

T.C.
GEBZE YÜKSEK TEKNOLOJİ ENSTİTÜSÜ
MÜHENDİSLİK VE FEN BİLİMLERİ
ENSTİTÜSÜ

GENEL RELATİVİTEDE BİR ÇALIŞMA

İbrahim BİLİR
YÜKSEK LİSANS TEZİ
MATEMATİK ANABİLİM DALI

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YÜKSEK LİSANS TEZİ JÜRİ ONAY SAYFASI

G.Y.T.E. Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun 15/06/2009 tarih ve 2009/13 sayılı kararıyla oluşturulan jüri tarafından 24/06/2009 tarihinde tez savunma sınavı yapılan İbrahim BİLİR'in tez çalışması Matematik Anabilim Dalında YÜKSEK LİSANS tezi olarak kabul edilmiştir.

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G.Y.T.E. Sosyal Bilimler Enstitüsü Yönetim Kurulu'nun
...../...../20... tarih ve/..... sayılı kararı.

İMZA/MÜHÜR

ÖZET

TEZ BAŞLIĞI: Genel Relativitede bir Çalışma

YAZAR ADI: İbrahim BİLİR

Bu tezde Einstein Alan Denklemlerini küresel simetrik bir uzayda çözmeye çalıştık. İlgili uzaydaki Enerji-Momentum Tensörünün madde-enerji yoğunluğunu ve basınç terimini sıfırdan farklı kabul ettik. Bu durumda bulunan uzay zamanının madde-enerji dağılımını veren fonksiyona bağlı olduğunu gördük. Bu fonksiyona örnek olayı seçerek Schwarzschild uzay zamanı ile karşılaştırılabilecek bir çözüm bulduk.

SUMMARY

THESIS TITLE : A Study in the General Relativity

THESIS AUTHOR: İbrahim BİLİR

In this thesis ,we tried to solve Einstein Field Equation in a spherical symmetric space. In this related space, we accepted matters- energy density and peressure term of Energy – Momentum Tensor nonzero. We found out that space time depends on the function which represent the matters – energy distribution. Choosing the sample occurrence for this function , we found a solution to compare Schwarzschild space time.

TEŞEKKÜR

Bu tez çalışmasında bilgisini ve yardımını benden esirgemeyen çok değerli danışmanım Yrd.Doç.Dr.Yücel ENGİNER'e ve Gebze Yüksek Teknoloji Enstitüsü Fen Fakültesi Matematik Bölümü'nün tüm öğretim elemanlarına teşekkür ederim.

Ayrıca çalışmalarımda beni destekleyen aileme de teşekkür ederim.

İÇİNDEKİLER DİZİNİ

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1.GİRİŞ

1.1. Einstein Alan Denklemleri

$$ds^2 = g_{ab} dx^a dx^b \quad 1.1$$

Bu denklemdeki ds^2 yay uzunluğunun karesidir kısaca metrik denir. Einstein alan denklemlerindeki g_{ab} bileşeni metrik tensördür. Dört boyutlu uzayda $a=b=0,1,2,3$ olmak üzere, x^0 koordinatı zamanı, x^1, x^2, x^3 koordinatları da uzayı oluşturur. Uzaysal kısım kartezyen, silindirik veya küresel koordinatlarda yazılabilir. Biz bu tezde küresel koordinatlarda çalışacağız. Küresel koordinatlardaki bu bileşenler (r, θ, ϕ) olarak yazılır.

g_{ab} , metrik tensörü Einstein Alan Denklemlerini kullanarak çözülür. Einstein Alan Denklemleri 4 boyutlu eğri uzay zamanı oluşturan Genel Rölativite Kuramını matematiksel olarak ifade eder. Einstein Alan Denklemleri

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab} \quad 1.2$$

şeklindedir. Bu eşitlikte

R_{ab} Ricci Tensörü

R Ricci Skaler Eğriliği

T_{ab} Enerji-Momentum Tensörüdür.

Einstein Alan Denklemlerinin çözümü 4 boyutlu eğri uzay zamanın madde ve enerji anlamında nasıl davrandığını bize verecektir. Bu konuyla ilgili detaylı çalışmalar Foster ¹, Ohanian ², Wold ³ de bulunabilir.

1.2. Einstein Alan Denklemlerinin Çözüm Algoritması

Çözüm için öncelikle g_{ab} ' ye bir form önerilir. g^{ab} , g_{ab} nin ters matrisi olarak tanımlanır.

g^{ab} ve g_{ab} den faydalanılarak Christoffel Sembolleri elde edilir. Christoffel Sembolleri

$$\Gamma_{bc}^a = \frac{1}{2} g^{ad} [\partial_b g_{dc} + \partial_c g_{bd} - \partial_d g_{bc}] \quad 1.3$$

formundadır. (1.3) ile verilen formüller Riemann Eğrilik Tensörünü bulmakta kullanılır. Riemann Eğrilik Tensörü

$$R_{bcd}^a = \partial_c \Gamma_{bd}^a - \partial_d \Gamma_{bc}^a - \Gamma_{bd}^e \Gamma_{ec}^a - \Gamma_{bc}^e \Gamma_{ed}^a \quad 1.4$$

a ve c indisleri eşitlenerek Ricci tensörleri bulunur. Ricci tensörleri aşağıdaki gibidir.

$$R_{00} = R_{000}^0 + R_{010}^1 + R_{020}^2 + R_{030}^3$$

$$R_{11} = R_{101}^0 + R_{111}^1 + R_{121}^2 + R_{131}^3$$

$$R_{22} = R_{202}^0 + R_{212}^1 + R_{222}^2 + R_{232}^3$$

$$R_{33} = R_{303}^0 + R_{313}^1 + R_{323}^2 + R_{333}^3$$

$$R = g^{ab} . R_{ab} \quad 1.5$$

Formülü kullanılarak Ricci Skaler Eğriliği bulunur. Bulunan bilgiler

$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}$ denkleminde yerine konularak ilgili diferansiyel denklem sistemi

elde edilir. Teorik olarak bu denklemin çözümünde g_{ab} metriğinin bileşenleri

bulunabilir. Bulunacak sonuçlar T_{ab} enerji-momentum tensörünün bileşenlerine bağlı

olacaktır.

2.EİNSTEİN ALAN DENKLEMLERİNİN ÇÖZÜMÜ

2.1.Metrik Form

ds^2 metrik formunu küresel koordinatlara uygularsak

$(0 < \theta < \pi)$ ve $(0 \leq \phi \leq 2\pi)$ olmak üzere aşağıdaki eşitlik elde edilir.

$$ds^2 = F(r)dt^2 + G(r)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2) \quad 2.1$$

Bu durumda metrik tensör g_{ab} aşağıdaki matris gibi ifade edilebilir.

$$g_{ab} = \begin{bmatrix} F(r) & 0 & 0 & 0 \\ 0 & G(r) & 0 & 0 \\ 0 & 0 & r^2G(r) & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta G(r) \end{bmatrix} \quad 2.2$$

Amaç $F(r)$ ve $G(r)$ yi bulmaktır.

2.2.Christoffel Sembolleri

g_{ab} metrik tensörünün tersi olarak g^{ab} yi tanımlamıştık. g^{ab} matrisi aşağıdaki gibidir.

$$g^{ab} = \begin{bmatrix} F^{-1}(r) & 0 & 0 & 0 \\ 0 & G^{-1}(r) & 0 & 0 \\ 0 & 0 & r^{-2}G^{-1}(r) & 0 \\ 0 & 0 & 0 & r^{-2}\sin^{-2}\theta G^{-1}(r) \end{bmatrix} \quad 2.3$$

$$g_{00} = F(r) , \quad g_{11} = G(r) , \quad g_{22} = r^2G(r) , \quad g_{33} = r^2\sin^2\theta G(r)$$

$$g^{00} = F^{-1}(r) , \quad g^{11} = G^{-1}(r) , \quad g^{22} = r^{-2}G^{-1}(r) , \quad g^{33} = r^{-2}\sin^{-2}\theta G^{-1}(r) \text{ dir.}$$

g_{ab} ve g^{ab} ler (1.3) denkleminde yerine yazılırsa aşağıdaki sonuçlar bulunur.

$$\Gamma_{00}^0 = 0$$

$$\Gamma_{10}^0 = \frac{1}{2}F^{-1}(r) \frac{\partial}{\partial r} F(r)$$

$$\Gamma_{01}^0 = \frac{1}{2}F^{-1}(r) \frac{\partial}{\partial r} F(r)$$

$$\Gamma_{11}^0 = 0$$

$$\Gamma_{02}^0 = 0$$

$$\Gamma_{12}^0 = 0$$

$$\Gamma_{03}^0 = 0$$

$$\Gamma_{20}^0 = 0$$

$$\Gamma_{21}^0 = 0$$

$$\Gamma_{22}^0 = 0$$

$$\Gamma_{23}^0 = 0$$

$$\Gamma_{30}^0 = 0$$

$$\Gamma_{31}^0 = 0$$

$$\Gamma_{32}^0 = 0$$

$$\Gamma_{33}^0 = 0$$

$$\Gamma_{00}^1 = -\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}F(r)$$

$$\Gamma_{01}^1 = 0$$

$$\Gamma_{02}^1 = 0$$

$$\Gamma_{03}^1 = 0$$

$$\Gamma_{10}^1 = 0$$

$$\Gamma_{11}^1 = \frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}G(r)$$

$$\Gamma_{12}^1 = 0$$

$$\Gamma_{13}^1 = 0$$

$$\Gamma_{00}^2 = 0$$

$$\Gamma_{01}^2 = 0$$

$$\Gamma_{02}^2 = 0$$

$$\Gamma_{03}^2 = 0$$

$$\Gamma_{20}^1 = 0$$

$$\Gamma_{21}^1 = 0$$

$$\Gamma_{22}^1 = -\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}[r^2 G(r)]$$

$$\Gamma_{23}^1 = 0$$

$$\Gamma_{13}^0 = 0$$

$$\Gamma_{30}^1 = 0$$

$$\Gamma_{31}^1 = 0$$

$$\Gamma_{32}^1 = 0$$

$$\Gamma_{33}^1 = -\frac{1}{2}G^{-1}(r)\frac{\partial}{\partial r}[r^2 \sin^2 \theta G(r)]$$

$$\Gamma_{20}^2 = 0$$

$$\Gamma_{21}^2 = \frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}[r^2 G(r)]$$

$$\Gamma_{22}^2 = 0$$

$$\Gamma_{23}^2 = 0$$

$$\Gamma_{10}^2 = 0$$

$$\Gamma_{11}^2 = 0$$

$$\Gamma_{12}^2 = \frac{1}{2}r^{-2}G^{-1}(r)\frac{\partial}{\partial r}[r^2 G(r)]$$

$$\Gamma_{13}^2 = 0$$

$$\Gamma_{00}^3 = 0$$

$$\Gamma_{01}^3 = 0$$

$$\Gamma_{02}^3 = 0$$

$$\Gamma_{03}^3 = 0$$

$$\Gamma_{10}^3 = 0$$

$$\Gamma_{11}^3 = 0$$

$$\Gamma_{12}^3 = 0$$

$$\Gamma_{30}^2 = 0$$

$$\Gamma_{31}^2 = 0$$

$$\Gamma_{32}^2 = 0$$

$$\Gamma_{13}^3 = \frac{1}{2}r^{-2} \sin^{-2} \theta G^{-1}(r)\frac{\partial}{\partial r}[r^2 \sin^2 \theta G(r)]$$

$$\Gamma_{20}^3 = 0$$

$$\Gamma_{21}^3 = 0$$

$$\Gamma_{22}^3 = 0$$

$$\Gamma_{23}^3 = \frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)]$$

$$\Gamma_{30}^3 = 0$$

$$\Gamma_{31}^3 = \frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)]$$

$$\Gamma_{32}^3 = \frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)]$$

$$\Gamma_{33}^3 = 0$$

2.4

2.3. Rieman Eğrilik Tensörünün Bileşenlerinin Hesabı

(2.4) de bulunan christoffel sembolleri (1.4) denkleminde yerine yazılarak aşağıdaki riemann eğrilik tensörleri hesaplanır.

$$R_{101}^0 = -\frac{\partial}{\partial r} \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] + \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] - \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right]$$

$$R_{110}^0 = \frac{\partial}{\partial r} \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] + \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right] - \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right]$$

$$R_{202}^0 = \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right]$$

$$R_{220}^0 = - \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} F^{-1}(r) \frac{\partial}{\partial r} F(r) \right]$$

$$R_{331}^1 = \frac{\partial}{\partial r} \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] - \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] +$$

$$\left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right]$$

$$R_{332}^1 = \frac{\partial}{\partial \theta} \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] - \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] +$$

$$\left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$R_{002}^2 = - \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$R_{020}^2 = \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$R_{103}^2 = \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]$$

$$R_{112}^2 = \frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] + \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$- \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$R_{121}^2 = - \frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] + \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$- \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$R_{203}^2 = \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]$$

$$R_{020}^2 = \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]$$

$$\begin{aligned}
R_{313}^2 = & -\frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{323}^2 = & -\frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{331}^2 = & -\frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{332}^2 = & -\frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right]
\end{aligned}$$

$$R_{030}^3 = \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} F(r) \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right]$$

$$\begin{aligned}
R_{113}^3 = & \frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} G(r) \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{231}^3 = & -\frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& + \left[\frac{1}{2} r^{-2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{232}^3 = & -\frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& + \left[-\frac{1}{2} G^{-1}(r) \frac{\partial}{\partial r} [r^2 G(r)] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \\
& - \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{312}^3 = & \frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right] \\
& - \frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

$$\begin{aligned}
R_{321}^3 = & \frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial r} [r^2 \sin^2 \theta G(r)] \right] \\
& - \frac{\partial}{\partial r} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta G^{-1}(r) \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta G(r)] \right]
\end{aligned}$$

2.5

2.4. Ricci Tensörleri

Einstein Alan Denklemlerinin kesin çözümlerini bulmak zordur. $\varepsilon \ll 1$, $F(r) = c_0 e^{e^f}$, $G(r) = c_1 e^{\varepsilon^g}$ seçilerek, çözüm için yaklaşık bir metod uygulayacağız.

Kesin çözümler Stephani⁵ den bakılabilir.

(2,5) de bulunan Riemann eğrilik tensörleri kullanarak aşağıdaki Ricci Tensörleri bulunur.

$$R_{00} = R_{000}^0 + R_{010}^1 + R_{020}^2 + R_{030}^3$$

$$R_{00} = \left[\frac{\partial}{\partial r} \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_1 e^{\varepsilon g(r)} \right] \right] \\ \left[-\left[\frac{1}{2} c_0^{-1} e^{-\varepsilon f(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \right] \\ \left[\left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \right] \\ \left[\left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \left[\frac{1}{2} r^{-2} \sin^2 \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right] \right] \right]$$

$$R_{00} = \frac{\partial}{\partial r} \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \\ + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_1 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \\ - \left[\frac{1}{2} c_0^{-1} e^{-\varepsilon f(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \\ + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} c_1 \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \\ + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} c_0 \varepsilon e^{\varepsilon f(r)} f'(r) \right] \\ \cdot \left[\frac{1}{2} r^{-2} \sin^2 \theta c_1^{-1} e^{-\varepsilon g(r)} \sin^2 \theta c_1 \frac{\partial}{\partial r} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right]$$

$$R_{00} = -\frac{1}{2} c_1^{-1} c_0 \varepsilon \left[\varepsilon e^{\varepsilon(f(r)-g(r))} (f(r)-g(r))' f'(r) + e^{\varepsilon(f(r)-g(r))} f''(r) \right] \\ + \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon r^{-1} e^{\varepsilon(f(r)-g(r))} f'(r) \right] \\ + \left[-\frac{1}{2} c_1^{-1} c_0 r^{-1} \varepsilon e^{\varepsilon(f(r)-g(r))} f'(r) \right]$$

$$R_{00} = -\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} [f''(r) + 2r^{-1} f'(r)]$$

$$\begin{aligned}
R_{11} &= -\frac{1}{2} \varepsilon f''(r) + r^{-2} - \frac{1}{2} \varepsilon g''(r) + \frac{1}{2} r^{-1} \varepsilon g'(r) - r^{-2} - r^{-1} \varepsilon g'(r) \\
&\quad + r^{-2} - \frac{1}{2} \varepsilon g''(r) + \frac{1}{2} r^{-1} \varepsilon g'(r) - r^{-2} - r^{-1} \varepsilon g'(r) \\
R_{11} &= -\frac{1}{2} \varepsilon f''(r) - \varepsilon g''(r) - r^{-1} \varepsilon g'(r)
\end{aligned}$$

$$R_{22} = R_{202}^0 + R_{212}^1 + R_{222}^2 + R_{232}^3$$

$$\begin{aligned}
R_{22} &= \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \left[\frac{1}{2} c_0^{-1} e^{-\varepsilon f(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \\
&\quad + \frac{\partial}{\partial r} \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \\
&\quad + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_1 e^{\varepsilon g(r)} \right] \\
&\quad - \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \\
&\quad - \frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right] \right] \\
&\quad + \left[-\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 c_1 e^{\varepsilon g(r)} \right] \right] \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right] \right] \\
&\quad - \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right] \right] \\
&\quad \cdot \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right] \right] \\
R_{22} &= \left[-\frac{1}{2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \left[\frac{1}{2} e^{-\varepsilon f(r)} \varepsilon e^{\varepsilon f(r)} f'(r) \right] \\
&\quad + \frac{\partial}{\partial r} \left[-\frac{1}{2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \\
&\quad + \left[-\frac{1}{2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \left[\frac{1}{2} e^{-\varepsilon g(r)} \varepsilon e^{\varepsilon g(r)} g'(r) \right] \\
&\quad - \left[\frac{1}{2} r^{-2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \left[-\frac{1}{2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \\
&\quad - \frac{\partial}{\partial \theta} \left[\frac{1}{2} \sin^{-2} \theta \ 2 \sin \theta \cdot \cos \theta \right] \\
&\quad + \left[-\frac{1}{2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \left[\frac{1}{2} r^{-2} e^{-\varepsilon g(r)} \left[2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right] \\
&\quad - \left[\frac{1}{2} \sin^{-2} \theta \ 2 \sin \theta \cdot \cos \theta \right] \left[\frac{1}{2} \sin^{-2} \theta \ 2 \sin \theta \cdot \cos \theta \right]
\end{aligned}$$

$$R_{22} = [-r] \left[\frac{1}{2} \varepsilon f'(r) \right] + \frac{\partial}{\partial r} \left[-r - \frac{1}{2} r^2 \varepsilon g'(r) \right] + [-r] \left[\frac{1}{2} \varepsilon g'(r) \right] \\ + [1 + r \varepsilon g'(r)] - \frac{\partial}{\partial \theta} [\cot \theta] - [1 + r \varepsilon g'(r)] - \cot^2 \theta$$

$$R_{22} = \left[-\frac{1}{2} \varepsilon r f'(r) \right] + \left[-1 - \frac{1}{2} \varepsilon [2r g'(r) + r^2 g''(r)] \right] - \left[\frac{1}{2} \varepsilon r g'(r) \right] \\ + [1 + r \varepsilon g'(r)] + [1 + \cot^2 \theta] - [1 + r \varepsilon g'(r)] - \cot^2 \theta$$

$$R_{22} = -\frac{1}{2} \varepsilon r f'(r) - \frac{3}{2} \varepsilon r g'(r) - \frac{1}{2} \varepsilon r^2 g''(r)$$

$$R_{33} = R_{303}^0 + R_{313}^1 + R_{323}^2 + R_{333}^3$$

$$R_{33} = \left[-\left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \left[\frac{1}{2} c_0^{-1} e^{-\varepsilon f(r)} \frac{\partial}{\partial r} c_0 e^{\varepsilon f(r)} \right] \right] \\ \left[-\frac{\partial}{\partial r} \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \right. \\ \left. - \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} c_1 e^{\varepsilon g(r)} \right] \right. \\ \left. + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \right] \\ \left[-\frac{\partial}{\partial \theta} \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \right. \\ \left. - \left[\frac{1}{2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial r} [r^2 c_1 e^{\varepsilon g(r)}] \right] \right. \\ \left. + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \left[\frac{1}{2} r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \frac{\partial}{\partial \theta} [r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)}] \right] \right]$$

$$R_{33} = \left[-\left[\frac{1}{2} e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \left[\frac{1}{2} e^{-\varepsilon f(r)} \varepsilon e^{\varepsilon f(r)} f'(r) \right] \right] \\ \left[-\frac{\partial}{\partial r} \left[\frac{1}{2} e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \right. \\ \left. - \left[\frac{1}{2} e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \left[\frac{1}{2} e^{-\varepsilon g(r)} \varepsilon e^{\varepsilon g(r)} g'(r) \right] \right. \\ \left. + \left[\frac{1}{2} r^{-2} \sin^{-2} \theta e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \left[\frac{1}{2} e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \right] \\ \left[-\frac{\partial}{\partial \theta} [\sin \theta \cos \theta] - \left[\frac{1}{2} e^{-\varepsilon g(r)} \sin^2 \theta [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \left[\frac{1}{2} r^{-2} e^{-\varepsilon g(r)} [2r e^{\varepsilon g(r)} + r^2 \varepsilon e^{\varepsilon g(r)} g'(r)] \right] \right. \\ \left. + [\sin^{-2} \theta \sin \theta \cos \theta] [\sin \theta \cos \theta] \right]$$

$$\begin{aligned}
R_{33} = & -\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \frac{\partial}{\partial r} \left[\sin^2 \theta \left[r + \frac{1}{2} r^2 \varepsilon g'(r) \right] \right] - \left[\frac{1}{2} \sin^2 \theta r \varepsilon g'(r) \right] \\
& + \left[r^{-1} + \frac{1}{2} \varepsilon g'(r) \right] \left[r \sin^2 \theta + \frac{1}{2} \sin^2 \theta r^2 \varepsilon g'(r) \right] \\
& - \cos 2\theta - \left[r \sin^2 \theta + \frac{1}{2} \sin^2 \theta r^2 \varepsilon g'(r) \right] \left[r^{-1} + \frac{1}{2} \varepsilon g'(r) \right] + \cos^2 \theta
\end{aligned}$$

$$\begin{aligned}
R_{33} = & -\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \sin^2 \theta - r \sin^2 \theta \varepsilon g'(r) - \frac{1}{2} r^2 \sin^2 \theta \varepsilon g''(r) - \frac{1}{2} \sin^2 \theta r \varepsilon g'(r) \\
& + \sin^2 \theta + \sin^2 \theta r \varepsilon g'(r) - \cos^2 \theta + \sin^2 \theta - \sin^2 \theta - r \sin^2 \theta \varepsilon g'(r) + \cos^2 \theta
\end{aligned}$$

$$R_{33} = -\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \frac{1}{2} r^2 \sin^2 \theta \varepsilon g''(r) - \frac{3}{2} \sin^2 \theta r \varepsilon g'(r) \quad 2.6$$

2.5. Ricci Skaler Eğriliği

(1.5) formülünde (2.6) ve (2.3) de bulunan sonuçlar yerine konularak R Ricci Sakaler Eğriliği bulunur.

$$R = g^{bd} R_{bd}$$

$$g^{bd} = [g_{bd}]^{-1}$$

$$g^{bd} = \begin{bmatrix} g^{00} & 0 & 0 & 0 \\ 0 & g^{11} & 0 & 0 \\ 0 & 0 & g^{22} & 0 \\ 0 & 0 & 0 & g^{33} \end{bmatrix} = \begin{bmatrix} F_{(r)}^{-1} & 0 & 0 & 0 \\ 0 & G_{(r)}^{-1} & 0 & 0 \\ 0 & 0 & r^{-2} G_{(r)}^{-1} & 0 \\ 0 & 0 & 0 & r^{-2} \sin^{-2} \theta G_{(r)}^{-1} \end{bmatrix}$$

$$R_{bd} = \begin{bmatrix} R_{00} & 0 & 0 & 0 \\ 0 & R_{11} & 0 & 0 \\ 0 & 0 & R_{22} & 0 \\ 0 & 0 & 0 & R_{33} \end{bmatrix}$$

$$R = g^{00} R_{00} + g^{11} R_{11} + g^{22} R_{22} + g^{33} R_{33}$$

$$\begin{aligned}
R = & \left[c_0^{-1} e^{-\varepsilon f(r)} \right] \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \left[f''(r) + 2r^{-1} f'(r) \right] \right] \\
& + \left[c_1^{-1} e^{-\varepsilon g(r)} \right] \left[-\frac{1}{2} \varepsilon f'''(r) - \varepsilon g''(r) - r^{-1} \varepsilon g'(r) \right] \\
& + \left[r^{-2} c_1^{-1} e^{-\varepsilon g(r)} \right] \left[-\frac{1}{2} \varepsilon r f'(r) - \frac{3}{2} \varepsilon r g'(r) - \frac{1}{2} \varepsilon r^2 g''(r) \right] \\
& + \left[r^{-2} \sin^2 \theta c_1^{-1} e^{-\varepsilon g(r)} \right] \left[-\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \frac{1}{2} r^2 \sin^2 \theta \varepsilon g''(r) - \frac{3}{2} \sin^2 \theta r \varepsilon g'(r) \right]
\end{aligned}$$

$$\begin{aligned}
R = & \left[-\frac{1}{2} c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) \right] + \left[-\frac{1}{2} c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2g''(r) + 2r^{-1} g'(r) \right] \\
& + \left[-\frac{1}{2} c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[r^{-1} f'(r) + 3r^{-1} g'(r) + g''(r) \right] \\
& + \left[-\frac{1}{2} c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[r^{-1} f'(r) + g''(r) + 3r^{-1} g'(r) \right]
\end{aligned}$$

$$R = \left[-\frac{1}{2} c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[2f''(r) + 4r^{-1} f'(r) + 4g''(r) + 8r^{-1} g'(r) \right]$$

$$R = \left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \quad 2.7$$

2.6.Einstein Alan Denklemlerinin Diferansiyel Denklem Sistemi Olarak Oluşturulması

Bulunan sonuçlar (1,2) denkleminde yerine yazılırsa

$$R_{ab} - \frac{1}{2} R g_{ab} = T_{ab}$$

$$R_{00} - \frac{1}{2} R g_{00} = T_{00}$$

$$\begin{aligned}
T_{00} = & \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[f''(r) + 2r^{-1} f'(r) \right] \\
& - \frac{1}{2} \left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \cdot \left[c_0 e^{\varepsilon f(r)} \right]
\end{aligned}$$

$$\begin{aligned}
T_{00} &= \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[f''(r) + 2r^{-1} f'(r) \right] \\
&\quad - \left[-\frac{1}{2} c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[f'''(r) + 2r^{-1} f''(r) + 2g''(r) + 4r^{-1} g'(r) \right] \\
T_{00} &= \left[c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[g''(r) + 2r^{-1} g'(r) \right] \\
R_{11} - \frac{1}{2} R \cdot g_{11} &= T_{11}
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
T_{11} &= \left[-\frac{1}{2} \varepsilon f''(r) - \varepsilon g''(r) - r^{-1} \varepsilon g'(r) \right] \\
&\quad - \frac{1}{2} \left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \cdot \left[c_1 e^{\varepsilon g(r)} \right]
\end{aligned}$$

$$\begin{aligned}
T_{11} &= \left[-\frac{1}{2} \varepsilon \right] \left[f''(r) + 2g''(r) + 2r^{-1} g'(r) \right] \\
&\quad + \left[-\frac{1}{2} \varepsilon \right] \left[-f''(r) - 2r^{-1} f'(r) - 2g''(r) - 4r^{-1} g'(r) \right]
\end{aligned}$$

$$T_{11} = \left[-\frac{1}{2} \varepsilon \right] \left[-2r^{-1} g'(r) - 2r^{-1} f'(r) \right]$$

$$T_{11} = \left[\varepsilon r^{-1} \right] \left[g'(r) + f'(r) \right] \tag{2.9}$$

$$R_{22} - \frac{1}{2} R \cdot g_{22} = T_{22}$$

$$\begin{aligned}
T_{22} &= \left[-\frac{1}{2} \varepsilon r f'(r) - \frac{3}{2} \varepsilon r g'(r) - \frac{1}{2} \varepsilon r^2 g''(r) \right] \\
&\quad - \frac{1}{2} \left[\left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \right] \cdot \left[c_1 r^2 e^{\varepsilon g(r)} \right]
\end{aligned}$$

$$\begin{aligned}
T_{22} &= \left[-\frac{1}{2} \varepsilon \right] \left[r f'(r) + 3r g'(r) + r^2 g''(r) \right] \\
&\quad + \left[\left[-\frac{1}{2} \varepsilon \right] \left[-r^2 f''(r) - 2r f'(r) - 2r^2 g''(r) - 4r g'(r) \right] \right]
\end{aligned}$$

$$T_{22} = \left[-\frac{1}{2} \varepsilon \right] \left[r f'(r) + 3r g'(r) + r^2 g''(r) - r^2 f''(r) - 2r f'(r) - 2r^2 g''(r) - 4r g'(r) \right]$$

$$T_{22} = \left[\frac{1}{2} r \varepsilon \right] \left[f'(r) + r f''(r) + g'(r) + r g''(r) \right] \tag{2.10}$$

$$R_{33} - \frac{1}{2}R.g_{33} = T_{33}$$

$$T_{33} = \left[-\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \frac{1}{2} r^2 \sin^2 \theta \varepsilon g''(r) - \frac{3}{2} \sin^2 \theta r \varepsilon g'(r) \right] \\ - \frac{1}{2} \left[\left[-c_1^{-1} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \right] \cdot \left[r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \right]$$

$$T_{33} = \left[-\frac{1}{2} \sin^2 \theta r \varepsilon f'(r) - \frac{1}{2} r^2 \sin^2 \theta \varepsilon g''(r) - \frac{3}{2} \sin^2 \theta r \varepsilon g'(r) \right] \\ - \left[\left[-\frac{1}{2} c_1^{-1} r^2 \sin^2 \theta c_1 e^{\varepsilon g(r)} \varepsilon e^{-\varepsilon g(r)} \right] \left[f''(r) + 2r^{-1} f'(r) + 2g''(r) + 4r^{-1} g'(r) \right] \right]$$

$$T_{33} = \left[-\frac{1}{2} r \varepsilon \sin^2 \theta \right] \left[f'(r) + r g''(r) + 3 g'(r) \right] \\ - \left[\left[-\frac{1}{2} r \sin^2 \theta \varepsilon \right] \left[r f''(r) + 2 f'(r) + 2 r g''(r) + 4 g'(r) \right] \right]$$

$$T_{33} = \left[-\frac{1}{2} r \varepsilon \sin^2 \theta \right] \left[f'(r) + r g''(r) + 3 g'(r) - r f''(r) - 2 f'(r) - 2 r g''(r) - 4 g'(r) \right]$$

$$T_{33} = \left[\frac{1}{2} r \varepsilon \sin^2 \theta \right] \left[r f''(r) + f'(r) + r g''(r) + g'(r) \right] \quad 2.11$$

$$R_{ab} - \frac{1}{2}R.g_{ab} \quad \text{nin esas köşegen haricindeki elemanları sıfırdır.}$$

3. DİFERANSİYEL DENKLEM SİSTEMİNİN ÇÖZÜMÜ

$$R_{00} - \frac{1}{2}R.g_{00} = T_{00}$$

$$R_{11} - \frac{1}{2}R.g_{11} = T_{11}$$

$$R_{22} - \frac{1}{2}R.g_{22} = T_{22}$$

$$R_{33} - \frac{1}{2}R.g_{33} = T_{33}$$

$$T_{00} = \left[c_1^{-1} c_0 \varepsilon e^{\varepsilon(f(r)-g(r))} \right] \left[g''(r) + 2r^{-1} g'(r) \right]$$

$$T_{00} = c_1^{-1} c_0 \varepsilon \{1 + \varepsilon f(r) - \varepsilon g(r)\} \left[g''(r) + 2r^{-1} g'(r) \right]$$

$$T_{00} = c_1^{-1} c_0 \varepsilon \left[g''(r) + 2r^{-1} g'(r) \right]$$

$$T_{11} = \left[\varepsilon r^{-1} \right] \left[g'(r) + f'(r) \right]$$

$$T_{22} = \left[\frac{1}{2} r \varepsilon \right] \left[f'(r) + r f''(r) + g'(r) + r g''(r) \right]$$

$$T_{33} = \left[\frac{1}{2} r \varepsilon \sin^2 \theta \right] \left[r f''(r) + f'(r) + r g''(r) + g'(r) \right]$$

$$T_{22} = T_{33} = 0 \quad \text{seçilir}$$

$$R_{33} - \frac{1}{2} R \cdot g_{33} = \sin^2 \theta \left[R_{22} - \frac{1}{2} R \cdot g_{22} \right] = 0$$

$\theta \neq 0$ ve $\theta \neq \pi$ ve $0 < \theta < \pi$ olduğundan $\sin \theta \neq 0$ olur.

Bu nedenle $R_{22} - \frac{1}{2} R \cdot g_{22} = 0$ olur.

(2.10) ve (2.11) den

$$\left[\frac{1}{2} r \varepsilon \right] \left[f'(r) + r f''(r) + g'(r) + r g''(r) \right] = 0$$

$$\left[f'(r) + r f''(r) + g'(r) + r g''(r) \right] = 0 \quad 3.1$$

4 denklem bu şekilde 3 denkleme indirgenir. $T_{00} = E(r)$, $T_{11} = P(r)$ seçilir.

$$\left[\varepsilon r^{-1} \right] \left[g'(r) + f'(r) \right] = P(r) \quad 3.2$$

$$c_1^{-1} c_0 \varepsilon \left[g''(r) + 2r^{-1} g'(r) \right] = E(r) \quad 3.3$$

$$(3.2) \text{ den } g' + f' = \frac{r}{\varepsilon} P$$

$$f' = \frac{r}{\varepsilon} P - g' \quad 3.4$$

r ye göre ikinci türev alınırsa $f'' = \frac{1}{\varepsilon} P + \frac{r}{\varepsilon} P' - g''$ elde edilir.

f' ve f'' (3.1). denkleminde yerine yazılır.

$$\left[\frac{r}{\varepsilon} P - g' + r \frac{1}{\varepsilon} P + \frac{r^2}{\varepsilon} P' - r g'' + g' + r g'' \right] = 0$$

$$\left[\frac{r}{\varepsilon} P + r \frac{1}{\varepsilon} P + \frac{r^2}{\varepsilon} P' \right] = 0$$

$$\frac{2r}{\varepsilon} P + \frac{r^2}{\varepsilon} P' = 0 \quad 3.5$$

$$2P + r P' = 0$$

$$2P + r \frac{dP}{dr} = 0$$

$$r \frac{dP}{dr} = -2P \quad 3.6$$

$$\frac{dP}{P} = -\frac{2}{r} dr$$

$$\int \frac{dP}{P} = -\int \frac{2}{r} dr$$

$$\ln P = -2 \ln r + \ln k \quad (k = \text{sabit})$$

$$\ln \frac{P}{k} = \ln \frac{1}{r^2}$$

$$P = \frac{k}{r^2} \quad 3.7$$

$$c_1^{-1} c_0 \varepsilon \left[g''(r) + 2r^{-1} g'(r) \right] = E(r)$$

$$g'' + 2r^{-1} g' = \frac{c_1}{c_0 \varepsilon} E$$

Bu denklemi çözmek için $g' = u$ dönüşümü yapılır.

$$u' + \frac{2}{r} u = \frac{c_1}{c_0 \varepsilon} E \quad 3.8$$

Bu denklem h integrasyon çarpanı ile çarpılırsa

$$u'h + \frac{2}{r} h u = \frac{c_1}{c_0 \varepsilon} h E \quad 3.9$$

$$h' = \frac{2}{r} h \quad \text{olmalı} \quad 3.10$$

$$\frac{h'}{h} = \frac{2}{r}$$

$$\int \frac{h'}{h} = \int \frac{2}{r}$$

$$\ln h = 2 \ln r$$

$$h = r^2 \quad \text{bulunur.} \quad 3.11$$

$$u'h + \frac{2}{r} h u = \frac{c_1}{c_0 \varepsilon} h E \quad 3.12$$

$$u'r^2 + \frac{2}{r} r^2 u = \frac{c_1}{c_0 \varepsilon} r^2 E \quad 3.13$$

$$\frac{d}{dr}(r^2 u) = \frac{c_1}{c_0 \varepsilon} r^2 E \quad 3.14$$

$$\int d(r^2 u) = \int \frac{c_1}{c_0 \varepsilon} r^2 E dr$$

$$r^2 u = \frac{c_1}{c_0 \varepsilon} \int r^2 E dr + l \quad (l = \text{sabit}) \quad 3.15$$

$$u = \frac{c_1}{c_0 \varepsilon r^2} \int r^2 E dr + \frac{l}{r^2}$$

$$g' = \frac{dg}{dr} = \frac{c_1}{c_0 \varepsilon r^2} \int r^2 E dr + \frac{l}{r^2} \quad 3.16$$

$$\int dg = \int \left[\frac{c_1}{c_0 \varepsilon r^2} \int r^2 E dr \right] dr + \int \frac{l}{r^2} dr + n \quad (n = \text{sabit}) \quad 3.17$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \left[\frac{1}{r^2} \int r^2 E dr \right] dr + l \int \frac{1}{r^2} dr + n \quad (n = \text{sabit})$$

$$(3.4) \text{ den } f' = \frac{r}{\varepsilon} \frac{k}{r^2} - g' \quad \text{şeklinde yazılır.} \quad 3.18$$

$$(3.16) \text{ ve } (3.18) \text{ den } f' = \frac{r}{\varepsilon} \frac{k}{r^2} - \left[\frac{c_1}{c_0 \varepsilon r^2} \int r^2 E dr + \frac{l}{r^2} \right] \text{ yazılır.} \quad 3.19$$

$E = \frac{B_0}{r^3}$ Olarak seçilip işlemlere devam edilir.

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int r^2 E dr - \frac{l}{r^2}$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int r^2 \frac{B_0}{r^3} dr - \frac{l}{r^2} \quad 3.20$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int \frac{B_0}{r} dr - \frac{l}{r^2}$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int \frac{B_0}{r} dr - \frac{l}{r^2} \quad \left(\frac{r}{B_0} = x \right) \quad (dr = B_0 dx) \quad 3.21$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int \frac{1}{\frac{r}{B_0}} dr - \frac{l}{r^2}$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} \int \frac{1}{x} B_0 dx - \frac{l}{r^2}$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} B_0 \ln x - \frac{l}{r^2} \quad 3.22$$

$$f' = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} B_0 \ln \frac{r}{B_0} - \frac{l}{r^2}$$

$$\frac{df}{dr} = \frac{k}{r\epsilon} - \frac{c_1}{c_0\epsilon r^2} B_0 \ln \frac{r}{B_0} - \frac{l}{r^2} \quad 3.23$$

$$df = \frac{k}{r\epsilon} dr - \frac{c_1}{c_0\epsilon r^2} B_0 \ln \frac{r}{B_0} dr - \frac{l}{r^2} dr \quad 3.24$$

$$\int df = \int \frac{k}{r\epsilon} dr - \int \frac{c_1}{c_0\epsilon r^2} B_0 \ln \frac{r}{B_0} dr - \int \frac{l}{r^2} dr \quad 3.25$$

$$f = \frac{k}{\epsilon} \int \frac{1}{r} dr - \frac{c_1}{c_0\epsilon} B_0 \int \frac{1}{r^2} \ln \frac{r}{B_0} dr - \int \frac{l}{r^2} dr \quad 3.26$$

$$f = \frac{k}{\epsilon} \int \frac{1}{B_0} \frac{1}{\frac{r}{B_0}} B_0 dx - \frac{c_1}{c_0\epsilon} B_0 \int \frac{1}{\frac{B_0^2 r^2}{B_0^2}} \ln \frac{r}{B_0} B_0 dx - \int \frac{l}{r^2} dr$$

$$f = \frac{k}{\epsilon} \int \frac{1}{\frac{r}{B_0}} dx - \frac{c_1}{c_0\epsilon} \int \frac{1}{\frac{B_0^2}{r^2}} \ln \frac{r}{B_0} dx - \int \frac{l}{r^2} dr \quad 3.27$$

$$f = \frac{k}{\varepsilon} \int \frac{1}{x} dx - \frac{c_1}{c_0 \varepsilon} \int \frac{1}{x^2} \ln x dx - \int \frac{l}{r^2} dr$$

$$f = \frac{k}{\varepsilon} \ln x + \frac{c_1}{c_0 \varepsilon} \left[\frac{1}{x} (1 + \ln x) \right] + \frac{l}{r} + m \quad 3.28$$

$$f = \frac{k}{\varepsilon} \ln \frac{r}{B_0} + \frac{c_1}{c_0 \varepsilon} \left[\frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \right] + \frac{l}{r} + m \quad (l = m = 0) \text{ seçilir} \quad 3.29$$

$$f = \frac{k}{\varepsilon} \ln \frac{r}{B_0} + \frac{c_1}{c_0 \varepsilon} \left[\frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \right] \quad 3.30$$

$$F(r) = c_0 e^{\varepsilon f} \approx c_0 (1 + \varepsilon f) \quad 3.31$$

$$F(r) = c_0 \left[1 + \varepsilon \left(\frac{k}{\varepsilon} \ln \frac{r}{B_0} + \frac{c_1}{c_0 \varepsilon} \left[\frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \right] \right) \right] \quad 3.32$$

$$F(r) = c_0 \left[1 + \left(k \ln \frac{r}{B_0} + \frac{c_1}{c_0} \left[\frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \right] \right) \right]$$

$$F(r) = c_0 + c_0 k \ln \frac{r}{B_0} + c_1 \left[\frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \right]$$

$$F(r) = c_0 + c_0 k \ln \frac{r}{B_0} + \frac{c_1 B_0}{r} + \frac{c_1 B_0}{r} \ln \frac{r}{B_0} \quad 3.33$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \left[\frac{1}{r^2} \int r^2 E dr \right] dr + 0 + n \quad (n = \text{sabit})$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \left[\frac{1}{r^2} \int \frac{B_0}{x} dx \right] dr + n \quad 3.34$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \frac{1}{r^2} B_0 \ln x dr + n$$

$$g = \frac{c_1}{c_0 \varepsilon} \int \frac{1}{x^2} \ln x dx + n$$

$$g = \frac{c_1}{c_0 \varepsilon} \left[\frac{-1}{x} (1 + \ln x) \right] + n \quad 3.35$$

$$g = -\frac{c_1}{c_0 \varepsilon} \frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) + n \quad 3.36$$

$$G(r) = c_1 e^{\varepsilon g} \approx c_1 (1 + \varepsilon g) \quad 3.37$$

$$G(r) = c_1 \left(1 - \frac{c_1}{c_0} \frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) + \varepsilon n \right) \quad n = 0 \quad \text{seçilir} \quad 3.38$$

$$G(r) = c_1 - \frac{c_1^2}{c_0} \frac{B_0}{r} \left(1 + \ln \frac{r}{B_0} \right) \quad 3.39$$

$$G(r) = c_1 - \frac{c_1^2}{c_0} \frac{B_0}{r} - \frac{c_1^2}{c_0} \frac{B_0}{r} \ln \frac{r}{B_0} \quad 3.40$$

(3.33) ve (3.40) da bulunan sonuçlar (2.1) denkleminde yerine yazılırsa aşağıdaki ifade elde edilir.

$$\begin{aligned} ds^2 = & \left[c_0 + c_0 k \ln \frac{r}{B_0} + \frac{c_1 B_0}{r} + \frac{c_1 B_0}{r} \ln \frac{r}{B_0} \right] dt^2 \\ & + \left[c_1 - \frac{c_1^2}{c_0} \frac{B_0}{r} - \frac{c_1^2}{c_0} \frac{B_0}{r} \ln \frac{r}{B_0} \right] \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right] \end{aligned} \quad 3.41$$

Yukarıdaki metrikte $c_0 = 1$, $c_1 = -1$, $B_0 = 2M$ seçilirse aşağıdaki eşitlik bulunur.

$$\begin{aligned} ds^2 = & \left[1 - \frac{2M}{r} + k \ln \frac{r}{2M} - \frac{2M}{r} \ln \frac{r}{2M} \right] dt^2 \\ & - \left[1 + \frac{2M}{r} + \frac{2M}{r} \ln \frac{r}{2M} \right] \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right] \end{aligned} \quad 3.42$$

Schwarzschild metriği aşağıdaki gibidir.

$$ds^2 = \left[1 - \frac{2M}{r} \right] dt^2 - \left[1 - \frac{2M}{r} \right]^{-1} \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right] \quad 3.43$$

$$\left(\frac{2M}{r} \ll 1 \right) \quad (1+x)^n \approx 1+nx \quad |x| \ll 1 \quad \text{ise}$$

Schwarzschild yaklaşık olarak aşağıdaki gibi olur.

$$ds^2 \approx \left[1 - \frac{2M}{r}\right] dt^2 - \left[1 + \frac{2M}{r}\right] dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \quad 3.44$$

Schwarzschild uzay zamanı küresel simetrik ve dönmeyen bir cismin etrafında yaydığı gravitasyon alanının tasviridir. Bu çözümün varlığını ve tekliğini Birkoff⁷ ispat etmiştir

4. SONUÇLAR VE DEĞERLENDİRMELER

Bu çalışmada özel bir fonksiyon seçerek tanımladığımız enerji momentum tensörüne bağlı olarak Einstein Alan Denklemlerinin bir çözümü yapıldı.

Bu çözüm bize 4 boyutlu uzay zamanına ait kütlelerin gravitasyonel anlamda nasıl davrandıkları hakkında bilgi verir. Elde edilen çözümün yapısı enerji momentum tensörünün nasıl seçildiği ile yakından ilgilidir. Bu tezde $E(r) = \frac{B_0}{r^3}$ seçildi. Bu şekildeki bir seçimin sonucunda oluşan uzay zamanının yapısının Schwarzschild metriği ile yaklaştığı görüldü.

$$ds^2 = \left[1 - \frac{2M}{r}\right] dt^2 - \left[1 - \frac{2M}{r}\right]^{-1} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \quad \text{Schwarzschild}$$

metriğinde $\frac{2M}{r} \ll 1$ olarak seçilirse $\left[1 - \frac{2M}{r}\right]^{-1} \approx 1 + \frac{2M}{r}$ şeklinde yazılabilir. Bunun sonucunda Schwarzschild metriği yaklaşık olarak

$$ds^2 \approx \left[1 - \frac{2M}{r}\right] dt^2 - \left[1 + \frac{2M}{r}\right] dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\varphi^2 \quad \text{şeklinde bulunur.}$$

Bu çalışmada bizim bulduğumuz

$$ds^2 = \left[1 - \frac{2M}{r} + k \ln \frac{r}{2M} - \frac{2M}{r} \ln \frac{r}{2M}\right] dt^2 - \left[1 + \frac{2M}{r} + \frac{2M}{r} \ln \frac{r}{2M}\right] \left[dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2\right]$$

metriğinin, Schwarzschild metriğine benzediği görülür.

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ÖZGEÇMİŞ

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