are shown on the schematic from **Figure 1.1**. If Analytical calculation is made for a/D : 0.05 shaft with square rotates 45° hole. The Results are shown in the **Figure 2.127**.

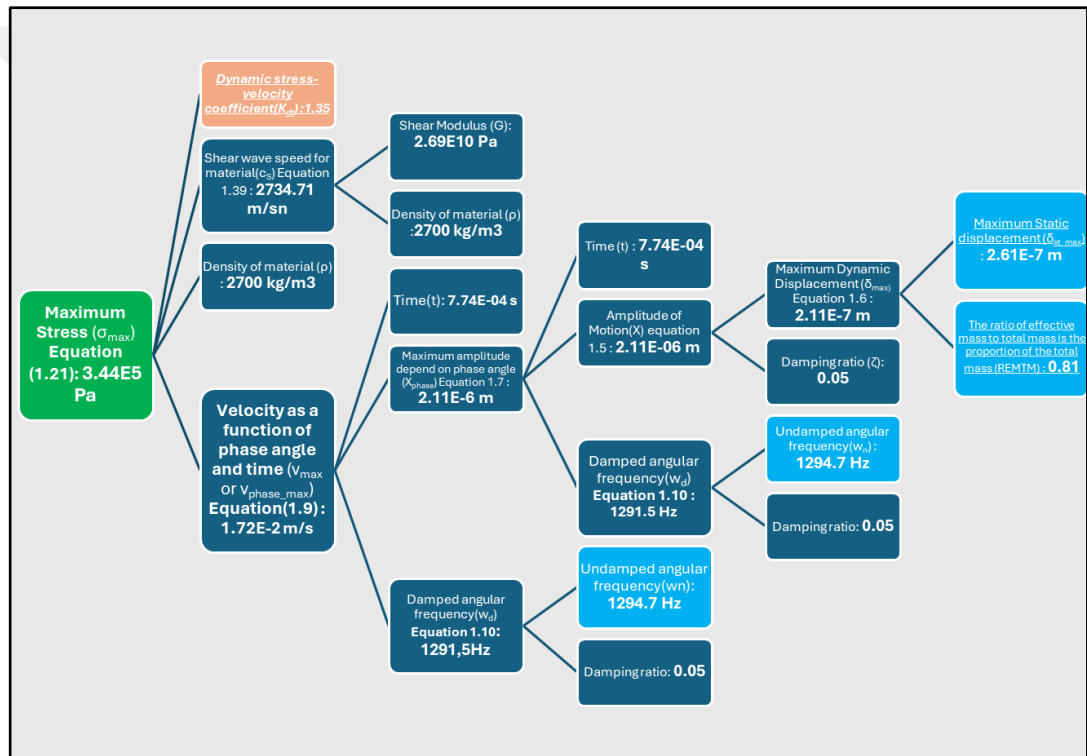


Figure 2.127: Analytical Calculation for a/D :0.05 on torsional load

If Analytical calculation is made for a/D : 0.25 shaft with square rotates 45° hole. The Results are shown in the **Figure 2.128**.

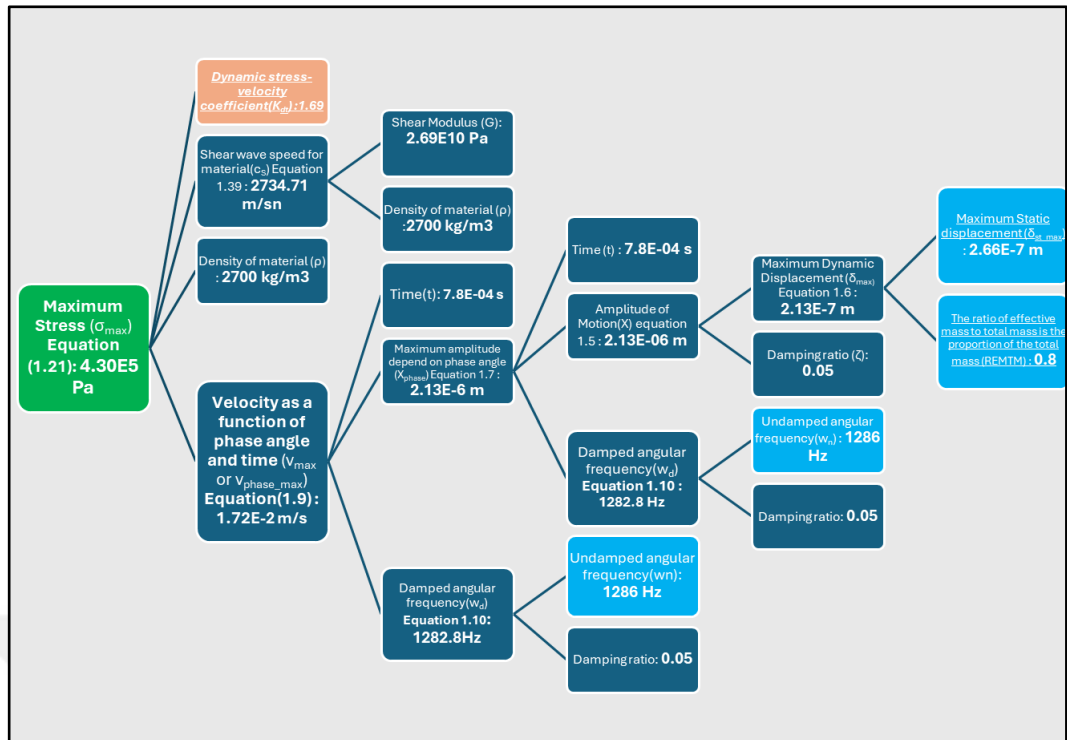


Figure 2.128: Analytical Calculation for a/D:0.25 on torsional load

Comparison of FEM and Analytical results are shared in the **Table 2.34**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.34: Comparison of FEM and Analytical Results

a/D	FEM Stress	Combined Method Stress	Comparison
0.05	3.4E+05	3.4E+05	~0%
0.25	4.39E+05	4.3E+05	~2%
	Velocity	Velocity	
0.05	1.72E-02	1.72E-02	~0%
0.25	1.76E-02	1.73E-2	~2%

2.7. Case Study-6-Round shaft with rectangular hole

2.7.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Geometry is shown in the **Figure 2.129**. There is a, b (2a) and r difference parameter between circle and rectangular hole.

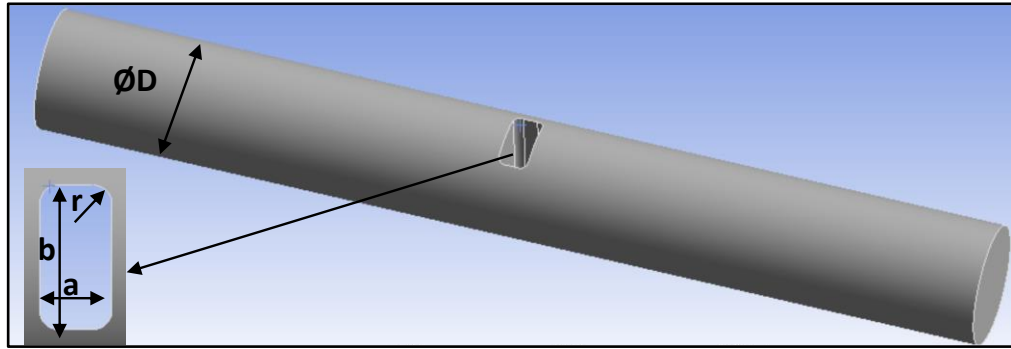


Figure 2.129: Round shaft with rectangular hole

The geometry dimensions are shared in the **Table 2.35**.

Table 2.35: Square hole dimensions

Parameter	Value		
L [m]	0.6096		
D [m]	0.0762		
a/D	0.05	0.15	0.25
a [m]	3.810E-03	1.143E-02	1.91E-02
b [m]	7.62E-03	2.29E-02	3.81E-02
r [m]	1E-03	3E-03	5E-03

Applied torsional load is shown in the **Figure 2.130**. Torsional unit load is applied on the variable geometries. 3 different geometries are used for torsional loading analysis.

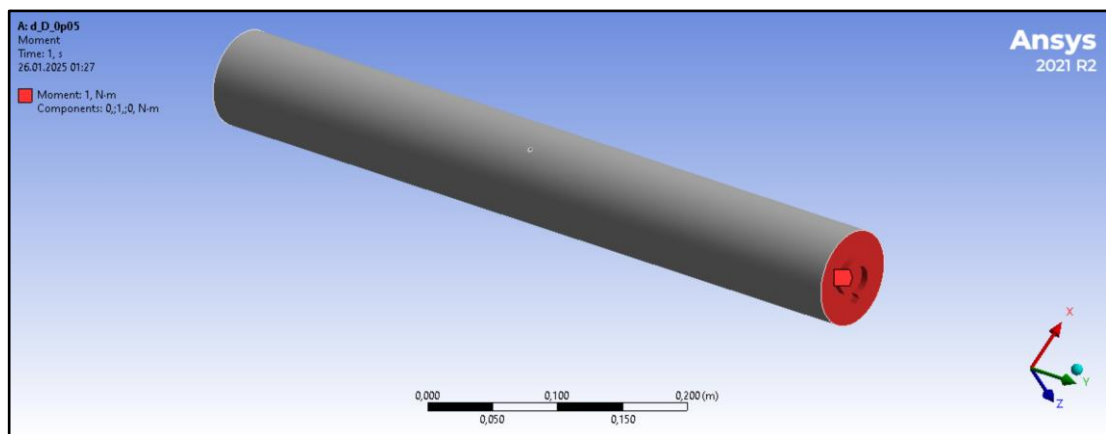


Figure 2.130: Torsional Load Direction

2.7.2. Static Analysis Result

Static analysis result for $a/D:0.05$ round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.131**. Maximum deformation is $2.61E-7$ m.

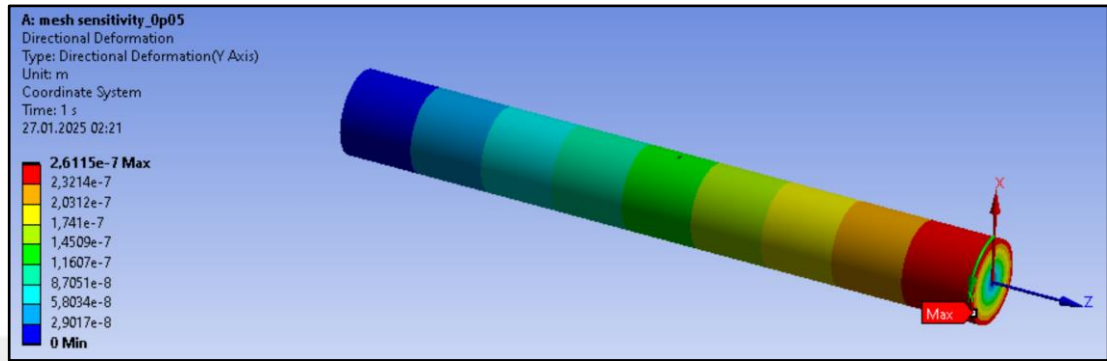


Figure 2.131: Torsional Deformation at RY direction for $a/D:0.05$

Static analysis result for $a/D:0.25$ round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.132**. Maximum deformation is $2.8E-7$ m.

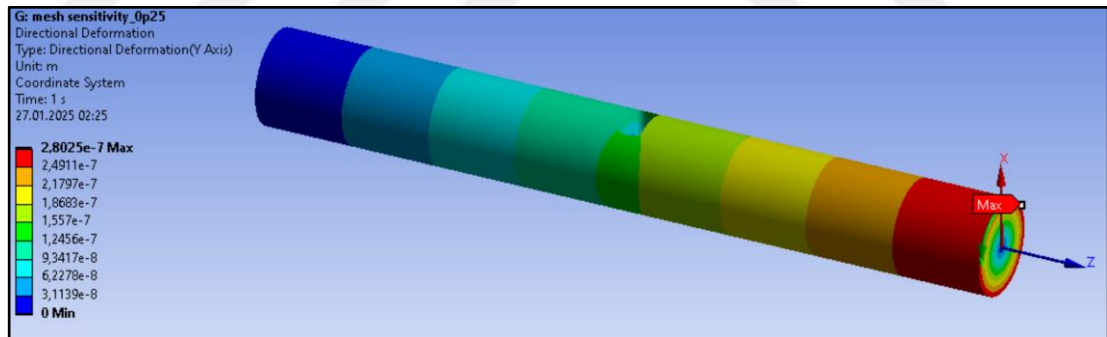


Figure 2.132: Torsional Deformation at RY direction for $a/D:0.25$

2.7.3. Harmonic Load Analysis Result

8 modes are calculated in the modal analysis. One critical mode shapes are selected for torsional modes. This is 1294.1 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.36**.

Table 2.36: REMTM for Round Shaft with rectangular hole

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	145,37	3,4E-05	1,5E-13	6,1E-01	9,7E-01	9,8E-15	5,5E-05
2	145,52	6,1E-01	9,4E-13	3,4E-05	5,5E-05	5,3E-15	9,7E-01
3	864,72	1,0E-07	5,6E-11	1,9E-01	2,5E-02	1,3E-11	1,3E-08
4	868,47	1,9E-01	5,9E-11	1,0E-07	1,4E-08	2,5E-12	2,5E-02
5	1294,1	1,1E-12	2,8E-13	7,3E-12	7,9E-13	8,1E-01	1,2E-13
6	2091,8	3,2E-10	8,1E-01	7,9E-11	1,3E-12	4,5E-13	5,5E-12
7	2269,3	6,8E-02	3,6E-09	1,1E-05	4,9E-07	1,7E-13	3,1E-03
8	2270,1	1,1E-05	1,7E-09	6,8E-02	3,1E-03	1,7E-15	4,9E-07
Sum		8,7E-01	8,1E-01	8,7E-01	1,0E+00	8,1E-01	1,0E+00

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, damping coefficient is defined as 0.05. For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.133-Figure 2.135**.

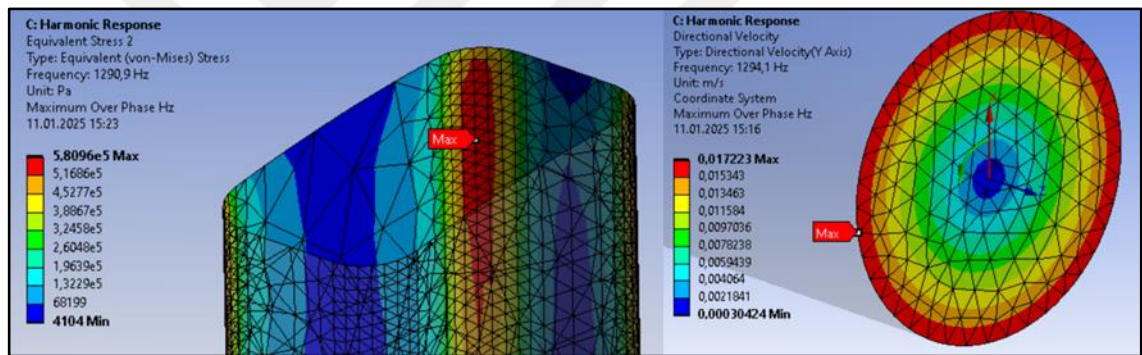


Figure 2.133: Round shaft with rectangular hole for a/D:0.05

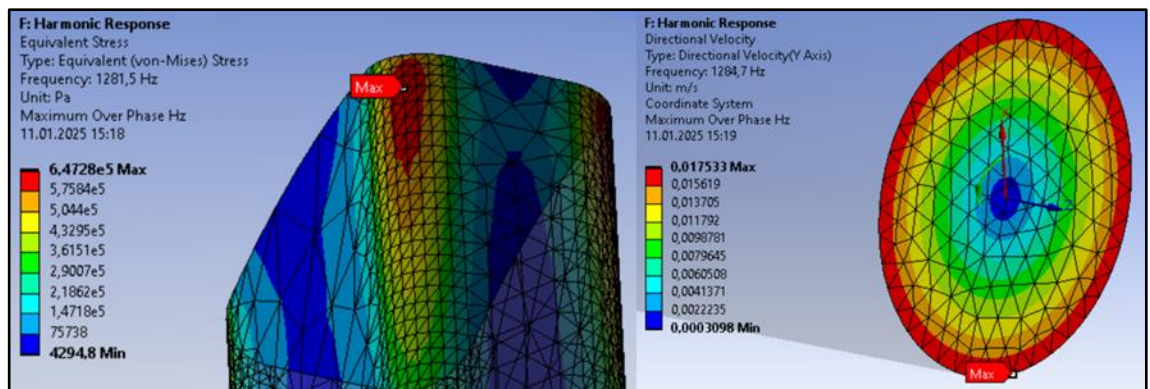


Figure 2.134: Round shaft with rectangular hole for a/D:0.15

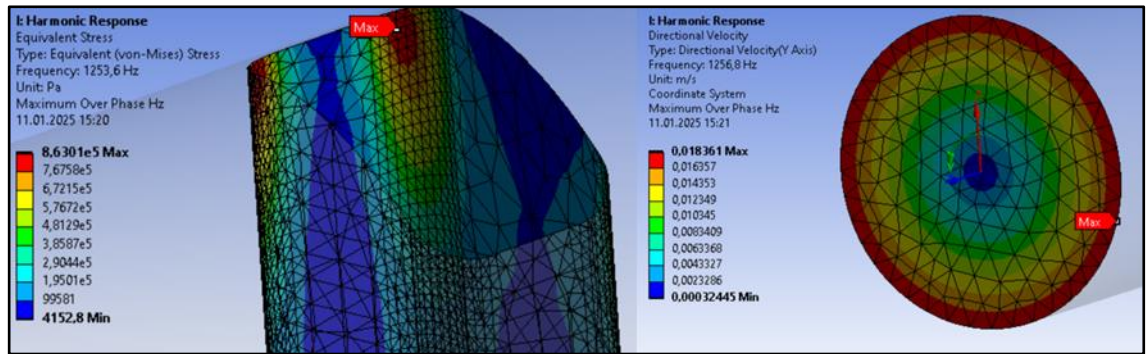


Figure 2.135: Round shaft with rectangular hole for $a/D:0.25$

The results are shared in the **Table 2.37**.

Table 2.37: Stress, velocity, K_d square and K_d circle results

a/D	0.05	0.15	0.25
Maksimum Stress Pa	5.81E+05	6.47E+05	8.63E+05
Maksimum Velocity m/sn	1.72E-02	1.75E-02	1.84E-02
$K_{dt_rectangular}$	2.28	2.50	3.18
K_{dt_circle}	1.50	1.51	1.60
$K_{drectangular}/K_{dcircle}$	0.66	0.61	0.50
Average	0.35		
RMS	0.46		
RMS error	0.19		

$K_{dt_rectangular}$ and K_{dt_circle} values are shown as graph in the **Figure 2.136**.

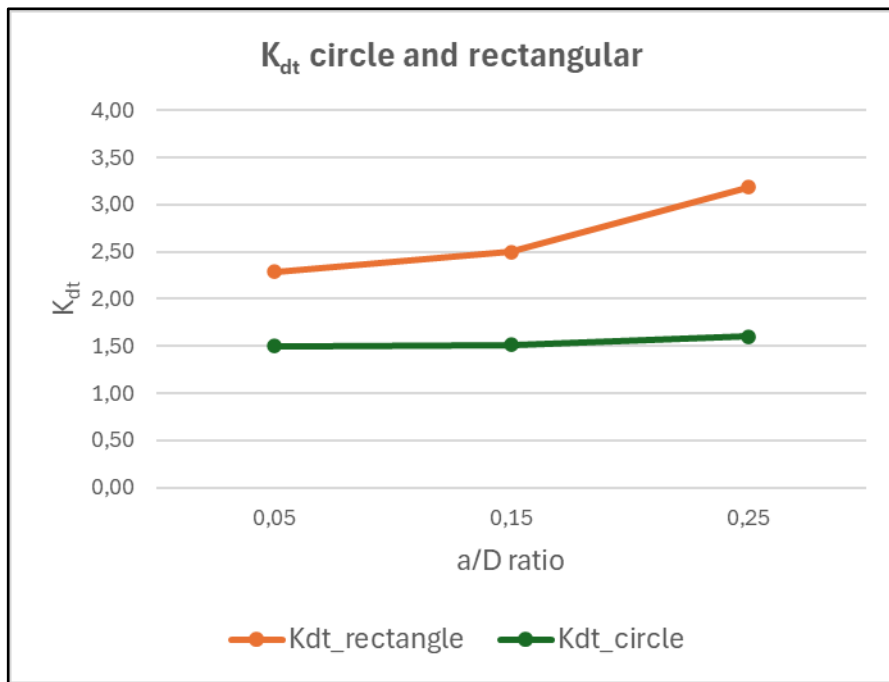


Figure 2.136: K_{dt} values for rectangular and circle holes

In Figure 2.136-Figure 2.112, it is shown that as the a/D and d/D ratios of square and circular holes increase proportionally, there is no constant coefficient relationship. When the graphs in Figure 2.137-Figure 2.138 are examined, it is observed that the stresses vary in parallel with the coefficients like square hole. However, the velocity exhibits variations at different rates.

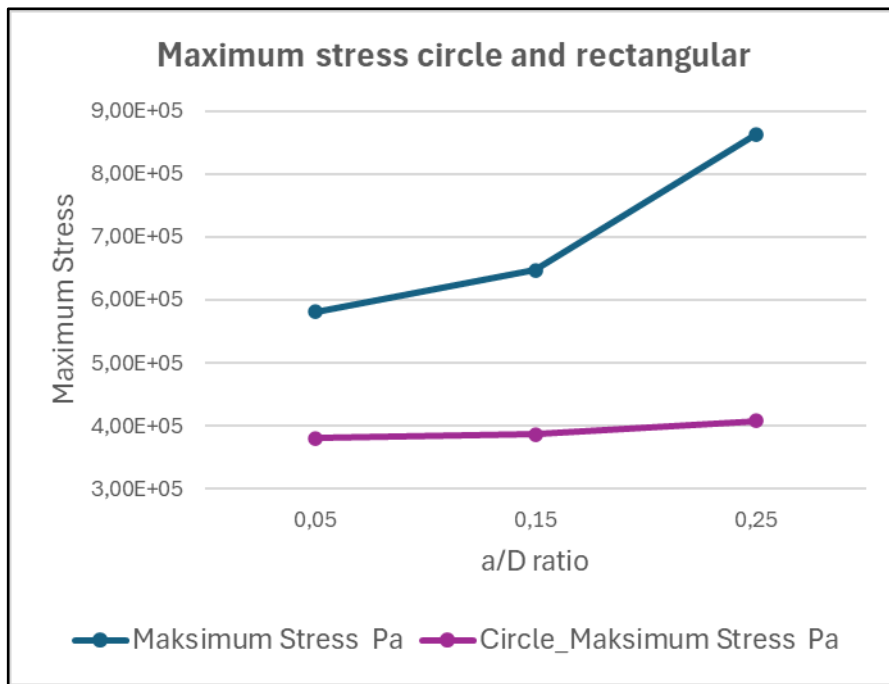


Figure 2.137: Maximum Stress of Circle and rectangular hole

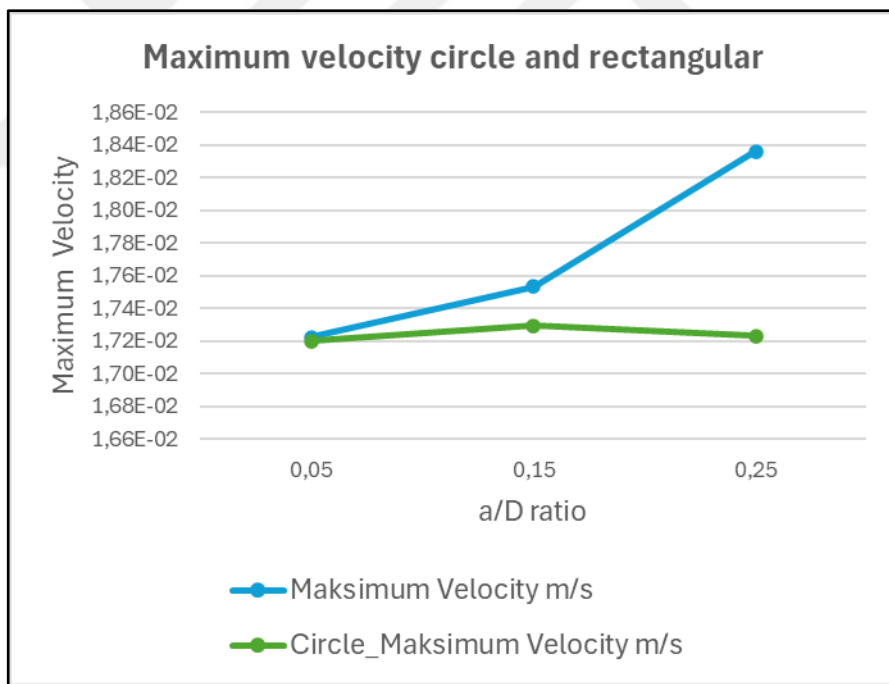


Figure 2.138: Maximum Velocity of Circle and rectangular hole

2.7.4. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity

and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.7.4.1. Maximum Velocity and Stress for Torsional Load

These calculation values are shown on the schematic from **Figure 1.1**. If Analytical calculation is made for a/D : 0.05 shaft with square hole. The Results are shown in the **Figure 2.139**.

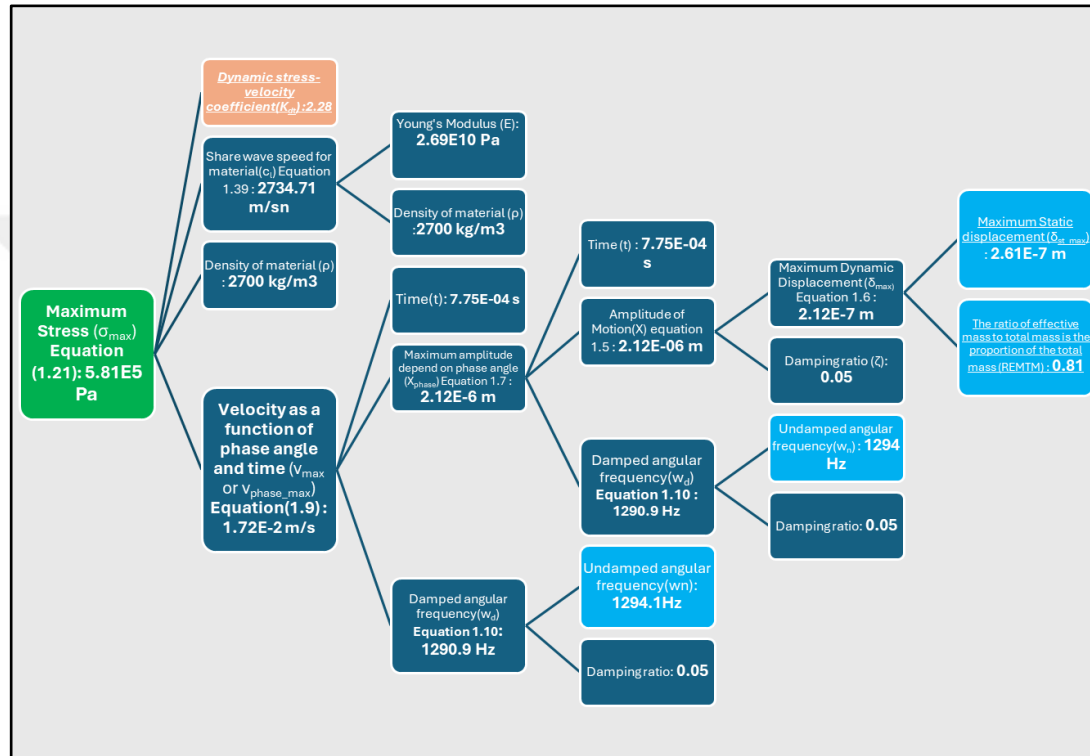


Figure 2.139: Analytical Calculation for a/D :0.05 on torsional load

If Analytical calculation is made for a/D : 0.25 shaft with square hole. The Results are shown in the **Figure 2.140**.

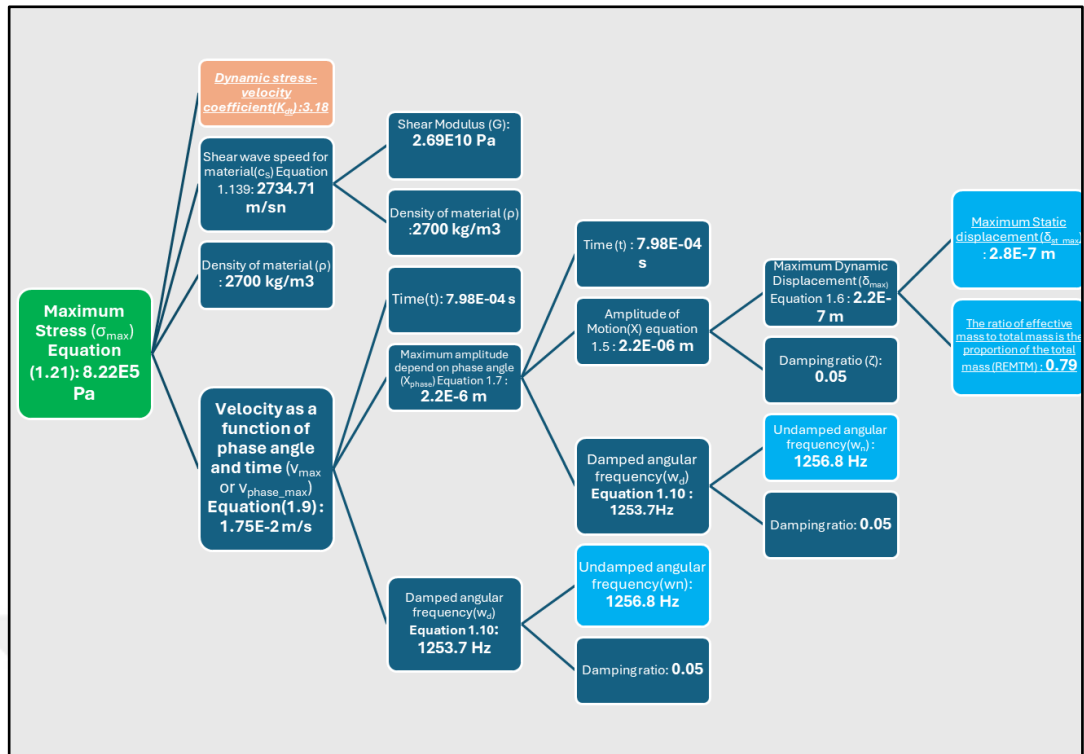


Figure 2.140: Analytical Calculation for a/D:0.25 on torsional load

A comparison of FEM and Analytical results are shared in the **Table 2.38**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.38: Comparison of FEM and Analytical Results

a/D	FEM Stress	Combined Method Stress	Comparison
0.05	5.81E+05	5.81E+05	~0%
0.25	8.63E+05	8.22E+05	~5%
	Velocity	Velocity	
0.05	1.72E-2	1.72E-2	~0%
0.25	1.75E-2	1.84E-2	~5%

2.8. Case Study-7-Round shaft with rectangular hole rotate 90°

2.8.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Geometry is shown in the **Figure 2.141**. There is a, b (2a) and rectangular hole is rotated 90°.

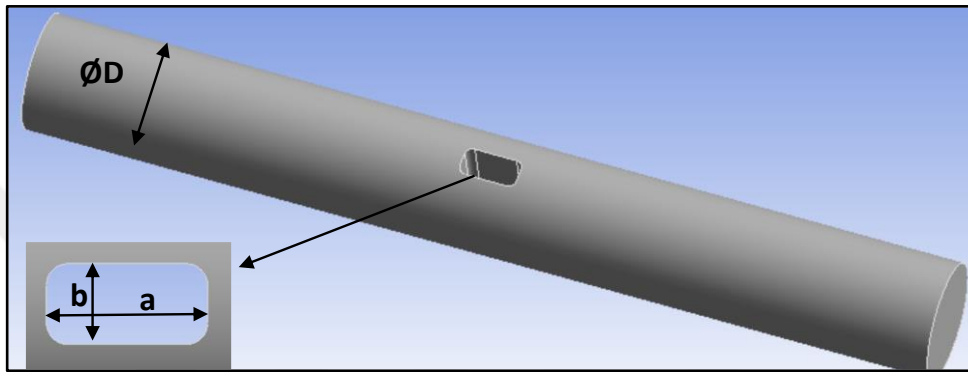


Figure 2.141: Round shaft with rectangular hole rotate 90°

The geometry dimensions are shared in the **Table 2.39**.

Table 2.39: Square hole dimensions

Parameter	Value		
L [m]	0.6096		
D [m]	0.0762		
a/D	0.05	0.15	0.25
a [m]	7.62E-03	2.29E-02	3.81E-02
b [m]	3.810E-03	1.143E-02	1.91E-02
r [m]	1E-03	3E-03	5E-03

Applied torsional load is shown in the **b**. Torsional unit load is applied on the variable geometries. 3 different geometries are used for torsional loading analysis.

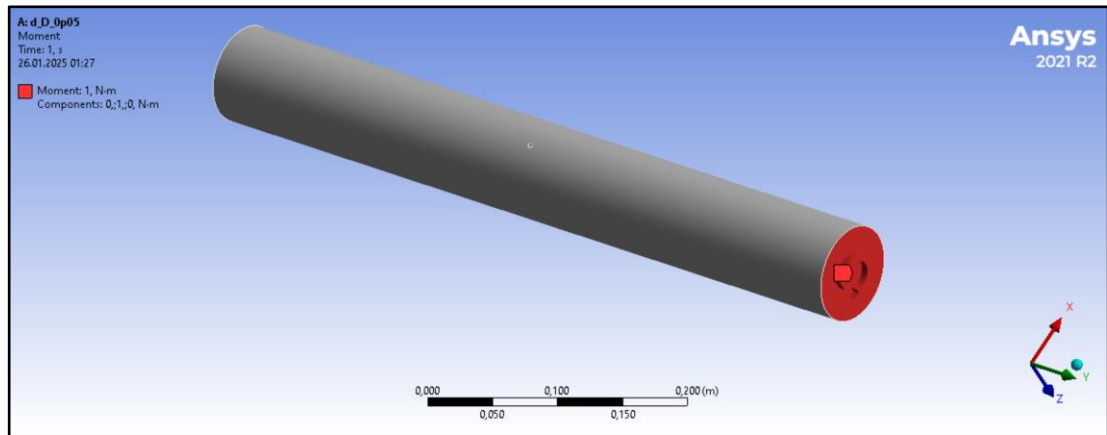


Figure 2.142: Torsional Load Direction

2.8.2. Static Analysis Result

Static analysis result for $a/D:0.05$ round shaft with transverse hole included deformation at Load direction RY directions is shown in **Figure 2.143**. Maximum deformation is $2.61E-7$ m.

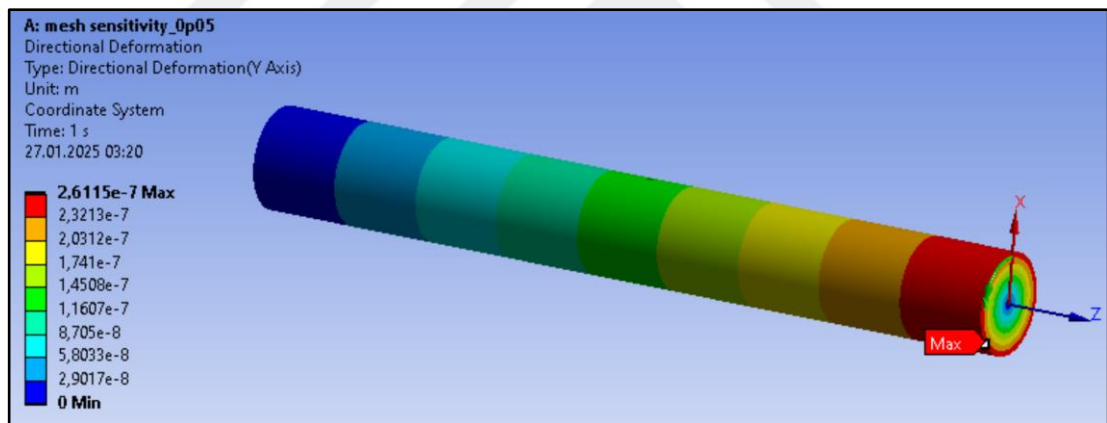


Figure 2.143: Torsional Deformation at RY direction for $a/D:0.05$

Static analysis result for $a/D:0.25$ round shaft with transverse hole included deformation at Load direction RY directions is shown **Figure 2.144**. Maximum deformation is $2.77E-7$ m.

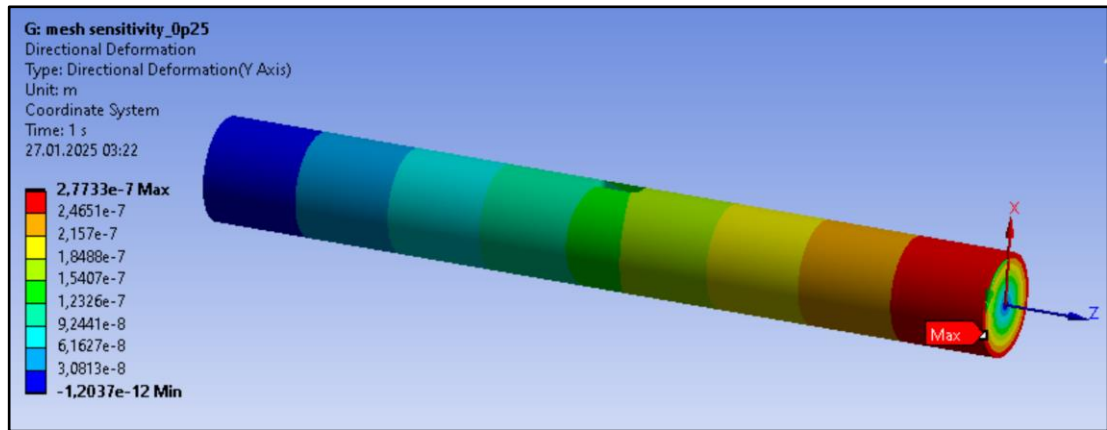


Figure 2.144: Torsional Deformation at RY direction for a/D:0.25

8 modes are calculated in the modal analysis. One critical mode shapes are selected for torsional modes. This is 1294.1 Hz for RY direction (Torsional). The Ratio of Effective Mass to Total Mass is shared in the **Table 2.40**.

Table 2.40: REMTM for rectangular hole rotate 90o

Mode	Frequency[Hz]	TX	TY	TZ	RX	RY	RZ
1	145,45	9,9E-06	2,4E-13	6,1E-01	9,7E-01	1,7E-13	1,6E-05
2	145,52	6,1E-01	6,0E-13	9,9E-06	1,6E-05	1,3E-13	9,7E-01
3	866,78	8,9E-11	1,0E-11	1,9E-01	2,5E-02	2,4E-13	1,5E-11
4	868,48	1,9E-01	1,2E-11	8,2E-11	8,7E-12	2,1E-11	2,5E-02
5	1294,1	4,0E-11	2,2E-10	7,8E-13	7,7E-15	8,1E-01	1,7E-12
6	2093,7	6,4E-10	8,1E-01	5,8E-10	1,9E-11	1,9E-10	4,0E-11
7	2269,4	6,8E-02	7,2E-09	6,9E-06	3,1E-07	1,0E-10	3,1E-03
8	2270,4	6,8E-06	6,0E-09	6,8E-02	3,1E-03	2,7E-11	3,1E-07
Sum		8,7E-01	8,1E-01	8,7E-01	1,0E+00	8,1E-01	1,0E+00

Harmonic response analysis is based on Modal analysis. In Harmonic response analysis, damping coefficient is defined as 0.05. For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.145-Figure 2.147**.

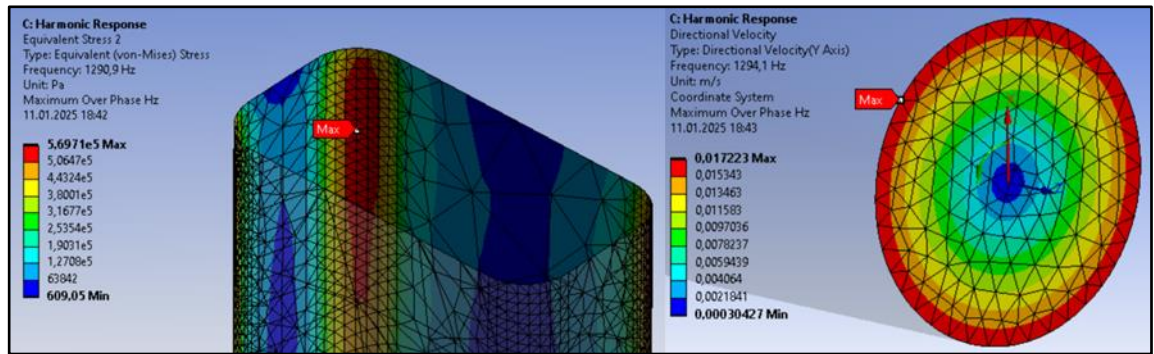


Figure 2.145: Round shaft with rectangular hole rotate 90^0 for $a/D:0.05$

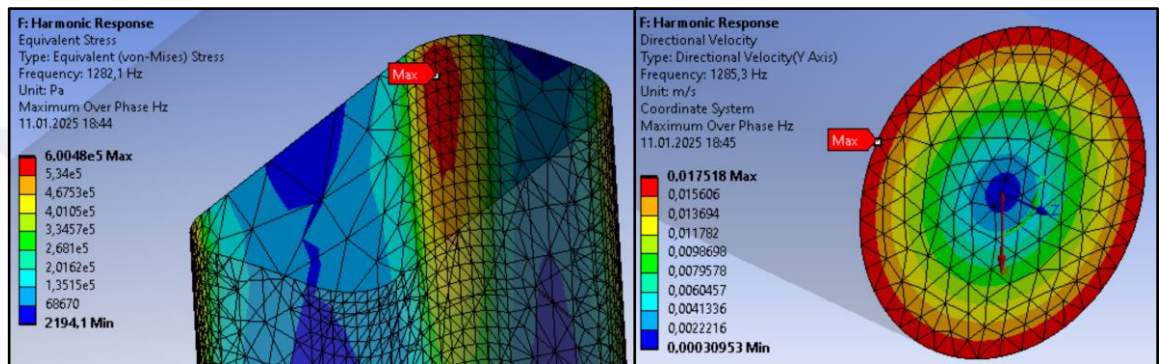


Figure 2.146: Round shaft with rectangular hole rotate 90^0 for $a/D:0.15$

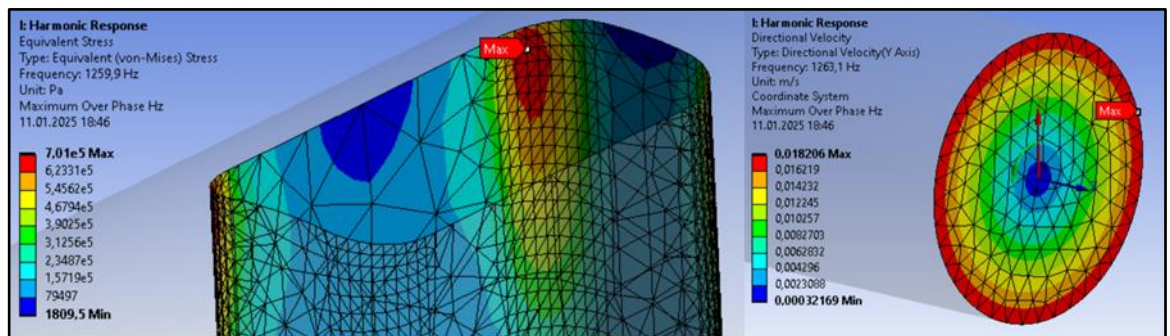


Figure 2.147: Round shaft with rectangular hole rotate 90^0 for $a/D:0.25$

The results are shared in the **Table 2.41**.

Table 2.41: Stress, velocity, Kd rectangular 900 and Kd circle results

a/D	0.05	0.15	0.25
Maksimum Stress Pa	5.69E+05	6.00E+05	7.01E+05
Maksimum Velocity m/sn	1.72E-02	1.75E-02	1.82E-02
K _d rectangular rotate 90	2.24	2.32	2.61
K _d circle	1.50	1.51	1.60
K _d rectangular rotate 90/K _d circle	0.66	0.61	0.50
Average	0.39		
RMS	0.50		
RMS error	0.20		

K_{dt_rectangular} and K_{dt_circle} values are shown as graph in the **Figure 2.148**.

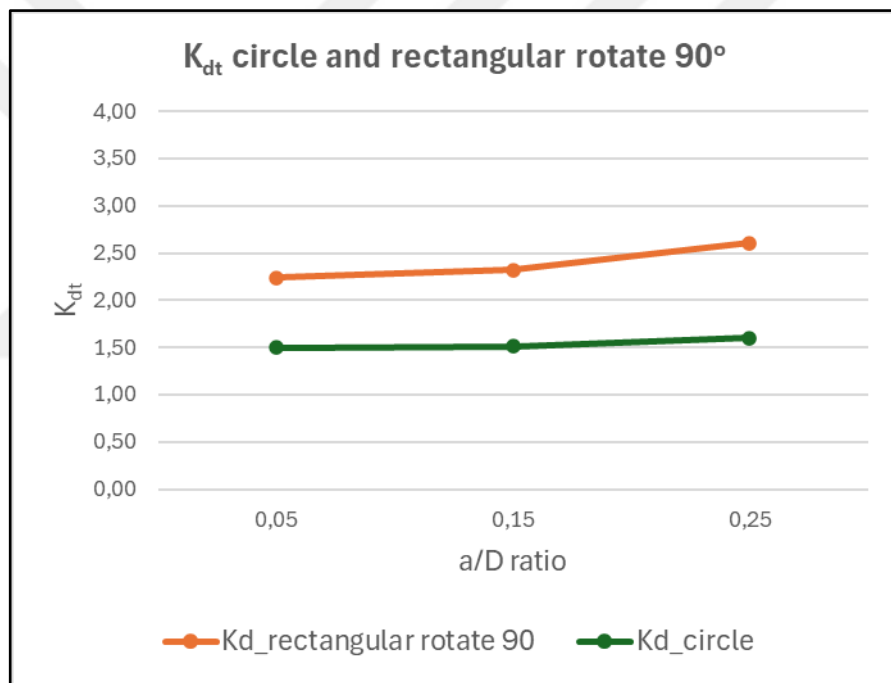


Figure 2.148: K_{dt} values for rectangular rotate 90° and circle holes

In **Figure 2.148**, it is shown that as the a/D and d/D ratios of square and circular holes increase proportionally, there is no constant coefficient relationship. When the graphs in **Figure 2.149**-**Figure 2.150** are examined, it is observed that the stresses vary in parallel with the coefficients like square and rectangular hole. However, the velocity exhibits variations at different rates.

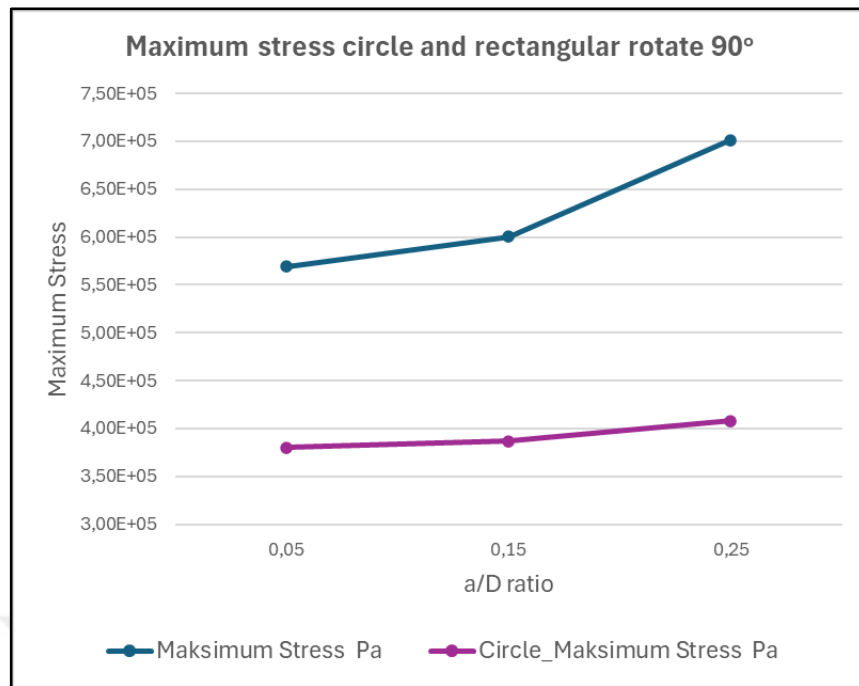


Figure 2.149: Maximum Stress of Circle and Rectangular rotate 90°

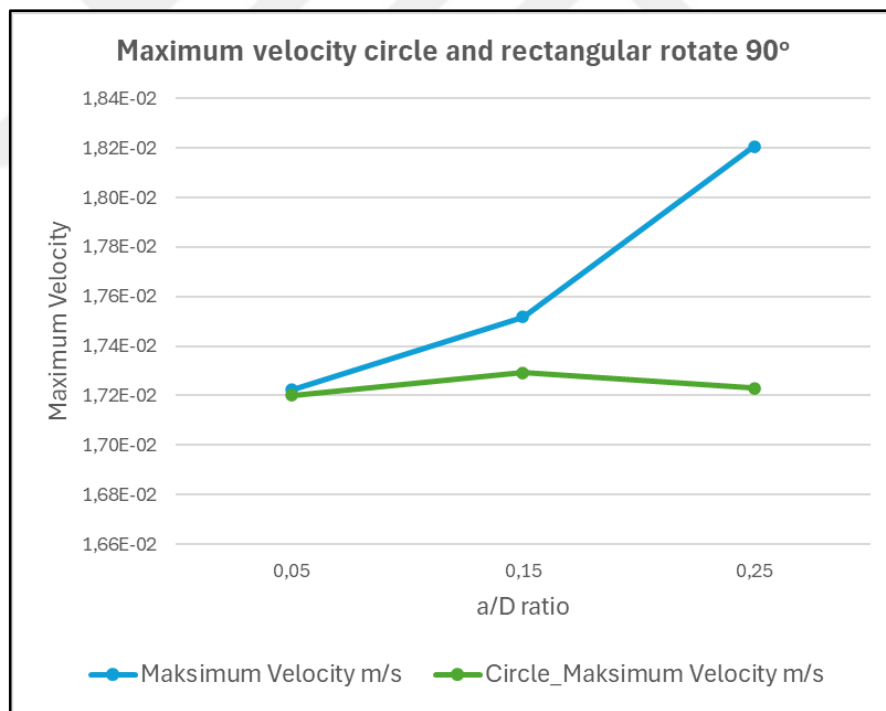


Figure 2.150: Maximum Velocity of Circle and Rectangular rotate 90°

2.8.3. Calculation with Analytical Equations and FEM parameters

In this section, analytical calculations are performed to validate the results of the FEM analysis. Additionally, by comparing FEM and analytical results (Maximum velocity

and Stress), this study contributes to the effective and efficient use of both methods in calculations. The methodology is shown in the **Figure 1.1**.

2.8.3.1. Maximum Velocity and Stress for Torsional Load

These calculation values are shown on the schematic from Figure 1.1. If Analytical calculation is made for a/D: 0.05 shaft with rectangular hole 90°. The Results are shown in the **Figure 2.151**.

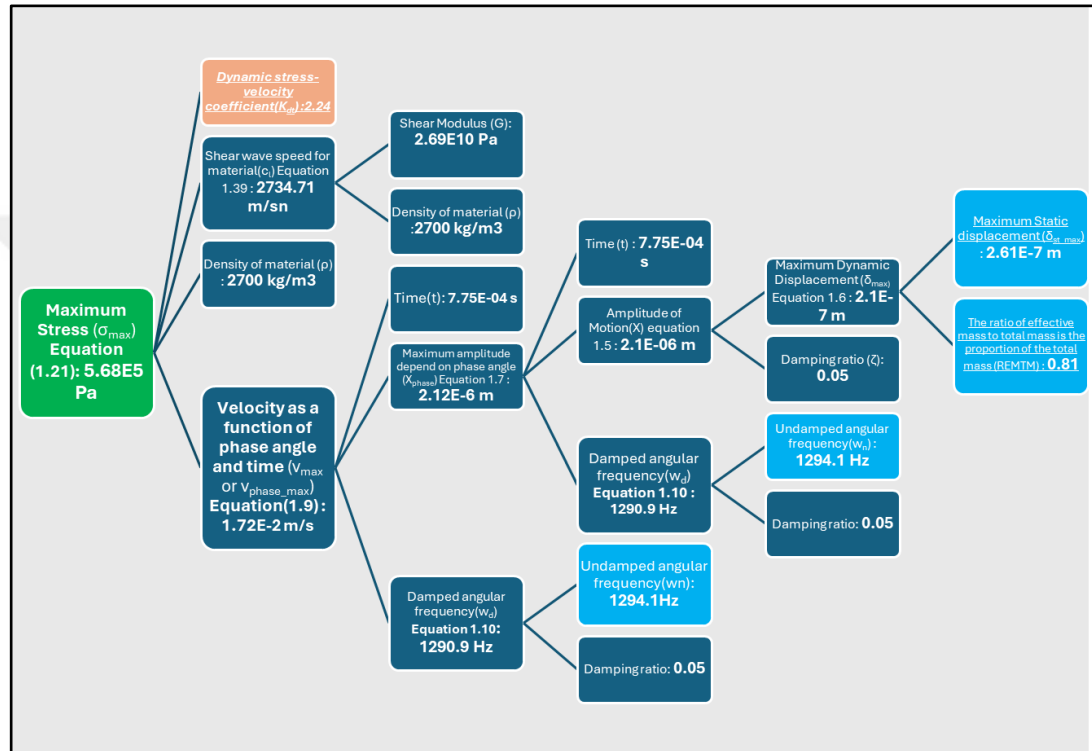


Figure 2.151: Analytical Calculation for a/D:0.05 on torsional load

If Analytical calculation is made for a/D: 0.25 shaft with rectangular rotate 90° hole. The results are shown in the **Figure 2.152**.

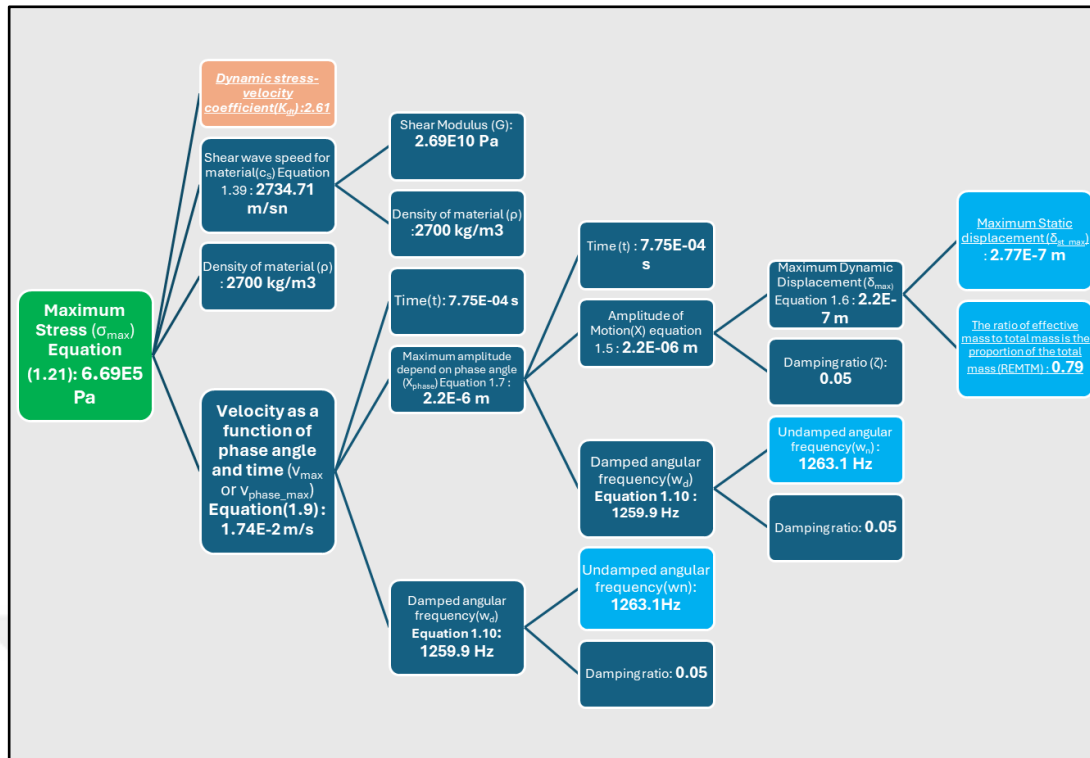


Figure 2.152: Analytical Calculation for a/D:0.25 on torsional load

Comparison of FEM and Analytical results are shared in the **Table 2.42**. The difference shows that some analytical parameters can be used instead of FEM parameters.

Table 2.42: Comparison of FEM and Analytical Results

a/D	FEM Stress	Combined Method Stress	Comparison
0.05	5.69E+05	5.68E+05	~0%
0.25	7.01E+05	6.69E+05	~5%
	Velocity	Velocity	
0.05	1.72E-2	1.72E-2	~0%
0.25	1.82E-2	1.74E-2	~5%

2.9. Case Study-8-Round shaft with plus hole

2.9.1. Geometry, Material Properties, Mesh Model and Boundary Conditions

Geometry is shown in the **Figure 2.153**. There is a and r difference parameter from circle hole.

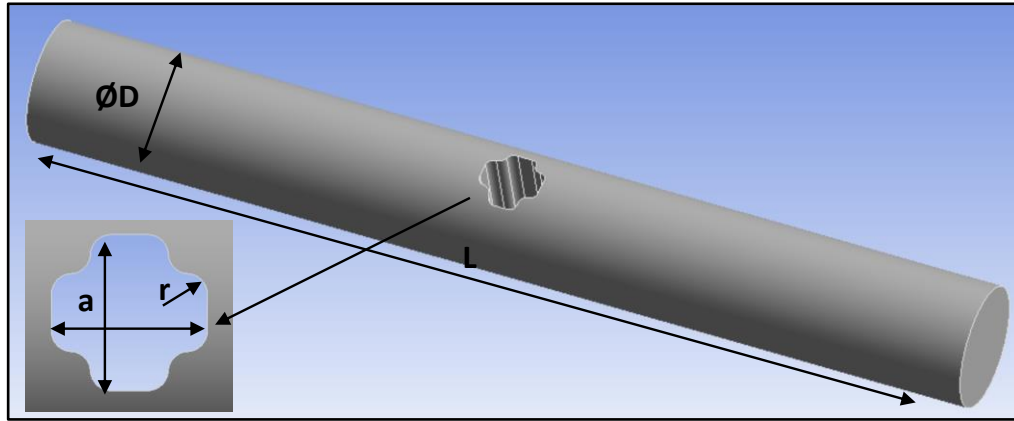


Figure 2.153: Round shaft with plus hole

The geometry dimensions are shared in the **Table 2.43**.

Table 2.43: Plus hole dimensions

Parameter	Value		
L [m]	0.6096		
D [m]	0.0762		
a/D	0.05	0.15	0.25
a [m]	7.62E-03	2.29E-02	3.81E-02
b [m]	3.810E-03	1.143E-02	1.91E-02
r [m]	1E-03	3E-03	5E-03

Plus hole is changed position to the right and left of the shaft in the **Figure 2.154**. The variations in K_{dt} , velocity, and stress based on the change in hole position will be calculated and evaluated.

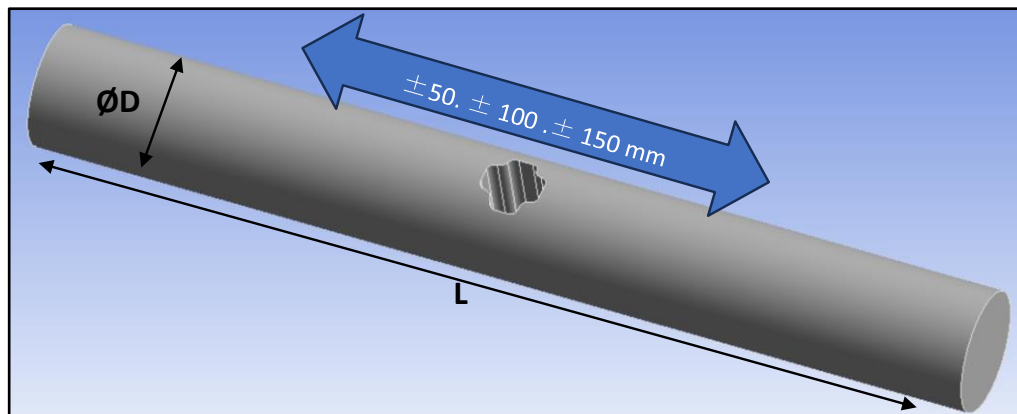


Figure 2.154: Round shaft with plus hole change position

The results of static and modal analyses were required for the deformation and mass participation ratio values in analytical calculations. However, no analytical calculations will be performed in this section; therefore, these parts have not been included.

2.9.2. Harmonic Load Analysis Result

For Harmonic analysis with torsional load velocity and maximum von mises stress results are shown in **Figure 2.155-Figure 2.161**.

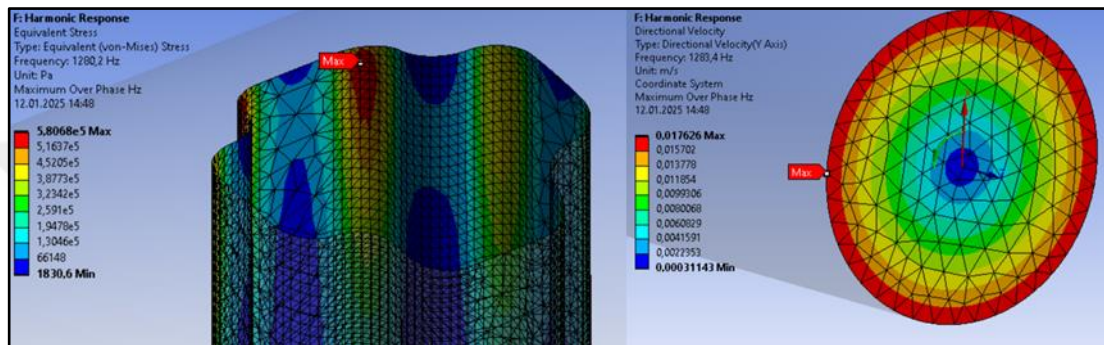


Figure 2.155: Round shaft with plus hole center for a/D:0,15

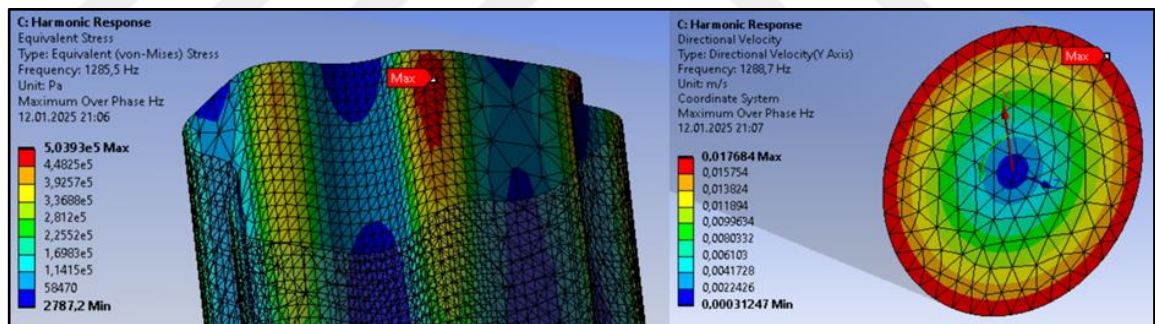


Figure 2.156: Round shaft with plus hole +50 mm close to applied load

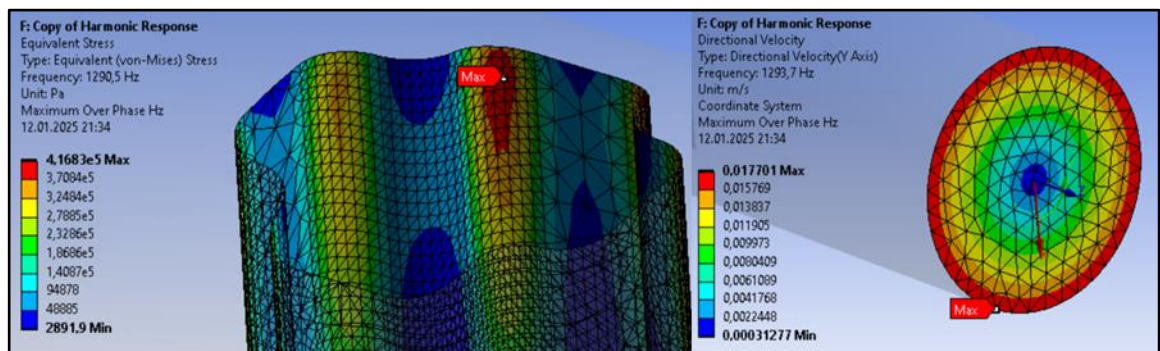


Figure 2.157: Round shaft with plus hole +100 mm close to applied load

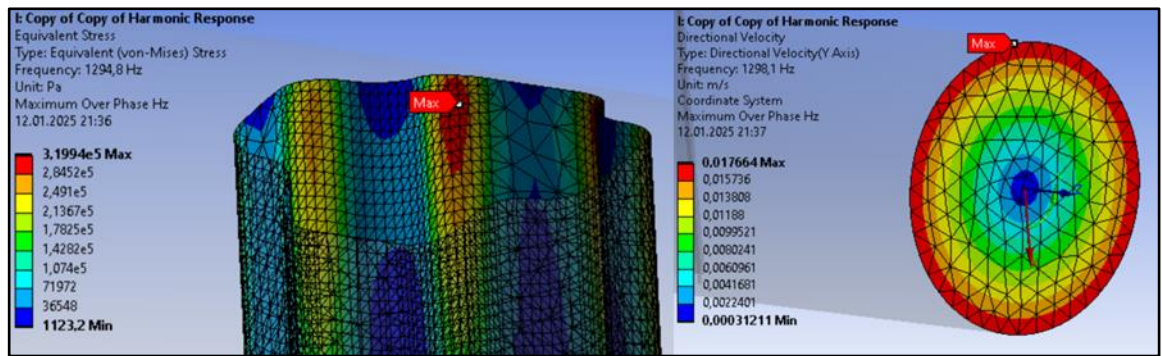


Figure 2.158: Round shaft with plus hole +150 mm close to applied load

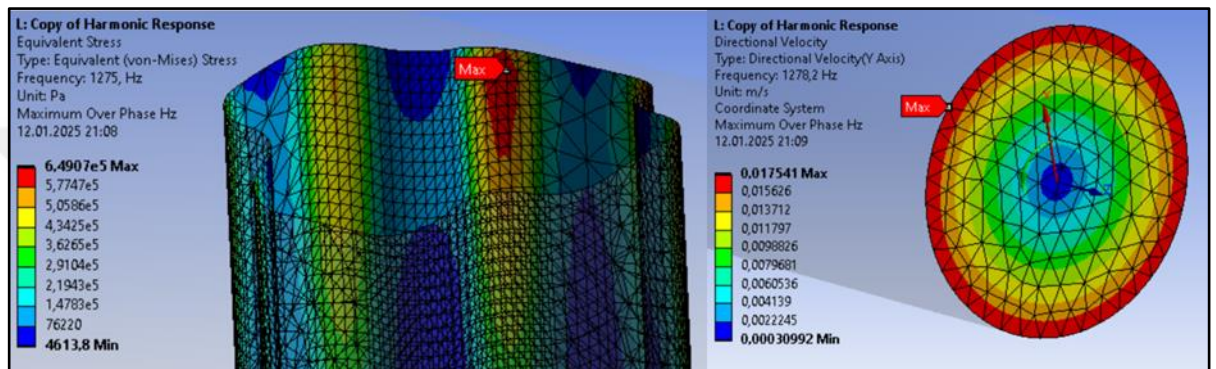


Figure 2.159: Round shaft with plus hole -50 mm close to applied load

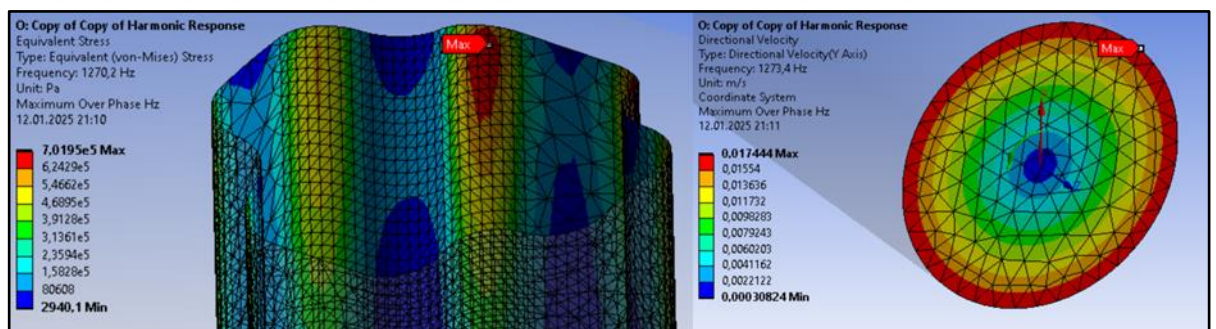


Figure 2.160: Round shaft with plus hole -100 mm close to applied load

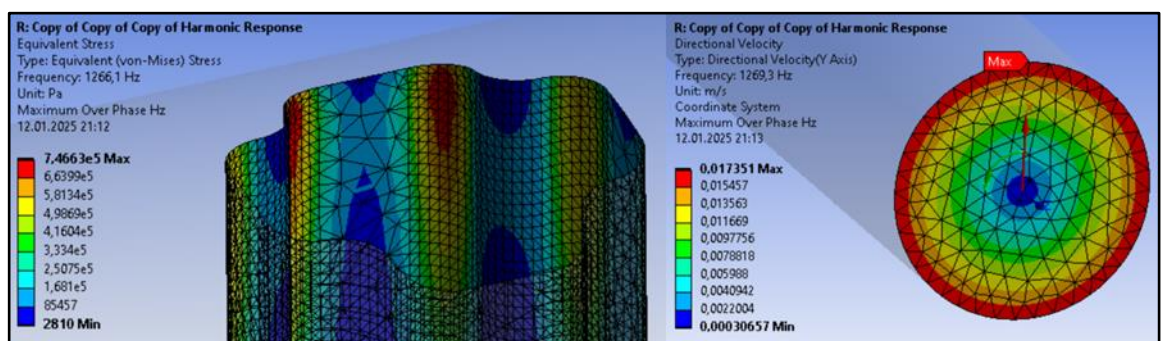


Figure 2.161: Round shaft with plus hole -150 mm close to applied load

The results are shared in the **Table 2.44**.

Table 2.44: Stress, velocity, Kdt plus depend on position on the round shaft

Position mm	-150	-100	-50	Center	50	100	150
Maksimum Stress Pa	7,47E+05	7,02E+05	6,49E+05	5,81E+05	5,04E+05	4,17E+05	3,20E+05
Maksimum Velocity m/s	1,74E-02	1,74E-02	1,75E-02	1,76E-02	1,77E-02	1,77E-02	1,77E-02
K _d plus	2,91	2,72	2,51	2,23	1,93	1,59	1,23

K_d plus change position on the round shaft values and maximum stress values are shown as graph in the **Figure 2.162**. As the plus-shaped hole moves from the fixed point toward the force application point, the K_{dt} and maximum stress values decrease.



Figure 2.162: K_{dt} values and Maximum stress for plus hole change position

When examining the graph in **Figure 2.163**. It is observed that as the hole moves closer to the loading point from the center, the increase in velocity slows down and eventually begins to decrease beyond a certain position. Meanwhile, the reduction in the K_{dt} and maximum stress continues.

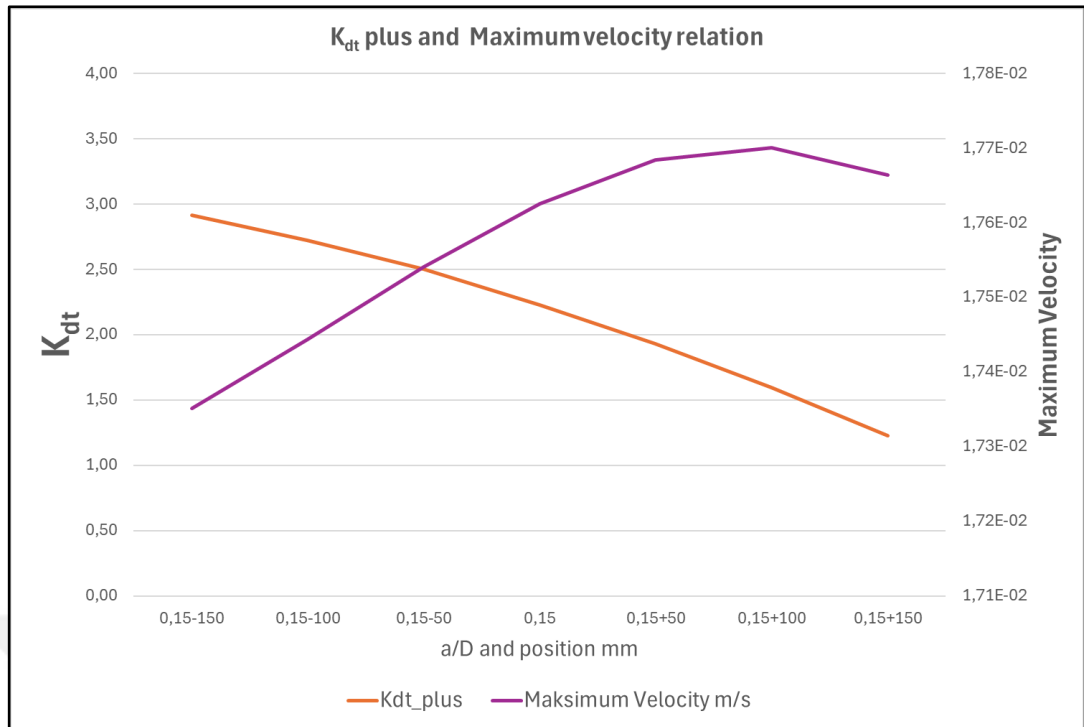


Figure 2.163: K_{dt} values and Maximum velocity for plus hole change position

In this study, all K_d , K_{db} , and K_{dt} coefficients range from the case studies are presented in the **Table 2.45**.

Table 2.45: K_d , K_{db} , and K_{dt} coefficients range

Parameter	Geometry	$K_d/K_{db}/K_{dt}$ Values
K_d range	Simple Shaft	1
K_{db} range	Simple Shaft	2
K_d range	Shoulder Fillet	1.18-1.97
K_{db} range	Shoulder Fillet	1.58-2.58
K_{db} range	Circle Hole	1.92-2.20
K_{dt} range	Circle Hole	1.50-1.60
K_{dt} range	Square Hole	1.91-2.07
K_{dt} range	Rotate 45° square hole	1.35-1.69
K_{dt} range	Rectangular Hole	2.28-3.18
K_{dt} range	Rotate 90° rectangular hole	2.24-2.61
K_{dt} range	Plus hole change position	1.23-2.91

3. CONCLUSION

In this study, the relationships between the K_d and K_t coefficients and their behaviors in different geometries have been examined in detail. While a constant relationship exists in the round shaft geometry with a fillet radius, no clear parallelism or relationship has been found between the K_d and K_t coefficients in other geometries. In the geometries examined in this study, it was observed that changes in radii and hole diameters led to consistent trends in K_d values: if these changes resulted in an increase in stress, K_d always increased; similarly, if the stress decreased, K_d also decreased. However, this behavior is not valid for K_t values.

According to the K_d values presented in the **Table 2.45**, the maximum K_d value was calculated as 3.18 for the round shaft with a rectangular hole geometry. The reason for this high value is that the hole does not have equal clearance in perpendicular directions in the horizontal plane, increasing the complexity of the geometry. Except for simple geometries, K_d calculations were performed for regions where stress concentration occurs in other geometries. The obtained K_d values range between 1.18 and 3.18. During the studies, the dimensions of the round shaft with a circular hole geometry were increased by 100 times, and K_d calculations were repeated. Although the analysis results and natural frequency values changed, it was observed that the K_d coefficient remained constant.

The analysis did not solely rely on finite element analysis (FEA) calculations but rather combined analytical calculations with FEA to maximize the advantages of both methods. When the method shared in **Figure 1.1** was compared with FEA-only results, a maximum difference of 5% was calculated. This method can be applied in multi-element models where analysis times would otherwise be prolonged. The 5% difference can be incorporated into the safety factor, compensating for its potential adverse effects.

Future studies should focus on preparing the graphs of K_d coefficients for specific geometries where K_t coefficients are already available. Incorporating these findings into engineering textbooks will enable more accurate and reliable calculations in

engineering designs. This approach is expected to make a significant contribution to achieving more precise and efficient calculations, leading to safer and more robust designs. Consequently, the accurate calculation of coefficients across a broader range of applications and their integration into design processes will ensure better engineering practices.



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BIOGRAPHY

Samet Göner graduated from Yıldız Technical University with a bachelor's degree in mechanical engineering in 2011. Following his graduation, he has worked for 13 years in various projects across the furniture, automotive, water transportation, electrical, and defense industries, specializing in structural analysis, calculations, and design optimization. He is currently continuing his career in the aerospace industry.



PUBLICATIONS AND PRESENTATIONS FROM THE THESIS

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